Assignment 4

MA2.101: Linear Algebra (Spring 2022)

April 21, 2022

Due date: May 2, 2022

General Instructions: All symbols have the usual meanings (example: F is an arbitrary field, $\mathbb{R}$ is the set of reals, $\mathbb{N}$ the set of natural numbers). Remember to prove all your intermediate claims, starting from basic definitions and theorems used in class to show whatever is being asked. You may use any other non-trivial theorems not used in class, as long as they are well known and a part of basic LA texts. Try proving everything from definitions and keep arguments mathematically well formed and concise.

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1. Let V be a vector space which is spanned by a finite set of vectors $\beta_1, \beta_2, \ldots, \beta_m$. Then, prove that any independent set of vectors in V is finite and contains no more than m elements.

2. If W_1 and W_2 are finite dimensional sub-spaces of vector space V then prove the following:

   - $W_1 + W_2$ is finite dimensional
   - $\dim W_1 + \dim W_2 = \dim(W_1 \cap W_2) + \dim(W_1 + W_2)$

Note: Some more problems might get added next week.