Assignment MA3.101: Linear Algebra
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1. Find all the values of k for which following matrices are invertible,
   a) $\begin{pmatrix} k & -k & 3 \\ 0 & k+1 & 1 \\ k & -8 & k-1 \end{pmatrix}$ b) $\begin{pmatrix} k & k & 0 \\ k^2 & 2 & k \\ 0 & k & k \end{pmatrix}$

2. If A is an $n \times n$ matrix , then prove that $det(adjA) = (detA)^{-1}$

3. Verify that if $r < s$ then rows r and s of a matrix can be interchanged by performing $2(s-r)-1$ interchanges of adjacent rows.

4. Prove that if A is a diagonalizable matrix such that every eigenvalue of A is either 0 or 1. Then A is idempotent (i.e $A^2 = A$).

5. For each of the following matrices, either find a matrix P such that $P^{-1}AP$ is a diagonal matrix. If such a matrix does not exist, give clear reasons for the same.

   (a) $A = \begin{bmatrix} -1 & 4 & 2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 5 & 0 & 0 \\ 1 & 5 & 0 \\ 0 & 1 & 5 \end{bmatrix}$

   (c) $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3 & 0 & 1 \end{bmatrix}$

6. Use Cramer's rule to solve the given linear system
   a) $x + y - z = 1$
   $x + y + z = 2$
   $x - y = 3.$

   b) $2x + y - 3z = 1$
   $y + z = 1$
   $z = 1.$

7. Let λ_i $(i = 1, 2, ..n)$ are complete set of eigenvalues (repetitions included) of the $n \times n$ matrix A. Prove that $det(A) = \Pi_{i=1}^{n} \lambda_i$

8. Prove that $det(AB) = det(BA)$