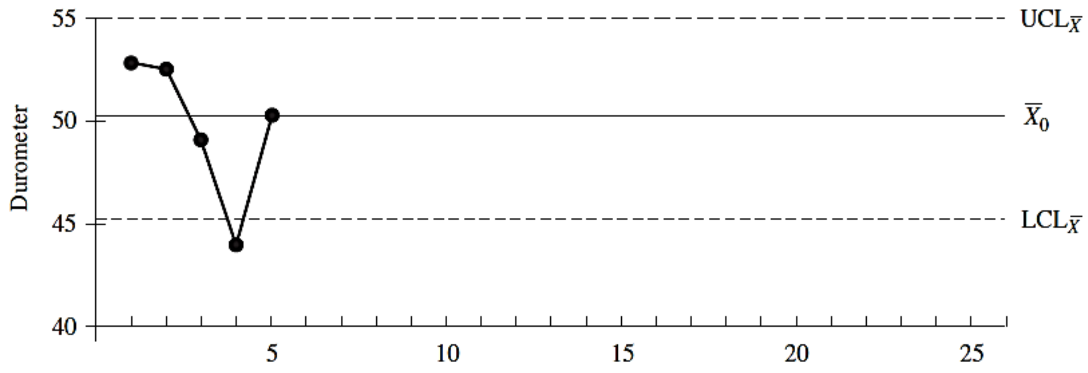
Quality Management

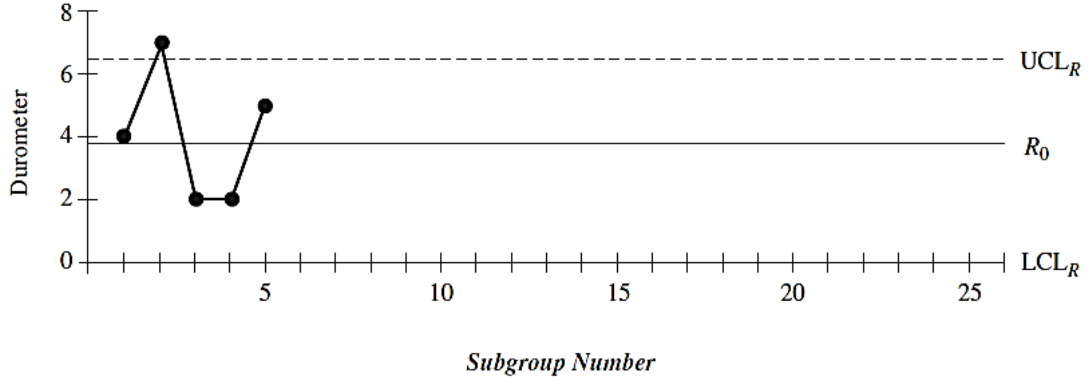
L_7
28/01/23

Variable Control Charts

– In practice, control charts are posted at individual machines or work centers to control a particular quality

– characteristic. Usually, an X-bar chart for the central tendency and an *R* chart for the dispersion are used together.





Variable Control Charts

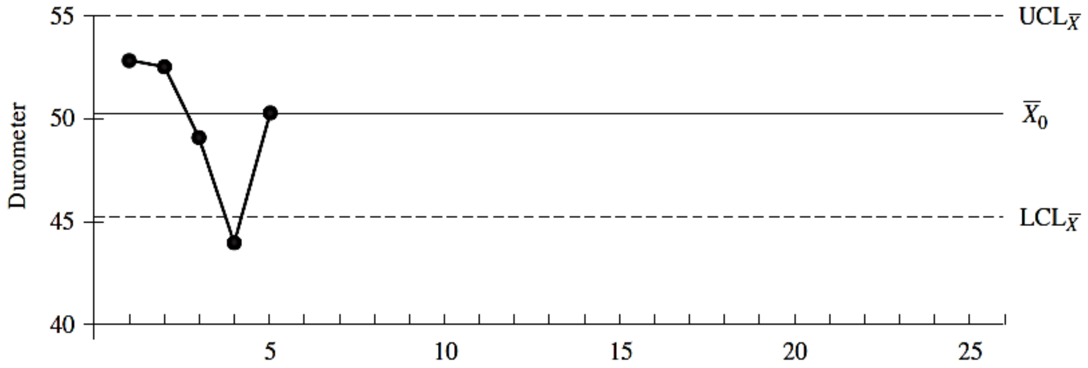
At work center number 365-2 at 8:30 A.M., the operator selects four items for testing and records the observations of 55, 52, 51, and 53 in the rows marked X_1, X_2, X_3, and X_4, respectively. A subgroup average value of 52.8 is obtained by summing the observation and dividing by 4, and the range value of 4 is obtained by subtracting the low value, 51, from the high value, 55. The operator places a small solid circle at 52.8 on the *X bar* chart and a small solid circle at 4 on the *R* chart and then proceeds with his other duties.

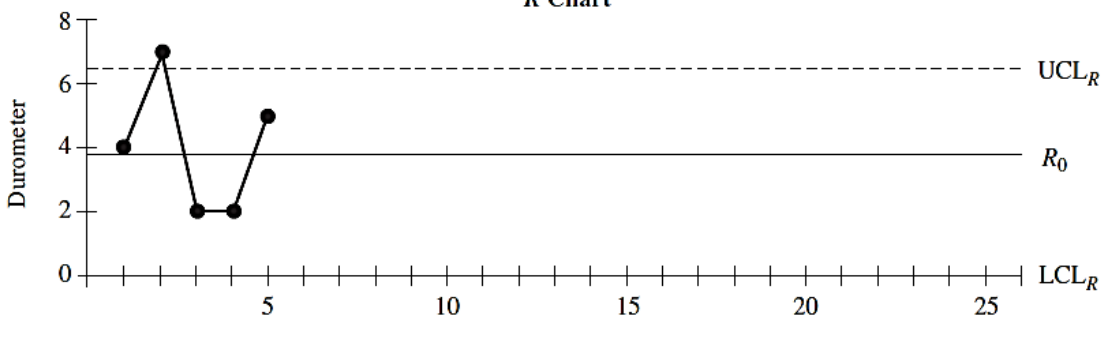
$\bar{X}$ and *R* Charts

Work Center Number 365-2

QualityCharacteristic Durometer Date 3/6/93

Time	8:30 AM	9:30 AM	10:40 AM	11:50 AM	1:30 PM								
Subgroup	1	2	3	4	5	6	7	8	9	10	11	12	13
X_1	55	51	48	45	53								
X_2	52	52	49	43	50								
X_3	51	57	50	45	48								
X_4	53	50	49	43	50								
Sum	211	210	196	176	201								
$\bar{X}$	52.8	52.5	49	44	50.2								
R	4	7	2	2	5								





Variable Control Charts

The frequency with which the operator inspects a product at a particular machine or work center is determined by the quality of the product.

Quality Characteristic

The variable that is chosen for the *X-bar* and *R* charts must be a quality characteristic that is measurable and can be expressed in numbers.

- Those quality characteristics affecting the performance of the product would normally be given first attention.
- These may be a function of the raw materials, component parts, subassemblies, or finished parts. In other
- words, high priority is given to the selection of those characteristics that are giving difficulty in terms of production
- problems and/or cost. An excellent opportunity for cost savings frequently involves situations where
- spoilage and rework costs are high. A Pareto analysis is also useful for establishing priorities. Another possibility
- occurs where destructive testing is used to inspect a product.
- A judicious selection.

Variable Control Charts: Subgroup Size and Method

Decisions on the size of the sample or subgroup require a certain amount of empirical judgment; however, some helpful guidelines are:

1. As the subgroup size increases, the control limits become closer to the central value, which makes the control chart more sensitive to small variations in the process average.

2. As the subgroup size increases, the inspection cost per subgroup increases. Does the increased cost of larger subgroups justify the greater sensitivity?

3. When costly and/or destructive testing is used and the item is expensive, a small subgroup size of two or three is necessary, because it will minimize the destruction of expensive product.

4. Because of the ease of computation, a sample size of five is quite common in industry; however, when inexpensive electronic hand calculators are used, this reason is no longer valid.

Variable Control Charts: Data Collection

Because of difficulty in the assembly of a gear hub to a shaft using a key and keyway, the project team recommends using *X bar* and *R* charts. The quality characteristic is the shaft keyway depth of 6.35 mm (0.250 in.). Using a rational subgroup of four, a technician obtains five subgroups per day for five days. The samples are measured, the subgroup average and range are calculated, and the results are recorded on the form as shown in Table. Additional recorded information includes the date, time, and any comments pertaining to the process. For simplicity, individual measurements are coded from 6.00 mm. Thus, the first measurement of 6.35 is recorded as 35.

Data on the Depth of the Keyway (millimeters)

Subgroup Number	Date	Time	MEASUREMENTS X_1	X_2	X_3	X_4	Average $\overline{X}$	Range R	Comment
1	7/23	8:50	35	40	32	37	6.36	0.08	
2		11:30	46	37	36	41	6.40	0.10	
3		1:45	34	40	34	36	6.36	0.06	
4		3:45	69	64	68	59	6.65	0.10	New, temporary operator
5		4:20	38	34	44	40	6.39	0.10	
.	.	.	.	.	.	.	.	.	
.	.	.	.	.	.	.	.	.	
.	.	.	.	.	.	.	.	.	
17	7/29	9:25	41	40	29	34	6.36	0.12	
18		11:00	38	44	28	58	6.42	0.30	Damaged oil line
19		2:35	35	41	37	38	6.38	0.06	
20		3:15	56	55	45	48	6.51	0.11	Bad material
21	7/30	9:35	38	40	45	37	6.40	0.08	
22		10:20	39	42	35	40	6.39	0.07	
23		11:35	42	39	39	36	6.39	0.06	
24		2:00	43	36	35	38	6.38	0.08	
25		4:25	39	38	43	44	6.41	0.06	
Sum							160.25	2.19	

where UCL = upper control limit

LCL = lower control limit

$\sigma_{\overline{X}}$ = population standard deviation of the subgroup averages

σ_R = population standard deviation of the range

Variable Control Charts: Data Collection

The central lines for the $\bar{X}$ and R charts are obtained using the equations

$$\bar{\bar{X}} = \Sigma \bar{X}_i / g \qquad \bar{R} = \Sigma R_i / g$$

where $\bar{\bar{X}}$ = average of the subgroup averages (read "X double bar")
$\bar{X}_i$ = average of the th subgroup
g = number of subgroups
$\bar{R}$ = average of the subgroup ranges
R_i = range of the ith subgroup

Trial control limits for the charts are established at $\pm 3\sigma$ from the central line, as shown by the equations

$$\mathrm{UCL}_{\bar{X}} = \bar{\bar{X}} + 3\sigma_{\bar{X}} \qquad \mathrm{UCL}_R = \bar{R} + 3\,\sigma_R$$

$$\mathrm{LCL}_{\bar{X}} = \bar{\bar{X}} - 3\sigma_{\bar{X}} \qquad \mathrm{LCL}_R = \bar{R} - 3\,\sigma_R$$

Variable Control Charts: Data Collection

In practice, the calculations are simplified by using the product of the average of the range ($\bar{R}$) and a factor A_2 to replace the three standard deviations ($A_2 \bar{R} = 3\sigma_{\bar{X}}$) in the equation for the $\bar{X}$ chart. For the R chart, the range is used to estimate the standard deviation of the range. Therefore, the derived equations are

$$\mathrm{UCL}_{\bar{X}} = \bar{\bar{X}} + A_2\bar{R} \qquad \mathrm{UCL}_R = D_4\bar{R}$$
$$\mathrm{LCL}_{\bar{X}} = \bar{\bar{X}} - A_2\bar{R} \qquad \mathrm{LCL}_R = D_4\bar{R}$$

where A_2, D_3, and D_4 are factors that vary with the subgroup size and are found in Appendix A. For the $\bar{X}$ chart, the upper and lower control limits are symmetrical about the central line. Theoretically, the control limits for an R chart should also be symmetrical about the central line. But, for this situation to occur, with subgroup sizes of six or less, the lower control limit would need to have a negative value. Because a negative

Factors for Computing Central Lines and 3 σ Control Limits for Variables Charts

Sample Size	Chart for Averages: Factors for Control Limits		Chart for Standard Deviations: Factor for Central Line	Chart for Standard Deviations: Factors for Control Limits	
n	A_2	A_3	C_4	B_3	B_4
2	1.880	2.659	0.7979	0	3.267
3	1.023	1.954	0.8862	0	2.568
4	0.729	1.628	0.9213	0	2.266
5	0.577	1.427	0.9400	0	2.089
6	0.483	1.287	0.9515	0.030	1.970
7	0.419	1.182	0.9594	0.118	1.882
8	0.373	1.099	0.9650	0.185	1.815

Sample Size	Chart for Ranges: Factor for Central Line	Chart for Ranges: Factors for Control Limits				Chart for Medians
n	d_2	D_3	D_4	D_5	D_6	A_5
2	1.128	0	3.267	0	3.865	2.224
3	1.693	0	2.574	0	2.745	1.265
4	2.059	0	2.282	0	2.375	0.829
5	2.326	0	2.114	0	2.179	0.712
6	2.534	0	2.004	0	2.055	0.562
7	2.704	0.076	1.924	0.078	1.967	0.520
8	2.847	0.136	1.864	0.139	1.901	0.441

Variable Control Charts: Data Collection

In order to illustrate the calculations necessary to obtain the trial control limits and the central line, the data concerning the depth of the shaft keyway will be used. From Table 15-4, $\Sigma\overline{X} = 160.25$, $\Sigma R = 2.19$, and $g = 25$; thus, the central lines are

$$\overline{\overline{X}} = \Sigma\,\overline{X}/g = 160.25/25 = 6.41 \text{ mm}$$
$$\overline{R} = \Sigma\, R/g = 2.19/25 = 0.0876 \text{ mm}$$

From Appendix Table A, the values for the factors for a subgroup size (n) of four are $A_2 = 0.729$, $D_3 = 0$, and $D_4 = 2.282$. Trial control limits for the $\overline{X}$ chart are

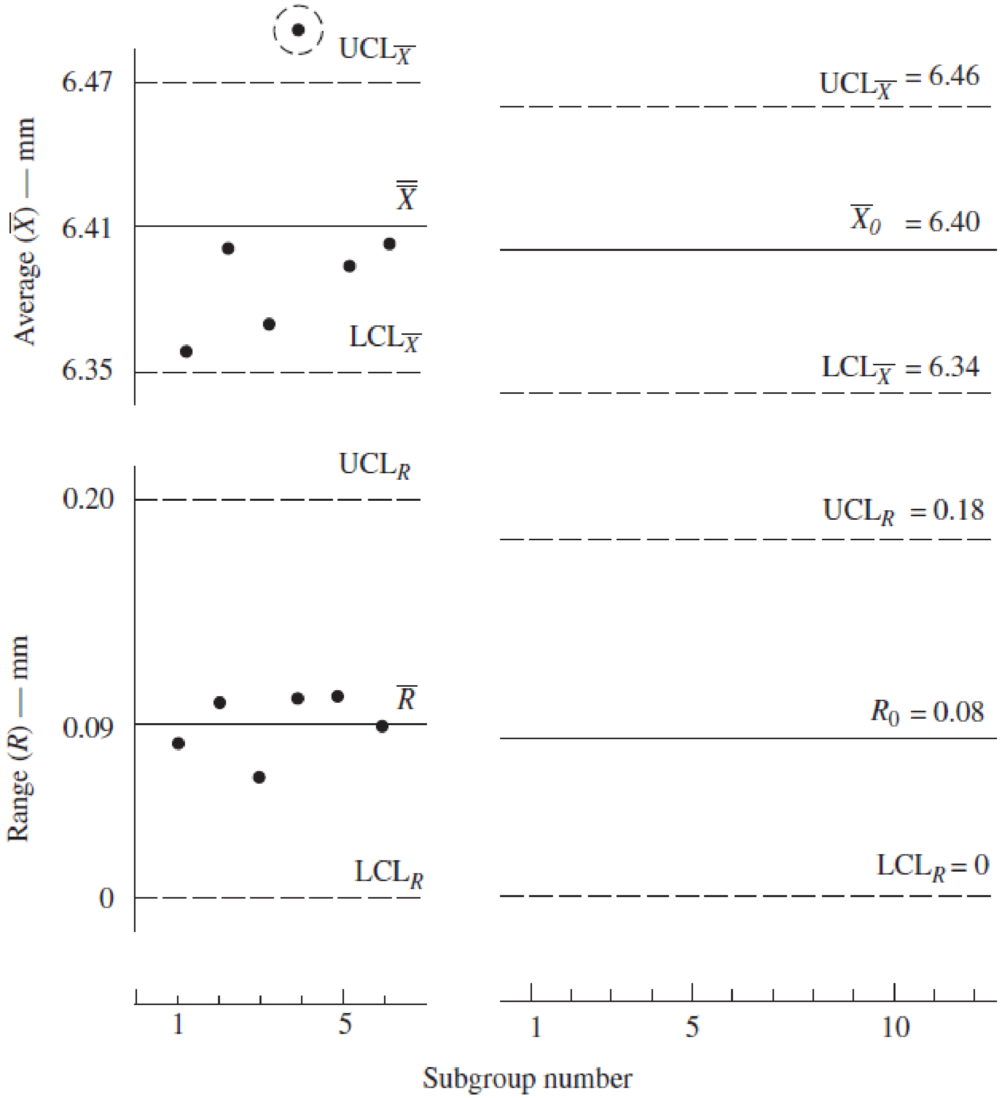
$$\begin{aligned} \text{UCL}_{\overline{X}} &= \overline{\overline{X}} + A_2\overline{R} \\ &= 6.41 + (0.729)(0.0876) \\ &= 6.47 \text{ mm} \end{aligned} \qquad \begin{aligned} \text{LCL}_{\overline{X}} &= \overline{\overline{X}} - A_2\overline{R} \\ &= 6.41 - (0.729)(0.0876) \\ &= 6.35 \text{ mm} \end{aligned}$$

Trial control limits for the R chart are

$$\begin{aligned} \text{UCL}_R &= D_4\overline{R} \\ &= (2.282)(0.0876) \\ &= 0.20 \text{ mm} \end{aligned} \qquad \begin{aligned} \text{LCL}_R &= D_3\overline{R} \\ &= (0)(0.0876) \\ &= 0 \text{ mm} \end{aligned}$$

Revised Central Lines and Control Limits

Revised central lines and control limits are established by discarding out-of-control points with assignable causes and recalculating the central lines and control limits. The *R* chart is analyzed first to determine if it is stable. Because the out-of-control point at subgroup 18 on the *R* chart has an assignable cause (damaged oil line), it can be discarded from the data. The remaining plotted points indicate a stable process.

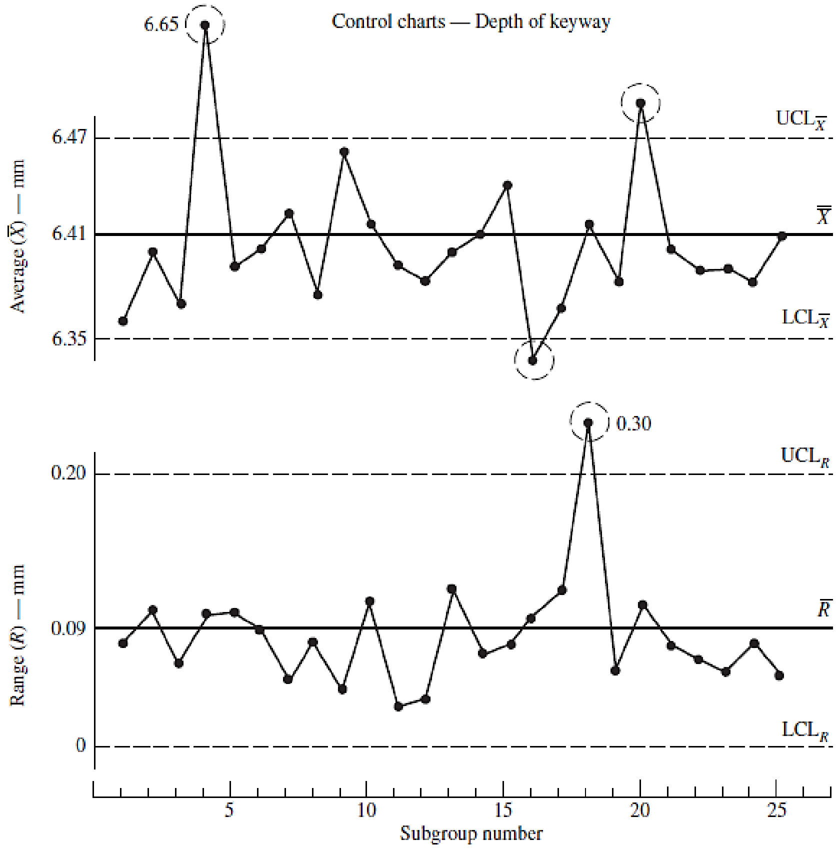


Revised Central Lines and Control Limits

Before proceeding to the action step, some final comments are appropriate. First, many analysts eliminate this step in the procedure because it appears to be somewhat redundant. However, by discarding out-of control points with assignable causes, the central line and control limits are more representative of the process. If this step is too complicated for operating personnel, its elimination would not affect the next step.

Second, the central line X_0 for the X chart is frequently based on the specifications. In such a case, the procedure is used only to obtain R_0. If, in our example problem, the nominal value of the characteristic is 6.38 mm, then $\overline{X}_0$ is set to that value and the upper and lower control limits are

$$\begin{aligned} UCL_{\overline{X}} &= \overline{X}_0 + A_2 R_0 \\ &= 6.38 + (0.729)(0.079) \\ &= 6.44 \text{ mm} \end{aligned} \qquad \begin{aligned} LCL_{\overline{X}} &= \overline{X}_0 - A_2 R_0 \\ &= 6.38 - (0.729)(0.079) \\ &= 6.32 \text{ mm} \end{aligned}$$



X-bar and R Charts for Preliminary Data with Trial Control Limits

Revised Central Lines and Control Limits

- The central line and control limits for the *R* chart do not change. This modification can be taken only if the
- process is adjustable. If the process is not adjustable, then the original calculations must be used.

Control Charts

Several control charts may be designed for different situations. Each chart has its own field of applications and its own advantages and disadvantages. However, all control charts have some characteristics in common and are interpreted in almost the same manner. Generally, quality characteristics are of two types (variables and attributes). So, control charts are broadly classified into two categories:

1. Control charts for variables (Control charts for measurable characteristics)
2. Control charts for attributes (Control charts for non-measurable characteristics)

Control Charts

We have learnt about control charts for variables, which are used to control the measurable quality characteristics. You have also learnt how to construct control charts for mean (X-chart) and control chart for variability (R-chart). However, there are many situations in which measurement is not possible, e.g., the number of failures in a production run, number of defects in a bolt of cloth, surface roughness of a cricket ball, etc. In such cases, we cannot use the control chart for variables.

Control Charts

There are different types of control charts for attributes for different situations. We classify the control charts for attributes into two groups as follows:

1. Control charts for defectives, and
2. Control charts for defects.

Control charts for defectives are mainly of two types as given below:

1. Control chart for fraction defective (p-chart), and
2. Control chart for number of defectives (np-chart).

Both control charts for defectives are based on the binomial distribution. Control charts for defects are also of two types as given below:

1. Control chart for number of defects (c-chart), and
2. Control chart for number of defects per unit (u-chart).

CONTROL CHARTS FOR FRACTION DEFECTIVE (p-CHART)

- The most widely used control chart for attributes is the fraction (proportion) defective chart, that is, the p-chart. The p-chart may be applied to quality characteristics, which cannot be measured or impracticable and uneconomical to measure it.
- These items/units are classified as defective or non-defective on the basis of certain criteria (defects). You have learnt that the control charts for variables are used to control only one quality characteristic at a time.
- If the quality controller would like to control two characteristics, he/she must use two control charts for variables. But a single p-chart may be applied to more than one quality characteristic, in fact, to as many quality characteristics as we want.
- Before describing the control chart for fraction defective (p-chart), we explain the meaning of fraction defective.

CONTROL CHARTS FOR FRACTION DEFECTIVE (p-CHART)

Fraction defective (p) is defined as the ratio of the number of defective terms/units/articles found in any inspection to the total number of items/units/articles inspected. Symbolically, we write

$$\text{Fraction defective } (\mathrm{p}) = \frac{\text{Number of defective items}}{\text{Total number of items inspected}} \quad \ldots (1)$$

For example, if 500 cricket balls are inspected and 20 balls are found defective, the fraction defective of the balls is given as:

$$Fraction\ defective\ (p) = \frac{No.of\ defective\ balls}{Total\ No.of\ balls\ inspected}$$

Fraction defective is always less than or equal to 1 and expressed as a decimal or fraction.

CONTROL CHARTS FOR FRACTION DEFECTIVE (p-CHART)

$$\text{Centre line} = E(p)$$

$$\text{Upper control limit (UCL)} = E(p) + 3SE(p)$$

$$\text{Lower control limit (LCL)} = E(p) - 3SE(p)$$

$$CL = E(p) = P$$

$$UCL = E(p) + 3SE(p) \quad = P + 3\sqrt{\frac{P(1-P)}{n}}$$

$$LCL = E(p) - 3SE(p) \quad = P - 3\sqrt{\frac{P(1-P)}{n}}$$

Reference:

- https://www.cimt.org.uk/projects/mepres/alevel/fstats_ch8.pdf
- https://web.mit.edu/2.810/www/files/readings/ControlChartConstantsAndFormulae.pdf
- Douglus C. and Montgoemery, Statistical quality control a modern introduction by Wiley.