MDL Assignment - 1

NAGA MANOHAR
2021101128

①

(a) The state is "wrong" as their could be situation ⇒ Raju is carrying an umbrella, but it didn't rain (since it's a sunny-day)

using truth Tables ↓

let P - the event of raining
Q - the event of Raju carrying an umbrella.

Now statement 1 is $P \to Q$

and
If both statements together are valid
If both statements together are valid thus
⇒ $P \to Q$ is true and Q is true
P has to be true

⇒ $P \to Q,\ Q \Rightarrow P$ — ①

Truth Tables

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T — ②
F	F	T

from ①, ②
$P \to Q$, Q — is True
P — is False
∴ The statement isn't valid

$S \Rightarrow T$ then there should no case when S = True & T = false

Thus, the 2 statements can't be valid simultaneously

(a) Thus, if statement 1 is true, then statement 2 need NOT be true.

(b) let
P - the event of weather is warm
Q - be the event of sky is clear.
R - be the event of we go swimming
S - be the event of we go boating

St 1: If the weather is warm and the sky is clear, then either we go swimming or we go boating. i.e.,

$$(P \wedge Q) \rightarrow (R \vee S) \quad — (1)$$

St2:- It is not the case that if we do not go swimming, then the sky is not clear. i.e.,

$\sim(\sim R \to \sim Q)$ — ②

St3:- Therefore, either the weather is warm or we go boating i.e.,

$(P \vee S)$ — ③

Now for the St1, St2, St3 to be true simultaneously.

①, ② ⟹ ③

Thus ① ⟹ $(P \wedge Q) \to (X \vee Y)$

⟹ $\sim(P \wedge Q) \vee (X \vee Y)$

⟹ $(\sim P \vee \sim Q) \vee (X \vee Y)$

② ⟹ $\sim(\sim(\sim R) \vee \sim Q)$

⟹ $\sim(R \vee \sim Q)$

⟹ $\sim R \wedge Q$

Thus, in Truth table if ①, ② are true we should get then ③ must be true

St1, St2 – are premise (1, 2)

St3 – conclusion

P	Q	R	S	St1	St2	St3 (conclusion)
T	T	T	T	T	F	T
T	T	T	F	T	F	T
T	T	F	T	T	T	T
T	T	F	F	F	T	T
T	F	T	T	T	F	T
T	F	T	F	T	F	T
T	F	F	T	T	F	T
T	F	F	F	T	F	T
F	T	T	T	T	F	T
F	T	T	F	T	F	F
F	T	F	T	T	T	T
$\boxed{F}$	$\boxed{T}$	$\boxed{F}$	$\boxed{F}$	$\boxed{T}$	$\boxed{T}$	$\boxed{F}$
F	F	T	T	T	F	T
F	F	T	F	T	F	F
F	F	F	T	T	F	T
F	F	F	F	T	F	F

For, $P = F$, $Q = T$, $R = F$, $S = R \Rightarrow$ St1, St2 (T, T) but St3 (F)

Hence, ①, ② $\not\Rightarrow$ ③

Thus St1, St2 ~~does~~ $\not\Rightarrow$ St3

Hence, the statements together are invalid.

(2)

Given,

A - true, X - False

B - true, Y - False

T - True

F - False

(a) $\sim(A \vee X) = \sim(T \vee F)$ $\quad (\because T \vee F = T)$

$= \sim(T)$ $\quad (\sim T = F)$

$= F$ = (False)

(b) $A \vee (X \wedge Y) = T \vee (F \wedge F)$ $\quad \left(\because \begin{matrix} F \wedge F = F \\ T \vee F = T \end{matrix}\right)$

$= T \vee F$

$= T$ (True)

(c) $A \wedge (X \vee (B \wedge Y)) = T \wedge (F \vee (T \wedge F))$

$= T \wedge (F \vee F)$

$= T \wedge F$

$= F$ (False)

(d) $((A \wedge X) \vee \underset{F}{\sim B}) \wedge \sim((A \wedge X) \vee \underset{T}{\sim B})$

$= ((T \wedge F) \vee F) \wedge \sim((T \wedge F) \vee F)$

$= (F \vee F) \wedge \sim(F \vee F)$

$= F \wedge \sim F$

$= F \wedge T$

$= F$ (False)

(e) $(P \wedge Q) \wedge (\sim A \vee X)$

$= (P \wedge Q) \wedge (\sim T \vee F)$

$= (P \wedge Q) \wedge (F \vee F)$

$= (P \wedge Q) \wedge F$ $(\because X \wedge F = F$ for $X = F, T)$

$= F$ (False)

(f) $((X \wedge Y) \to A) \to (X \to (Y \to A))$

$= (F \to T) \to (F \to (F \to T))$

$= (T) \to (F \to T)$

$= T \to T$

$= T$ (True)

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

3(a)

Formal Proof Method :-

Given,

$P \to \sim Q, \sim Q \to R \Rightarrow P \to R$

① $P \to \sim Q$ (premise)

② $\sim Q \to R$ (premise 2)

③ $P \to R$ (∵ ①, ② ⇒ Hypothetical syllogism
$P \to Q, Q \to R \Rightarrow P \to R$)

$\boxed{\therefore P \to \sim Q, \sim Q \to R \Rightarrow P \to R}$ is true

Resolution Method :-

① Convert the premise's and goal to CNF (conjunction of disjunctions)

⇒

$P \to \sim Q = \sim P \vee \sim Q$ — P_1

$\sim Q \to R = \sim(\sim Q) \vee R = Q \vee R$ — P_2

~~$P \to R = \sim P \vee R$~~ $P \to R = \sim P \vee R$

Now to prove that P follows from S (S ⇒ P)

using refutation,

start with S and ~P in clausal form &

derive a contradiction

and if empty clause derived ⇒ S, ~P are never true together

⇒ S are True, atleast one clause of P is false

⇒ ~P is false

⇒ P is true.

Thus consider

$\sim(3)$ $\cancel{\sim(P\to Q)}$ $\sim(P\to R) = \sim(\sim P \vee R)$

$= P \wedge \sim R$

$\Downarrow$

$P, \sim R$

$C_1.\ \sim P \vee \sim Q$ (premise)

$C_2.\ Q \vee R$ (premise)

$C_3.\ P$ (from negation of conclusion (3))

$C_4.\ \sim R$ (" ")

$C_5.\ Q$ (C_2, C_4 resolution)

$C_6.\ \sim P$ (C_1, C_5 resolution)

C_7 Empty clause (C_3, C_6 resolution)

$S, \sim P\ (S \Rightarrow \sim P) \Rightarrow$ empty class is derived

$\Rightarrow \sim P$ is false

$\Rightarrow P$ is true

Hence, $\sim P \vee \sim Q,\ Q \vee R \Rightarrow \sim P \vee R$

$\Rightarrow \boxed{P \to \sim Q,\ \sim Q \to R \Rightarrow P \to R}$ is true

(b) (Here LHS is Empty clause)

consider

$R.H.S = ((P \vee Q) \wedge \sim P) \to Q$

$= ((P \wedge \sim P) \vee (Q \wedge \sim P)) \to Q$

$= (F \vee (Q \wedge \sim P)) \to Q$ ($\because F \vee X = X$)

$= (Q \wedge \sim P) \to Q$

$=$

$= \sim((Q \wedge \sim P)) \vee Q$

$= (\sim Q \vee \sim(\sim P)) \vee Q$

$= (\sim Q \vee P) \vee Q$

$= (Q \vee \sim Q) \vee P$

$= T \vee P$

$= T$ (True)

$=$ R.H.S

Formal Proof method:

from the above observations conclusion (R.H.S) is always true (Tautology)

Thus, is true

$\Rightarrow ((P \vee Q) \wedge \sim P) \rightarrow Q$

and valid.

Resolution method:

again in $S \Rightarrow P$ consider $\sim P$ and prove resolution of $S, \sim P$ empty clause

$\Rightarrow$ R.H.S $= ((P \vee Q) \wedge \sim P) \rightarrow Q$

$(A \to B = \sim A \vee B)$

$(\sim P)$ (R.H.S)

$\Rightarrow$ $\sim((P \vee Q) \wedge (\sim P)) \to Q)$

$= \sim(\sim((P \vee Q) \wedge \sim P)) \vee Q)$

$= (\sim(P \vee Q) \wedge P)) \vee Q)$

$= ((P \vee Q) \wedge \sim P \wedge \sim Q)$

$\Rightarrow$ clausal formst

C_1: $P \vee Q$ from negation of conclusion (R.H.S)

C_2. $\sim P$ " " "

C_3. $\sim Q$ " " "

C_4. P (C_1, C_3 Resolution)

C_5. EMPTY clause (C_2, C_4 Resolution)

$\therefore S \to \sim P \Rightarrow$ empty clause (resolution)

$\Rightarrow S \Rightarrow P$ is true

$\therefore \Rightarrow ((P \vee Q) \wedge \sim P) \to Q$ is true

Hence, proved