Assignment 1MA3.101: Linear Algebra
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1. Find the orthogonal basis of $\mathcal{R}^3$ that contains the vector
   (i) $v = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, (ii) $v = \begin{pmatrix} 3 \\ 1 \\ 5 \end{pmatrix}$.

2. Find the orthogonal basis of $\mathcal{R}^4$ that contains the vector
   (i) $v = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 0 \end{pmatrix}$.

3. Apply Gram Schmidt process to construct an orthonormal basis for the subspace $W = \text{span}(x_1, x_2, x_3)$ where $x_1 = \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix}$, $x_2 = \begin{pmatrix} 2 \\ 1 \\ 0 \\ 1 \end{pmatrix}$, $x_3 = \begin{pmatrix} 2 \\ 2 \\ 1 \\ 2 \end{pmatrix}$.

4. In the following given are the vectors from $\mathcal{R}^2$ and $\mathcal{R}^3$. Apply Gram Schmidt process to obtain the orthogonal basis. Then normalize the basis to obtain orthonormal basis.

   (a) $x_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $x_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

   (b) $x_1 = \begin{pmatrix} 3 \\ -3 \end{pmatrix}$, $x_2 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

   (c) $x_1 = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$, $x_2 = \begin{pmatrix} 0 \\ 3 \\ 3 \end{pmatrix}$, $x_3 = \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix}$

   (d) $x_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $x_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, $x_3 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

5. Show that in an inner product space there can not be unit vectors u and v with $< u, v >$ ¡-1.

6. Let u and v are two vectors in the inner product space V. Then show that $||u + v|| \leq ||u|| + ||v||$.

7. In $\mathcal{P}_2$, let $p(x) = a_0 + a_1x + a_2x^2$ and $q(x) = b_0 + b_1x + b_2x^2$. Show that $< p(x), q(x) >= a_0b_0 + a_1b_1 + a_2b_2$ defines an inner product on P_2

8. Prove that $d(u, v)\sqrt{||u||^2||v||^2}$ iff u and v are orthogonal

9. Prove that $||u + v|| = ||u - v||$ iff u and v are orthogonal.

10. Verify that if W is a subspace of an inner product space V and $v \in V$, then $perp_w(v)$ is orthogonal to all w in W.