International Institute of Information Technology Hyderabad

Discrete Structures (MA5.101)

Assignment 1
Deadline: December 26, 2021 (Sunday), 23:55 PM
Total Marks: 70
Instructions: Submit ONLY handwritten scanned pdf file
in the course moodle under Assignments directory.

1. Let us consider the structure $\langle \mathcal{P}(X), - \rangle$, where the operation $-$ (set difference) is defined by $A - B = \{x \mid x \in A \text{ and } x \notin B\} = A \cap B'$, where $\mathcal{P}(X)$ is the power set of X.
(a) Show that the operation is neither commutative nor associative. [5]
(b) Verify whether $A \cup (B \Delta C) = (A \cup B) \Delta (A \cup C)$ holds or not, where 'Δ' is the usual symmetric difference operator defined over sets. [5]
(c) Show that the following properties hold for all $A, B, C \in \mathcal{P}(X)$:
(i) $A \Delta (B \Delta C) = (A \Delta B) \Delta C$,
(ii) $A \Delta B = B \Delta A$,
(iii) $A \Delta \emptyset = A$,
(iv) $A \Delta X = A'$,
(v) $A \Delta A' = X$, where $A \Delta B$ is the usual symmetric difference between A and B, and A' the complement of A. [5 × 5 = 25]

2. Given two sets S and T and $S - T = S \cap T'$, prove that $S \Delta T = (S \cup T) - (S \cap T)$. [5]

3. A binary relation on a set that is reflexive and symmetric is called a *compatible relation*. Let $\mathcal{A}$ be a set of English words and R be a binary relation on $\mathcal{A}$ such that two words w_1 and w_2 in $\mathcal{A}$ are related if they have one or more letters in common. Show that R is a compatible relation. [5]

4. For a given set $\mathcal{A}$, consider the relation

$$R = \{(x, y) \mid x \in \mathcal{P}(A), y \in \mathcal{P}(B), \text{ and } x \subseteq y\}$$

where $\mathcal{P}(X)$ is the power set of X. Show that R is a partial order relation. [5]

5. Let R and R' be two equivalence relations on a set A. Prove that

(i) $R \cap R'$ is an equivalence relation in A. [5]

(ii) $R \cup R'$ is not necessarily an equivalence relation in A. [5]

6. On $R = R \times R$, the following relation ρ is defined, where R is the set of real numbers. Check whether it is an equivalence relation or not. If yes, find the equivalence classes.
   $(a, b)\rho(c, d)$ iff both the points lie on the same curve: $9x^2 + 16y^2 = k^2$, for some $k \in R$. [10]

All the best!!!