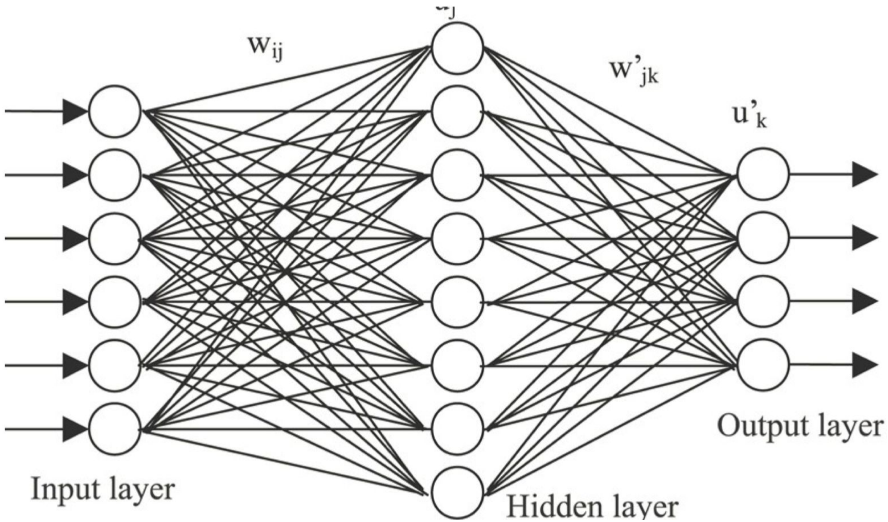
Lecture - 3

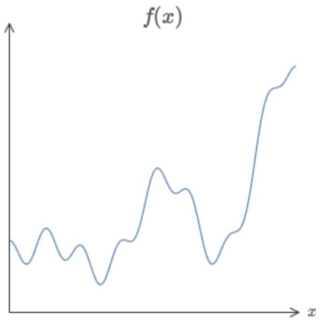
Training a neural Network - 1

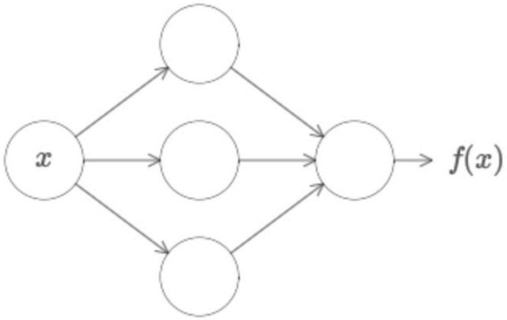
So far …

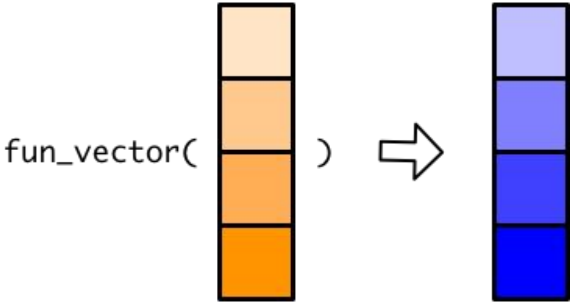


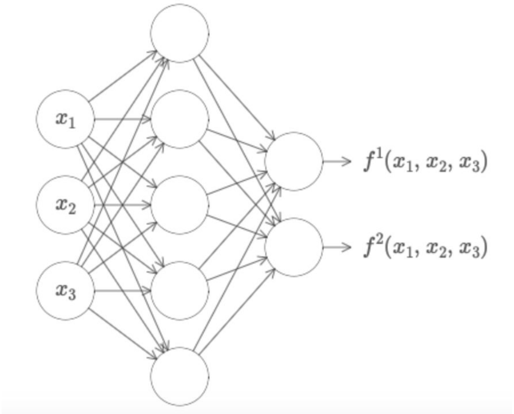
- Neural network (Network of Perceptron/MLP) can:
  - model any Boolean function
  - Model any decision boundary
  - Model any continuous valued function (how?)

Neural Network can compute any func.



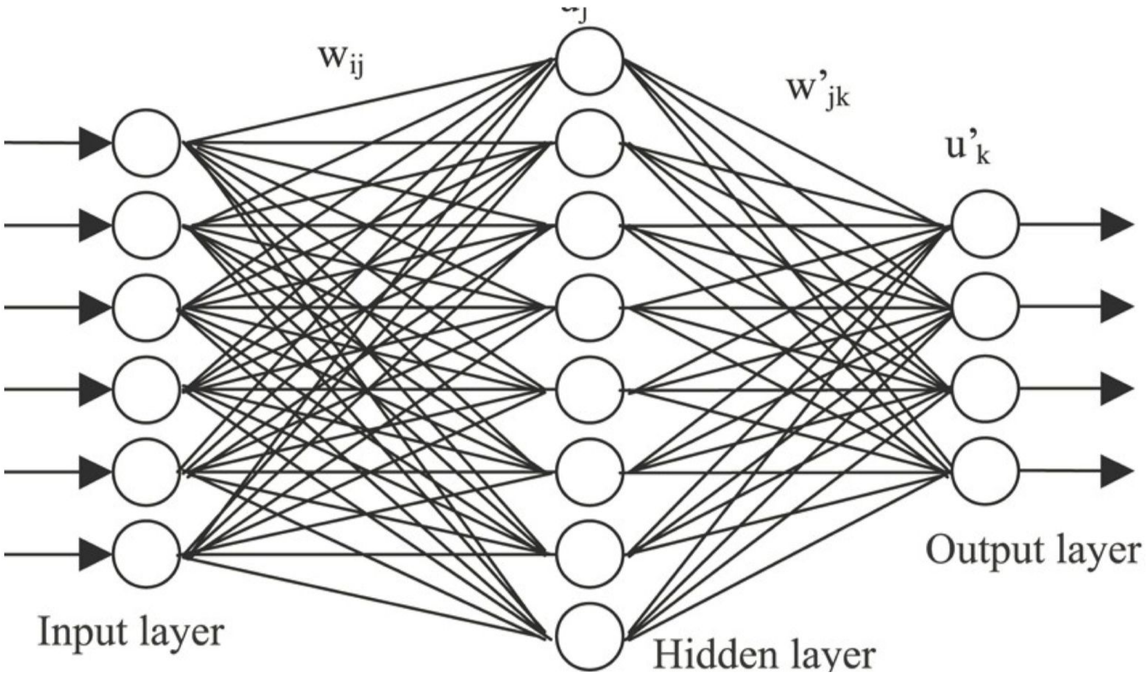






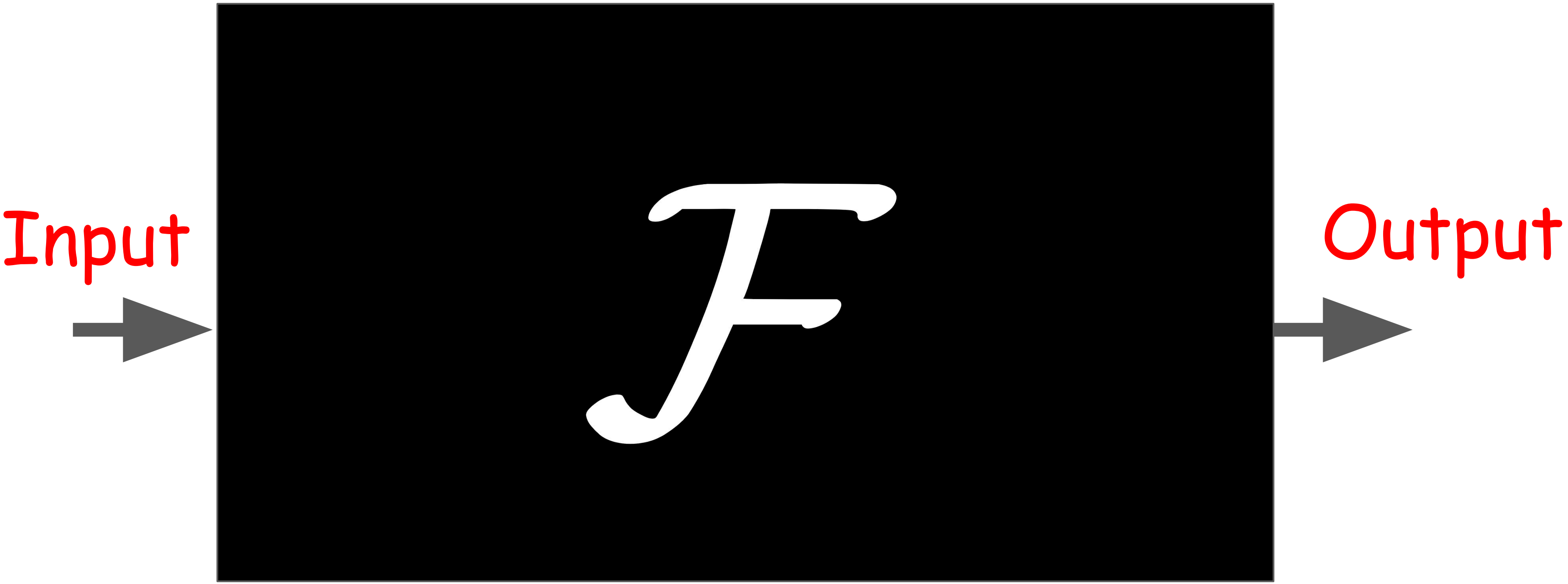
Neural Network

Input

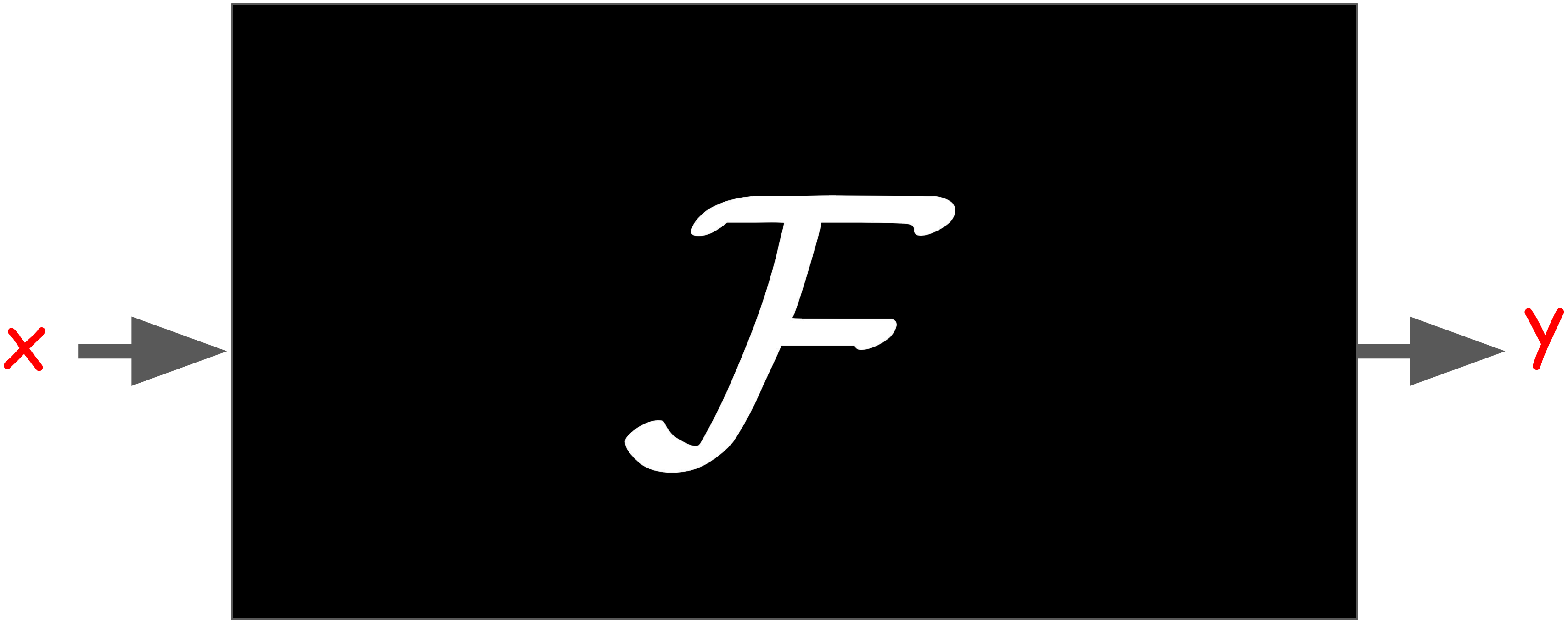


Output

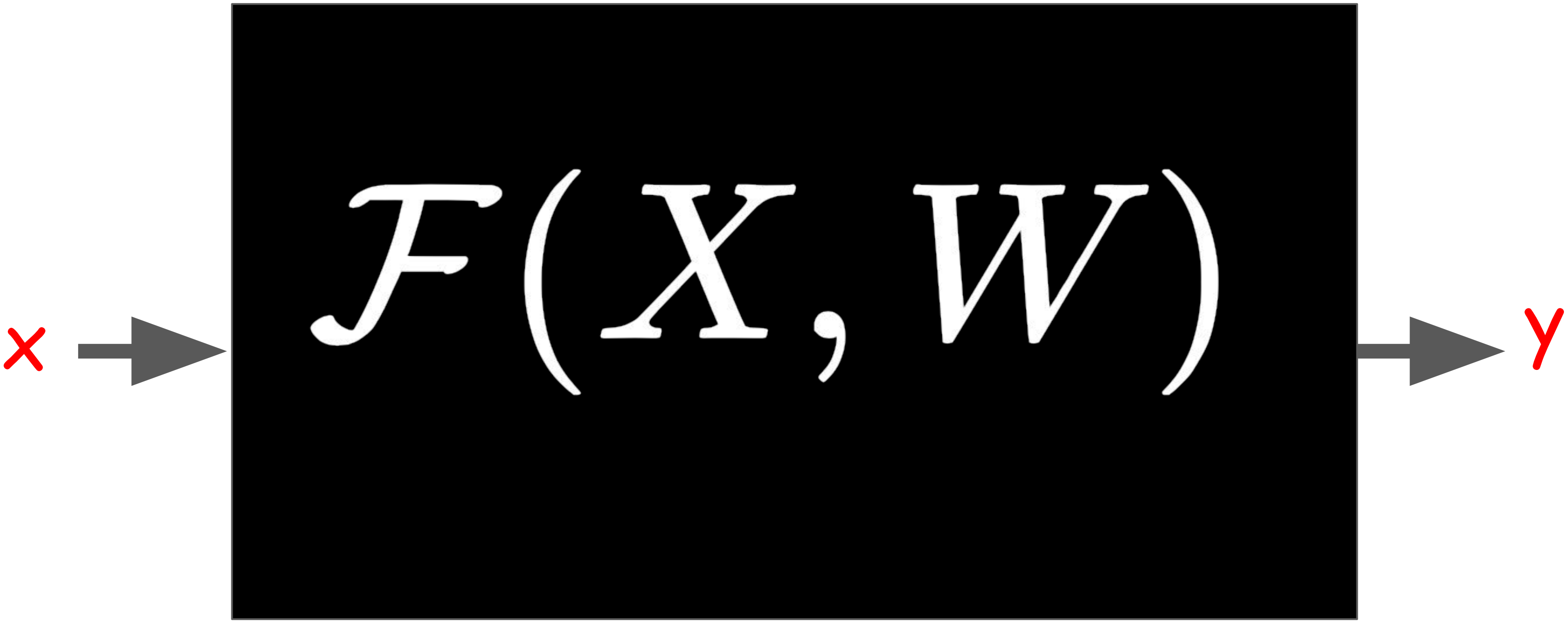
Neural Network as a function



Neural Network as a function



Neural Network as a function

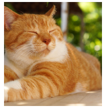


Neural Network as a function

$$\mathcal{F}(X, W)$$

dog

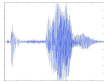
Neural Network as a function



$$\mathcal{F}(X, W)$$

cat

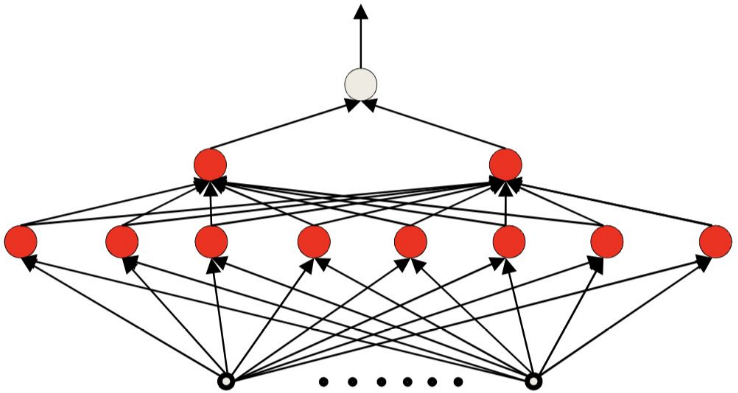
Neural Network as a function

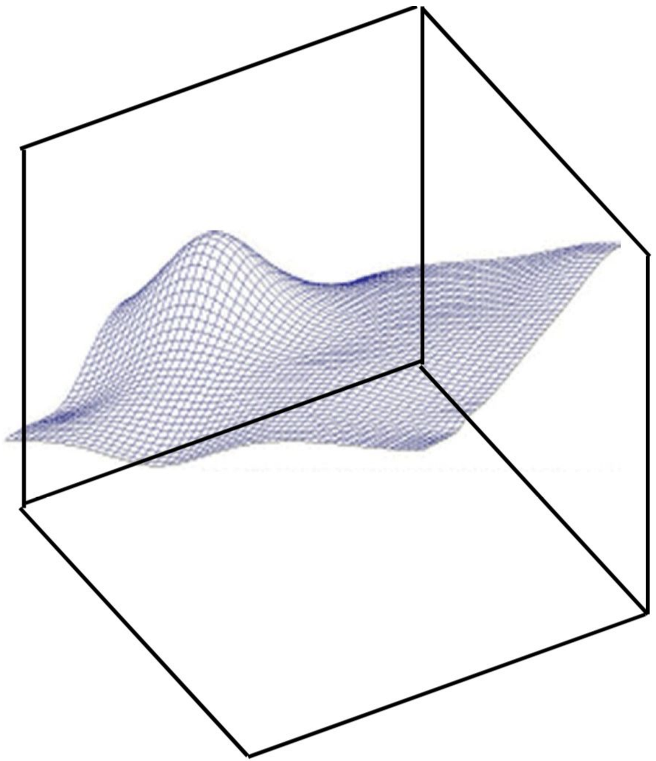


$$\mathcal{F}(X, W)$$

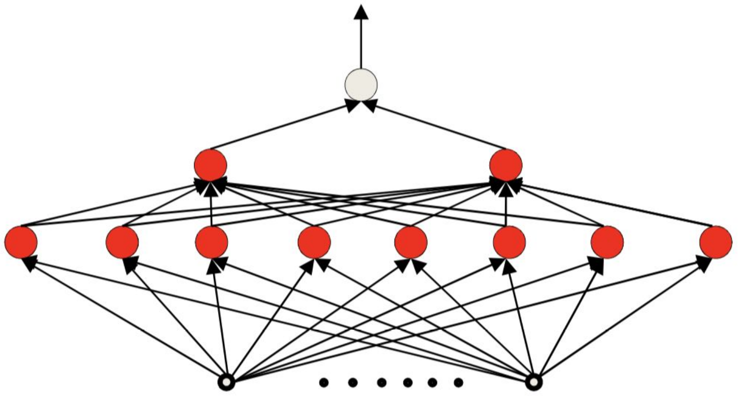
I love
Neural
Network

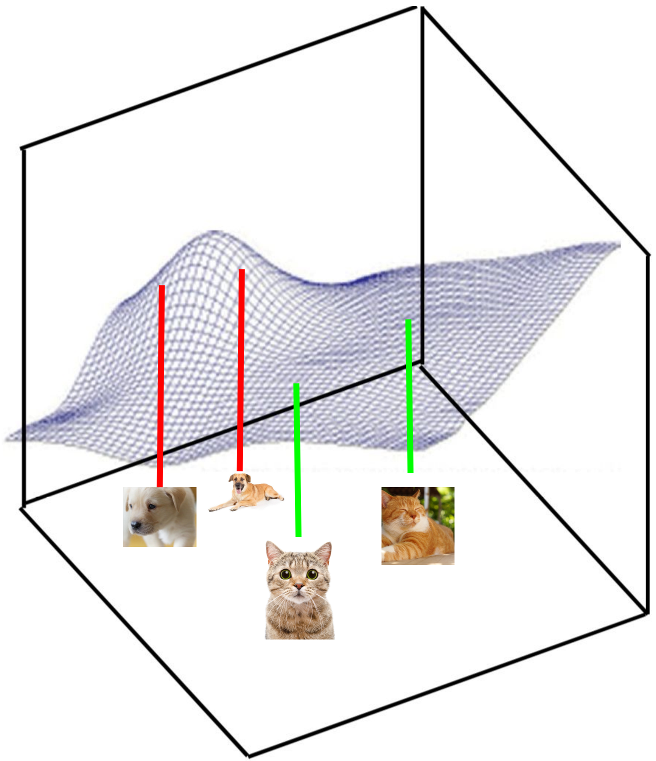
MLP Can represent anything





MLP Can represent anything





MLP Can represent anything

Option 1: Follow Lecture 2 and construct by hand

Option 2: Automatic estimation of an MLP

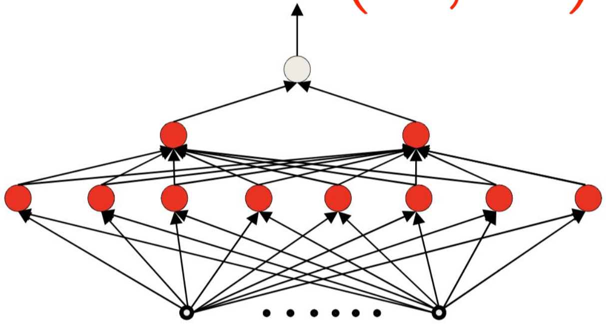
MLP Can represent anything

~~Option 1: Follow Lecture 2 and construct by hand~~

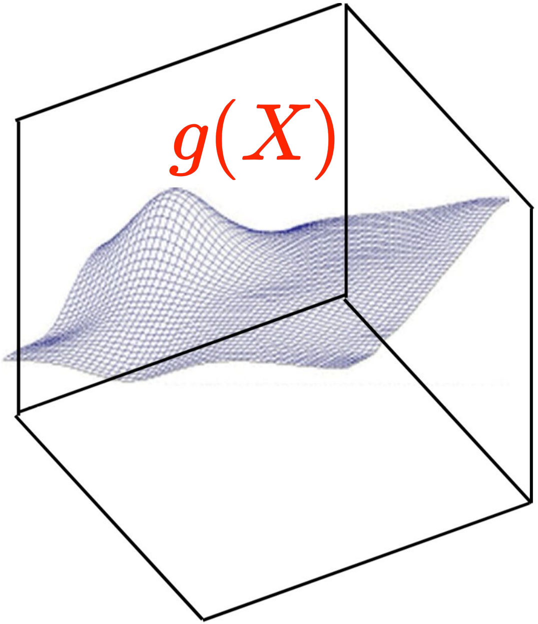
Option 2: Automatic estimation of an MLP

Automatic estimation of MLP

$$Y = \mathcal{F}(X, W)$$

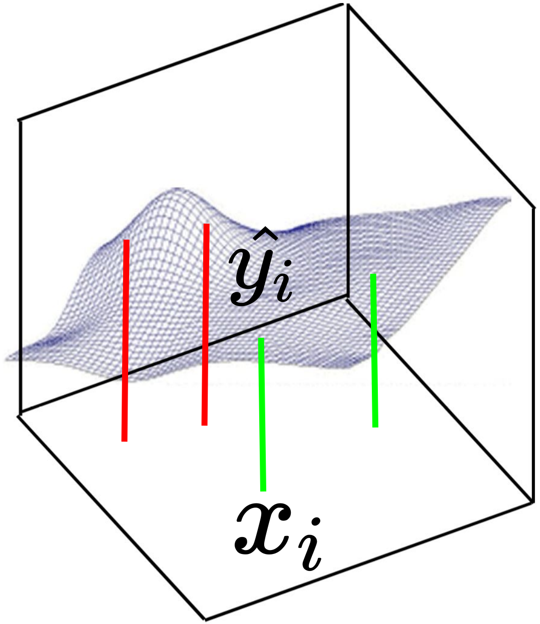


X

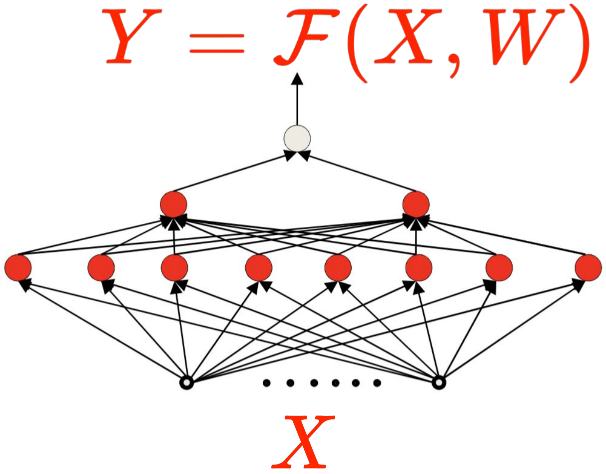


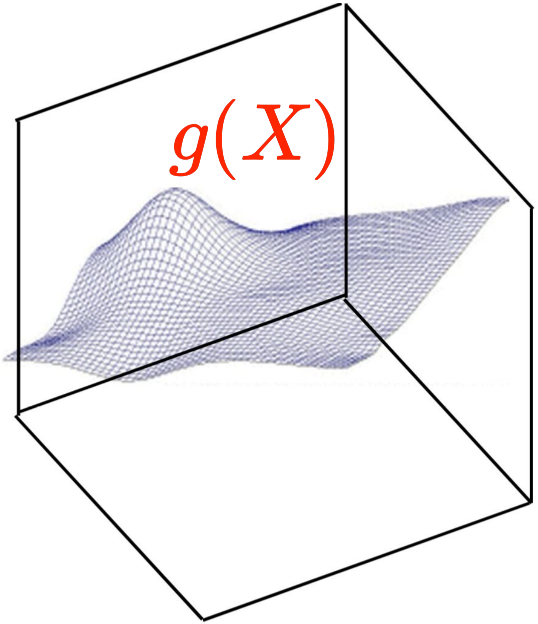
$$W^* = \arg\min_W \int_X \mathcal{DIV}(\mathcal{F}(X, W), g(X))$$

Problem: g(x) is not known everywhere



Automatic estimation of MLP





$$W^* = \arg\min_W \sum_{i=1}^{n} \mathcal{DIV}\left(\mathcal{F}(x_i, W), \hat{y}_i\right)$$

Module 1: Learning Algorithm

Supervised Learning Setup

Data: $\mathcal{D} = \{\mathbf{x}_i, \hat{y}_i\}_{i=1}^{n}$, *where* $\mathbf{x}_i = \{x_{i1}, x_{i2}, \cdots, x_{im}\}$ is a sample, and $\hat{y}_i \in \{+1, -1\}$ is a class label.

Supervised Learning Setup

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$$y = \mathcal{F}(\mathbf{x}, \mathbf{w})$$

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Model: $y = \mathcal{F}(\mathbf{x}, \mathbf{w})$ where $\mathcal{F}$ is a neural network.

Parameters: $\mathbf{w}$ needs to be learnt.

Learning Algo: Perceptron learning algo., gradient descent

Loss function: To guide learning algorithm

The problem

$$W^* = \arg\min_W \sum_{i=1}^{n} \mathcal{DIV}\big(\mathcal{F}(x_i, W), \hat{y}_i\big)$$

The problem

$$\mathcal{L}(W) = \boxed{\mathcal{DIV}}(\mathcal{F}(W, x), \hat{y})$$

Divergence (intuitively a function f(a,b) which has lower value when a = b)

Example of Divergence function

- Least Mean Square

$$DIV(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

- Cross-entropy

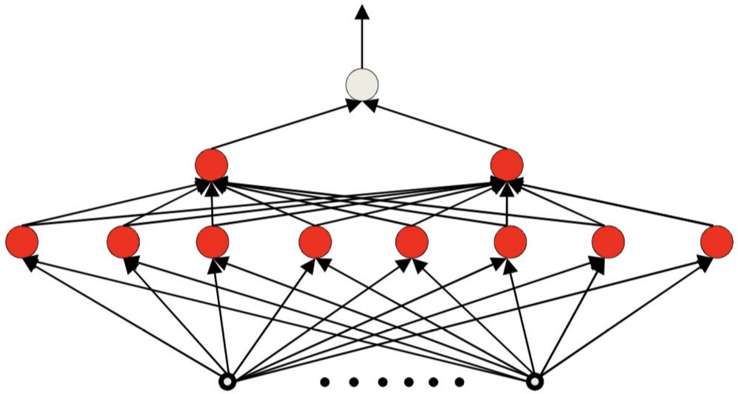
$$\mathcal{DIV}(y, \hat{y}) = -ylog(\hat{y}) - (1 - y)log(1 - \hat{y}))$$

The Problem

$$\mathcal{L}(W) = \mathcal{DIV}(\boxed{\mathcal{F}}(W, x), \hat{y})$$

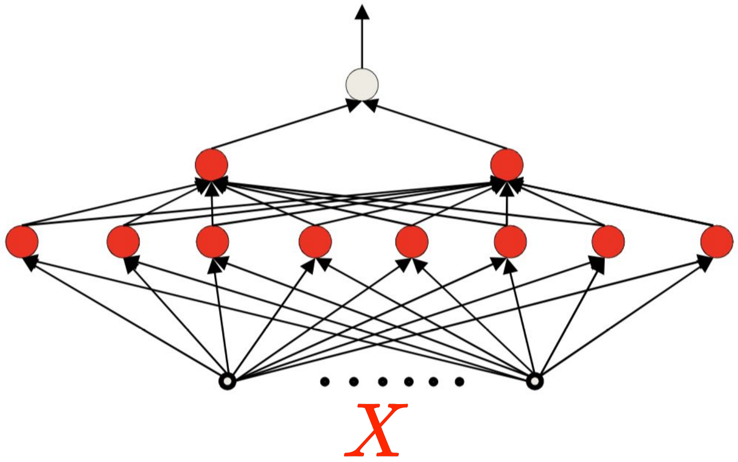
Is a neural network parameterized by W and takes input x

What is F?



What is F?

$$Y = \mathcal{F}(X, W)$$



What is F?

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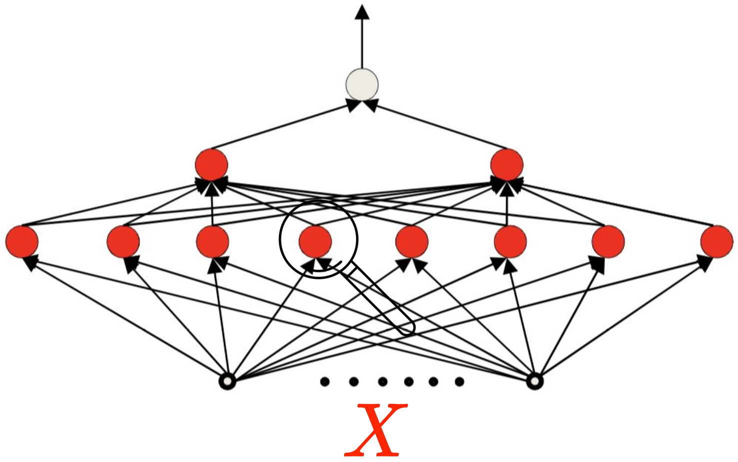
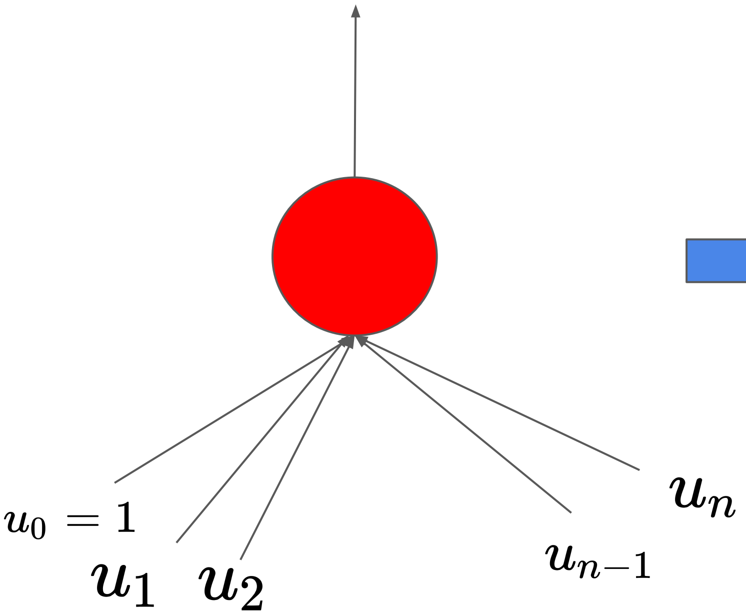
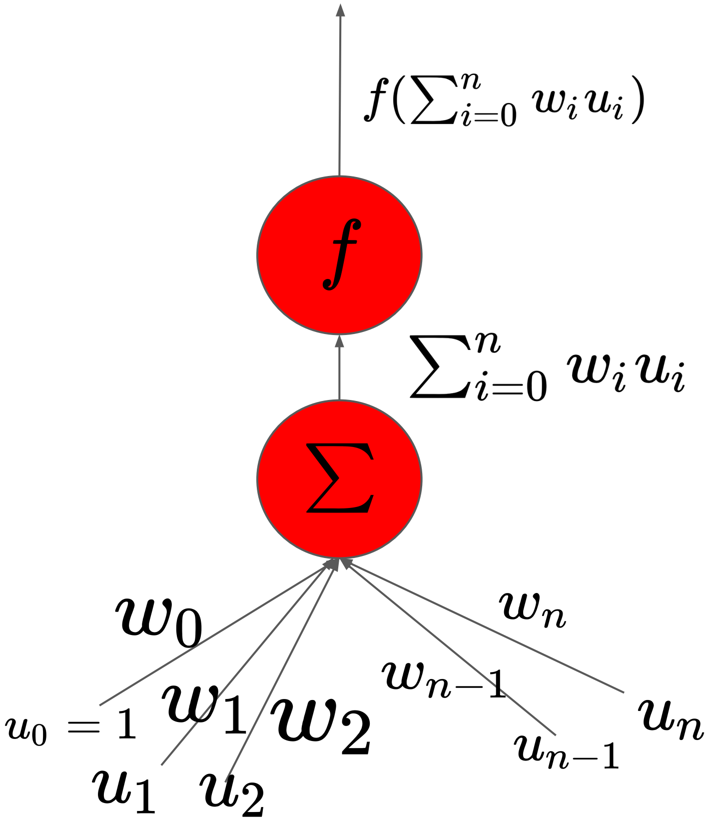
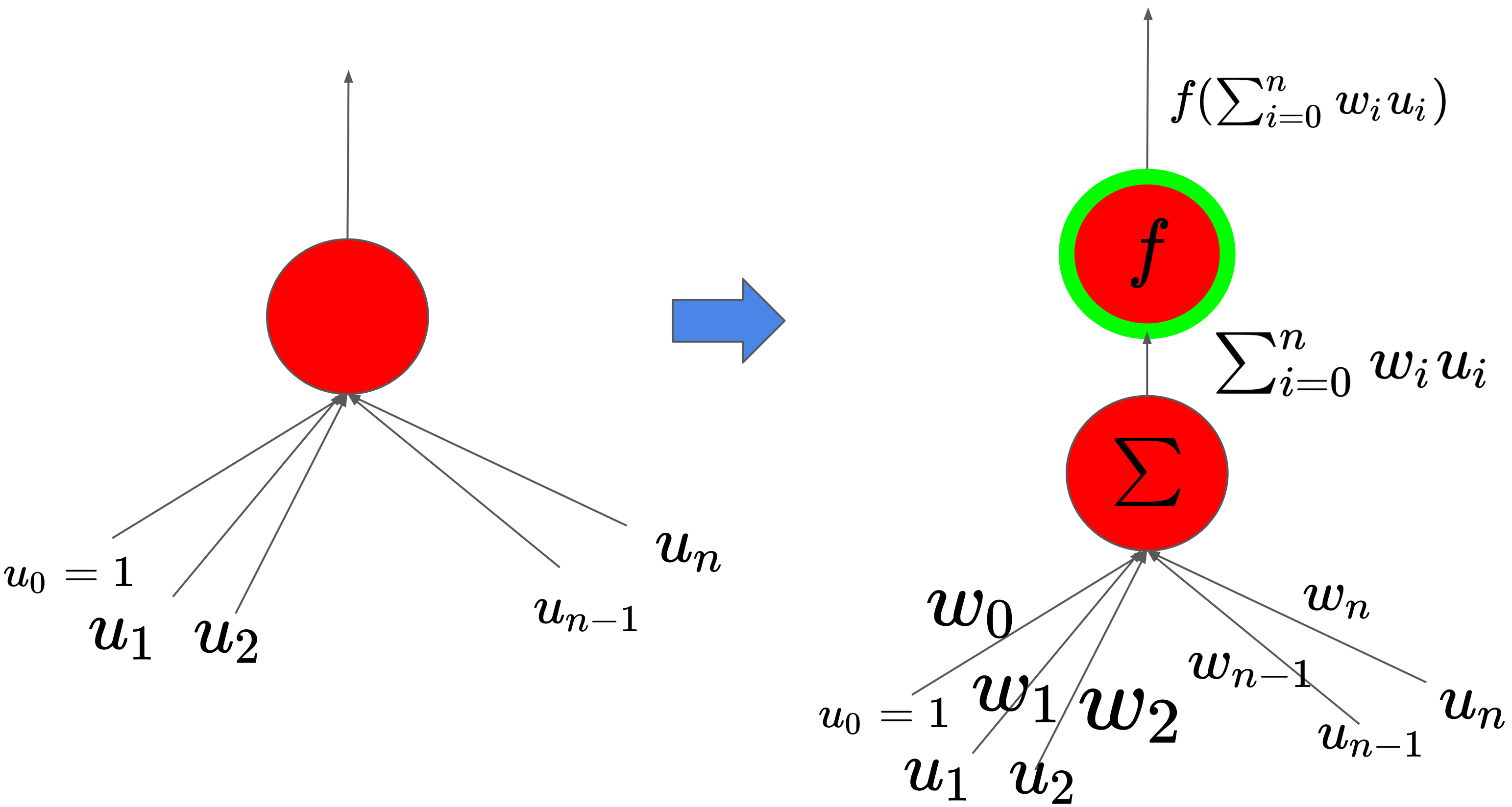


How does each neuron look?

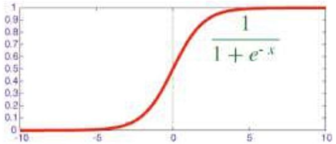




How does each neuron look?



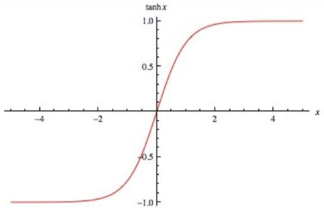
What is f ?



$$f(z) = \frac{1}{1+\exp(-z)}$$

$$f'(z) = f(z)(1 - f(z))$$

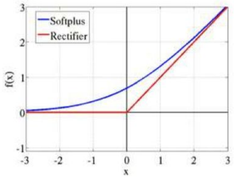
Sigmoid



$$f(z) = \tanh(z)$$

$$f'(z) = (1 - f^2(z))$$

tanh



$$f(z) = \begin{cases} 0, & z < 0 \\ z, & z \geq 0 \end{cases}$$

[*] $$f'(z) = \begin{cases} 1, z \geq 0 \\ 0, z < 0 \end{cases}$$

ReLU

$$f(z) = \log(1 + \exp(z))$$

$$f'(z) = \frac{1}{1+\exp(-z)}$$

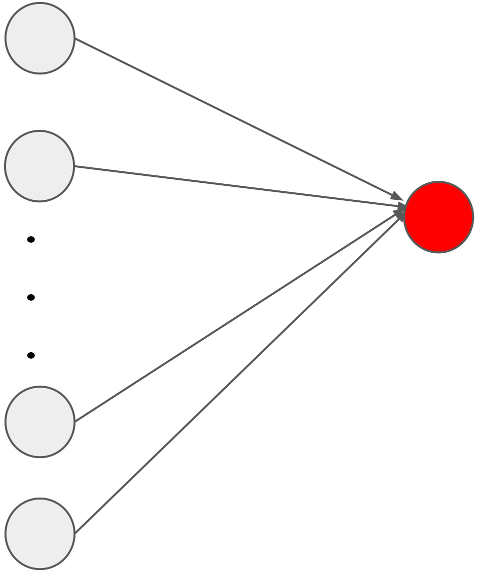
Log likelihood

Back to the problem ...

$$\mathcal{L}(W) = \mathcal{DIV}(\mathcal{F}(W, x), \boxed{\hat{y}})$$

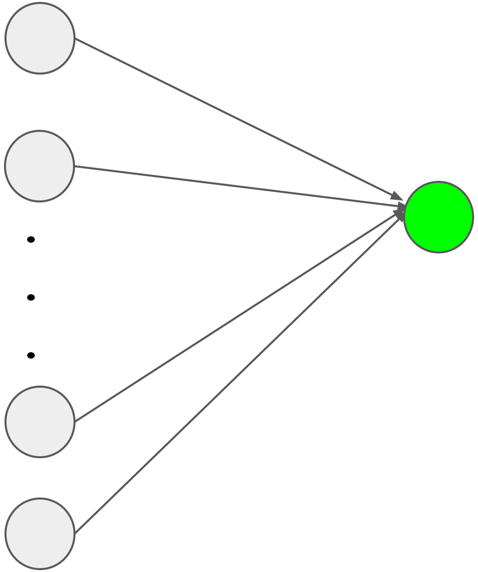
Groundtruth label

Example of $\hat{y}$: Cat vs dog Classifier



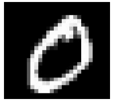
$$\hat{y} = 0$$

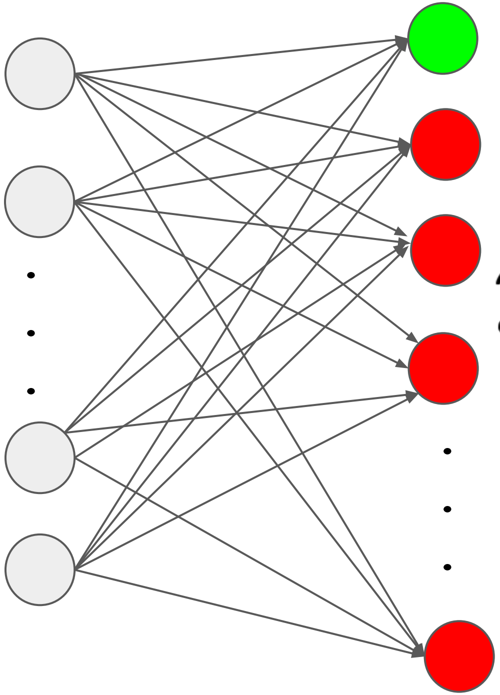
Example of $\hat{y}$: Cat vs dog Classifier



$$\hat{y} = 1$$

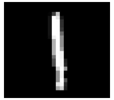
Example of $\hat{y}$: Digit classification

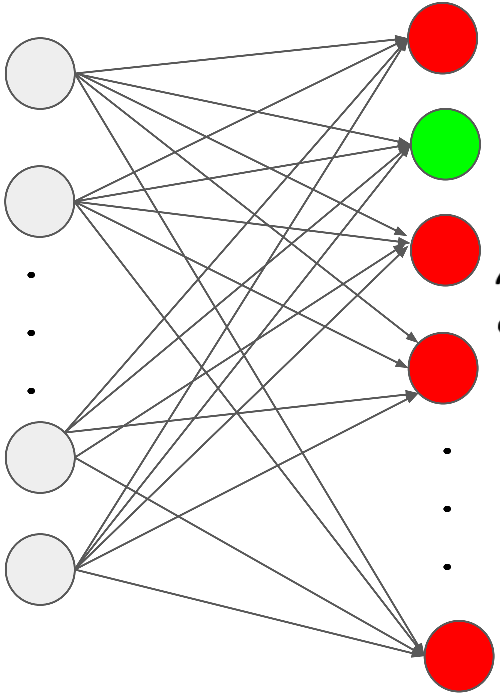




$$\hat{y} = [1, 0, 0, 0, \cdots, 0]$$

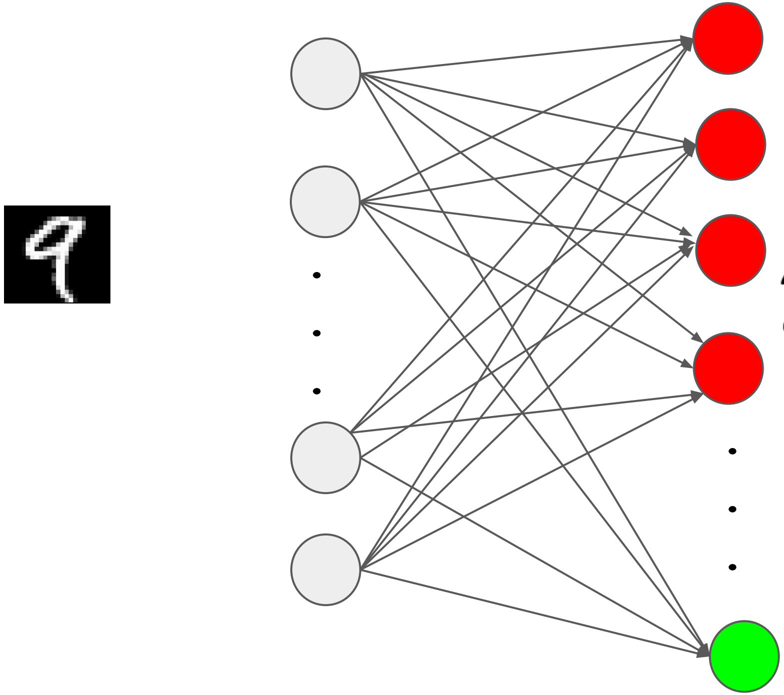
Example of $\hat{y}$: Digit classification





$$\hat{y} = [0, 1, 0, 0, \cdots, 0]$$

Example of $\hat{y}$: Digit classification



$$\hat{y} = [0, 0, 0, 0, \cdots, 1]$$

The problem

$$W^* = \arg\min_{W} \sum_{i=1}^{n} \mathcal{DIV}\big(\mathcal{F}(x_i, W), \hat{y}_i\big)$$

Back to high school ...

Let us understand?

1. What is a derivative?
2. What is gradient?
3. How to minimize a function?
4. What does direction of the gradient says?

On paper and pen!

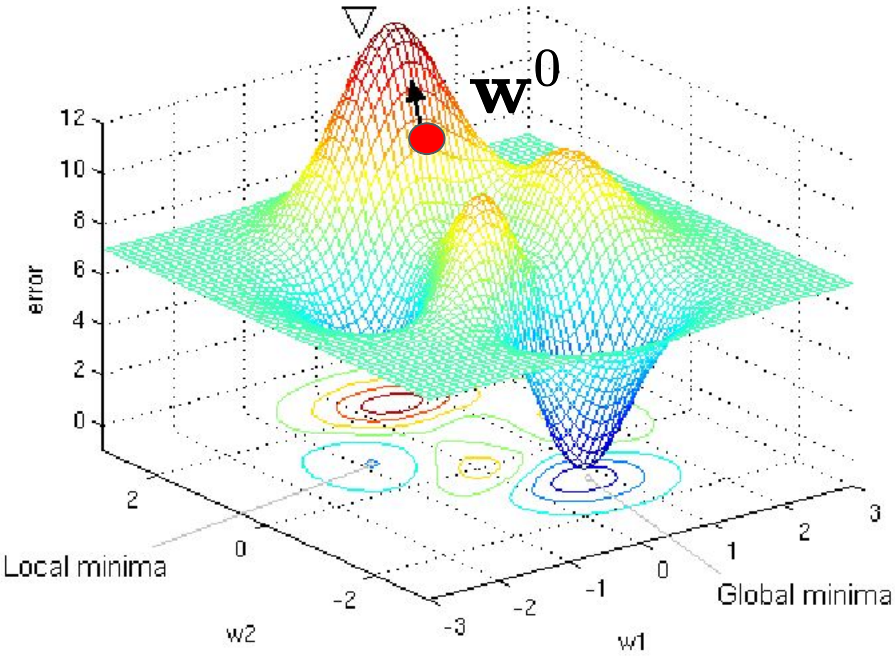
Gradient Descent

1. Initialize $iteration(t) \leftarrow 0, \eta, \mathbf{w}^0$
2. While $|f(\mathbf{w}^t) - f(\mathbf{w}^{t+1})| > \epsilon$

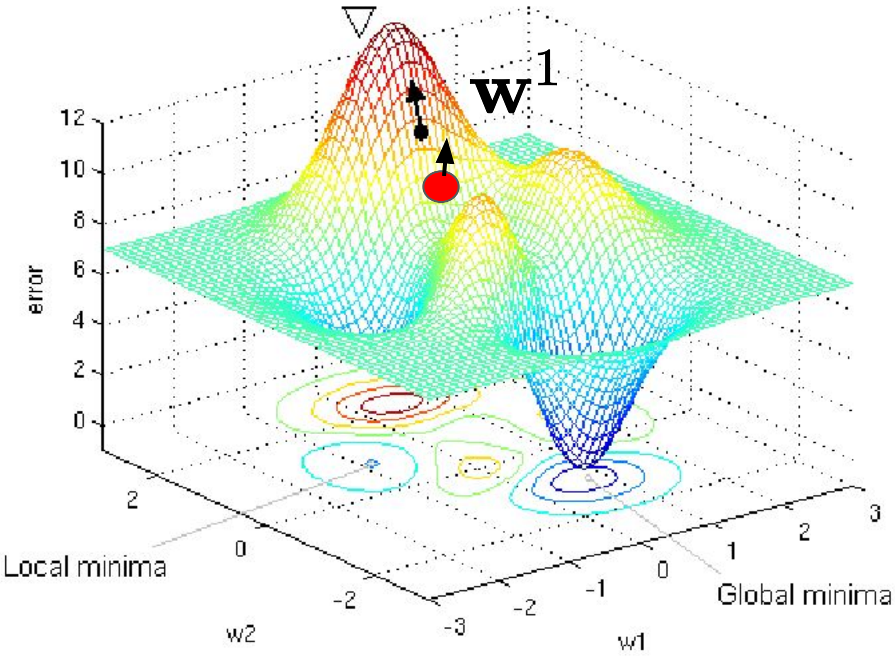
   (i) $\mathbf{w}^{t+1} = \mathbf{w}^t - \eta \nabla f(\mathbf{w^t})^T$

   (ii) $t = t + 1$

Gradient Descent in action



Gradient Descent in action



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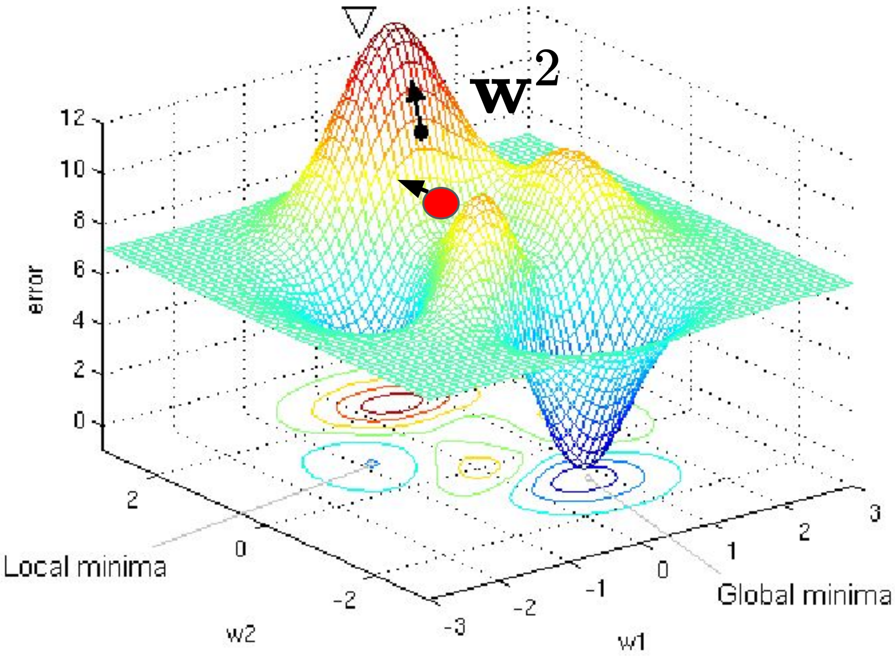

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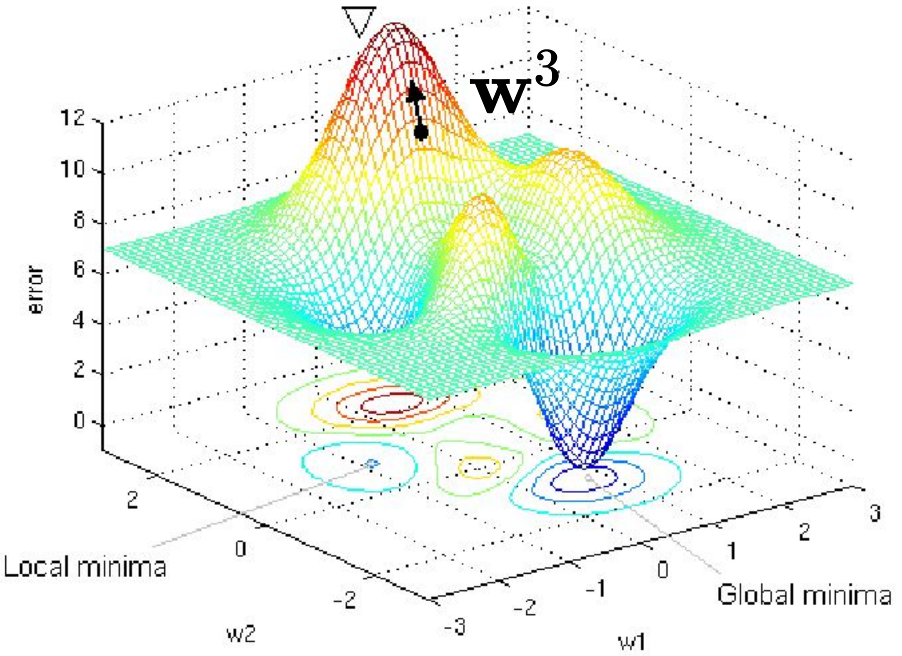

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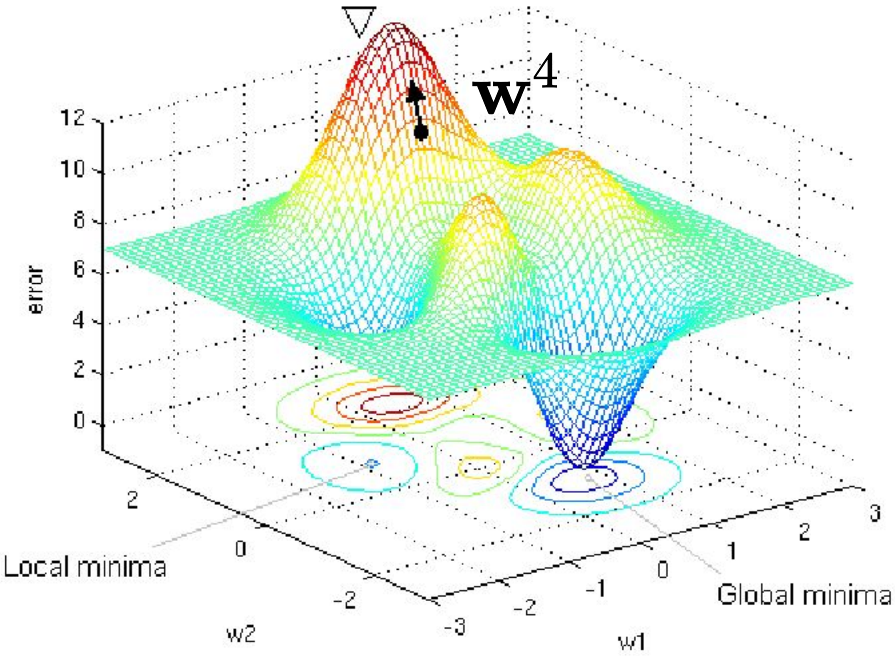

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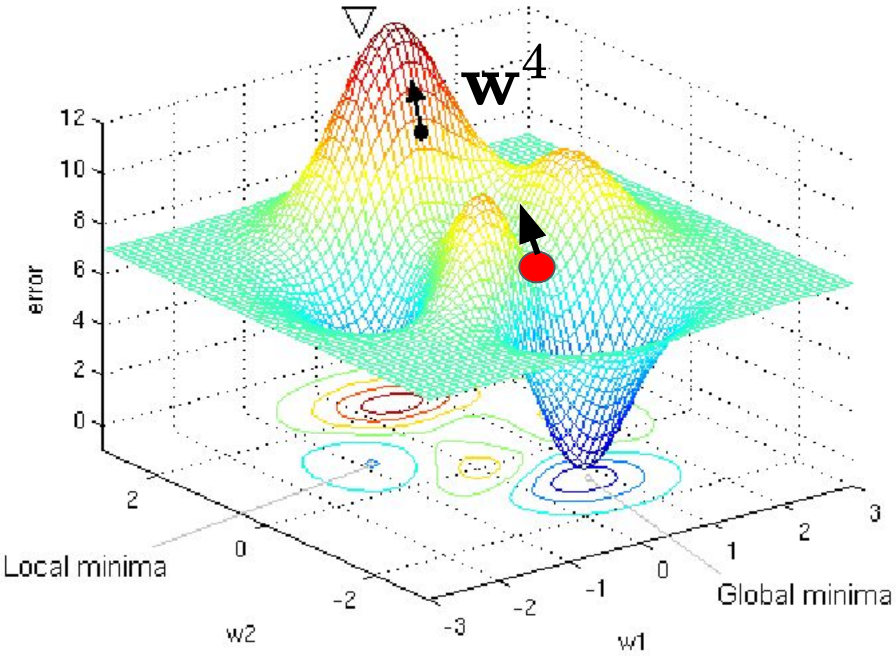
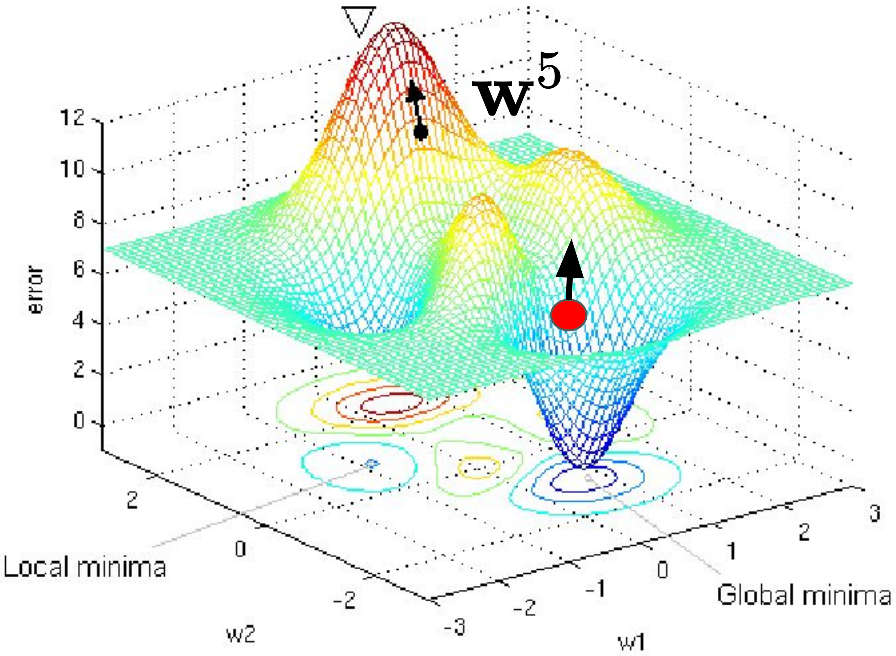


Gradient Descent in action



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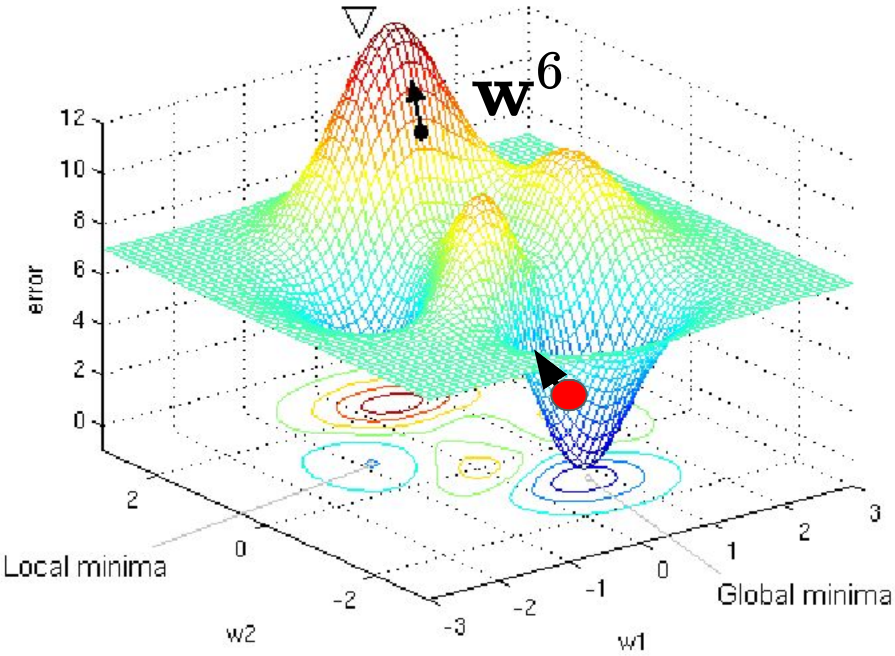

Gradient Descent in action