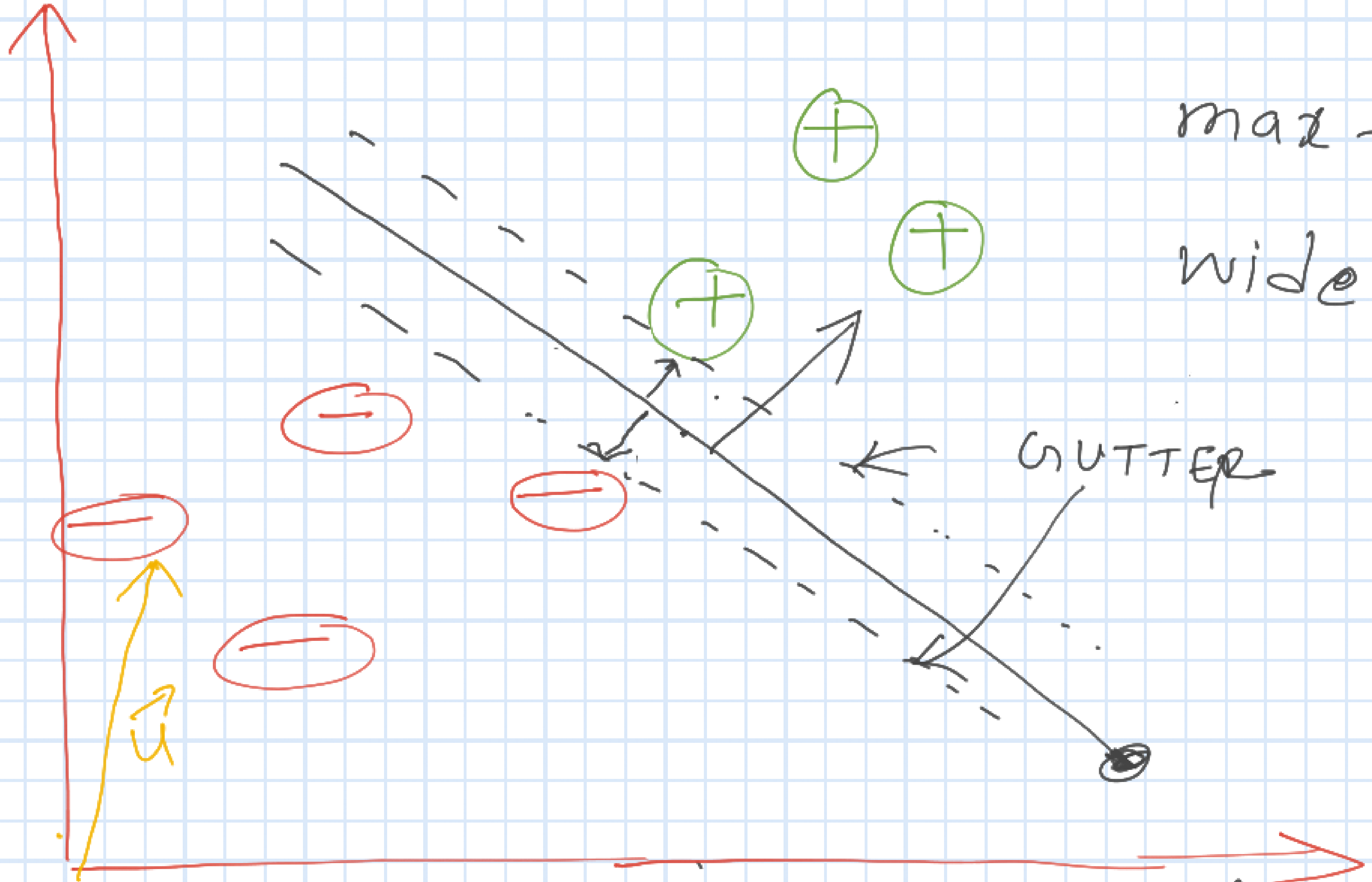
Support Vector Machine

(SVM)

max-margin Classifier

wide-street



$$\vec{w} \cdot \vec{u} \geq C$$

$$c = -b$$

$$\boxed{\vec{w} \cdot \vec{u} + b \geq 0}$$

~~then +ve~~

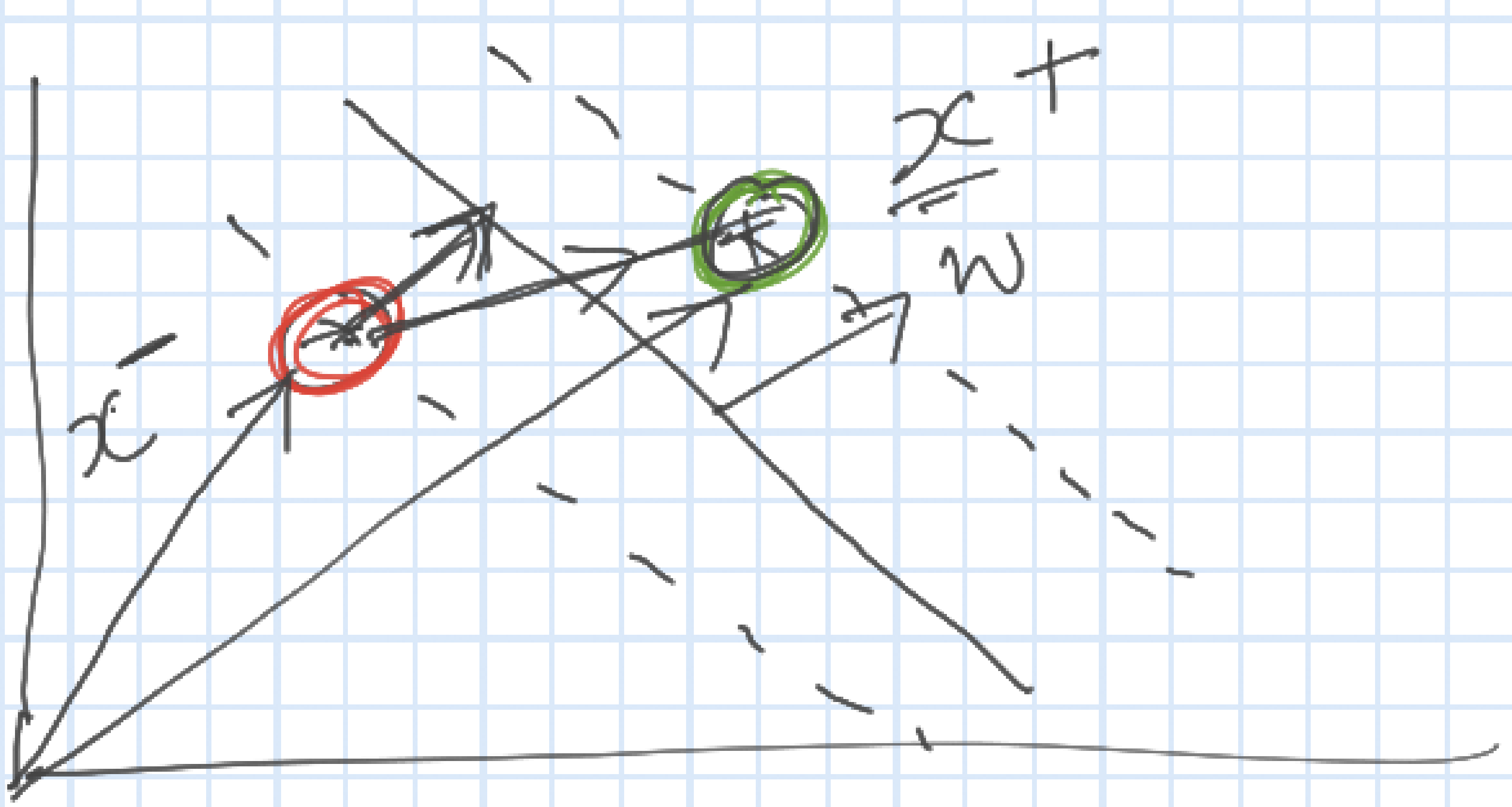
Decision RULE

$$\left.\begin{aligned}\vec{w}\cdot\vec{x}_+ + b &\geq 1, \quad y = +1 \\ \vec{w}\,\vec{x}_- + b &\leq -1 \quad y = -1\end{aligned}\right\} \quad \begin{aligned}&— a \\ &— b\end{aligned}$$

$y_i = \pm 1$ (class label)

$$y_i(\vec{w}\cdot\vec{x_i} + b) \geq 1$$

$$\Rightarrow \boxed{y_i(\vec{w}\,\vec{x_i} + b) - 1 \geq 0} \quad — i$$



$x^+ - x^-$

Unit vector

width of the street $= (x^+ - x^-) \cdot \dfrac{\vec{w}}{\|\vec{w}\|}$

$= \dfrac{\vec{x}^+ \cdot \vec{w} - x^- \vec{w}}{\|\vec{w}\|}$

$-1(\vec{x}^- \vec{w} + b) - 1 = 0$

$+1(\vec{x}^+ \cdot \vec{w} + b) - 1 = 0$

$= \dfrac{(1-b) - (-b-1)}{\|\vec{w}\|}$

$$\text{width} = \frac{2}{\|\vec{w}\|}$$

$$\frac{d}{dx} x^2 = 2x$$

maximize width

$\Downarrow$

$$\max \frac{2}{\|\vec{w}\|}$$

$$\min. \|\vec{w}\|$$

$$\min. \frac{1}{2}\|\vec{w}\|^2$$

$$\text{s.t. } y_i(\vec{x}_i \cdot \vec{w} + b) - 1 \geq 0$$

$$L(\vec{w}, b, \alpha) = L = \underline{\underline{\frac{1}{2}\|\vec{w}\|^2}} - \sum_i \alpha_i \left[\underline{y_i(\vec{x_i}\vec{w} + b) - 1} \right]$$

$\alpha_i \geq 0 \rightarrow$ langragian — Ⓐ

$$\frac{\partial L}{\partial \vec{w}} = 0$$

$$\Rightarrow \frac{\partial L}{\partial w} = \vec{w} - \sum_i \alpha_i y_i x_i = 0$$

$$\Rightarrow \boxed{\vec{w} = \sum_i \alpha_i y_i \vec{x_i}, \ \alpha_i \geq 0} \quad — ①$$

$$\frac{\partial L}{\partial b} = 0 - \sum_i \alpha_i y_i = 0$$

$$\Rightarrow \boxed{\sum_i \alpha_i y_i = 0} \quad — ②$$

$$L = \frac{1}{2}\left(\sum_i \alpha_i y_i \vec{x}_i\right)\left(\sum_j \alpha_j y_j \vec{x}_j\right)$$

$$- \sum \alpha_i y_i x_i \cdot \sum \alpha_j y_j x_j$$

$$- \sum \alpha_i y_i b \quad + \sum \alpha_i$$

$$= \sum \alpha_i \quad - \frac{1}{2}\sum_i \sum_j \alpha_i \alpha_j y_i y_j \left(\boxed{\vec{x}_i \cdot \vec{x}_j}\right)$$

$$\vec{w} \cdot \vec{u} + b \geq 0 \text{ Then +ve}$$

$$\Rightarrow \sum_i \alpha_i y_i \left(\boxed{x_i \cdot \vec{u}}\right) + b \geq 0$$

$$x_i \xrightarrow{\phi} \phi(x_i)$$

$$x_j \xrightarrow{\phi} \phi(x_j)$$

maximize $\phi(\vec{x_i}) \cdot \phi(\vec{x_j})$

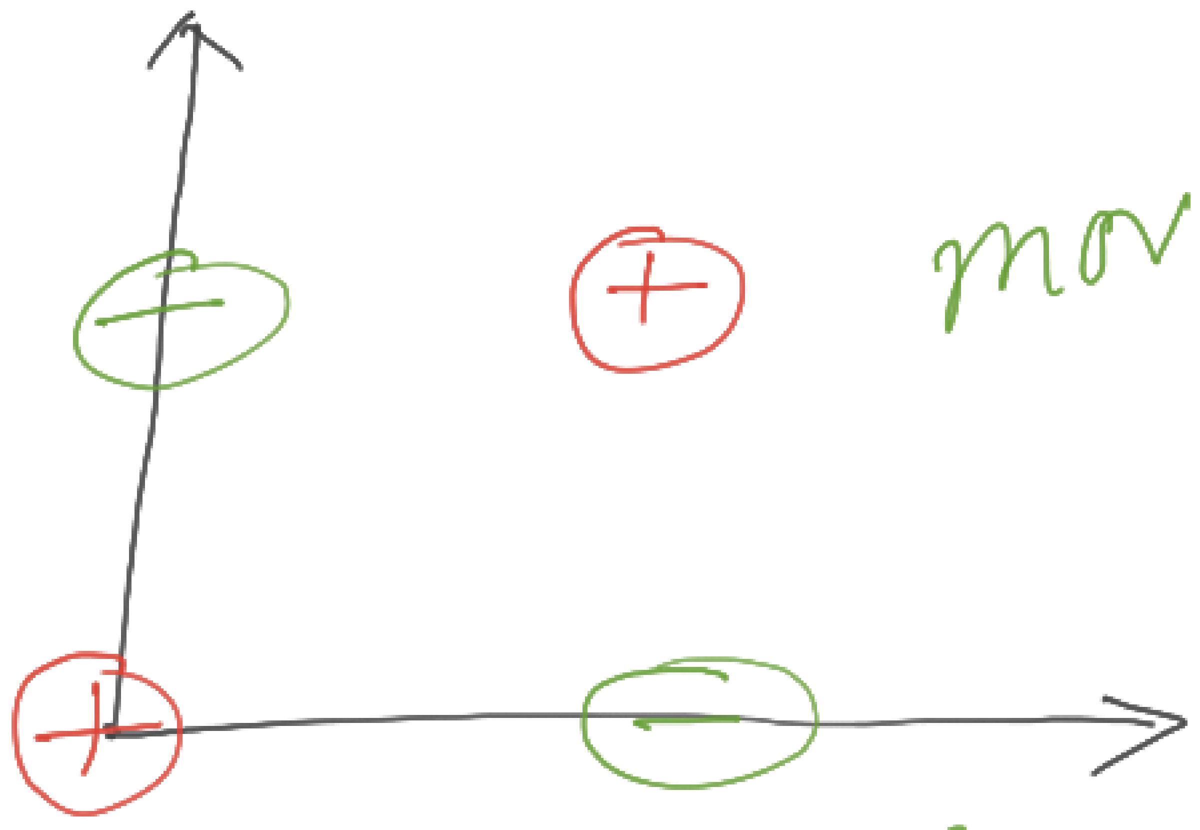
during test time

$$\phi(\vec{x_i}) \cdot \phi(\vec{q})$$

$$K(x_i, x_j) = \underbrace{\phi(x_i) \cdot \phi(x_j)}$$

Kernel

$$K = \begin{bmatrix} \; \end{bmatrix}_{n \times n}$$

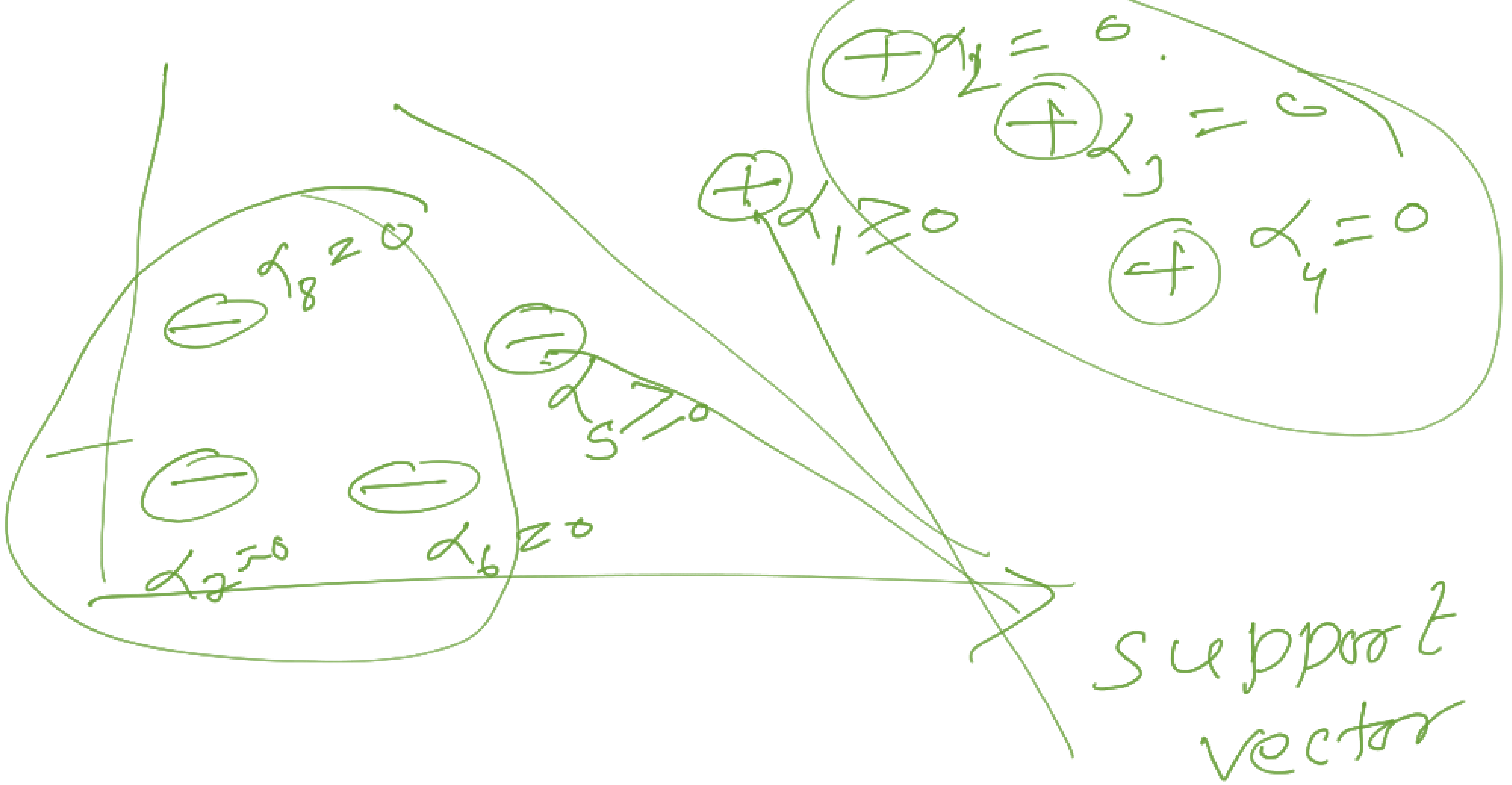


moving plus points up is ϕ

Different kernels

① Linear : $(\vec{u} \cdot \vec{v} + 1)^n = f(\vec{u}, \vec{v})$

② RBF $= e^{\frac{\|\vec{u} - \vec{v}\|}{\sigma}}$



α8 = 0
α2 = 0
α6 = 0
α5 > 0
α1 > 0
α3 = 0
α4 = 0
support vector

$\oplus \quad \oplus \quad \oplus \quad \oplus$

$\ominus \quad \ominus \quad \ominus \quad \ominus$

$$\frac{\partial}{\partial t} t^2$$

$$\frac{1}{2}\frac{\partial}{\partial \vec{x}} \|\vec{x}\|^2$$

$$\vec{x} \cdot$$

$$=$$

End-sem

50 points
- 20 MCQ 10×2 40 min
- 30 subjective 5×6 Rest

2 hours

those who missed Minor/minor + 1 hour paper

$(224-2)/2+1$

$222/2+1$

112

$(112-2)/2+1$

$110/2+1$

$55+1=56$