

Problem Set 5

MA2.101: Linear Algebra (Spring 2022)

May 5, 2022

Due date: May 8, 2022

General Instructions: All symbols have the usual meanings. Remember to prove all your intermediate claims, starting from basic definitions and theorems used in class to show whatever is being asked. Try proving everything from definitions and keep arguments mathematically well formed and concise.

1. Let V be a vector space over field \mathbf{F} . For $U, T_1, T_2 \in L(V, V)$ and $c \in \mathbf{F}$, prove the following:
 - $\mathbb{I} \cdot U = U \cdot \mathbb{I} = U$
 - $U \cdot (T_1 + T_2) = U \cdot T_1 + U \cdot T_2$
 $(T_1 + T_2) \cdot U = T_1 \cdot U + T_2 \cdot U$
 - $c(U \cdot T_1) = (cU) \cdot T_1 = U \cdot (cT_1)$
2. Given a linear transformation $T : V \mapsto W$ for vector spaces V, W over field \mathbf{F} , prove that if T is invertible then its inverse $f : T^{-1}$ is a linear transformation from W to V , i.e. $T^{-1} : W \mapsto V$.