LINEAR ALGEBRA

Assignment - 5

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(Q1) Answer:-

Given,

a vector space V over Field F.

$U, T_1, T_2 \in L(V, V)$

and $c \in F$

RTP:-

(1) $I \cdot U = U \cdot I = U$

(2) $U \cdot (T_1 + T_2) = U \cdot T_1 + U \cdot T_2$

$(T_1 + T_2) \cdot U = T_1 \cdot U + T_2 \cdot U$

(3) $c(U \cdot T_1) = (cU) \cdot T_1 = U \cdot (cT_1)$

(1) $I \cdot U = U \cdot I = U$

Let $\alpha \in V$ and $U(\alpha) = \beta \in V$ [$\because$ U is linear operator on V]

Now,

we have

$I \cdot U(\alpha) = I \cdot \beta = \beta$

but $(U(\alpha) = \beta)$

$\Rightarrow \; I \cdot U(\alpha) = U(\alpha)$

$\Rightarrow \boxed{I.U = U}$ — (i)

Now, again

$U.I(\alpha) \Rightarrow I(\alpha) = \alpha$

$= U(\alpha)$

$\boxed{\therefore U.I = U}$ — (ii)

Hence, from (i) & (ii)

$I.U = U = U.I = U$

$\Rightarrow \boxed{I.U = U.I = U}$

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(2) (i) $U.(T_1 + T_2) = U.T_1 + U.T_2$

(ii) $(T_1 + T_2).U = T_1U + T_2.U$

Let $\alpha \in V$

(i) $\Rightarrow$ $[U.(T_1 + T_2)](\alpha)$

$= U((T_1 + T_2)(\alpha))$

$= U(T_1.\alpha + T_2.\alpha)$

$= U.(T_1).\alpha + U.(T_2).\alpha$

$= (UT_1)\alpha + (UT_2)\alpha$

SO, $U(T_1 + T_2)(\alpha) = (UT_1 + UT_2)(\alpha)$

$$\boxed{\therefore U.(T_1+T_2) = U.T_1 + U.T_2}$$

(ii) $[(T_1+T_2)U](\alpha) = [T_1+T_2].U(\alpha)$

$= T_1(U(\alpha)) + T_2(U(\alpha))$

$= T_1.U(\alpha) + T_2.U(\alpha)$

$= (T_1.U + T_2.U)(\alpha)$

$$\boxed{\therefore (T_1+T_2).U = T_1.U + T_2.U}$$

Hence, proved

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(3) $c(U.T_1) = (cU).T_1 = U.(cT_1)$

Scalar $c \in$ Field F (given)

$c(UT_1)(\alpha) = c.U.T_1.(\alpha)$ — (a)

W.K.T $(cT).(\alpha) = c.T(\alpha)$

(a) $\Rightarrow (c.U).T_1(\alpha)$

$\Rightarrow c(U(T_1) = (c.U).T_1$

$$\boxed{c.(UT_1) = (cU).T_1}$$

Now Let $T(\alpha) = \beta$,

$\Rightarrow T(c\alpha) = c\beta$ ($\because$ T is a linear Transformation)

Now

for

$c(UT_1).(\alpha)$

if $T_1(\alpha) = \beta$ and $U(\beta) = \gamma \Rightarrow c\gamma = U(c\beta)$

let

$$U(cT_1)(\alpha) = U.(cT_1(\alpha))$$
$$= U(c\beta)$$
$$= c\gamma$$

Thus, we have proved that

$$\boxed{c.(U.T_1) = (cU).T_1 = U.(cT_1)}$$

$= U(k_R)$

(Q2) Answer:

Given

$T: V \to W$, where V, W are vector spaces over field F.

RTP:

If T is an invertible then its inverse func T^{-1} is a linear transformation from W to V i.e., $T^{-1}: W \to V$

proof:

let $T: V \to W$,

then by definition of inverse

T^{-1} has its domain = codomain (Range) of T

~~&~~ Range = Domain of T

then

$\exists\, T^{-1}: W \to V$ (since T is a bijection)

Now we have to prove that T^{-1} is a linear transformation from W to V.

consider

2 vectors $\alpha_1, \alpha_2 \in V$ and $\beta_1, \beta_2 \in W$

and

let $T(\alpha_1) = \beta_1 \Rightarrow T^{-1}(\beta_1) = \alpha_1$

$T(\alpha_2) = \beta_2 \Rightarrow T^{-1}(\beta_2) = \alpha_2$

As, T is a linear Transformation

$T(c\alpha_1 + \alpha_2) = c.T(\alpha_1) + T(\alpha_2)$

$= c\beta_1 + \beta_2$

$\Rightarrow T^{-1}(c\beta_1 + \beta_2) = c\alpha_1 + \alpha_2$

$= c.T^{-1}(\beta_1) + T^{-1}(\beta_2)$

$\therefore T^{-1}$ is also a linear Transformation

which maps from W to V

Hence, proved