Problem Set 3

MA2.101: Linear Algebra (Spring 2022)

April 11, 2022

Due date: April 18, 2022

General Instructions: All symbols have the usual meanings (example: F is an arbitrary field, $\mathbb{R}$ is the set of reals, $\mathbb{N}$ the set of natural numbers, and so on.) Remember to prove all your intermediate claims, starting from basic definitions and theorems used in class to show whatever is being asked. You may use any other non-trivial theorems not used in class, as long as they are well known and a part of basic linear algebra texts. It is always best to try to prove everything from definitions. Arguments should be mathematically well formed and concise.

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1. On $\mathbb{R}^n$, define two properties: $\bar{\alpha} \oplus \bar{\beta} = \bar{\alpha} - \bar{\beta}$ and $c\bar{\alpha} = -c\bar{\alpha}$,
   Which of the axioms for the vector space are satisfied by $(\mathbb{R}^n, \oplus, \cdot)$?

2. Let $\mathbf{V}$ be the set of pairs (x, y) of real numbers and let $\mathbf{F}$ be the field of real numbers.

$$\begin{aligned} (x, y) + (x_1, y_1) &= (x + x_1, 0) \\ c(x, y) &= (cx, 0) \end{aligned}$$

   Is $\mathbf{V}$ a vector space?

3. Let $\mathbf{V}$ be the set of all complex valued functions f on real line such that $\forall t \in \mathbb{R}$, $f(-t) = f((t))^* = f^*(t)$ Note: *f*(t) denotes the complex conjugation of f(t). For given problem, f(-t) is equal to the complex conjugation of f(t)*
   Show that $\mathbf{V}$ with operations $(f + g)(t) = f(t) + g(t)$ and $(cf)(t) = cf(t)$ is a vector space over the field $\mathbb{R}$.
   Give an example of a function f^n in $\mathbf{V}$ which is not real valued.

4. Prove the given theorems:

   1. A non empty subset $\mathbf{W}$ of vector space $\mathbf{V}$ is a subspace of $\mathbf{V}$ if and only if, for each pair of vectors $\bar{\alpha}, \beta \in \mathbf{W}$ and each scalar $c \in \mathbf{F}$, the vector $c\bar{\alpha} + \beta \in \mathbf{W}$
   2. Let $\mathbf{V}$ be a vector space over the field $\mathbf{F}$. The intersection of any collection of subspaces of the $\mathbf{V}$ is a subspace of $\mathbf{V}$.