LA - Assign 2's

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202101128

(1) Given the matrices are Invertible

Find: All possible values of 'k'

(a) $\begin{bmatrix} k & -k & 3 \\ 0 & k+1 & 1 \\ k & -8 & k-1 \end{bmatrix}_{3\times3} = A_{3\times3}$

'A' is Invertible

$\Rightarrow \det(A) \neq 0$

$\Rightarrow k((k+1)(k-1) - (-8)) + k(0-k) + 3(0-k(k+1))$

$\Rightarrow k(k^2-1+8) + k(-k) - 3k(k+1) \neq 0$

$\Rightarrow k^3 + 7k - k^2 - 3k^2 - 3k \neq 0$

$\Rightarrow k^3 - 4k^2 + 4k \neq 0$

$\Rightarrow k(k^2-4k+4) \neq 0$

$k \neq 0 \quad \Big| \quad (k-2)^2 \neq 0$

$\qquad\qquad\quad k \neq 2$

$\Rightarrow \boxed{k \in \mathbb{R} \setminus \{0,2\}}$

(b)

$$A = \begin{bmatrix} k & k & 0 \\ k^2 & 2 & k \\ 0 & k & k \end{bmatrix}$$

A is invertible

$\Rightarrow \det(A) \neq 0$

$\Rightarrow k(2k-k^2) - k(k^3-0) + 0 \neq 0$

$\Rightarrow -k^3 + 2k^2 - k^4 \neq 0$

$\Rightarrow -k^2(k-2+k^2) \neq 0$

$\Rightarrow k^2(k^2+k-2) \neq 0$

$k \neq 0$, $k \neq 0$ | $(k+2)(k-1) \neq 0$, $k \neq -2, 1$

$\Rightarrow \boxed{k \in R \setminus \{0, 1, -2\}}$

(2)

Given A is an $n \times n$ matrix

$\underline{\underline{PTPT}}$ $\det(\text{Adj}A) = (\det(A))^{n-1}$

W.K.T $A^{-1} = \dfrac{\text{Adj} A}{\det(A)}$

$A^{-1} \cdot \det(A) = \text{adj} A$

$\Rightarrow \det(A^{-1} \cdot \det(A)) = \det(\text{adj} A)$

$\Rightarrow \det(\text{adj} A) = \det(\underline{\det(A)} \cdot A^{-1})$

[$\because$ d

$\Rightarrow \det(a$

$A \cdot A^{-1} = I$

$\Rightarrow \det(A \cdot A$

$\det(A)$

$\Rightarrow \det$

$\Rightarrow$ [det

Hence,

$[\because \det(\lambda A) = \lambda^n \det(A)$ where $\lambda \in \mathbb{R}$ is constant and A is an $n \times n$ matrix $]$

$$\Rightarrow \det(\operatorname{adj} A) = [\det(A)]^n \cdot (\det A^{-1})$$

$$A \cdot A^{-1} = I$$

$$\Rightarrow \det(A \cdot A^{-1}) = \det(I)$$

$$\det(A)\det(A^{-1}) = 1$$

$$\boxed{\det(A^{-1}) = \frac{1}{\det(A)}}$$

$$\Rightarrow \det(\operatorname{adj} A) = [\det(A)]^n \cdot \frac{1}{\det(A)}$$

$$\Rightarrow \boxed{\det(\operatorname{adj} A) = [\det(A)]^{n-1}}$$

Hence, proved

③

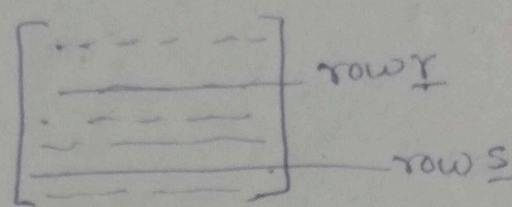
Given,

r, s are rows of a matrix s/t

$r < s$

RTP:- Number of Interchanges of adjacent rows to interchange r & s $= 2(s-r)-1$

consider the matrix

$A =$ 

No. of ~~in~~ rows between s & r $= s-r-1$ (Excluding s, r)

Now

Interchange row r with row $r+1$ — 1

then Interchange row $r+1$ with row $r+2$ — 2

So on

Interchange rows $s-1$ with row s — $\boxed{s-r}$

Thus we need $\boxed{s-r}$ interchanges to get row r to row s

But now row s (actual) is at row $s-1$

Now

Interchange row $s-1$ with row $s-2$ — 1

Interchange row $s-2$ with row $s-3$ — 2

so on

Interchange row $r+1$ with row r — $\boxed{s-r-1}$

Thus

we need ~~$(r-1)$~~ $\boxed{s-r-1}$ interchanges

to get rows to row r

$\therefore$ Total No. of Interchanges

= Move row r to row s + Move row s to row r

$= (s-r) + \quad + (s-r-1)$

(one swap is done when moving r to s)

$= 2(s-r)-1$

$= \boxed{2(s-r)-1}$

Hence, proved

(4)

Given,

$\underline{A}$ is diagonalizable matrix with eigen values $\lambda = 0, 1$

RTP: A is Idempotent i.e, $A^2 = A$

$\underline{A}$ is diagonalizable $\Rightarrow \exists D$ (let A be $n \times n$)

$D = diag\{d_1, d_2 \cdots, d_n\}$

s/t $D = P^{-1}AP$ ($A \sim D$)

$A \sim D$

$\Rightarrow$ Eigenvalues of A = entries in $D = \{0, 1\}$

$\Rightarrow D^2 = diag\{d_1^2, d_2^2, \cdots d_n^2\}$ [$\because d_1, d_2 \cdots d_n \in \{0, 1\}$ and $0^2 = 0$, $1^2 = 1$]

$D^2 = diag\{d_1, d_2, \cdots d_n\}$

$\Rightarrow \boxed{D^2 = D}$ —— (1)

W.K.T

$D^2 = P^{-1}A^2P$

$D = P^{-1}AP$

$\Rightarrow P^{-1}A^2P = P^{-1}AP$ ($\because$ (1))

$\Rightarrow \underline{P.P^{-1}}.A^2.\underline{P.P^{-1}} = \underline{P.P^{-1}}.A.\underline{P.P^{-1}}$ ($\because P.P^{-1} = P^{-1}.P = I$)

$\Rightarrow I.A^2.I = I.A.I$

$\Rightarrow \boxed{A^2 = A}$

Hence, A is $\underline{\text{Idempotent}}$

5(a)

Given a matrix A.

Find matrix P s/t $P^{-1}AP$ is a diagonal.

matrix $A = \begin{bmatrix} -1 & 4 & 2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}$

$|A - \lambda I| = 0$

+0 +3

$\Rightarrow \begin{vmatrix} -1-\lambda & 4 & 2 \\ -3 & 4-\lambda & 0 \\ -3 & 1 & 3-\lambda \end{vmatrix} = 0$

+0

$\Rightarrow (-1-\lambda)\big((4-\lambda)(3-\lambda)-0\big) - 4\big(-3(3-\lambda)\big) +$

$2\big(-3+3(4-\lambda)\big) = 0$

$\Rightarrow (3-\lambda)\big((-1-\lambda)(4-\lambda)+12\big) + 6(-1+4-\lambda) = 0$

$\Rightarrow (3-\lambda)\big((\lambda+1)(\lambda-4)+12\big) + 6(3-\lambda) = 0$

$\Rightarrow (3-\lambda)(\lambda^2-3\lambda-4+12+6) = 0$

$\Rightarrow (3-\lambda)(\lambda^2-3\lambda+14) = 0$

$\lambda = 3 \quad \Big| \quad \lambda = \dfrac{3 \pm \sqrt{47}\,i}{2}$

$\Rightarrow \lambda_1 = 3,\ \lambda_2 = \dfrac{3+\sqrt{47}\,i}{2},\ \lambda_3 = \dfrac{3-\sqrt{47}\,i}{2}$ are

the eigen values.

Eigen space for $\lambda = 2$; ①

$\Rightarrow (A - \lambda I)(x) = 0$

$$\Rightarrow \begin{bmatrix} -4 & 4 & 2 \\ -3 & 1 & 0 \\ -3 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow -3x_1 + x_2 = 0$

$-4x_1 + 4x_2 + 2x_3 = 0$

let $x_1 = t$

$\Rightarrow x_2 = 3x_1 = 3t$

$\Rightarrow x_3 = -4t$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} t \\ 3t \\ -4t \end{bmatrix} = t \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix}$$

Eigen vector (P_1)

Eigen space for $\lambda = \frac{3 + \sqrt{47}i}{2}$; ②

$(A - \lambda I)(x) = 0$

$$\begin{bmatrix} -\frac{5-\sqrt{47}i}{2} & 4 & 2 \\ -3 & \frac{5-\sqrt{47}i}{2} & 0 \\ -3 & 1 & \frac{3-\sqrt{47}i}{2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-\cancel{3x_1} + \left(\frac{5-\sqrt{47}i}{2}\right)x_2 = 0$

$-\cancel{3x_1} \pm x_2 \pm \left(\frac{3-\sqrt{47}i}{2}\right)x_3 = 0$

$+$

$$\left(\frac{3-\sqrt{47}i}{2}\right)x_2 - \left(\frac{3-\sqrt{47}i}{2}\right)x_3 = 0$$

$$x_2 - x_3 = 0$$

$$\boxed{x_2 = x_3}$$

$-\frac{5-\sqrt{47}i}{2}x_1 + 4x_2 + 2x_2 = 0$

$\Rightarrow x_1 = \frac{12x_2}{5+\sqrt{47}i} \times \frac{5-\sqrt{47}i}{5-\sqrt{47}i} = \frac{1}{6}.(5-\sqrt{47}i).x_2$

$x_2 = x_3$

let $x_2 = t$

3t

$\Rightarrow x_1 = \frac{1}{6}.(5-\sqrt{47}i)t$

$x_2 = t$

$x_3 = t$

Eigenspace

$\Rightarrow (A-\lambda I)(x) = 0$

$\Rightarrow x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{6}.(5-\sqrt{47}i)t \\ t \\ t \end{bmatrix} = t.\begin{bmatrix} \frac{1}{6}.(5-\sqrt{47}i) \\ 1 \\ 1 \end{bmatrix}$

(P_2) eigen vector

Eigen space for $\lambda = \frac{3-\sqrt{47}i}{2}$ ③

$(A-\lambda I)(x) = 0$

$\Rightarrow \begin{bmatrix} \frac{-5+\sqrt{47}i}{2} & 4 & 2 \\ -3 & \frac{5+\sqrt{47}i}{2} & 0 \\ -3 & 1 & \frac{3+\sqrt{47}i}{2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\Rightarrow -3x_1 + \left(\frac{5+\sqrt{47}i}{2}\right)x_2 = 0$

$\mp 3x_1 \pm x_2 \pm \left(\frac{3+\sqrt{47}i}{2}\right)x_3 = 0$

$\left(\frac{3+\sqrt{47}i}{2}\right)x_2 - \left(\frac{3+\sqrt{47}i}{2}\right)x_3 = 0$

$x_2 - x_3 = 0 \Rightarrow$

$\boxed{x_2 = x_3}$

5(a)

continued)

$(\because x_2 = x_3)$ $\boxed{25+47=72}$

$$-\frac{5+\sqrt{47}i}{2}x_1 + 6x_2 = 0$$

$$\Rightarrow x_1 = \frac{12x_2}{5-\sqrt{47}i} \times \frac{5+\sqrt{47}i}{5+\sqrt{47}i} = \frac{\cancel{12}^{1}(5+\sqrt{47}i)x_2}{\cancel{72}_{6}}$$

$$x_1 = \frac{1}{6}(5+\sqrt{47}i)x_2$$

let $x_2 = t$

$$\Rightarrow x_1 = \frac{1}{6}(5+\sqrt{47}i)t$$

$$x_2 = t$$

$$x_3 = t$$

$\therefore$ Eigen space $\Rightarrow$ $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{6}(5+\sqrt{47}i)t \\ t \\ t \end{bmatrix}$

$$= t\begin{bmatrix} \frac{1}{6}(5+\sqrt{47}i) \\ 1 \\ 1 \end{bmatrix}$$

Hence, matrix $(P = [P_1\ P_2\ P_3])$ P_3 (Eigen vector)

$$\Rightarrow P = \begin{bmatrix} 1 & \frac{1}{6}(5-\sqrt{47}i) & \frac{1}{6}(5+\sqrt{47}i) \\ 3 & 1 & 1 \\ -4 & 1 & 1 \end{bmatrix}$$

Hence $\boxed{P^{-1}A P = D}$ holds $D = \mathrm{diag}\{\lambda_1, \lambda_2, \lambda_3\} = \{3, \frac{3+\sqrt{47}i}{2}, \frac{3-\sqrt{47}i}{2}$

$\therefore$ A is a diagonalisable matrix nges

ove rows to rows

-r-1)

swap is done when moving r to s)

5(b)

Given matrix,

$$A = \begin{bmatrix} 5 & 0 & 0 \\ 1 & 5 & 0 \\ 0 & 1 & 5 \end{bmatrix}$$

Find: P s/t $P^{-1}AP$ is a diagonal matrix

$$|A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} 5-\lambda & 0 & 0 \\ 1 & 5-\lambda & 0 \\ 0 & 1 & 5-\lambda \end{vmatrix} = (5-\lambda)(5-\lambda)(5-\lambda) = 0$$

$$\lambda = 5, 5, 5$$

($\because$ determinant of Triangular matrix is product of its main diagonal elements)

Now $(A - \lambda I)x = 0$

$\lambda = 5$,

$$\Rightarrow \left(\begin{bmatrix} 5 & 0 & 0 \\ 1 & 5 & 0 \\ 0 & 1 & 5 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} \right) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow x_1 = 0,\ x_2 = 0,$ let $x_3 = k$

$\therefore$ Eigenspace for $\lambda = 5$

$$\Rightarrow \quad v = k \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

eigen vector

As there is only one eigen vector $\begin{bmatrix}0\\0\\1\end{bmatrix}$ (distinct)

the columns of matrix $\underline{P}$

for $P^{-1}A.P = D$ cannot be formed.

as if we take columns $P_1 = P_2 = P_3 = \begin{bmatrix}0\\0\\1\end{bmatrix}$

then Linearly dependent. $\Rightarrow$ 'P' is not invertible

Hence 'A' is <u>not diagonalisable</u> ($P^{-1}.A.P$ can't be formed)

(c) Given matrix,

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3 & 0 & 1 \end{bmatrix}$$

$\Rightarrow$ characteristic Eq^n $\Rightarrow$ $|A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} -\lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ 3 & 0 & 1-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda^2(1-\lambda) = 0$$

$$\Rightarrow \lambda = \underset{\lambda_1}{0}, \underset{\lambda_2}{0}, \underset{\lambda_3}{1}$$

for $\lambda = 0$: $\quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

Eigen Space

$$\Rightarrow (A - \lambda I)(x) = 0$$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow 3x_1 + x_3 = 0$

let $x_1 = t_1$, $x_2 = t_2$ $\Rightarrow$ $x_1 = t_1$, $x_2 = t_2$

$\Rightarrow 3t_1 + x_3 = 0$

$\Rightarrow x_3 = -3t_1$

$\Rightarrow$ Eigen space is

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} t_1 \\ t_2 \\ -3t_1 \end{bmatrix} = t_1 \underbrace{\begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}}_{P_1} + t_2 \underbrace{\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}}_{P_2}$$

eigenvectors

Eigen space for $\lambda = 1$ $(A - \lambda I)[x] = [0]$

$$\Rightarrow \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 3 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow x_1 = 0$

$x_2 = 0$

$x_3 = t$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ t \end{bmatrix} = t \cdot \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{P_3}$$

P_3 – eigen vector

Hence, the 3 eigen vectors form columns of matrix P $\Rightarrow$

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} = [P_1 \; P_2 \; P_3]$$

$P_1, P_2, P_3 \Rightarrow$ are Linearly Independent $\Rightarrow$ P is Invert-ible

$\Rightarrow P^{-1}A.P$ exists and $P^{-1}A.P = D = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$

$$\Rightarrow \quad D = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

Thus $\boxed{P^{-1} \cdot A \cdot P = D}$ holds

Hence $\underline{A}$ is Diagonalisable

6

CRAMER'S RULE:

(a) Given, system of linear equations

$$x+y-z=1$$
$$x+y+z=2$$
$$x-y\quad=3$$

$$\Delta = \begin{vmatrix} 1 & 1 & -1 \\ 1 & 1 & 1 \\ 1 & -1 & 0 \end{vmatrix} = 1(0-(-1))-1(0-1)-1(-1-1)$$
$$= 1+1+2 = 4$$

$$\Delta_1 = \begin{vmatrix} 1 & 1 & -1 \\ 2 & 1 & 1 \\ 3 & -1 & 0 \end{vmatrix} = 1(0-(-1))-1(0-3)-1(-2-3)$$
$$= 1+3+5 = 9$$

$$\Delta_2 = \begin{vmatrix} 1 & 1 & -1 \\ 1 & 2 & 1 \\ 1 & 3 & 0 \end{vmatrix} = 1(0-3)-1(-1)-1(3-2) = -3$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & -1 & 3 \end{vmatrix} = 1(3-(-2))-1(3-2)+(-1-1) = 5-1-$$
$$= 2$$

$$\therefore x = \frac{\Delta_1}{\Delta} = \frac{9}{4}$$

$$y = \frac{\Delta_2}{\Delta} = \frac{-3}{4} \quad \boxed{(1,4,2)} \quad \boxed{\left(9/4, -3/4, 1/2\right)}$$

$$z = \frac{\Delta_3}{\Delta} = \frac{2}{4} = \frac{1}{2}$$

(6) Given,
(b) System of linear equations :

$2x+y-3z=1$
$y+z=1$
$z=1$

CRAMER's RULE :

$$\Delta = \begin{vmatrix} 2 & 1 & -3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 2(0)-1(0-0)-3(0-0) = 2$$

$$\Delta_1 = \begin{vmatrix} 1 & 1 & -3 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = 1(1-0)-1(1-1)-3(0-1) = 1+0+3$$
$$= 4$$

$$\Delta_2 = \begin{vmatrix} 2 & 1 & -3 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix} = 2(1-1)-1(0-0)-3(0-0) = 0+0+0$$
$$= 0$$

$$\Delta_3 = \begin{vmatrix} 2 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 2(1-0)-1(0-0)+1(0-0)$$
$$= 2+0+0 = 2$$

$\therefore x = \frac{\Delta_1}{\Delta} = \frac{4}{2} = 2$

$y = \frac{\Delta_2}{\Delta} = \frac{0}{2} = 0$ $\boxed{(x, y, z) = (2, 0, 1)}$

$z = \frac{\Delta_3}{\Delta} = \frac{2}{2} = 1$

(7)

Given λ_i ($i=1,2,3 \ldots n$) are complete set of eigen values (with repetitions) of matrix $A_{n \times n}$

RTP is $\boxed{\det(A) = \prod_{i=1}^{n} \lambda_i}$

W.K.T $\det(A - \lambda I) = 0$ — (1) characteristic eqn

where soln's (roots) of this eqn (1) are $\lambda_1, \lambda_2 \ldots \lambda_n$

$\therefore \det(A - \lambda I) = k(\lambda - \lambda_1)(\lambda - \lambda_2) \ldots (\lambda - \lambda_n)$

where k be the leading coefficient of characteristic polynomial

let $A = I_{n \times n}$

(1) $\Rightarrow |I - \lambda I| = 0 \Rightarrow |(1-\lambda) I| = 0$

$\Rightarrow (1-\lambda)^n = 0$

$\therefore \lambda_1 = \lambda_2 = \lambda_3 = \ldots = \lambda_n = 1$

$\det((1-\lambda)I) = k.(\lambda-1).(\lambda-1) \ldots (\lambda-1)$

$\Rightarrow (1-\lambda)^n = k(\lambda-1)^n$

$\Rightarrow \quad k = \frac{(1-\lambda)^n}{(\lambda-1)^n} = (-1)^n$

$\boxed{k = (-1)^n}$

Now consider th

$\det(A - \lambda I) = (-1)^n$

let $\lambda = 0$

$\Rightarrow \det(A - 0 \cdot$

$\Rightarrow \det(A) =$

$\boxed{\det(A)$

Hence, prove

Now consider the eqn

$$\det(A-\lambda I) = (-1)^n (\lambda-\lambda_1)(\lambda-\lambda_2)\cdots(\lambda-\lambda_n)$$

let $\lambda = 0$

$$\Rightarrow \det(A-0.I) = (-1)^n (0-\lambda_1)(0-\lambda_2)\cdots(0-\lambda_n)$$

$$\Rightarrow \det(A) = (-1)^n.(-\lambda_1).(-\lambda_2)\cdots(-\lambda_n)$$

$$= (-1)^n.(-1)^n.(\lambda_1.\lambda_2\cdots\lambda_n)$$

$$\boxed{\det(A) = \lambda_1.\lambda_2\cdots\lambda_n}$$

Hence, proved

(6) Given, 2 matrices A, B

RTP: $\det(AB) = \det(BA)$

W.K.T $\quad \det(EA) = (\det E)(\det A)$

(Here, $E \Rightarrow$ Elementary matrix)

Now we can express matrix $\underline{A}$ as the product of Elementary matrices

$\Rightarrow A = E_k E_{k-1} \cdots E_1$

$\Rightarrow \det(AB) = \det(E_k \cdot E_{k-1} \cdots E_1 . B)$

$= \det E_k \det E_{k-1} \cdots \det E_1 . \det B$

$(\because \det(EA) = (\det E).\det A)$

$= \det(E_k E_{k-1}) \det E_{k-2} \cdots \det E_1 . \det B$

$= \det(E_k . E_{k-1} . E_{k-2}) \cdots \det E_1 . \det B$

$= \det(E_k . E_{k-1} . E_{k-2} \cdots E_1) . \det B$

$= (\det A)(\det B)$

$= (\det B)(\det A)$

$\boxed{\therefore \det(AB) = (\det A).(\det B)}$

$\det(BA) = (\det B)(\det A)$

$= (\det A).(\det B)$ ($\because$ multiplication is commutative)

$= \det(AB)$

$\Rightarrow \det(BA) = \det(AB)$

$$\boxed{\therefore \det(AB) = \det(BA)}$$

Hence, proved.