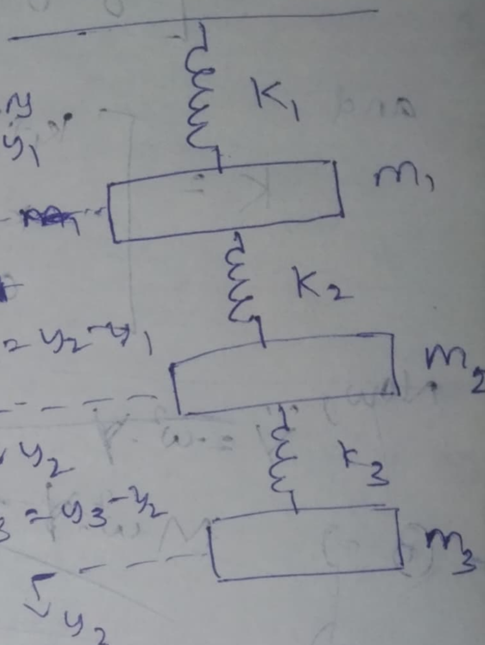
(2)

Given,

masses - m_1, m_2, m_3

vertical displacement - y_1, y_2, y_3

as shown in the below figure



W.K.T

by Hooke's law

$F = -kx$

where

x = extension in spring's

and gravity (mg) is acting downwards.

Thus,

Equation's of Motion's - for m_1

$$\cancel{-k_2(y_2 - y_1) + k_1 y_1 = -m_1 \ddot{y}_1}$$

$$\cancel{= -m_1 \ddot{y}_2}$$

$$k_2(y_2 - y_1) - k_1 y_1 = m_1 \ddot{y}_1$$

$$k_3 \cdot (y_3 - y_2) - k_2(y_2 - y_1) = m_2 \cdot \ddot{y}_2$$

$$-k_3(y_3 - y_2) = m_3 \cdot \ddot{y}_3$$

Thus,

$$-(k_1 + k_2)y_1 + k_2 y_2 = m_1 \cdot \ddot{y}_1$$

$$k_2 y_1 - (k_2 + k_3) \cdot y_2 + k_3 y_3 = m_2 \cdot \ddot{y}_2$$

$$k_3 y_2 - k_3 y_3 = m_3 \cdot \ddot{y}_3$$

Bonus

Thus, $F = -kx$

$\Rightarrow M\ddot{Y} = -KY$ — (1)

where $M = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}$, $\ddot{Y} = \begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \\ \ddot{y}_3 \end{bmatrix}$

and $K = \begin{bmatrix} k_1+k_2 & -k_2 & 0 \\ -k_2 & k_2+k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix}$

~~Now, $\ddot{Y} = -\omega^2 \cdot Y$~~ ~~Y = displacement~~

~~(1) $\Rightarrow M\omega^2 Y = -KY$~~

~~$\Rightarrow$~~

Now let $Y = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} e^{i\omega t} = A \cdot e^{i\omega t}$ $A = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$

$\Rightarrow \ddot{Y} = \frac{d^2}{dt^2} \cdot Ae^{i\omega t} = A(i\omega)(i\omega) \cdot e^{i\omega t} = -A\omega^2 \cdot e^{i\omega t}$

(1) $\Rightarrow M\ddot{Y} = -KY$

$\Rightarrow M \cdot (-A\omega^2 e^{i\omega t}) = -K \cdot Ae^{i\omega t}$

$\Rightarrow (K - M\omega^2) Ae^{i\omega t} = 0$

$\Rightarrow (K - M\omega^2) \cdot A = 0$

Now, ignore the trivial solution $A = 0$ (~~A 0~~ r

Then $\boxed{|K - M\omega^2| = 0}$

Given,

$k_1 = k_2 = k_3 = 1,$

$m_1 = m_2 = 2,$

$m_3 = 4.$

Now

$$K - M\omega^2 = \cancel{\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}} * \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} - \omega^2 \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2-2\omega^2 & -1 & 0 \\ -1 & 2-2\omega^2 & -1 \\ 0 & -1 & 1-4\omega^2 \end{bmatrix}$$

$$|K - M\omega^2| = 0$$

$$\Rightarrow (2-2\omega^2)\left((2-2\omega^2).(1-4\omega^2) - 1\right) + \left((4\omega^2-1)\right) + 0 = 0$$

$$\Rightarrow (2-2\omega^2)(2 - 8\omega^2 - 2\omega^2 + 8\omega^4 - 1) + 4\omega^2 - 1 = 0$$

$$\Rightarrow (2-2\omega^2)(8\omega^4 - 10\omega^2 + 1) + 4\omega^2 - 1 = 0$$

$$\Rightarrow 16\omega^4 - 20\omega^2 + 2 - 16\omega^6 + 20\omega^4 - 2\omega^2 + 4\omega^2 - 1 = 0$$

$$\Rightarrow -16\omega^6 + 36\omega^4 - 18\omega^2 + 1 = 0$$

$$\Rightarrow 16\omega^6 - 36\omega^4 + 18\omega^2 - 1 = 0$$

On solving we get

$$\cancel{\omega = \pm 1.24 \pm\sqrt{\frac{3}{2}}, \pm\sqrt{\frac{3}{4}}, \pm\sqrt{\frac{1}{2}}}$$

$$\omega = \cancel{\pm 1.24}, \pm 1.24, \pm 0.79, \pm 0.25$$

$$\boxed{\therefore \omega = \cancel{\pm} 1.24, \cancel{\pm} 0.79, \cancel{\pm} 0.25}$$

thus we get 3 frequencies $\Rightarrow \omega_f = 1.24, 0.79, 0.25$