Assignment 3 MA3.101: Linear Algebra
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1. Verify the Hermitian property of the following matrices and find unitary matrix U such that U^*AU is diagonal (where * denotes the hermitian operation). Finally, compute A^{100} (answer should be in the form of a single matrix. Products of numbers need not be reduced).

   (a) $A = \begin{bmatrix} 0 & 3+i \\ 3-i & -3 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 6 & 2+2i \\ 2-2i & 4 \end{bmatrix}$

   (c) $A = \begin{bmatrix} 2 & \frac{i}{\sqrt{2}} & \frac{-i}{\sqrt{2}} \\ \frac{-i}{\sqrt{2}} & 2 & 0 \\ \frac{i}{\sqrt{2}} & 0 & 2 \end{bmatrix}$

2. Verify the Hermitian property of the following matrices and find unitary matrix U such that U^*AU is diagonal (where * denotes the hermitian operation).

   (a) $A = \begin{bmatrix} 4 & 1-i \\ 1+i & 5 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 3 & -i \\ i & 3 \end{bmatrix}$

   (c) $A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & -1 & -1+i \\ 0 & -1-i & 0 \end{bmatrix}$

3. If A and B be orthogonally diagonalizable and $AB = BA$, then show that AB is orthogonally diagonalizable.

4. Orthogonally diagonalize the following matrices

   (a) $A = \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$

(b) $A = \begin{bmatrix} -1 & 3 \\ 3 & -1 \end{bmatrix}$

(c) $A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 3 & 1 \end{bmatrix}$

5. Show that every Hermitian Matrix, every unitary matrix is normal

6. Prove that if a square complex matrix is unitarily diagonalizable then it must be normal

7. Find a singular value decomposition of the following matrices:

   (a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 0 & 0 \\ 0 & 3 \\ -2 & 0 \end{bmatrix}$

8. Show that A and A^T have the same singular values.