Theory assignment

Automata Theory Monsoon 2022, IIIT Hyderabad

August 19, 2022

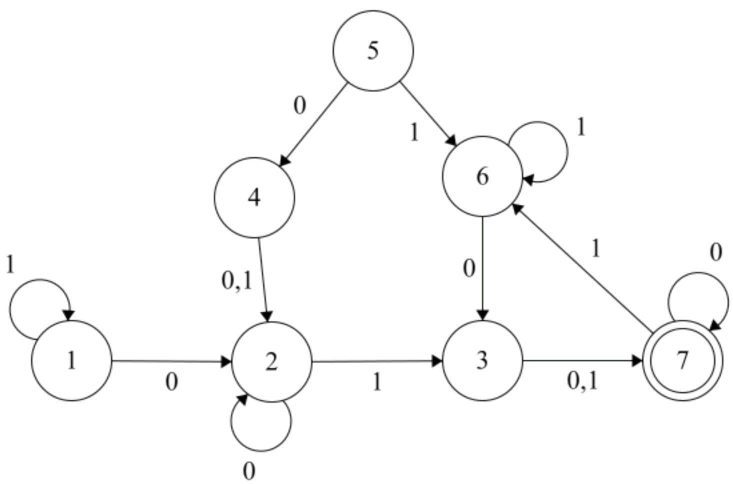
Total Marks: 55 points Due date: **24/08/22 11:59 pm** and **13/09/22 11:59 pm**

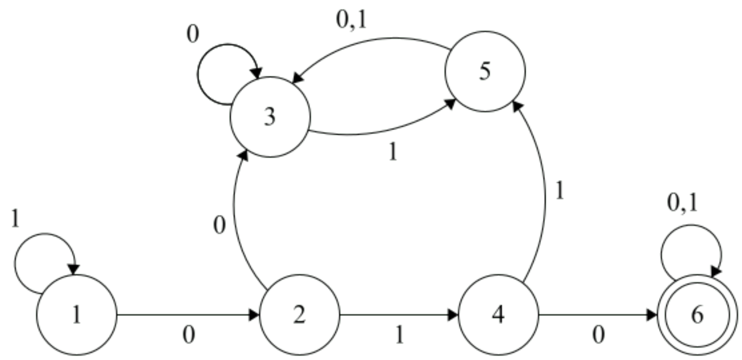
<u>**General Instructions:**</u> All symbols have the usual meanings (example: $\mathbb{R}$ is the set of reals, $\mathbb{N}$ the set of natural numbers, and so on.) FSM stands for finite state machine. DFA stands for deterministic finite automata. NFA stands for non-deterministic finite automata. a^* is the Kleene Star operation.

Section 1 (29 points) (due 24/08/22 11:59 pm)

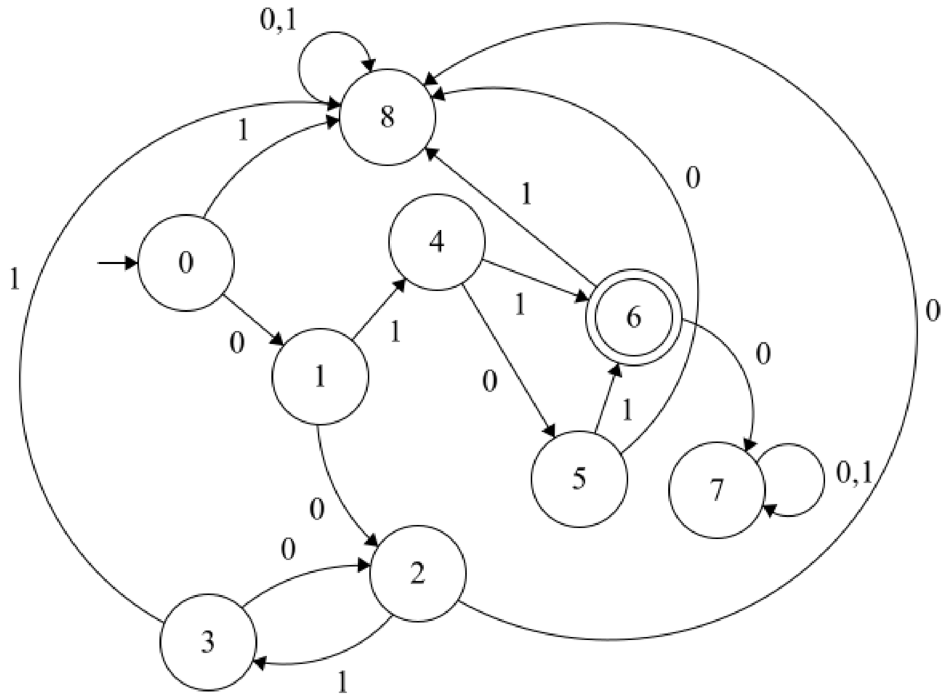
1. [3 points] A **dead state** is defined as a state from which there exists no path to any of the final states. Similarly an **unreachable state** is defined as a state, which cannot be reached from the initial state using any string S. Identify the set of **spurious states** which are either dead or unreachable or both) in the following DFAs with proper reasoning [Assume that the state numbered 1 is the start/ initial state]

(a)



(b) 

2. [3 points] Minimize the following DFA.



3. [3 points] For any language L over Σ, the prefix closure of L is defined as:

$$Pre(L) = \{x \in \Sigma^* \mid \exists\ y \in \Sigma^*\ such\ that\ xy \in L\} \tag{1}$$

Prove that Pre(L) is regular whenever L is regular.

4. [3 points] Prove that regular languages are closed under Dropout* operation,

$$\text{Dropout*}(A) = \{xz \mid xyz \in A \text{ and } x, y, z \in \Sigma^*\} \tag{2}$$

5. [1 point] Write a regular expression for your IIIT email address
(It should accept student and faculty email addresses, e.g. *shchakra@iiit.ac.in*, *zeeshan.ahmed@research.iiit.ac.in*, *amul.agarwal@students.iiit.ac.in* are valid email addresses)

6. [3 points] Provide an algorithm for converting

   (a) regular expression into right linear grammar

   (b) right linear grammar to a left linear grammar.

7. [4 points] Prove that the languages A and L are not regular

   (a) $A = \{\text{bits}(n) \mid len(bits(n)) \text{ is prime } , n \in \mathrm{N}\}$, where $bits(n)$ is the binary string representation of the number n and $len(s)$ is the length of a string s. E.g., $bits(6) = 110$ and $len(110) = 3$.

   (b) $L = \{a^{n!} \mid n \geq 0\}$.

8. [2 points] Is the given language L context free? If yes, provide a proof and convert it to Chomsky normal form. Otherwise, provide a counter example.

$$L = \{a, b\}^* - \{\text{palindromes}\}$$

9. [2 points] Write a context-free grammar for the complement of $L = \{a^n b^n \ \cup b^n a^n \mid n \geq 0\}$

10. [3 points] For any languages A, B over Σ $mix(A, B)$ is defined as

$$\{w | w = a_1 b_1 a_2 b_2 \ldots a_k b_k \text{ where } a_i, b_i \in \Sigma \ \forall i,\ a_1 a_2 \ldots a_k \in A \ \text{ and } b_1 b_2 \ldots b_k \in B\} \tag{3}$$

    Prove that $mix(A, B)$ is regular when A, B are regular.

11. [2 points] Construct a PDA for the language $L = \{a^i b^j c^k \mid i, j, k \geq 0,\ i = j \ or j = k\}$

Section 2 (26 points) (due 13/09/22 11:59 pm)

Note: Submit solutions for this section separately. First section's solution file will NOT be checked while checking this section's solutions.

1. [2 points] Show that if L is a CFL over a one-symbol alphabet, then L is regular.

2. [3 points] Let A be an infinite regular language. Provide a general way to split A into 2 infinite regular languages A_1, A_2 such that $A_1 \cup A_2 = A, A_1 \cap A_2 = \phi$.

3. [4 points] Prove the stronger form of the pumping lemma where in the $s = uvxyz$ decomposition the $|vy| > 0$ constraint is replaced by $|v| > 0$ as well as $|y| > 0$.

4. [5 points] Construct a Turing machine that given a string $w \in \{0, 1\}^n,\ n \in \mathbb{N}$ outputs the reverse string.

5. [3 points] Consider the language $L = \{a^n b^n c^n | n \in \mathbb{N}\}$. Is it Context Free? Is it Recursive? Is it recursively enumerable?

6. [2 points] Give an example of a language that is recursively enumerable but not recursive. Give an example of a language that is recursive but not recursively enumerable. Which one is not possible and why?

7. [4 points] Let $\Sigma = \{0, 1\}$. Prove that the set of all strings is *countably infinite* but the set of all languages is *uncountably infinite*.

8. [3 points] Prove that the language $L = \{M | M \text{ is a Turing Machine and M does not accept } \langle M \rangle\}$ is undecidable. Is it recursively enumerable?