Instructions:

1. **Write your name and roll number on the question paper.**
2. **Use of any electronic device or reading material is not allowed.**
3. **Write appropriate explanation/justification/steps wherever necessary.**
4. **If anything is not clear, or there is a mistake, or some question is incomplete, make appropriate assumptions, mention them and proceed.**

1. Write the objective function of SVM with L_2-regularization and hinge-loss.
   (a) Calculate its first derivative w.r.t. the learnable parameter vector $\mathbf{w}$. [2 **Marks**]
   (b) Derive the update rule for optimizing the objective function using gradient descent. [2 **Marks**]
   (c) Write a pseudo-code for learning $\mathbf{w}$ based on the above assuming sparse updates (i.e., the features are sparse). Write appropriate explanation wherever necessary. [5 **Marks**]

2. In Google's page-rank algorithm, we have discussed the idea of teleporting where the random walker can jump to some other node (page) in a uniformly random manner. Now, let us consider a special scenario where the teleporting is restricted to a fixed (given) subset of nodes. E.g., suppose we are given a graph with five nodes $\{A, B, C, D, E\}$, then teleporting will always result in jumping to one of the nodes in a given subset of nodes $\tau = \{A, B\}$, in a uniformly random manner.
   (a) Write/Derive the formula for calculating the entries of the 'A' matrix for this case (assuming 'β' and 'M' are given). Briefly explain an interpretation of this. [4 **Marks**]
   (b) Suppose we are given a graph with four nodes $\{A, B, C, D\}$, with $\tau = \{A\}$ and $\beta = 0.8$. The transition probabilities (corresponding to matrix M) are given by: 0.5 for $A \to B$, 0.5 for $A \to C$, 1 for $C \to A$, 1 for $B \to D$, and 1 for $D \to B$. We start with equal ranks for all the four nodes (i.e., 0.25). First calculate the A matrix using the given information based on the formula you derived in part-(a), and then calculate the rank vector obtained after performing three iterations of the power iteration method using it. [6 **Marks**]

3. Consider the utility matrix given in the following figure. Estimate the rating of movie '3' by user '3' (the cell marked by star) using both user-user collaborative filtering and item-item collaborative filtering for $k = 2$ nearest neighbours using Pearson correlation as the similarity measure. Show all the necessary steps. [10 **Marks**]