UNIT-V
MAGNETIC CIRCUITS

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INTRODUCTION

Although the lines of magnetic flux have no physical existence, they do form a very convenient and useful basis for explaining various magnetic effects and to calculate the magnitudes of various magnetic quantities. The complete closed path followed by any group of magnetic flux lines is referred as magnetic circuit. The lines of magnetic flux never intersect, and each line forms a closed path. Whenever a current is flowing through the coil there will be magnetic flux produced and the path followed by the magnetic flux is known as magnetic circuit. The operation of all the electrical devices like generators, motors, transformers etc. depend upon the magnetism produced by this magnetic circuit. Therefore, to obtain the required characteristics of these devices, their magnetic circuits have to be designed carefully.

Magneto Motive Force (MMF)

The magnetic pressure which sets up or tends to set up magnetic flux in a magnetic circuit is known as MMF.

1. Magneto motive force is the measure of the ability of a coil to produce flux.
2. The magnetic flux is due to the existence of the MMF caused by a current flowing through a coil having no. of turns.
3. ∴A coil with 'N' turns carrying a current of 'I' amperes represents a magnetic circuit producing an MMF of NI MMF=NI
4. Units of MMF = Ampere turns(AT)

Magnetic Flux:

1. The amount of magnetic lines of force set-up in a magnetic circuit is called magnetic flux.
2. The magnetic flux, that is established in a magnetic circuit is proportional to the MMF and the proportional constant is the reluctance of the magnetic circuit.

Magnetic flux α MMF

$$\text{Magnetic flux}=\frac{MMF}{RELUCTANCE}=\frac{NI}{S}$$

3. The unit of magnetic flux is Weber.

Reluctance:

1. The opposition offered to the flow of magnetic flux in a magnetic circuit is called reluctance

2. Reluctance of a magnetic circuit is defined as the ratio of magneto motive force to the flux established.
3. Reluctance depends upon length(l), area of cross-section(a) and permeability of the material that makes up the magnetic circuit.(S l, S a, S l/a)

$$S=\frac{l}{\mu a}$$

$$RELUCTANCE=\frac{MMF}{FLUX}$$

4. The unit of reluctance is AT/ Wb

Magnetic field strength(H)

1. If the magnetic circuit is homogeneous, and of uniform cross-sectional area, the magnetic field strength is defined as the magneto motive force per unit length of magnetic circuit.

$$H=\frac{MMF}{LENGTH}=\frac{NI}{l}$$

2. The unit of magnetic field strength is AT/m

Magnetic flux density(B)

1. The magnetic flux density in any material is defined as the magnetic flux established per unit area of cross-section.

$$B=\frac{FLUX}{AREA\ OF\ CROSS\ SECTION}=\frac{\emptyset}{A}$$

2.The unit of magnetic flux density is wb/m^2 or TESLA

Relative permeability

1. It is defined as the ratio of flux density established in magnetic material to the flux density established in air or vacuum for the same magnetic field strength.

INTRODUCTION TO ELECTROMAGNETIC INDUCTION::

When a conductor moves in a magnetic field, an *EMF* is generated; when it carries current in a magnetic field, a force is produced. Both of these effects may be deduced from one of the most fundamental principles of electromagnetism, and they provide the basis for a number of devices in which conductors move freely in a magnetic field. It has already been mentioned that most electrical machines employ a different form of construction.

FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION:

In 1831, Michael Faraday, an English physicist gave one of the most basic laws of electromagnetism called **Faraday's law of electromagnetic induction**. This law explains the working principle of most of the electrical motors, generators, electrical transformers and inductors. This law shows the relationship between electric circuit and magnetic field.

FARADAY'S FIRST LAW

Any change in the magnetic field of a coil of wire will cause an emf to be induced in the coil. This emf induced is called induced emf and if the conductor circuit is closed, the current will also circulate through the circuit and this current is called induced current.

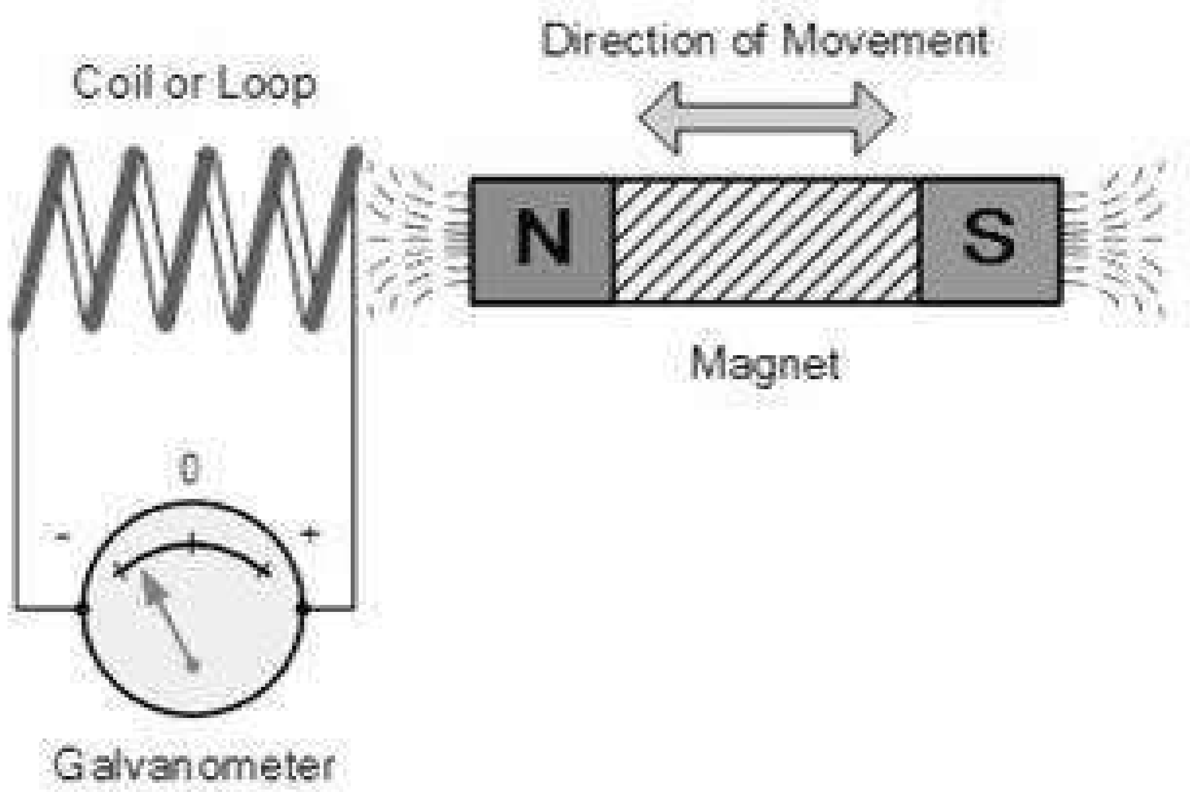
Method to change magnetic field:

- By moving a magnet towards or away from the coil
- By moving the coil into or out of the magnetic field.
- By changing the area of a coil placed in the magnetic field
- By rotating the coil relative to the magnet.

Faraday's Second Law

It states that the magnitude of emf induced in the coil is equal to the rate of change of flux that linkages with the coil. The flux linkage of the coil is the product of number of turns in the coil and flux associated with the coil.

Faraday Law Formula



Consider, a magnet is approaching towards a coil. Here we consider two instants at time T_1 and time T_2.

Flux linkage with the coil at time,

$$T_1 = N\phi_1 \ wb$$

Flux linkage with the coil at time,

$$T_2 = N\phi_2 \ wb$$

Change in flux linkage,

$$N(\phi_2 - \phi_1)$$

Let this change in flux linkage be,

$$\phi = (\phi_2 - \phi_1)$$

So, the Change in flux linkage

$$N\phi$$

Now the rate of change of flux linkage

$$\frac{N\phi}{t}$$

Take derivative above equation we will get the rate of change of flux linkage

$$N\frac{\mathrm{d}\phi}{\mathrm{d}t}$$

But according to Faraday's law of electromagnetic induction, the rate of change of flux linkage is equal to induced emf.

$$E = N\frac{d\phi}{dt}$$

$$E = -\ N\frac{d\phi}{dt}$$

Where, flux Φ in Wb = B*A
B = magnetic field strength
A = area of the coil

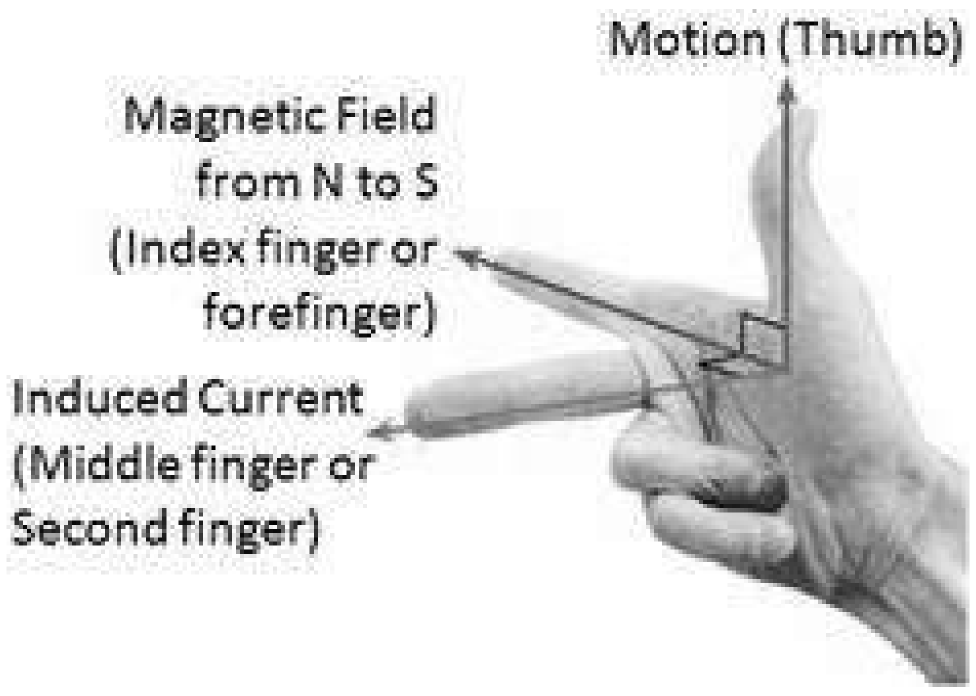
Lenz's law obeys Newton's third law of motion (i.e. to every action there is always an equal and opposite reaction) and the conservation of energy (i.e. energy may neither be created nor destroyed and therefore the sum of all the energies in the system is a constant).

Lenz's law : It states that when an emf is generated by a change in magnetic flux according to Faraday's Law, the polarity of the induced emf is such, that it produces a current that's magnetic field opposes the change which produces it.

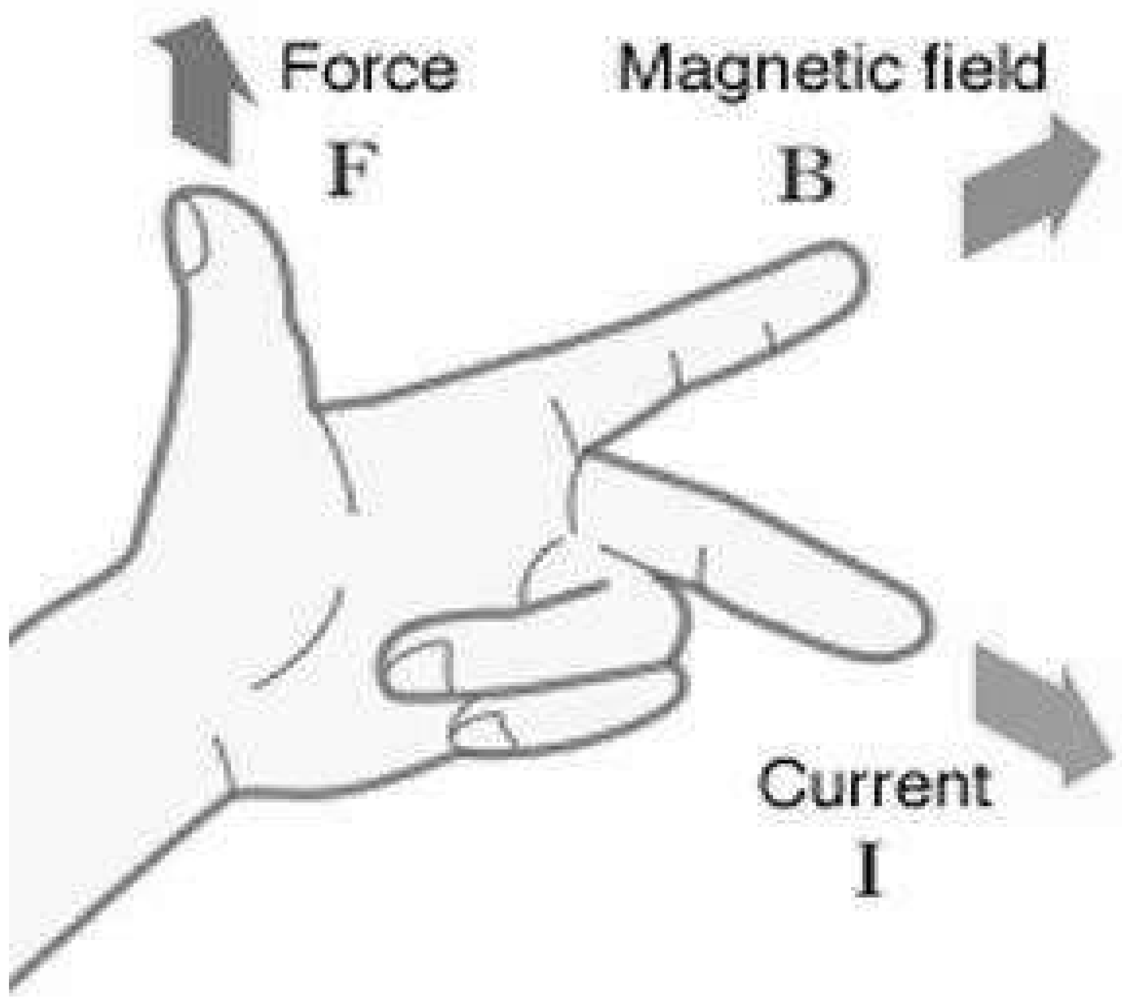
$$E = -\,N\frac{d\phi}{dt}$$

The negative sign used in Faraday's law of electromagnetic induction, indicates that the induced emf and the change in magnetic flux have opposite signs.

- There exists a definite relation between the direction of the induced current, the direction of the flux and the direction of motion of the conductor. The direction of the induced current may be found easily by applying either *Fleming's Right-hand Rule*



- There exists a definite relation between the direction of the induced current, the direction of the flux and the direction of motion of the conductor. The direction of the induced current may be found easily by applying either *Fleming's Left-hand Rule*



HOW TO INCREASE EMF INDUCED IN A COIL:

- By increasing the number of turns in the coil i.e. N, from the formulae derived above it is easily seen that if number of turns in a coil is increased, the induced emf also gets increased.
- By increasing magnetic field strength i.e. B surrounding the coil- Mathematically, if magnetic field increases, flux increases and if flux increases emf induced will also get

increased. Theoretically, if the coil is passed through a stronger magnetic field, there will be more lines of force for coil to cut and hence there will be more emf induced.

- By increasing the speed of the relative motion between the coil and the magnet - If the relative speed between the coil and magnet is increased from its previous value, the coil will cut the lines of flux at a faster rate, so more induced emf would be produced.

APPLICATIONS OF FARADAY'S LAW:

Faraday law is one of the most basic and important laws of electromagnetism. This law finds its application in most of the electrical machines, industries and medical field etc.

- Electrical Transformers work on Faraday's law of mutual induction.
- The basic working principle of electrical generator is Faraday's law of electromagnetic induction.
- The Induction cooker is a fastest way of cooking. It also works on principle of mutual induction. When current flows through the coil of copper wire placed below a cooking container, it produces a changing magnetic field. This alternating or changing magnetic field induces an emf and hence the current in the conductive container, and we know that flow of current always produces heat in it.
- Electromagnetic Flow Meter is used to measure velocity of certain fluids. When a magnetic field is applied to electrically insulated pipe in which conducting fluids are flowing, then according to Faraday's law, an electromotive force is induced in it. This induced emf is proportional to velocity of fluid flowing.
- Form the bases of Electromagnetic theory; Faraday's idea of lines of force is used in well known Maxwell's equations. According to Faraday's law, change in magnetic field gives rise to change in electric field and the converse of this is used in Maxwell's equations.
- It is also used in musical instruments like electric guitar, electric violin etc.

SELF INDUCTANCE:

Inductance is the property of electrical circuits containing coils in which a change in the electrical current induces an electromotive force (emf). This value of induced emf opposes the change in current in electrical circuits and electric current 'I' produces a magnetic field which generates magnetic flux acting on the circuit containing coils. **The ratio of the magnetic flux to the current is called the self-inductance.**

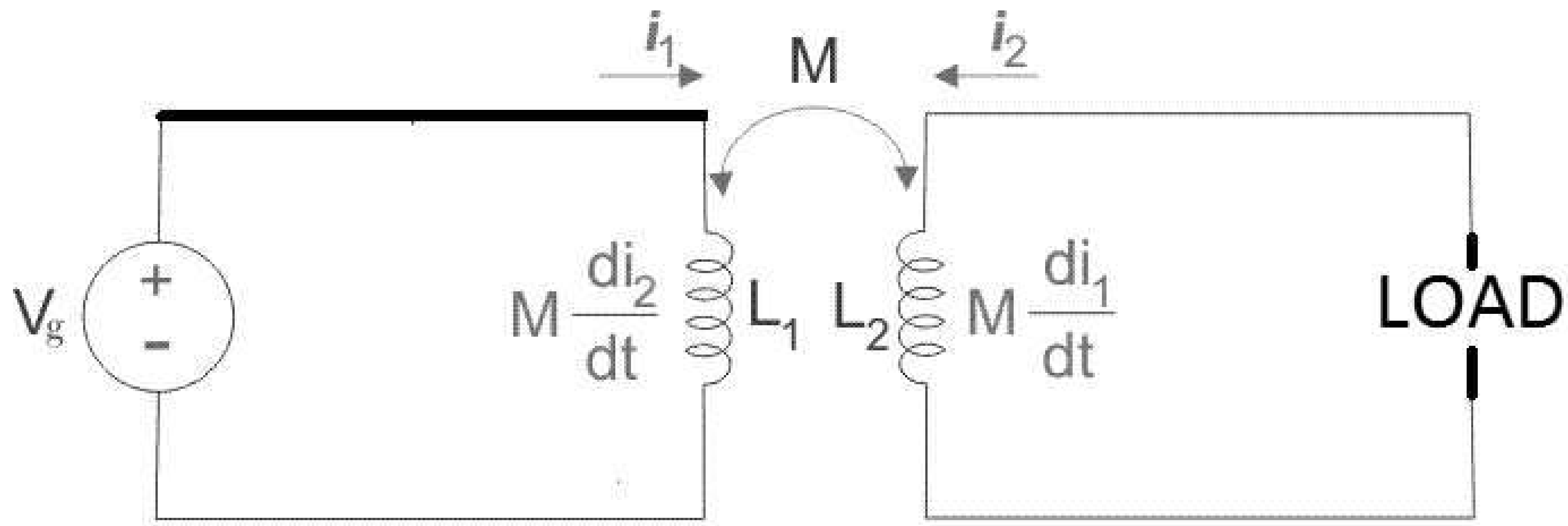
$$L = \frac{\psi = N\emptyset}{I}$$

The phenomenon of inducing an emf in a coil whenever a current linked with coil changes is called induction. Here units of L are Weber per ampere which is equivalent to Henry.

'Ø' denotes the magnetic flux through the area spanned by one loop, 'I' is the current flowing through the coil and N is the number of loops (turns) in the coil.

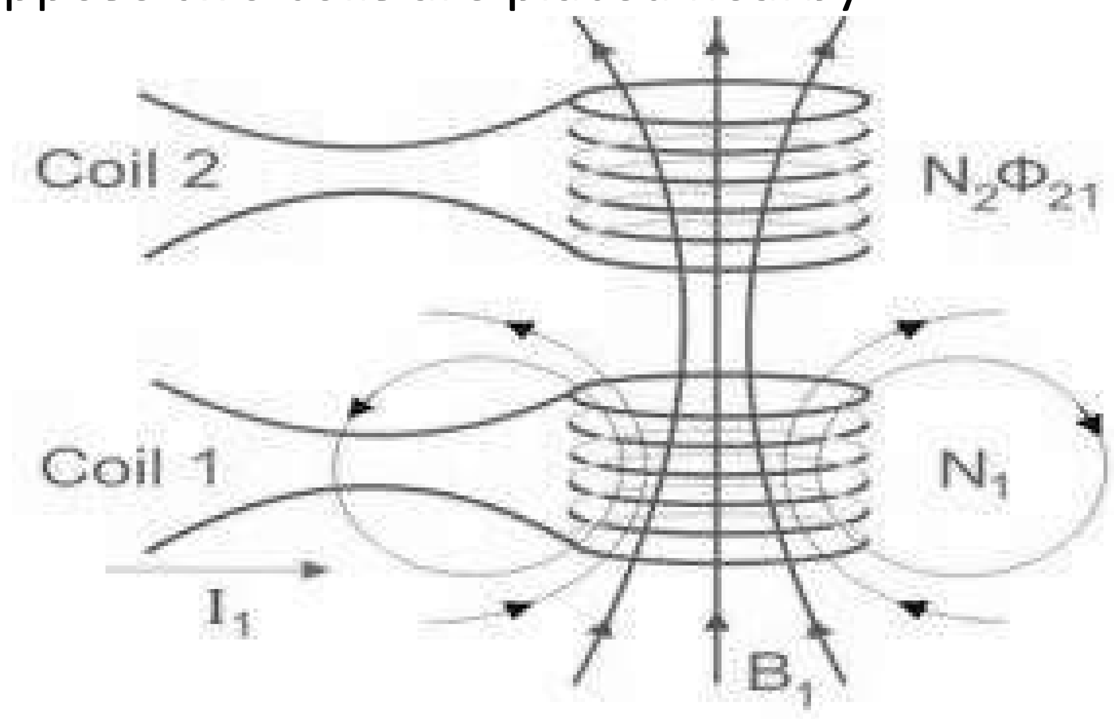
MUTUAL INDUCTANCE:

Mutual Inductance is the ratio between induced Electro Motive Force across a coil to the rate of change of current of another adjacent coil in such a way that two coils are in possibility of flux linkage. Mutual induction is a phenomenon when a coil gets induced in EMF across it due to rate of change current in adjacent coil in such a way that the flux of one coil current gets linkage of another coil. Mutual inductance is denoted as (M), it is called co-efficient of Mutual Induction between two coils



Mutual inductance for two coils gives the same value when they are in mutual induction with each other. Induction in one coil due to its own rate of change of current is called self inductance (L), but due to rate of change of current of adjacent coil it gives **mutual inductance** (M)

From the above figure, first coil carries current i_1 and its self inductance is L_1. Along with its self inductance it has to face mutual induction due to rate of change of current i_2 in the second coil. Same case happens in the second coil also. Dot convention is used to mark the polarity of the mutual induction. Suppose two coils are placed nearby



Coil 1 carries I_1 current having N_1 number of turn. Now the flux density created by the coil 1 is B_1. Coil 2 with N_2 number of turn gets linked with this flux from coil 1. So flux linkage in coil 2 is N_2.

ϕ_{21} [ϕ_{21} is called leakage flux in coil 2 due to coil 1].

$$\varepsilon_2 = -N_2 . \frac{d\varphi_{21}}{dt} \ volt.$$

$$Again, \ \varepsilon_2 = -M_{21} . \frac{dI_1}{dt} \ volt.$$

Now it can be written from these equations,

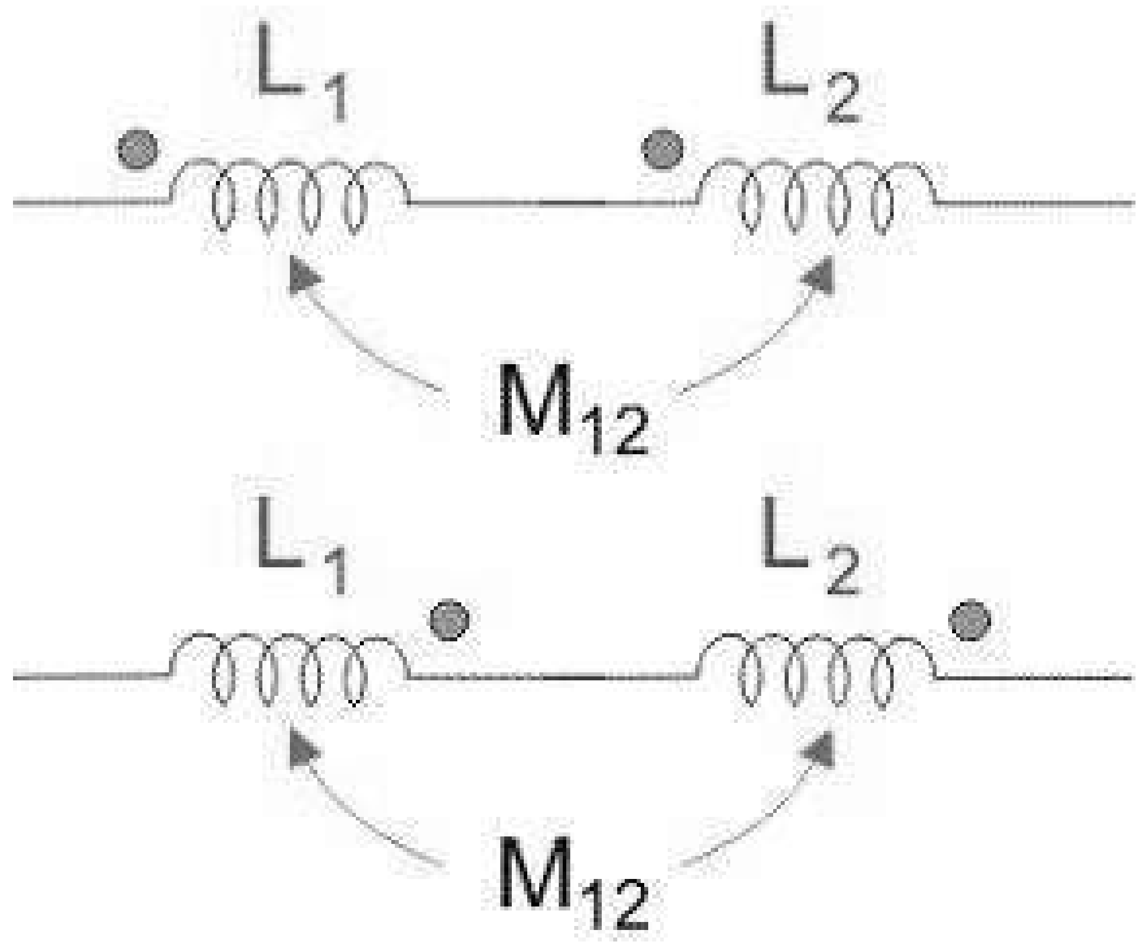
$$M_{21} = \frac{\varphi_{21} N_2}{I_1}$$

DOT CONVENTION:

- Dot convention is used to determine the polarity of a magnetic coil in respect of other magnetic coil.
- Dot convention is normally used to determine the total or equivalent inductance (**L_{eq}**).

SERIES AIDING:

- Suppose two coils are in series with same place dot.
- When 2 dots are at the same place of both inductors(while at entering place or leaving place)as shown in below figure i.e. the total mutual inductance gets aided(added)

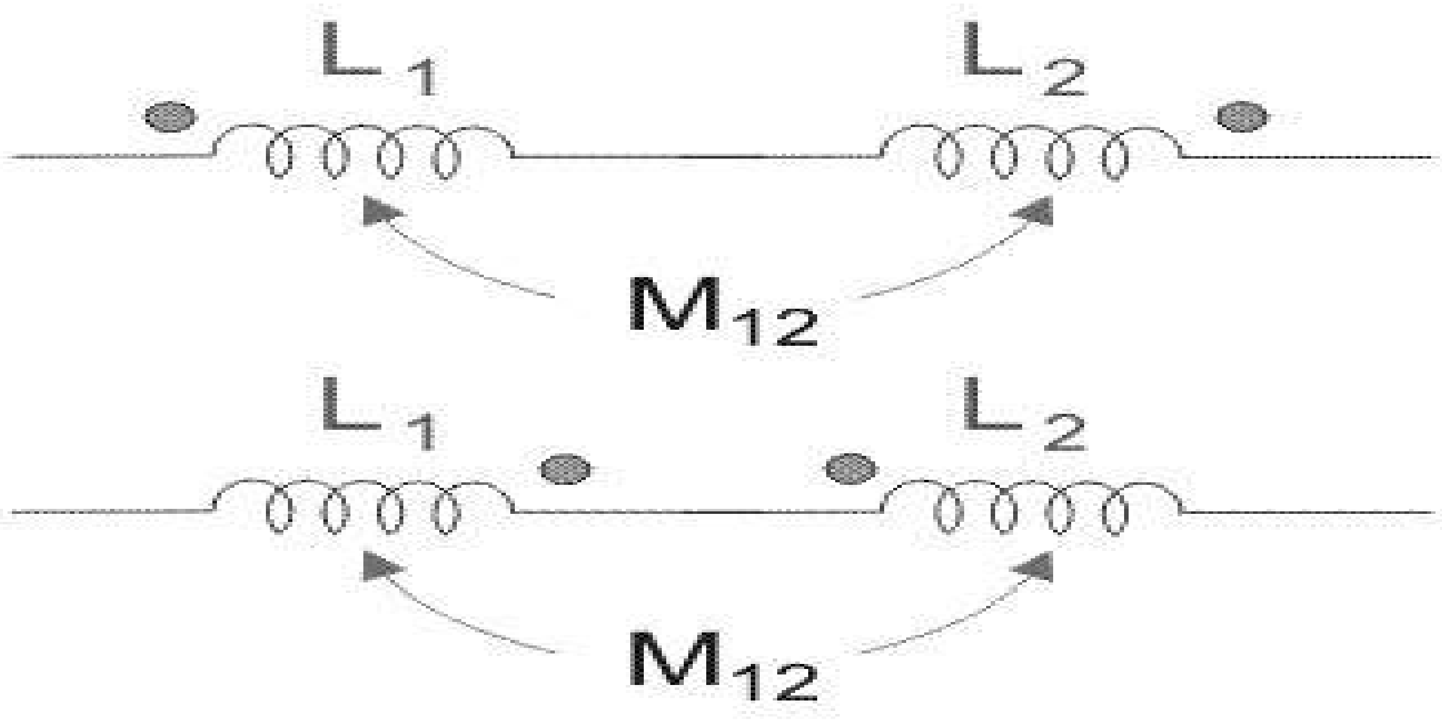


Mutual inductance between them is positive.

$$So, \quad L_{eq} = L_1 + L_2 + 2M_{12}$$

SERIES OPPOSING:

- Suppose two coils are in series with opposite place dot.
- When 2 dots are at the opposite place of both inductors(while one at entering place and other at leaving place)as shown in below figure i.e. the total mutual inductance gets differed

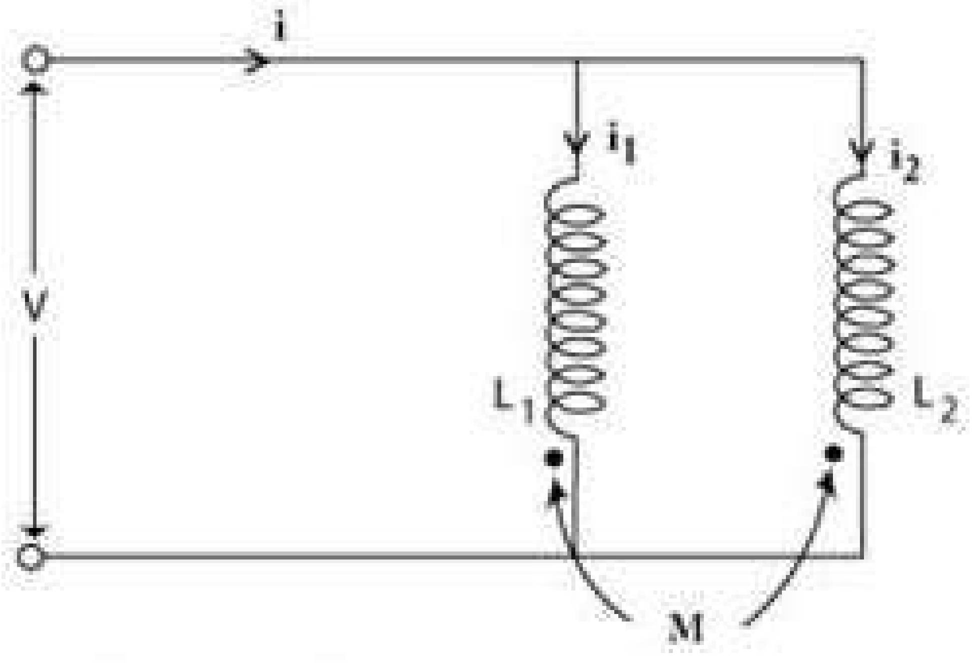


Mutual inductance between them is negative.

$$So, \quad L_{eq} = L_1 + L_2 - 2M_{12}$$

PARALLEL AIDING:

- Suppose two coils are in parallel with same place dot.
- When 2 dots are at the same place of both inductors(while at entering place or leaving place)as shown in below figure i.e. the total mutual inductance gets aided(added)



$V=L_1\frac{dI_1}{dt}+M\frac{dI_2}{dt}$

$V=L_2\frac{dI_2}{dt}+M\frac{dI_1}{dt}$

$$\begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix}\begin{bmatrix} \frac{di_1}{dt} \\ \frac{di_2}{dt} \end{bmatrix}=\begin{bmatrix} V \\ V \end{bmatrix}$$

$$\Delta=\begin{vmatrix} L_1 & M \\ M & L_2 \end{vmatrix}=L_1L_2-M^2$$

$$\frac{di_1}{dt}=\frac{\begin{vmatrix} V & M \\ V & L_2 \end{vmatrix}}{\Delta}=\frac{V(L_2-M)}{\Delta}\quad,\quad \frac{di_2}{dt}=\frac{\begin{vmatrix} L_1 & V \\ M & V \end{vmatrix}}{\Delta}=\frac{V(L_1-M)}{\Delta}$$

From the above figure,

$i=i_1+i_2$

$$\frac{di}{dt}=\frac{di_1}{dt}+\frac{di_2}{dt}=\frac{V(L_2-M)}{\Delta}+\frac{V(L_1-M)}{\Delta}=\frac{V(L_1+L_2-2M)}{\Delta}=\frac{V(L_1+L_2-2M)}{L_1L_2-M^2}$$

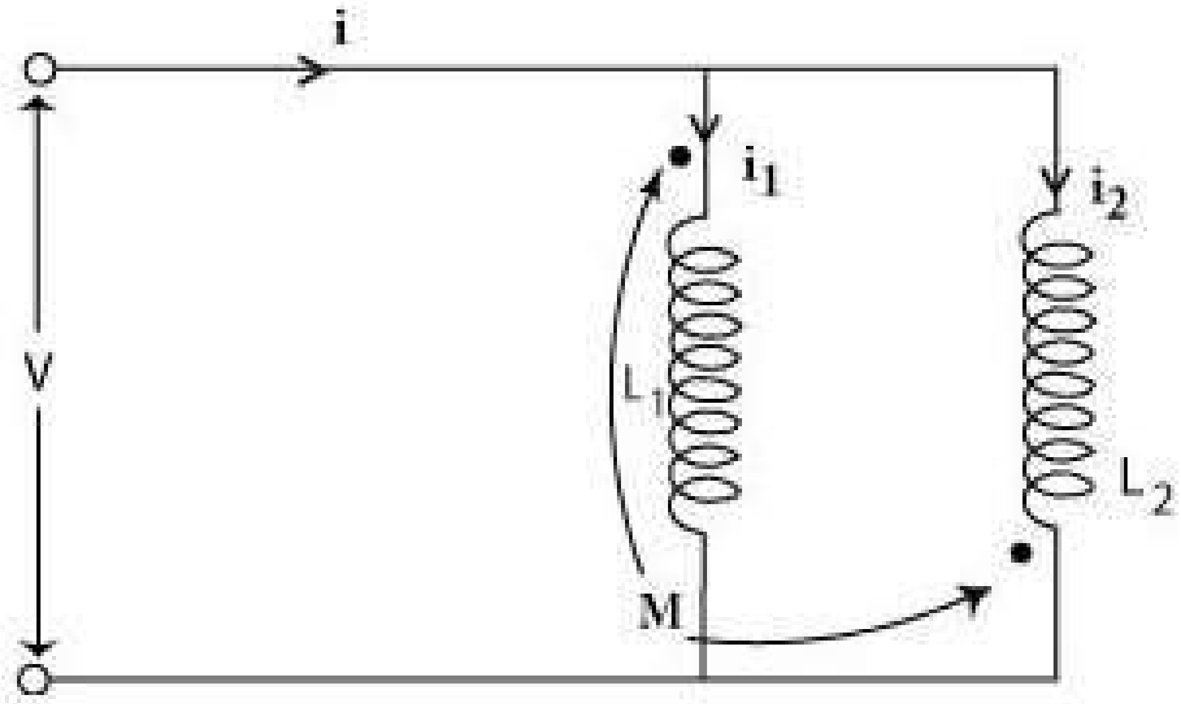
$$V=\frac{L_1L_2-M^2}{(L_1+L_2-2M)}\frac{di}{dt}$$

Therefore total inductance is given by,

$$\mathbf{L_{eq}}=\frac{L_1L_2-M^2}{(L_1+L_2-2M)}$$

PARALLEL OPPOSING:

- Suppose two coils are in parallel with opposite place dot.
- When 2 dots are at the opposite place of both inductors(while one at entering place and other at leaving place)as shown in below figure i.e. the total mutual inductance gets differed



$$V=L_1\frac{dI_1}{dt}-M\frac{dI_2}{dt}$$

$$V=L_2\frac{dI_2}{dt}-M\frac{dI_1}{dt}$$

$$\begin{bmatrix} L_1 & -M \\ -M & L_2 \end{bmatrix}\begin{bmatrix} \frac{di_1}{dt} \\ \frac{di_2}{dt} \end{bmatrix}=\begin{bmatrix} V \\ V \end{bmatrix}$$

$$\Delta=\begin{vmatrix} L_1 & -M \\ -M & L_2 \end{vmatrix}=L_1L_2-M^2$$

$$\frac{di_1}{dt}=\frac{\begin{vmatrix} V & -M \\ V & L_2 \end{vmatrix}}{\Delta}=\frac{V(L_2+M)}{\Delta}, \qquad \frac{di_2}{dt}=\frac{\begin{vmatrix} L_1 & V \\ -M & V \end{vmatrix}}{\Delta}=\frac{V(L_1+M)}{\Delta}$$

From the above figure,

$$i=i_1+i_2$$

$$\frac{di}{dt}=\frac{di_1}{dt}+\frac{di_2}{dt}=\frac{V(L_2+M)}{\Delta}+\frac{V(L_1+M)}{\Delta}=\frac{V(L_1+L_2+2M)}{\Delta}=\frac{V(L_1+L_2+2M)}{L_1L_2-M^2}$$

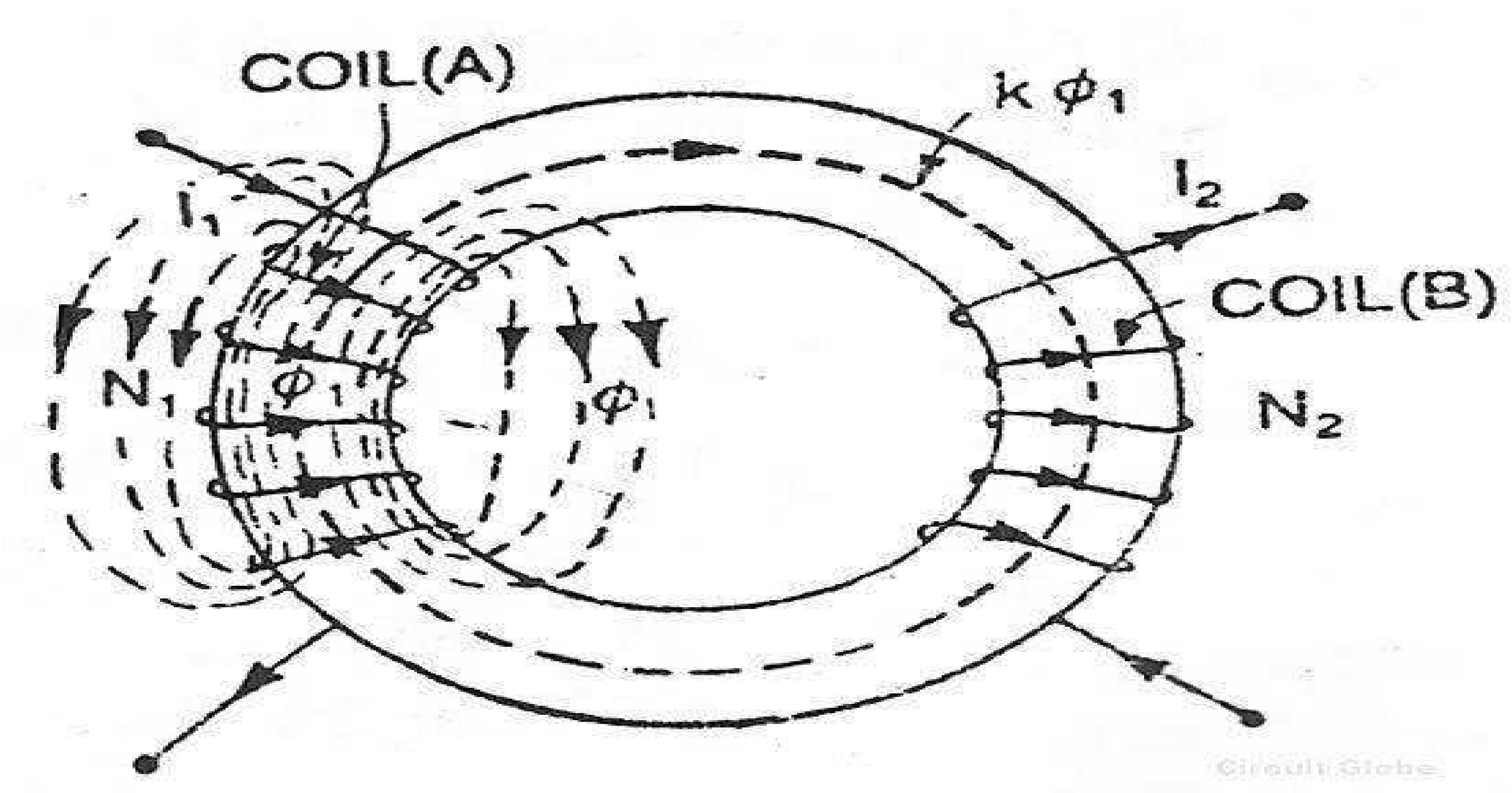
$$V=\left(\frac{L_1L_2-M^2}{L_1+L_2+2M}\right)\frac{di}{dt}$$

Therefore total inductance is given by,

$$L_{eq} = \frac{L_1 L_2 - M^2}{(L_1 + L_2 + 2M)}$$

COEFFICIENT OF COUPLING:

The fraction of magnetic flux produced by the current in one coil that links with the other coil is called **coefficient of coupling** between the two coils. It is denoted by (k).
Two coils are taken coil A and coil B, when current flows through one coil it produces flux; the whole flux may not link with the other coil coupled, and this is because of leakage flux by a fraction (k) known as **Coefficient of Coupling.**



k=1 when the flux produced by one coil completely links with the other coil and is called magnetically tightly coupled.

k=0 when the flux produced by one coil does not link at all with the other coil and thus the coils are said to be magnetically isolated.

DERIVATION:

Consider two magnetic coils A and B.
When current I_1 flows through coil A.

$$L_1 = \frac{N_1\varphi_1}{I_1} \text{ and } M = \frac{N_2\varphi_{12}}{I_1} \ldots\ldots(1) \text{ as } (\varphi_{12} = k\varphi_1)$$

Considering coil B in which current I_2 flows

$$L_2 = \frac{N_2\varphi_2}{I_2} \text{ and } M = \frac{N_1\varphi_{21}}{I_2} = \frac{N_1 k\varphi_2}{I_2} \ldots\ldots(2) \quad \text{as } (\varphi_2 = k\varphi_2)$$

Multiplying equation (1) and (2)

$$MxM = \frac{N_{2k\varphi_1}}{I_1} x \frac{N_{1k\varphi_2}}{I_2}$$

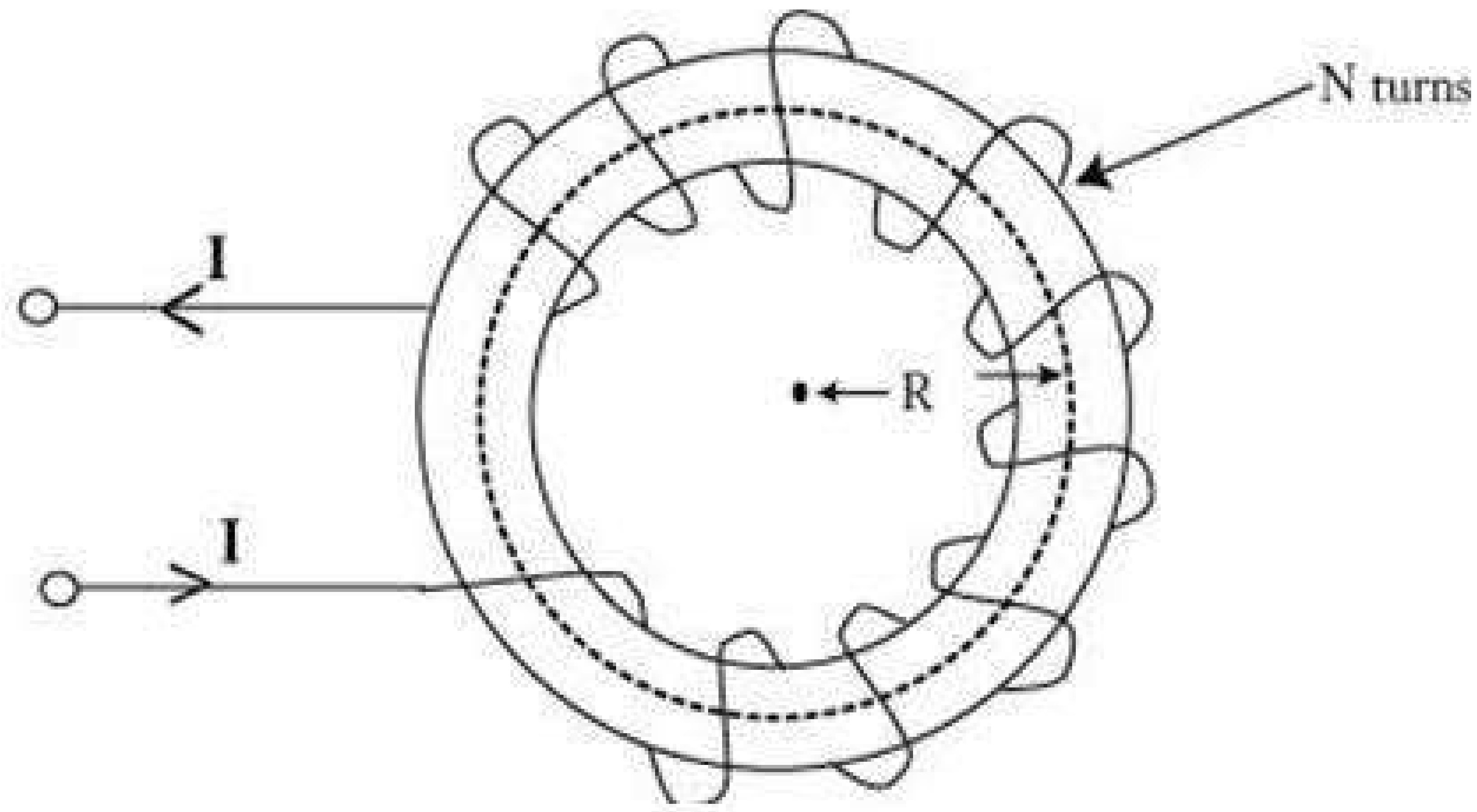
$$M^2 = k^2 \frac{N_{1\varphi_1}}{I_1} x \frac{N_{2\varphi_2}}{I_2} = k^2 L_1 L_2$$

$$\mathrm{M} = \sqrt{L_1 L_2} \quad \dots\dots\dots . (A)$$

The above equation (A) shows the relationship between mutual inductance and self inductance between two the coils

SERIES MAGNETIC CIRCUIT:

- A series magnetic circuit is analogous to a series electric circuit. A magnetic circuit is said to be series, if the same flux is flowing through all the elements connected in a magnetic circuit. Consider a circular ring having a magnetic path of 'l' meters, area of cross section 'a' m^2 with a mean radius of 'R' meters having a coil of 'N' turns carrying a current of 'I' amperes wound uniformly as shown in below fig



The flux produced by the circuit is given by

$$\text{Magnetic flux}=\frac{MMF}{RELUCTANCE}=\frac{NI}{S}=\frac{NI}{l/\mu a}$$

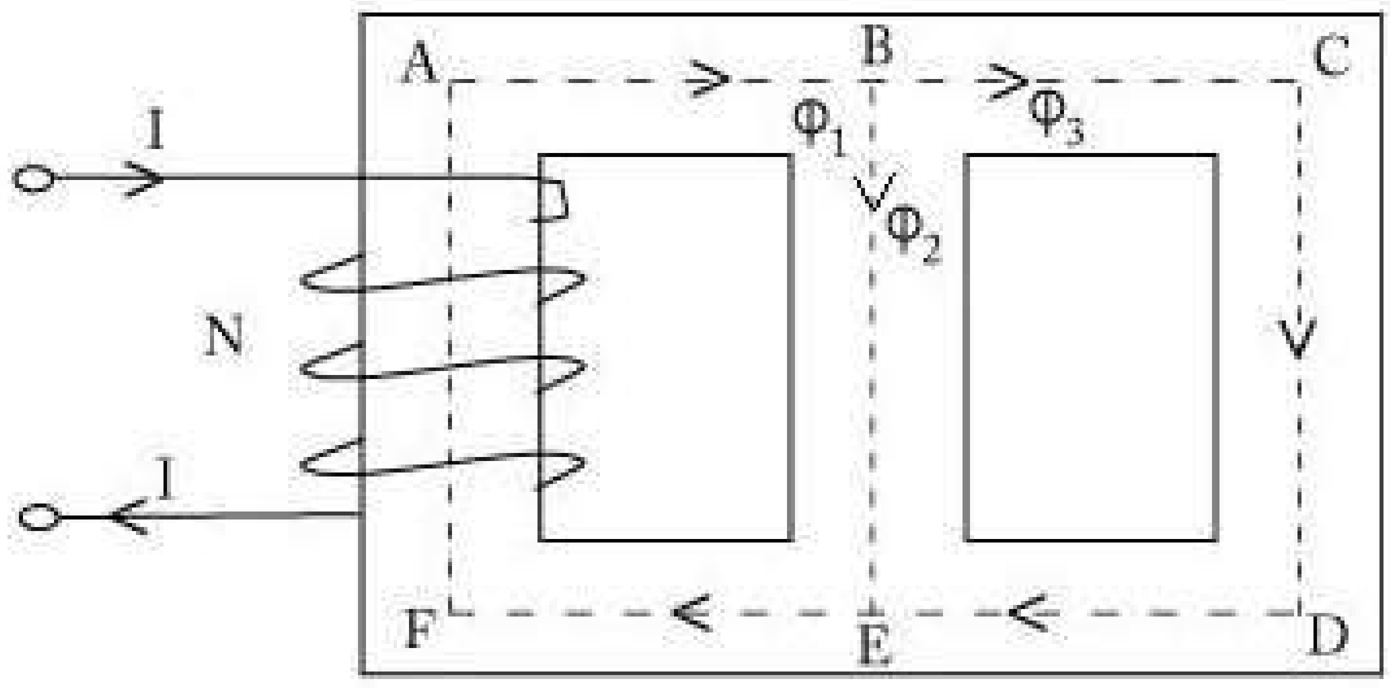
In the above equation NI is the MMF of the magnetic circuit, which is analogous to EMF in the electrical circuit.

PARALLEL MAGNETIC CIRCUIT

- A magnetic circuit which has more than one path for magnetic flux is called a parallel magnetic circuit. It can be compared with a parallel electric circuit which has more than one path for electric current. The concept of parallel magnetic circuit is illustrated in fig. 2. Here a coil of 'N' turns wounded on limb 'AF' carries a current of 'I' amperes. The magnetic flux 'ϕ_1' set up by the coil divides at 'B' into two paths namely

  Magnetic flux passes 'ϕ_2' along the path 'BE'

  Magnetic flux passes 'ϕ_3' along the path 'BCDE' i.e $\phi_1 = \phi_2 + \phi_3$



The magnetic paths 'BE' and 'BCDE' are in parallel and form a parallel magnetic circuit. The AT required for this parallel circuit is equal to AT required for any one of the paths. Let S_1= reluctance of path EFAB

Let, S_1= reluctance of path EFAB

S_2= reluctance of path BE

S_3= reluctance of path BCDE

Total MMF= MMF for path EFAB + MMF for path BE or path BCD

$$NI=\Phi_1 S_1+ \Phi_2 S_2= \Phi_1 S_1+ \Phi_3 S_3$$

COMPOSITE MAGNETIC CIRCUIT:

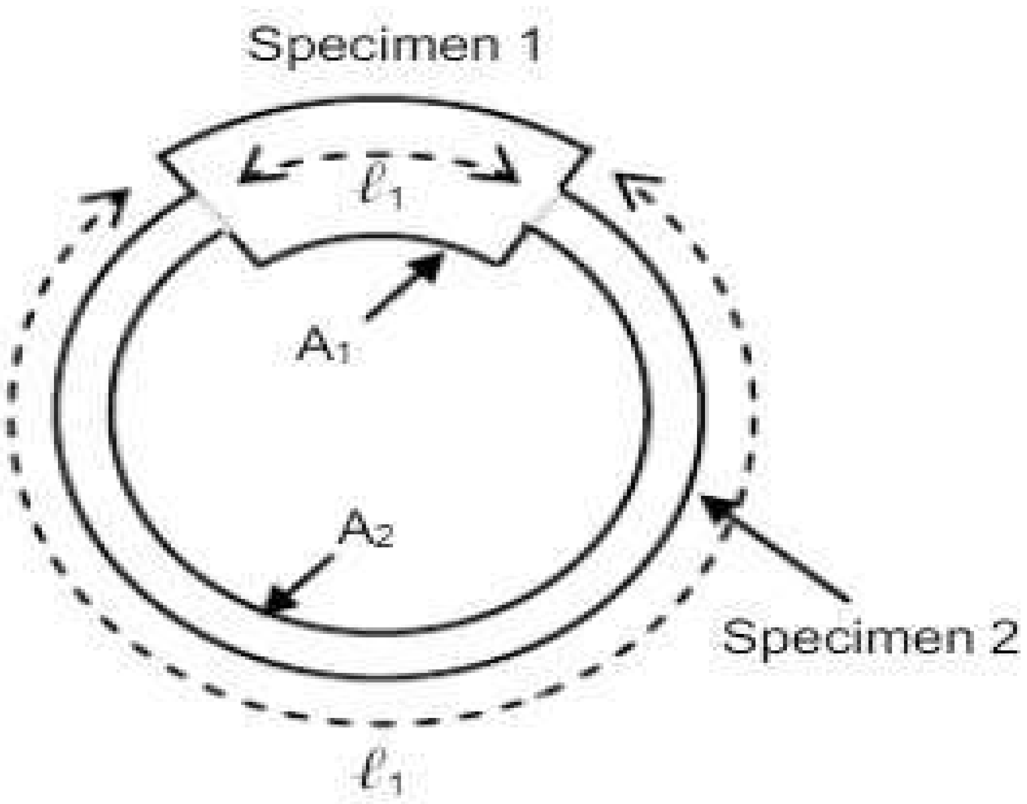
Consider a magnetic circuit which consists of two specimens of iron arranged as shown in figure. Let ℓ_1 and ℓ_2 be the mean lengths of specimen 1 and specimen2 in meters, A_1 and A_2 be their respective cross sectional areas in square meters, and $_1$ and $_2$ be their respective relative permeability's.

The reluctance of specimen 1 is given as

$$S_1 = \frac{\ell_1}{\mu_0 \mu_1 A_1} \quad (AT/Wb)$$

and that for specimen 2 is

$$S_2 = \frac{\ell_2}{\mu_0 \mu_2 A_2} \quad (AT/Wb)$$



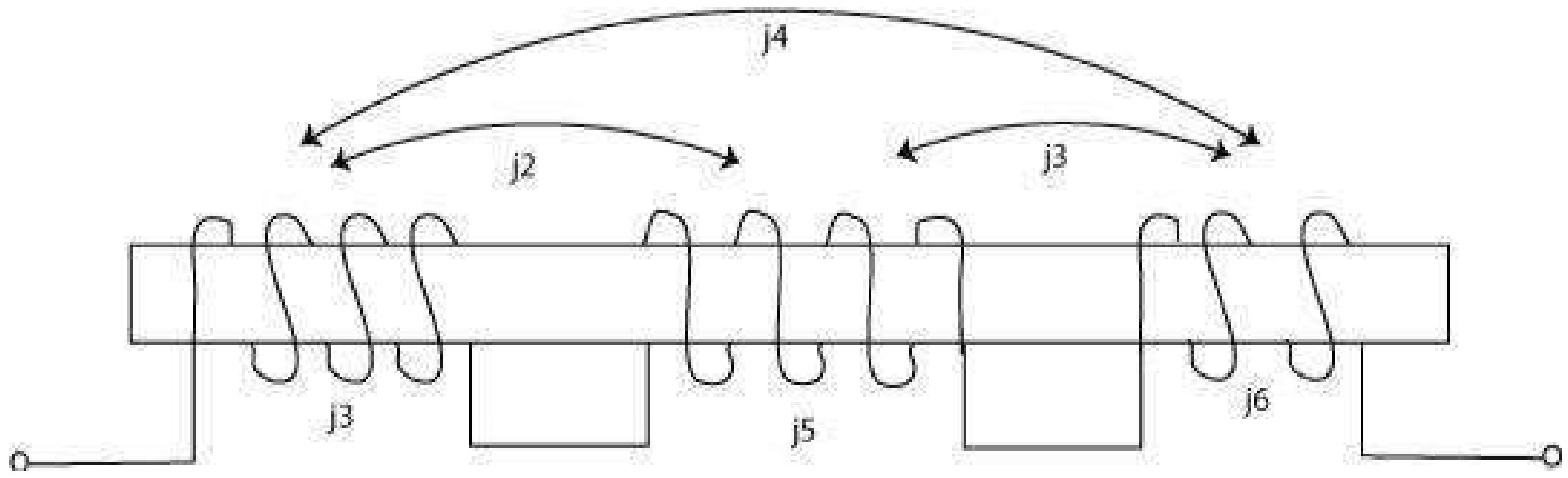
If a coil of N turns carrying a current I is wound on the specimen 1 and if the magnetic flux is assumed to be confined to iron core then the total reluctance is given by the sum of the individual reluctances S_1 and S_2. This is equivalent to adding the resistances of a series circuit. Thus the total reluctance is given by

$$S = S_1 + S_2 = \frac{\ell_1}{\mu_0 \mu_1 A_1} + \frac{\ell_2}{\mu_0 \mu_2 A_2} \quad AT/Wb$$

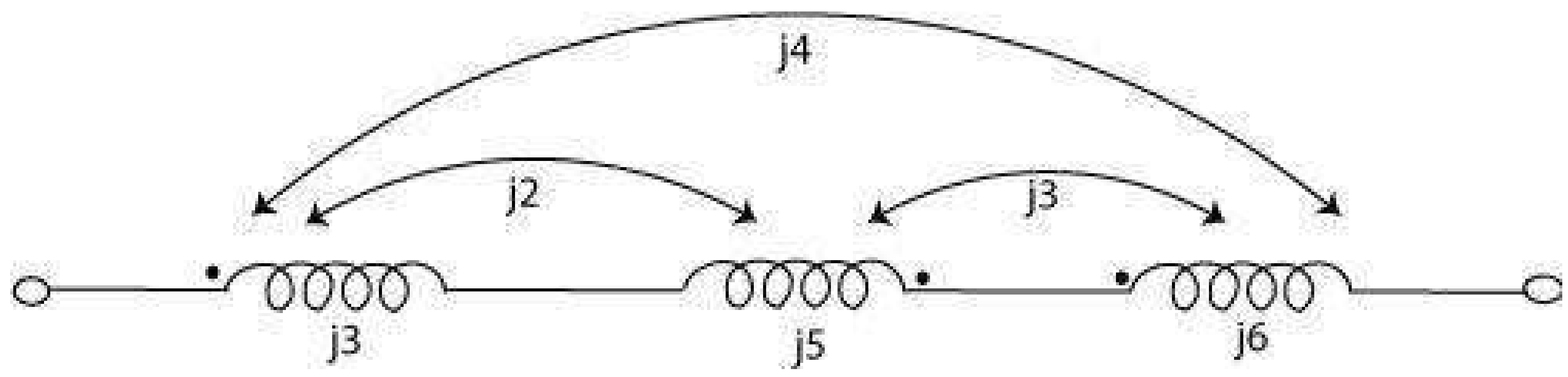
And the total flux is given by

$$\Phi = \frac{\text{mmf}}{\text{S}} = \frac{\text{NI}}{\dfrac{\ell_1}{\mu_0 \mu_1 A_1} + \dfrac{\ell_2}{\mu_0 \mu_2 A_2}} \quad \left(\frac{\text{AT}}{(\text{AT}/\text{Wb})} \Rightarrow \text{Wb} \right)$$

Problem : Sketch the dotted equivalent circuit for the coupled coil shown in the fig. and find the equivalent inductive?



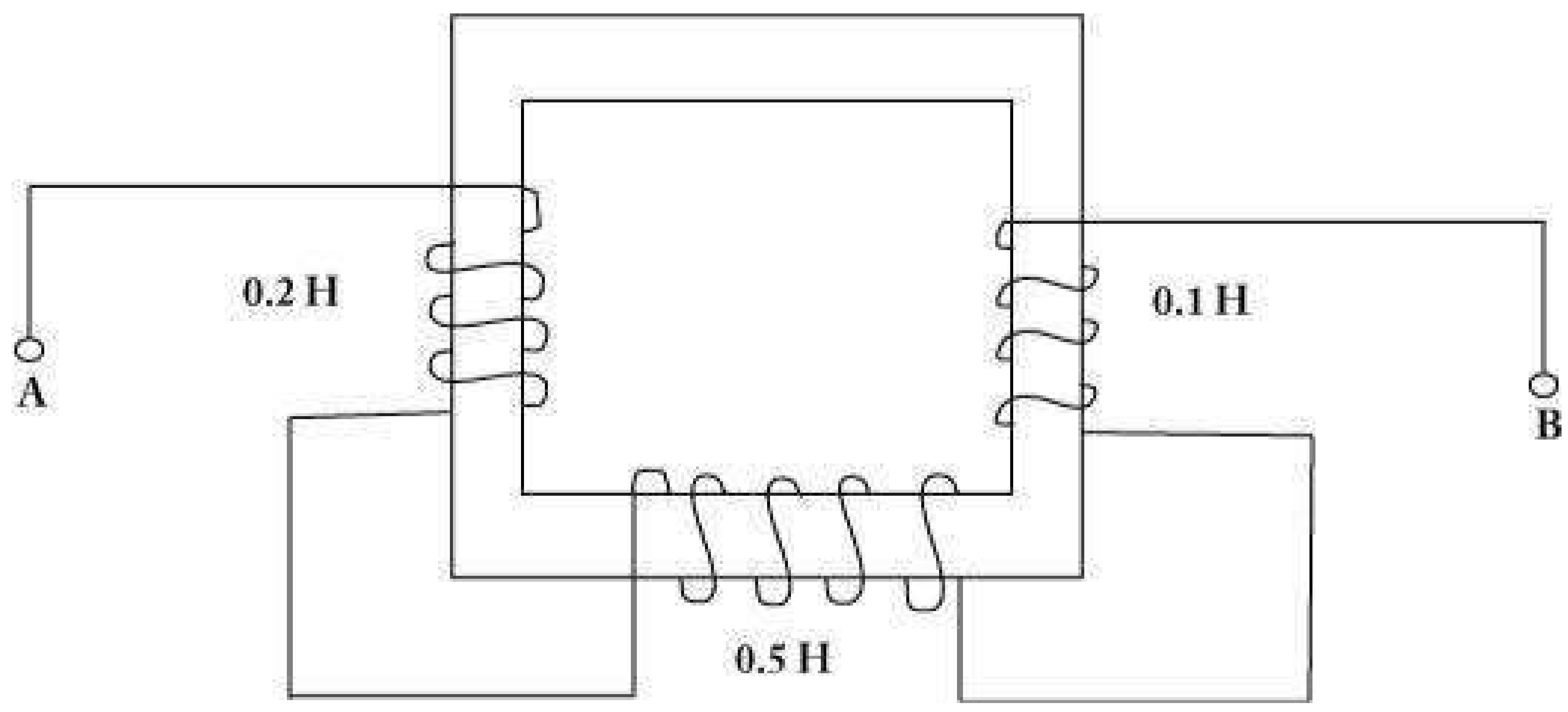
Solution: The dotted equivalent circuit is



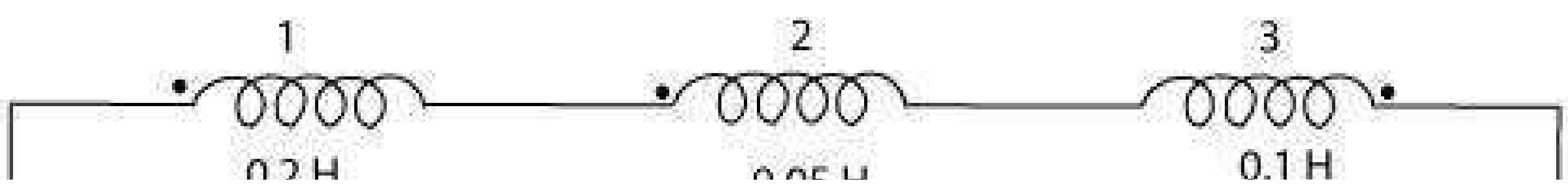
The equivalent inductive reactance is

j Xeq = j3 + j5 + j6 - 2 x j2 -2x j3+ 2x j4) = j14 - j2 = j12

Problem: Sketch the dotted equivale1lt circuit for the coupled coils shown in figure and find the equivalent inductance at the terminals AB. All coupling coefficie1lt are 0.5.



Solution: The dotted equivalent circuit is



$M_{12} = 0.5\sqrt{0.2 \times 0.05} = 0.05$

$= \sqrt{\quad \times \quad} =$

$M_{23} \quad 0.5\sqrt{0.05 \;\; 0.1} \quad 0..0035$

$= \sqrt{\quad \times \quad} =$

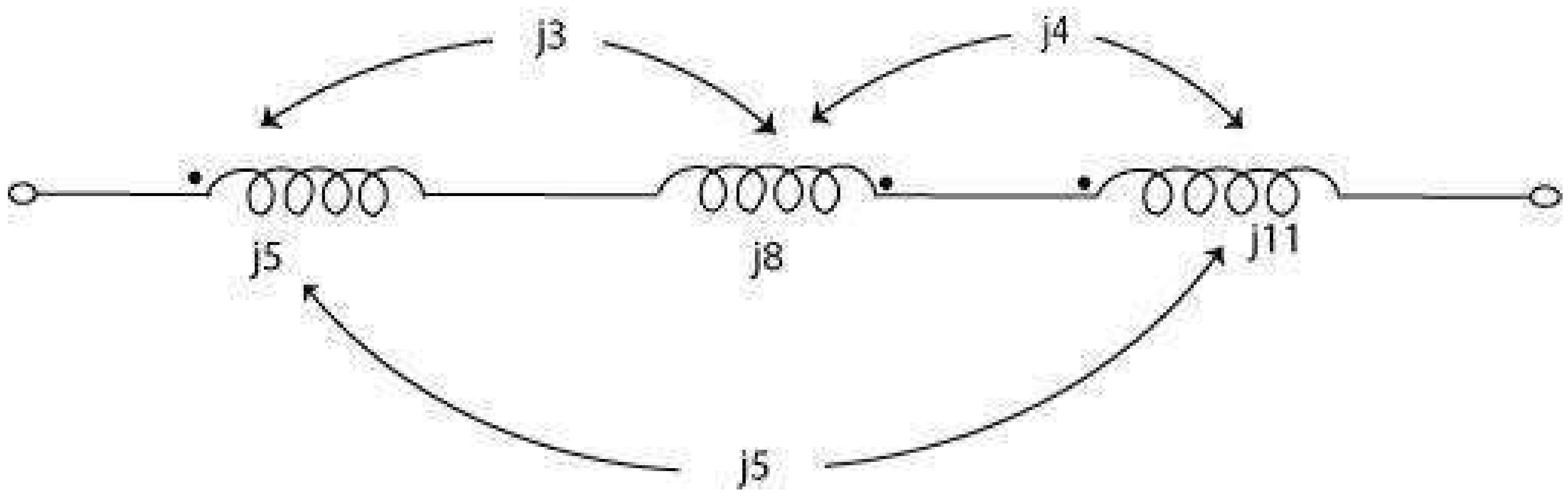
$M_{31} \quad 0.5\sqrt{0.2 \;\; 0.1} \quad 0..0707$

Equivalent inductance between terminals *A* and *B* L_{AB} = 0.2 + 0.05+ 0.1 + 2(0.05 - 0.035 - 0.0707)

=0.2386H

Problem: For the given circuit as shown in figure

a) Find the equivalent reactance
b) Draw the transformer coupling circuit

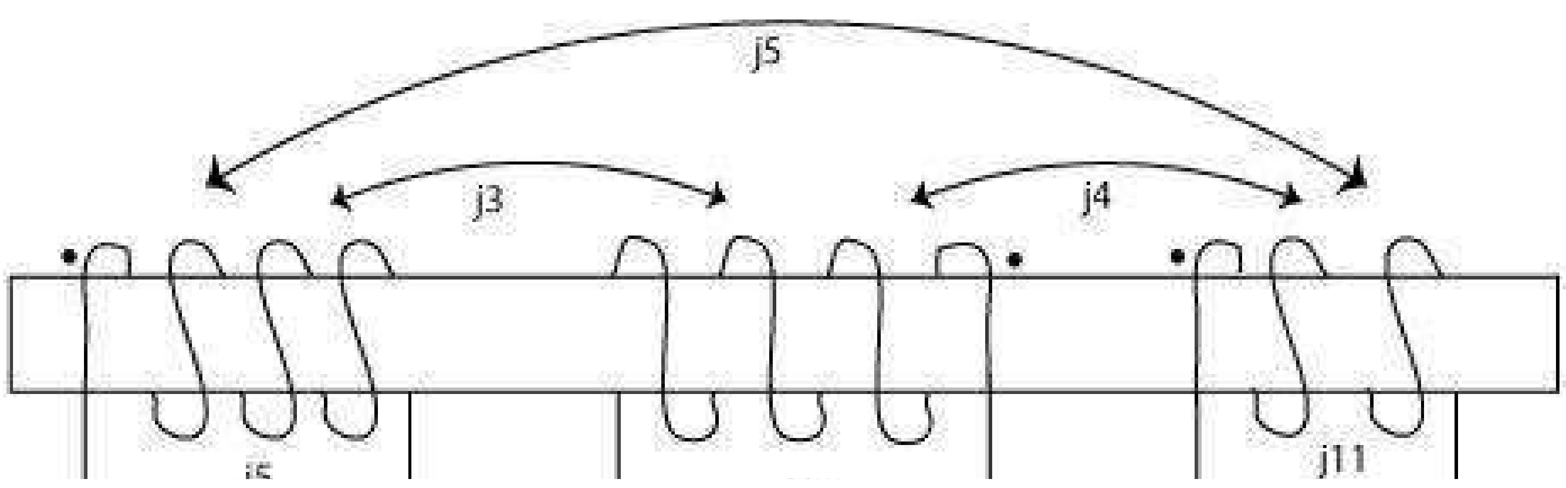


Solution

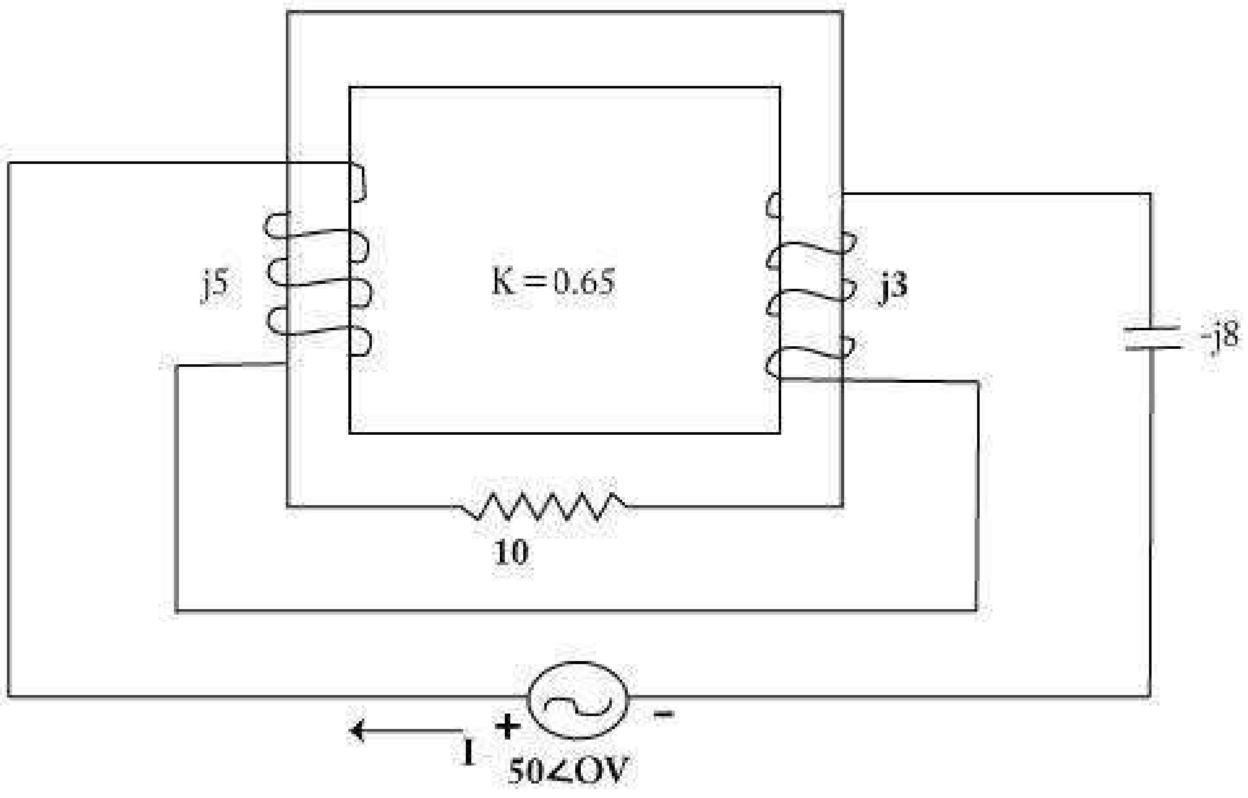
i) The equivalent reactance is

$j\,X_{eq} = j5 + j8 + j11 + 2(-j3 - j4 + j5) = j24 - j4 = j20$

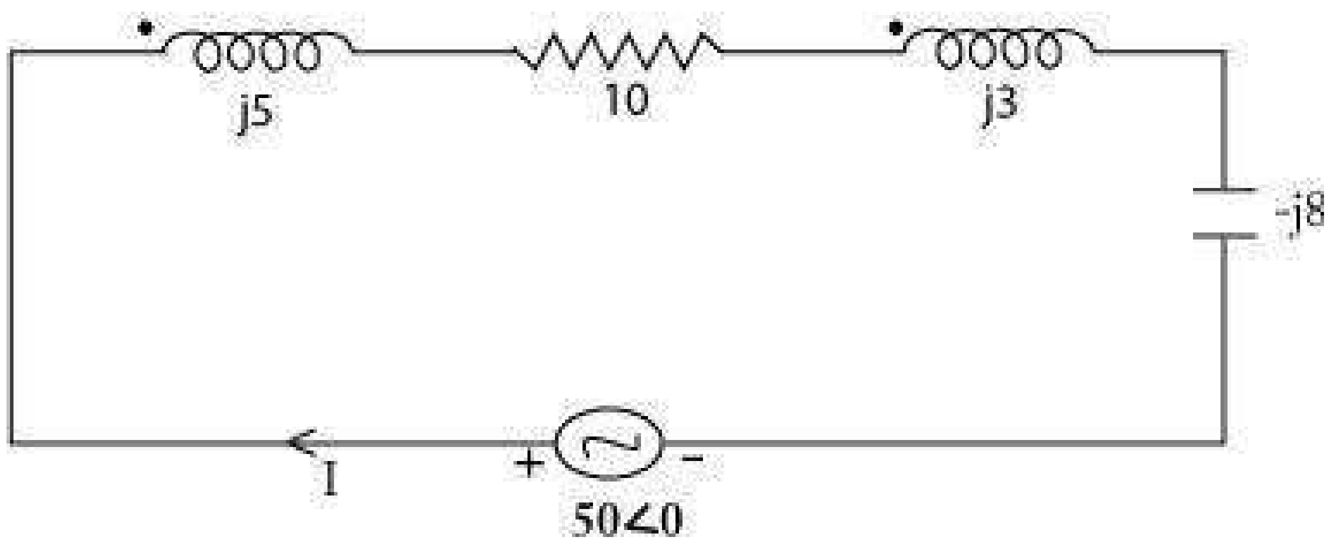
ii) The T/F coupling circuit is



Problem: Sketch the dotted equivalent circuit for the coupled coils shown in figure and find the current I?



Solution: The dotted equivalent circuit is



$$jX_M = jK\sqrt{X_{L1} \times X_{L2}} = j0.65\sqrt{5 \times 3} = j2.5$$

By applying KVL to the circuit

10I- j8I + (j5 + j3 + 2 x j2.5)I =50

I(10- j8 + j3) =50

I = 4.47 at an angle of 26.56 A