Science2 - Assignment-1

NAGA.MANOHAR
2021101128

① Given,

$p(t) = a_1t^2 + a_2t + a_3 = y$

t	-1	-0.5	0	0.5	1
y	1	0.5	0	0.5	2

Find: a_1, a_2, a_3?

Now we form A as Vandermonde matrix

$Ax = b$

$$\Rightarrow Ax = \underbrace{\begin{bmatrix} 1 & t_1 & t_1^2 \\ 1 & t_2 & t_2^2 \\ 1 & t_3 & t_3^2 \\ 1 & t_4 & t_4^2 \\ 1 & t_5 & t_5^2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} a_3 \\ a_2 \\ a_1 \end{bmatrix}}_{x} \cong \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} = b$$

$$\Rightarrow Ax = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -0.5 & 0.25 \\ 1 & 0 & 0 \\ 1 & 0.5 & 0.25 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} a_3 \\ a_2 \\ a_1 \end{bmatrix} \cong \begin{bmatrix} 1 \\ 0.5 \\ 0 \\ 0.5 \\ 2 \end{bmatrix} = b$$

Now as the system is overdetermined ($m > n$)

we use LLS to find $x = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$

Normal eqn $\Rightarrow$ $A^T b = A^T A x$

$$\Rightarrow x = (A^T A)^{-1} \cdot A^T \cdot b$$

From the code implementation we get

$$x = \begin{bmatrix} a_3 \\ a_2 \\ a_1 \end{bmatrix} = \begin{bmatrix} 0.086 \\ 0.4 \\ 1.4 \end{bmatrix}$$

Thus

$$\boxed{p(t) = 1.4t^2 + 0.4t + 0.086}$$