Discrete Structures (MA5.101)

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International Institute of Information Technology, Hyderabad
Assignment Set 2 (Monsoon 2021)
Functions, Countable Set, Propositional Logic,
Permutations, Pigeonhole Principle and Mathematical Induction
Deadline: January 27, 2022 (Thursday), 23:55 PM
Total Marks: 100
Instructions: Submit ONLY handwritten scanned pdf file
in the course moodle under Assignments directory.

January 16, 2022

Functions

1. (a) Let an injective mapping f from a finite set A to itself, that is, $f : A \to A$. Prove or disprove f is bijective with proper justification.

   (b) Let a function f is defined by $f : A \to B$. The function f is invertible iff f is bijective.

   [5 + 5 = 10]

2. (a) Prove or disprove that for a non-empty set A, there is no surjection $g : A \to P(A)$, where $P(A)$ is a power set of A.

   (b) Are the two functions f and g equal? Give reasons

   (i) f, $g : \mathbb{D} \to \mathbb{R}$ defined by $f(x) = \sin x - \cos x$, $x \in \mathbb{D}$, $g(x) = \sqrt{1 - \sin 2x}, x \in \mathbb{D}$; and $\mathbb{D} = \{x \in \mathbb{R} : 0 \leq x \leq \frac{\pi}{2}\}$.

   (ii) f, $g : \mathbb{D} \to \mathbb{R}$ defined by $f(x) = 2\tan^{-1} x$, $x \in \mathbb{D}$, $g(x) = \tan^{-1} \frac{2x}{1-x^2}, x \in \mathbb{D}$; and $\mathbb{D} = \{x \in \mathbb{R} : x > 1\}$.

   [5 + (2.5 + 2.5) = 10]

Countable Sets

3. Prove or disprove a subset of a countable set is countable or finite.

   [5]

4. Determine whether or not the following set is countable:
   the set $A = \{a^2 \mid a \in \mathcal{N}\}$ where $\mathcal{N}$ is the set of natural numbers.

   [5]

Propositional Logic

5. (a) Show that if p, q, and r are compound propositions such that p and q are logically equivalent and q and r are logically equivalent, then p and r are logically equivalent.

   (b) Show that $\neg p \to (q \to r)$ and $q \to (p \vee r)$ are logically equivalent by a series of logical equivalences.

[5 + 5 = 10]

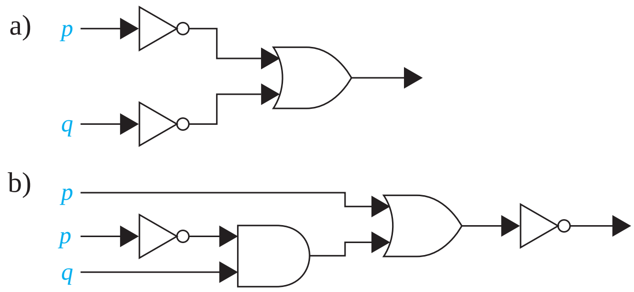

Figure 1: Combinatorial circuits

6. (i) Find the output of each of these combinatorial circuits shown in Figure 1.

   (ii) (a) Show that $(p \to q) \wedge (q \to r) \to (p \to r)$ is a tautology.

   (b) Show that $(p \vee q) \wedge (\neg p \vee r) \to (q \vee r)$ is a tautology.

[5 + (2.5 + 2.5) = 10]

Permutations

7. Let f, g be permutations on the set $\{1, 2, 3, 4, 5, 6\}$. Find fg, gf, f^{-1} and g^{-1}, when $f = (1\,2\,4\,5\,6)$, $g = (2\,6\,3\,4\,5)$.

[4 * 2.5 = 10]

8. The order of a permutation f is the least positive integer n such that $f^n = I$, I being the identity permutation.
   Prove that
   (i) the order of an r-cycle permutation is r.
   (ii) the order of a permutation f is the lcm (least common multiple) of the lengths of its disjoint cycles.
   Find the the order of the following permutation using the result (ii)

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 2 & 1 \end{pmatrix}$$

[5 + 5 = 10]

Mathematical Induction

9. Prove by induction that the sum of the cubes of three consecutive integers is divisible by 9.

[5]

10. It is known that for any positive integer $n \geq 2$

$$\frac{1}{n+1} + \frac{1}{n+2} + \ldots + \frac{1}{2n} - A > 0,$$

where A is a constant. How large can A be?

[5]

11. Let A and B be square matrices. If $AB = BA$, then prove that $(AB)^n = A^n B^n$, for $n \geq 1$.

[5]

Pigeonhole Principle

12. Show that if any 30 people are selected, then one may choose a subset of five so that all five were born on the same day of the week.

[5]

13. Show that if any eight positive integers are chosen, two of them will have the same remainder when divided by 7.

[5]

14. A chess player wants to prepare for a championship match by playing some practice games in 77 days. She wants to play at least one game a day but no more than 132 games altogether. Show that no matter how she schedules the games there is a period of consecutive days within which she plays exactly 21 games.

[5]

**************** **End of Question Paper** *******************