Ensemble Learning

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Slides are prepared from several information sources on the web and books: Kamalika Chaudhury and Kilien Weinberger

Decision Tree – Limitations and Advantages

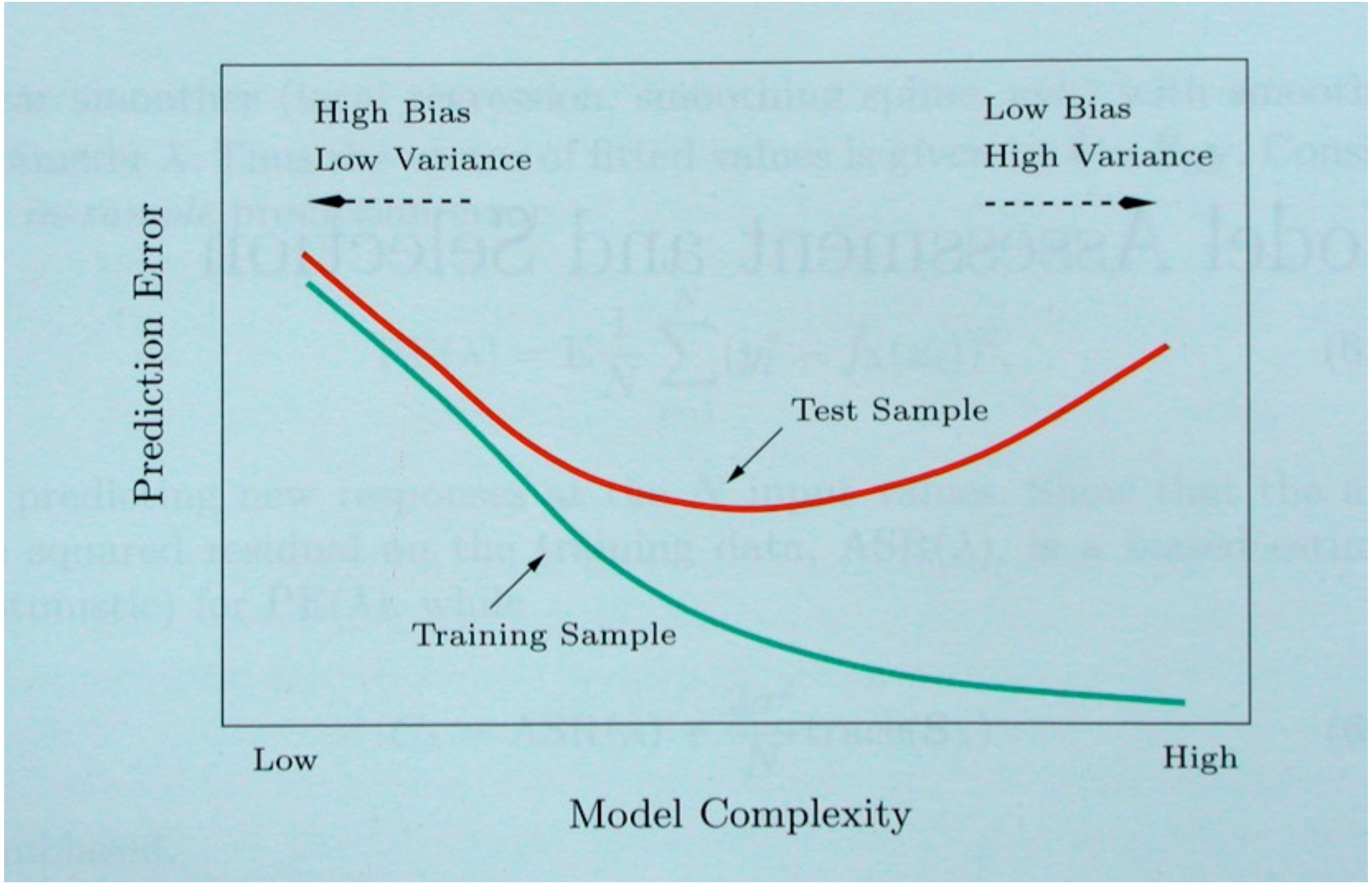
- Trees are flexible → good expressiveness
- Trees are flexible → poor generalization

- Options:
  - Pruning
  - Early stopping

- CART: Classification and Regression Trees

- Can we combine information from multiple decision tress to improve the performance?

Bias/Variance Tradeoff



Hastie, Tibshirani, Friedman "Elements of Statistical Learning" 2001

Ensemble Learning

- Simple learners:
  - Are good: Low variance, don’t usually overfit
  - Are bad: High bias, can’t solve hard learning problems

- Instead of learning a single (weak) classifier, learn many weak classifiers that are good at different parts of the input space

- How to combine multiple classifiers into a single one
- Works well if the classifiers are complementary

- Two types of ensemble methods:

  Bagging

  Boosting

Reduce Variance Without Increasing Bias

- Averaging reduces variance:

$$Var(\bar{X}) = \frac{Var(X)}{n}$$

(when predictions are **independent**)

Average models to reduce model variance

One problem:

only one training set

where do multiple models come from?

Bagging: Bootstrap Aggregation

- Leo Breiman (1994)
- Take repeated bootstrap samples from training set D.
- *Bootstrap sampling*: Given set D containing N training examples, create D' by drawing N examples at random with replacement from D.

- Bagging:
  - Create k bootstrap samples $D_1 \ldots D_k$.
  - Train the classifier on each D_i.
  - Classify new instance by majority vote / average.

Bagging (Bootstrap AGgregatING)

Input: n labelled training examples $(x_i, y_i), i = 1,..,n$

Algorithm:

Repeat k times:

Select m samples out of n **with replacement** to get training set S_i

Train classifier (decision tree, k-NN, perceptron, etc) h_i on S_i

Output: Classifiers $h_1, .., h_k$

Classification: On test example x, output majority$(h_1, .., h_k)$

Example

Input: n labelled training examples $(x_i, y_i), i = 1,..,n$

Algorithm:

Repeat k times:

Select m samples out of n **with replacement** to get training set S_i

Train classifier (decision tree, k-NN, perceptron, etc) h_i on S_i

How to pick m?

Popular choice: $m = n$

Still different from working with entire training set. Why?

Bagging

Input: n labelled training examples $S = \{(x_i, y_i)\}, i = 1,..,n$

Suppose we select n samples out of n **with replacement** to get training set S_i

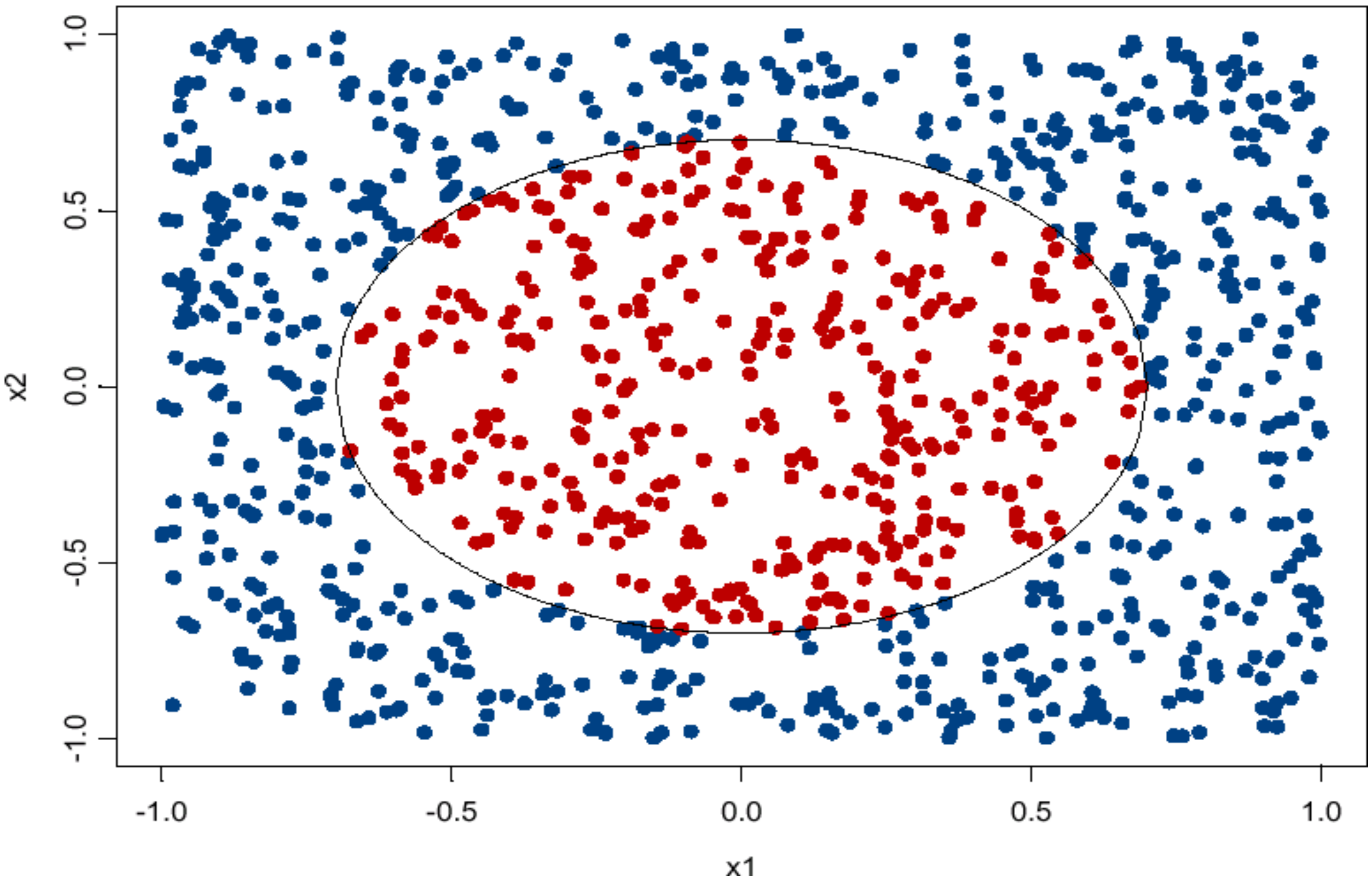
Still different from working with entire training set. Why?

$$\Pr(S_i = S) = \frac{n!}{n^n}$$ **(tiny number, exponentially small in n)**

$$\Pr((x_i, y_i) \text{ not in } S_i) = \left(1 - \frac{1}{n}\right)^n \approxeq e^{-1}$$

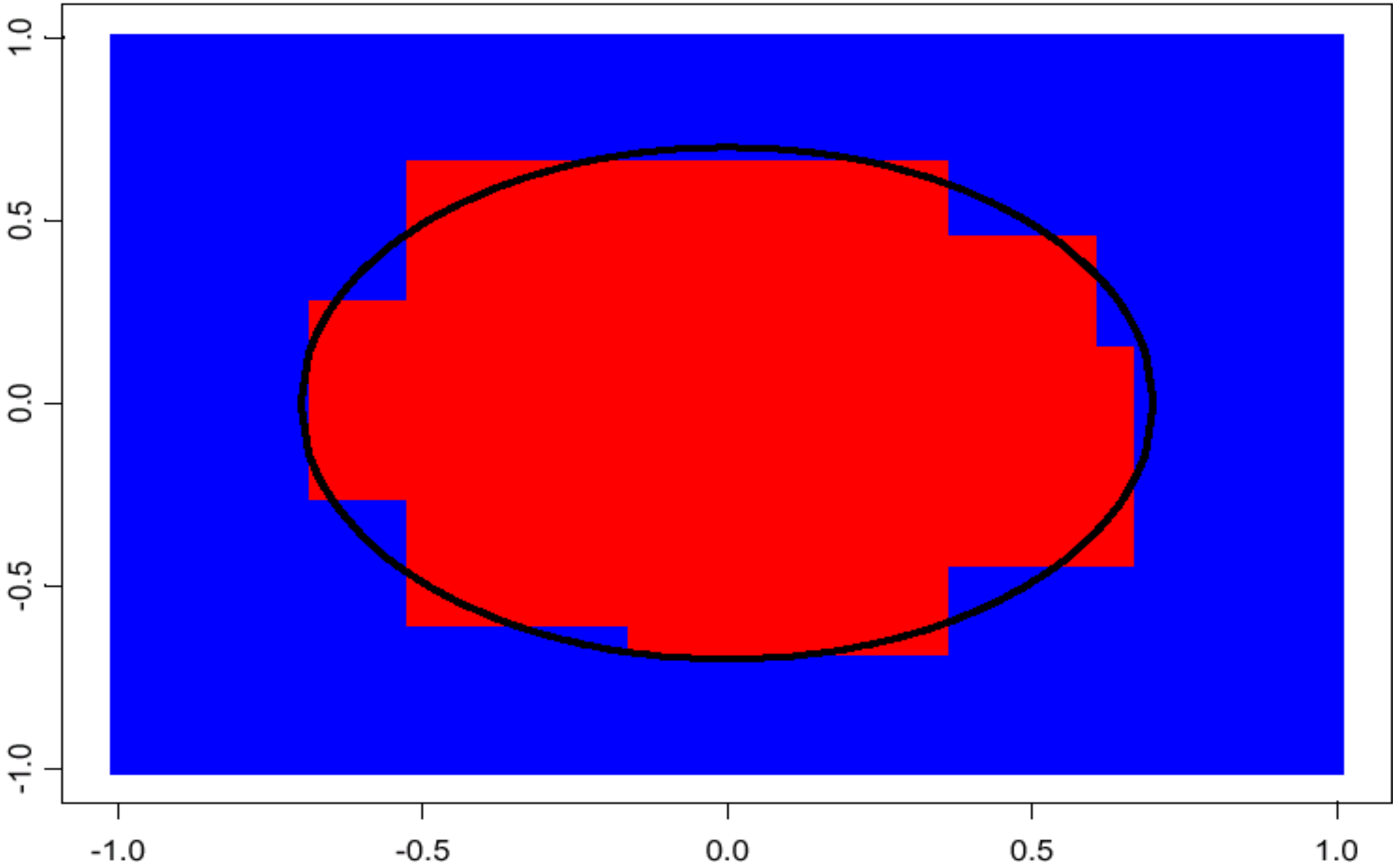
For large data sets, about 37% of the data set is left out!

Example



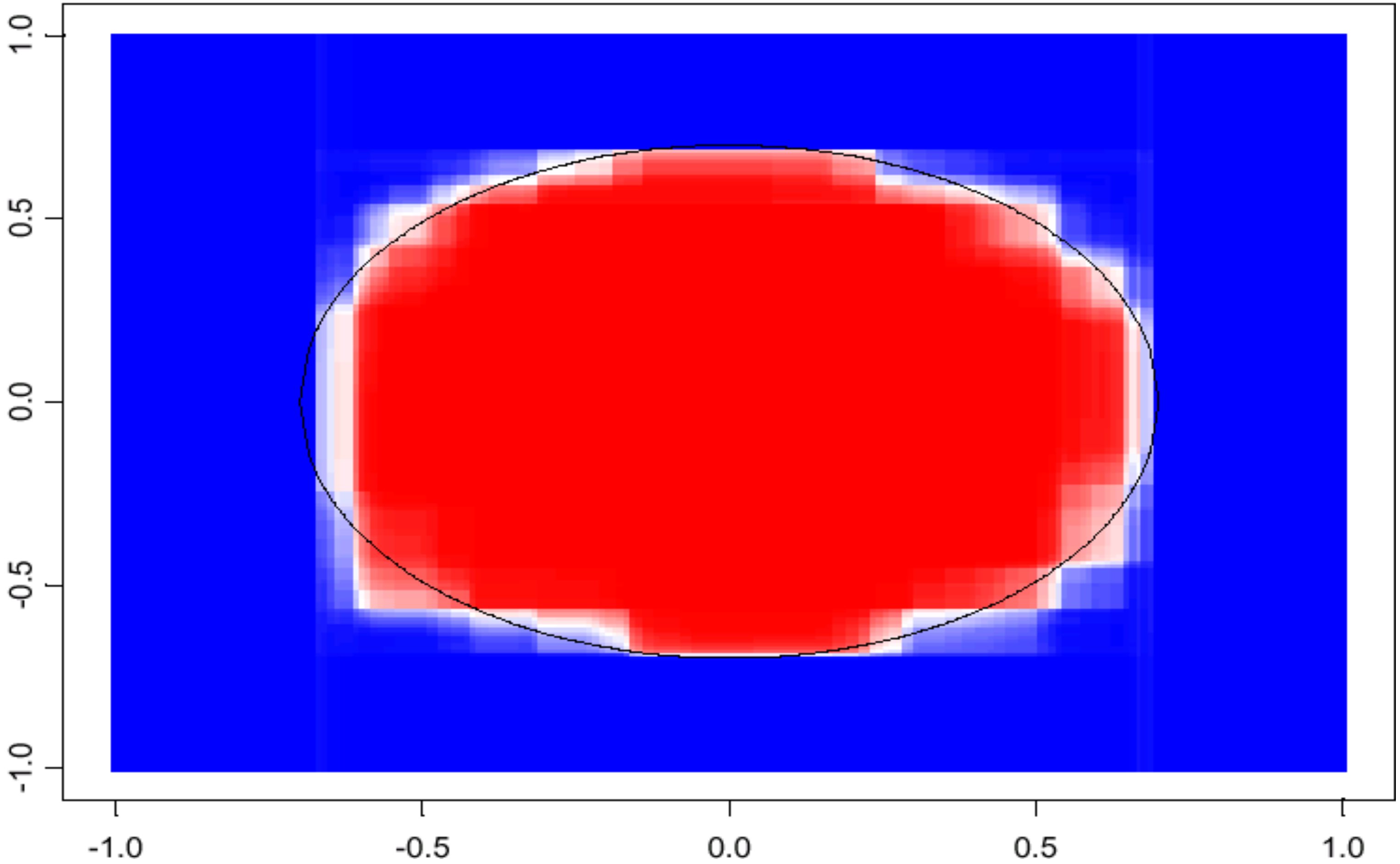
decision tree learning algorithm; very similar to ID3

CART decision boundary



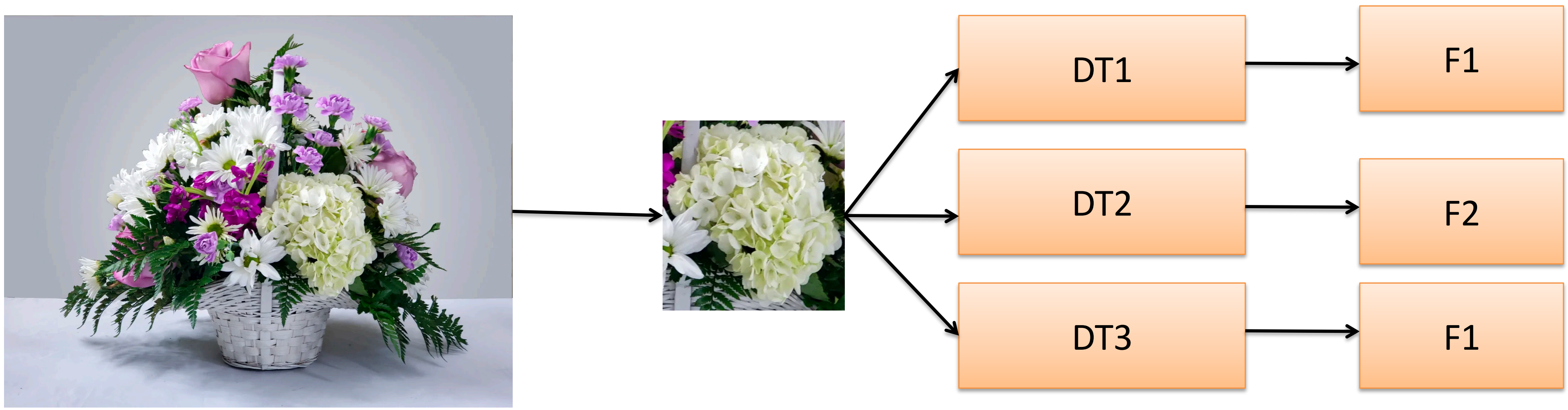
8

100 bagged trees



shades of blue/red indicate strength of vote for particular classification

Random Decision Forest



Majority voting: F1

Questions?

Ensemble Learning

How to combine multiple classifiers into a single one

Works well if the classifiers are complementary

This class: two types of ensemble methods:

- Bagging
- Boosting

Boosting

Goal: Determine if an email is spam or not based on text in it

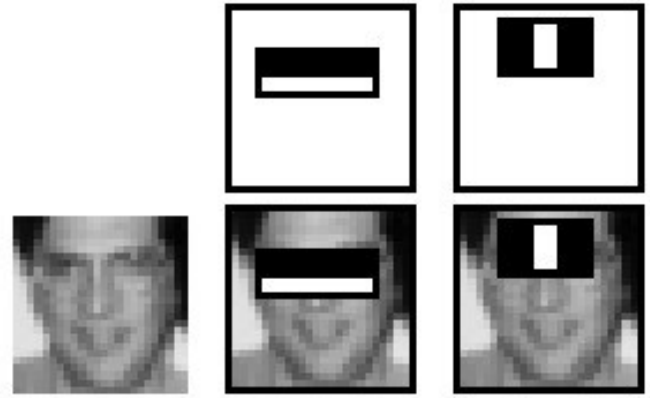
From: Yuncong Chen **Text:** 151 homeworks are all graded...	Not Spam
From: Work from home solutions **Text:** Earn money without working!	Spam

Sometimes it is:

- Easy to come up with simple rules-of-thumb classifiers,
- Hard to come up with a single high accuracy rule

Boosting

Goal: Detect if an image contains a face in it



= Is the black region darker on average than the white?

Sometimes it is:

- Easy to come up with simple rules-of-thumb classifiers,
- Hard to come up with a single high accuracy rule

Boosting

Weak Learner: A simple rule-of-the-thumb classifier that doesn’t necessarily work very well

Strong Learner: A good classifier

Boosting: How to combine many weak learners into a strong learner?

Boosting

Procedure:

1. Design a method for finding a good rule-of-thumb
2. Apply method to training data to get a good rule-of-thumb
3. Modify the training data to get a 2nd data set
4. Apply method to 2nd data set to get a good rule-of-thumb
5. Repeat T times...

Boosting

1. How to get a good rule-of-thumb?

   Depends on application e.g, single node decision trees

2. How to choose examples on each round?

   Focus on the **hardest examples** so far - namely, examples misclassified most often by previous rules of thumb

3. How to combine the rules-of-thumb to a prediction rule? Take a weighted majority of the rules

Some Notations

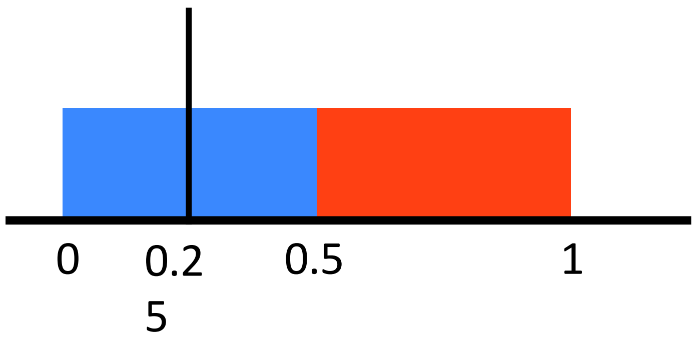
Let D be a distribution over examples, and h be a classifier

Error of h with respect to D is:

$$err_D(h) = Pr_{(X,Y)\sim D}(h(X) \neq Y)$$

Example:

Below X is uniform over [0, 1], and Y = 1 if X > 0.5, 0 otherwise



$err_D(h) = 0.25$

Some Notation

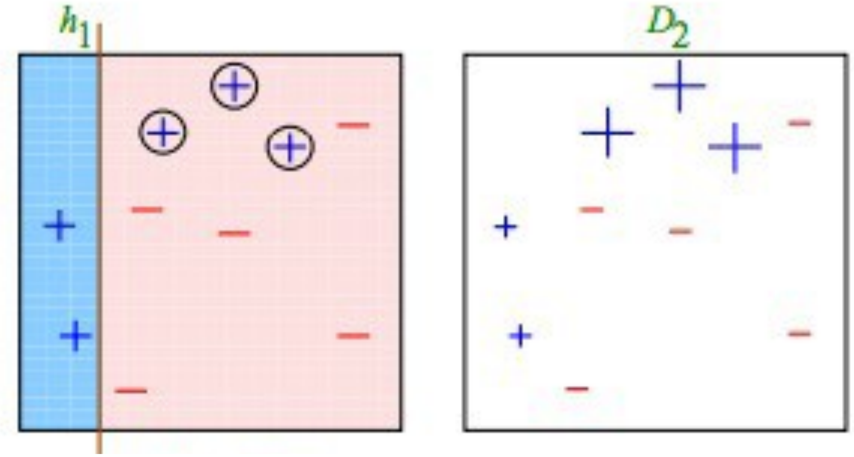
Let D be a distribution over examples, and h be a classifier
Error of h with respect to D is:

$$err_D(h) = Pr_{(X,Y)\sim D}(h(X) \neq Y)$$

h is called a **weak learner** if $err_D(h) < 0.5$

If you guess completely randomly, then the error is 0.5

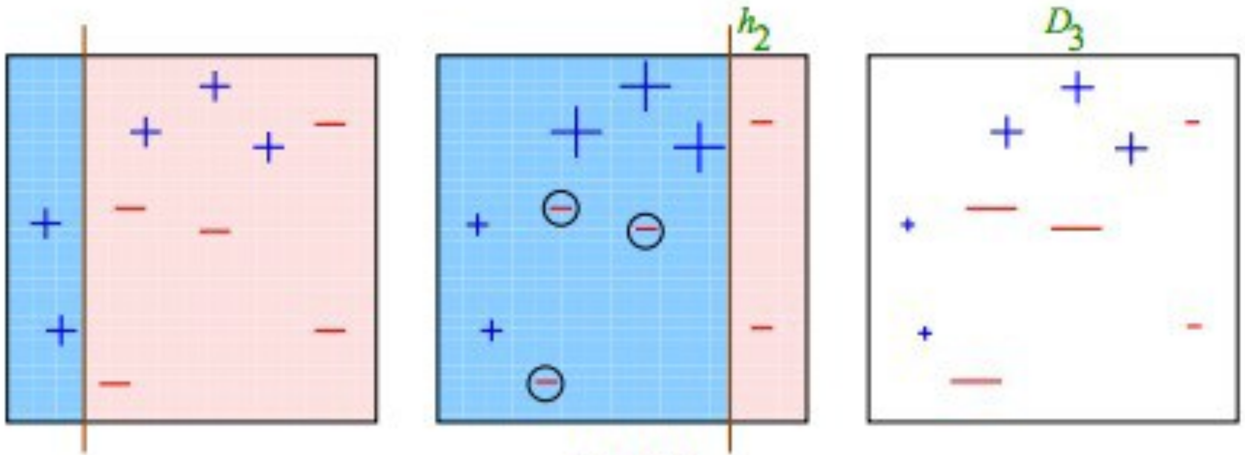
Boosting: Example



Schapire, 2011

weak classifiers: horizontal or vertical half-planes

Boosting: Example



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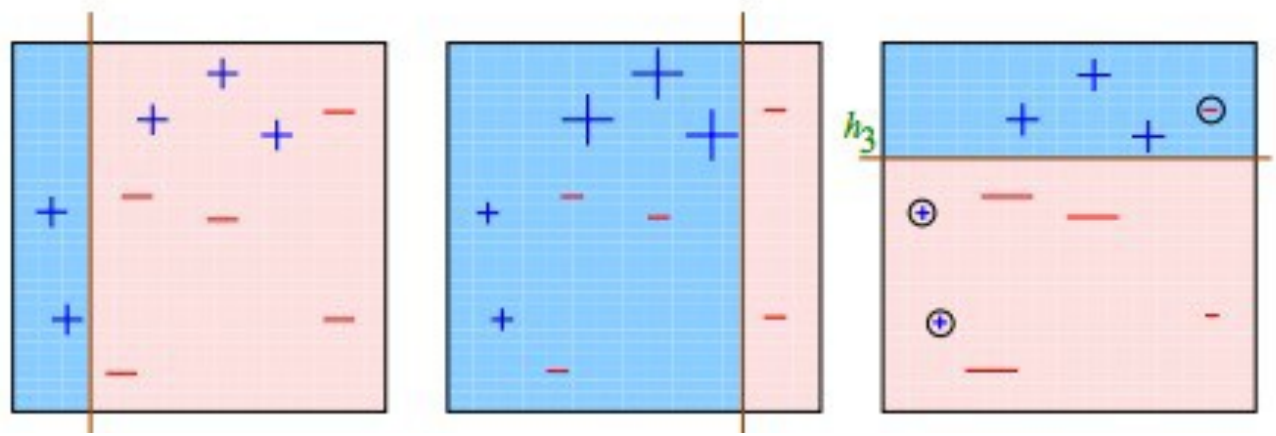
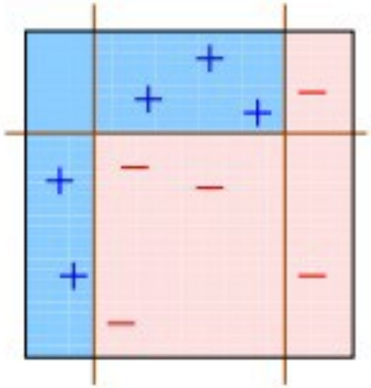
Boosting: Example



Schapire, 2011

weak classifiers: horizontal or vertical half-planes

The Final Classifier



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weak classifiers: horizontal or vertical half-planes

Some Notation

Given training examples $\{(x_i, y_i)\}$, i = 1,..,n, we can assign weights w_i to the examples. If the w_is sum to 1, then we can think of them as a distribution W over the examples.

The error of a classifier h wrt W is:

$$err_W(h) = \sum_{i=1}^{n} w_i 1(h(x_i) \neq y_i)$$

Note: 1 here is the indicator function

Boosting

Given training set S = {(x_1,y_1),...,(x_n,y_n)}, y in {-1, 1}

For t = 1,...,T

Construct distribution D_t on the examples

Find weak learner h_t which has small error $err_{D_t}(h_t)$ wrt D_t

Output final classifier

Initially, $D_1(i)$ = 1/n, for all i (uniform)

Given D_t and h_t:

$$D_{t+1}(i) = \frac{D_t(i)}{Z_t} exp(-\alpha_t y_i h_t(x_i))$$

Weight update rule

where:

$$\alpha_t = \frac{1}{2} \ln \left(\frac{1 - err_{D_t}(h_t)}{err_{D_t}(h_t)} \right)$$

Z_t = normalization constant

Boosting

Given training set $S = \{(x_1, y_1), ..., (x_n, y_n)\}$, y in $\{-1, 1\}$

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$D_{t+1}(i)$ goes down if x_i is classified correctly by h_t, up otherwise

High $D_{t+1}(i)$ means hard example

Boosting

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Higher if h_t has low error wrt D_t, lower otherwise. >0 if $err_{D_t}(h_t) < 0.5$

Z_t = normalization constant

Boosting

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where:

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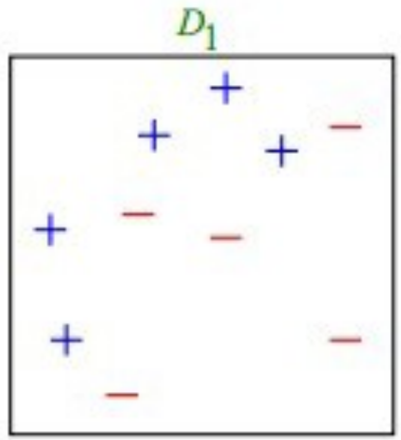
Z_t = normalization constant

Final classifier:

$$sign(\sum_{t=1}^{T} \alpha_t h_t(x))$$

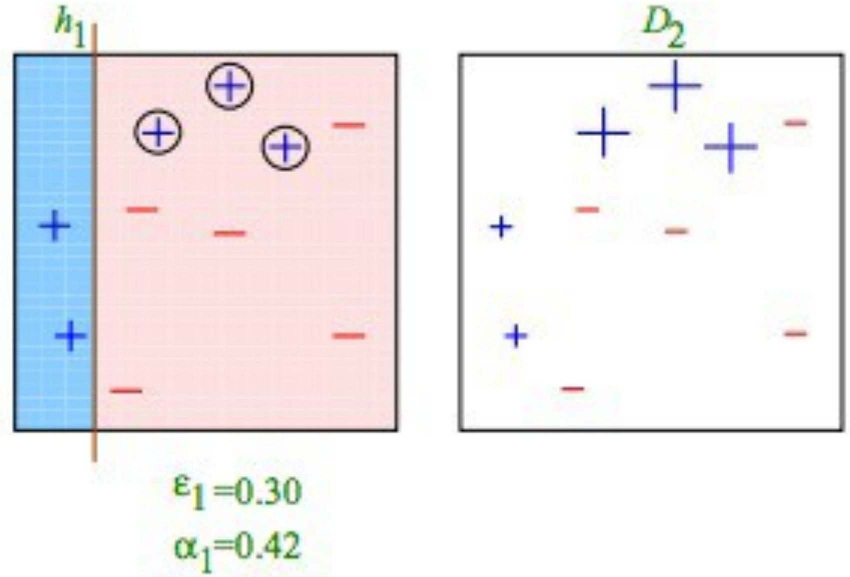
Boosting: Example



Schapire, 2011

weak classifiers: horizontal or vertical half-planes

Boosting: Example



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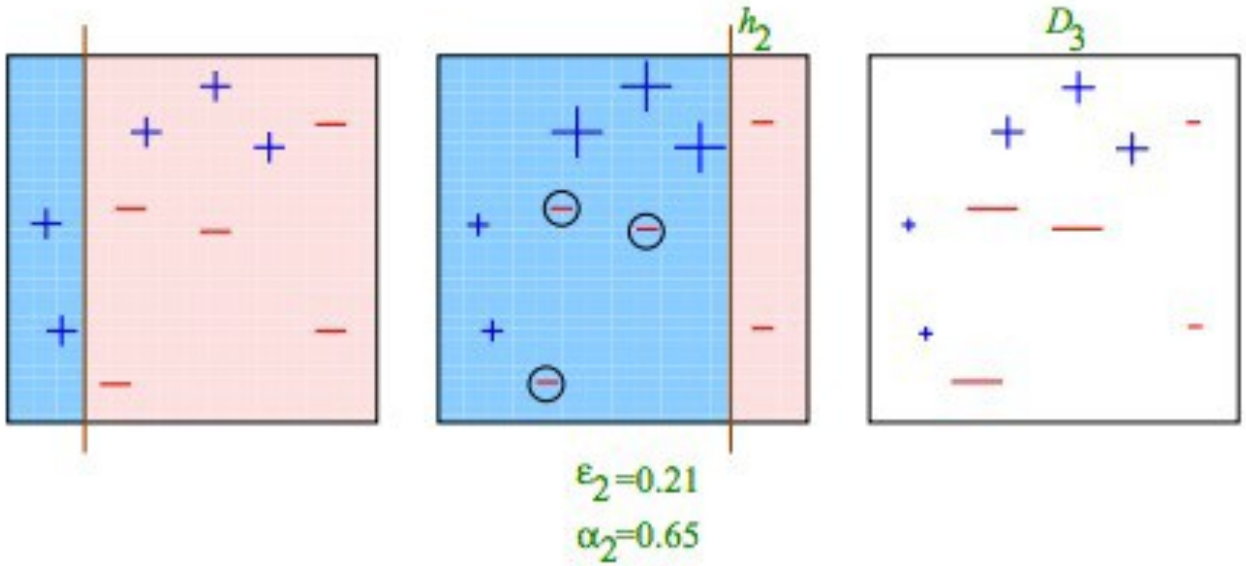

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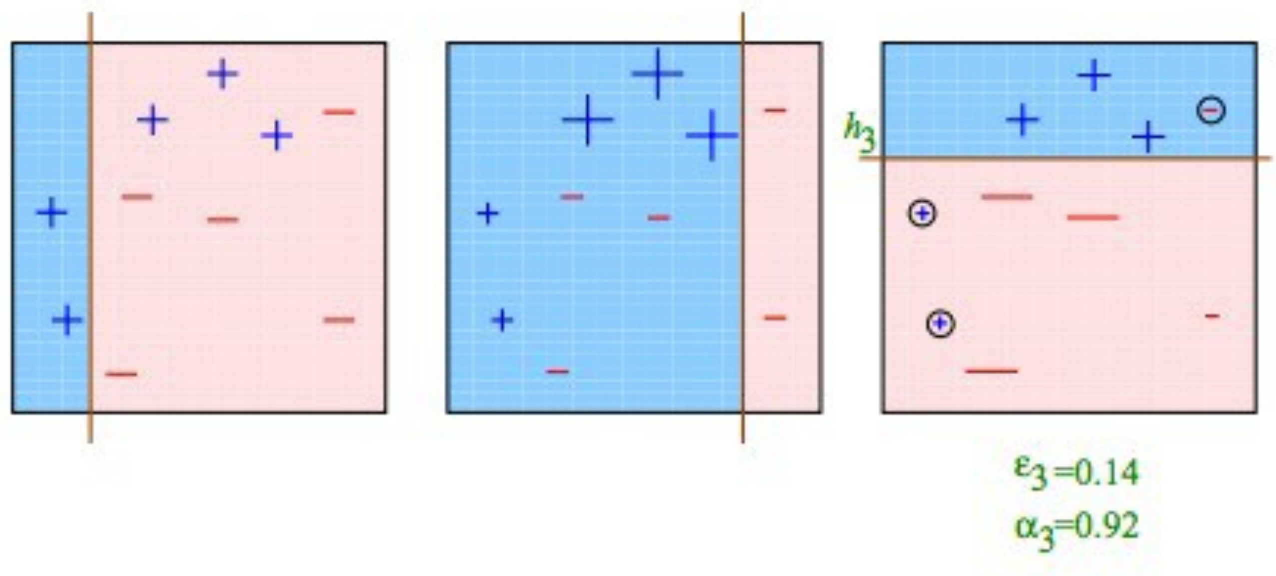

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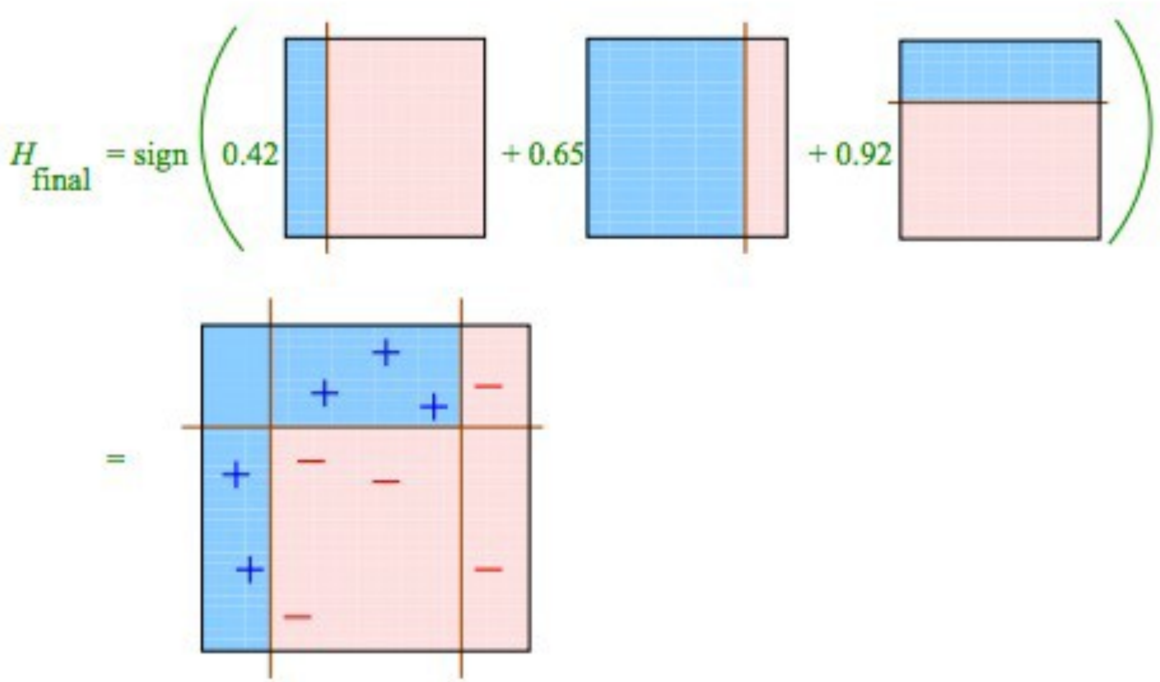
The Final Classifier



Schapire, 2011

weak classifiers: horizontal or vertical half-planes

How to Find Stopping Time

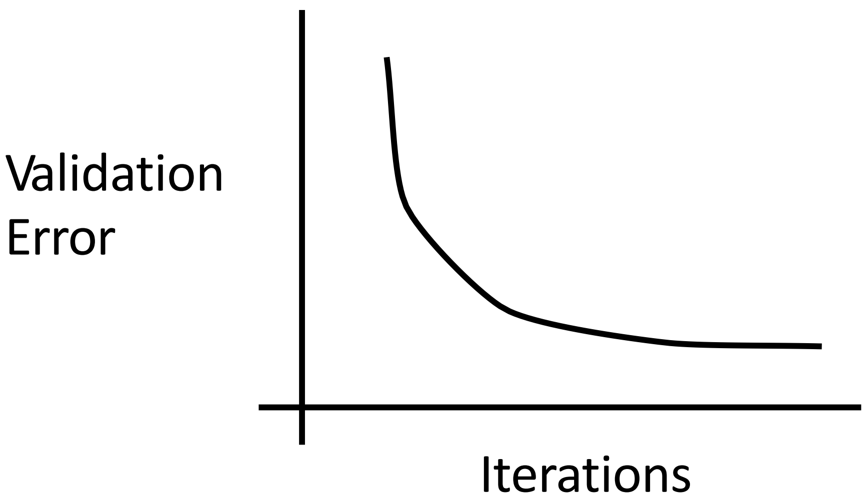
Given training set $S = \{(x_1, y_1), \ldots, (x_n, y_n)\}$, y in $\{-1, 1\}$

For $t = 1, \ldots, T$

Construct distribution D_t on the examples

Find weak learner h_t which has small error $err_{D_t}(h_t)$ wrt D_t

Output final classifier



To find stopping time, use a **validation dataset**.
Stop when the error on the validation dataset stops getting better, or when you can't find a good rule of thumb.

Questions?