

BT6270 Assignment 1: Hodgkin-Huxley Model

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I. THE HH MODEL

The Hodgkin-Huxley model describes how the membrane potential of a neuron evolves due to ionic currents through voltage-gated channels. Based on a circuit analogy, it accounts for membrane capacitance in parallel with sodium (Na^+), potassium (K^+), and leak channels.

$$C_m \frac{dV}{dt} = I_{\text{app}} - I_{\text{Na}} - I_{\text{K}} - I_{\text{L}}, \quad (1)$$

where C_m is the membrane capacitance, I_{app} is the externally applied current, and I_{Na} , I_{K} , I_{L} are sodium, potassium, and leak currents, respectively, given by:

$$I_{\text{Na}} = \bar{g}_{\text{Na}} m^3 h (V - E_{\text{Na}}), \quad (2)$$

$$I_{\text{K}} = \bar{g}_{\text{K}} n^4 (V - E_{\text{K}}), \quad (3)$$

$$I_{\text{L}} = \bar{g}_{\text{L}} (V - E_{\text{L}}), \quad (4)$$

where \bar{g}_{Na} , \bar{g}_{K} , \bar{g}_{L} are maximal conductances. m , h , n are gating variables (probabilities of channel subunits being open/closed), each of which follow:

$$\frac{dx}{dt} = \alpha_x(V)(1-x) - \beta_x(V)x, \quad x \in \{m, h, n\}, \quad (5)$$

with voltage-dependent transition rates $\alpha_x(V)$, $\beta_x(V)$:

$$\alpha_m = \frac{0.1(V+40)}{1 - e^{-(V+40)/10}}, \quad \beta_m = 4e^{-0.0556(V+65)}, \quad (6)$$

$$\alpha_h = 0.07e^{-0.05(V+65)}, \quad \beta_h = \frac{1}{1 + e^{-0.1(V+35)}}, \quad (7)$$

$$\alpha_n = \frac{0.01(V+55)}{1 - e^{-(V+55)/10}}, \quad \beta_n = 0.125e^{-(V+65)/80}, \quad (8)$$

(when V is expressed in mV)

II. SIMULATION

All parameters were initialized to the default values provided in the assignment's base code. The simulation was implemented in Python using the Hodgkin-Huxley equations with an Euler integration scheme (time step $\Delta t = 0.01$ ms, total duration $T = 500$ ms). The firing rate versus applied current (F vs. I_{app}) curve was obtained by sweeping I_{app} in steps of $0.005 \mu\text{A}/\text{mm}^2$. The threshold values I_1 , I_2 , and I_3 were then determined to four significant figures from the corresponding V - n phase plane diagrams. A spike was defined as an event where the membrane potential V crossed $V_{th} = 10$ mV from below.

The code used to generate the results and figures in this report is provided in the accompanying Jupyter notebook.

III. RESULTS

The simulation results are organized according to the dynamical regimes of the Hodgkin-Huxley model under varying applied current.

A. Dynamical Regimes

Regime	Behavior	I_{app} Range
R1	No APs (resting state)	$I < I_1$
R2	Finite number of APs	$I_1 \leq I < I_2$
R3	Continuous periodic spiking	$I_2 \leq I < I_3$
R4	Distorted APs	$I_3 \leq I$

TABLE I. Dynamical regimes of the HH model for different applied currents which show different behavior.

From numerical simulations, the estimated threshold values are: $I_1 \approx 0.0224 \mu\text{A}/\text{mm}^2$, $I_2 \approx 0.0621 \mu\text{A}/\text{mm}^2$, and $I_3 \approx 0.4630 \mu\text{A}/\text{mm}^2$. These values are also justified later in the report by looking at V - n phase-plane diagrams.

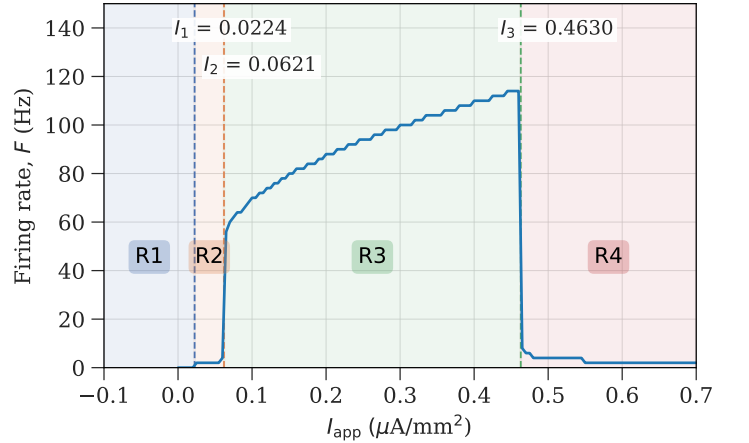


FIG. 1. Firing rate as a function of applied current.

In R4, the system initially crosses V_{th} for a few cycles, which are counted as action potentials, before settling into a limit cycle that remains entirely below V_{th} .

B. Voltage, Conductance, and Gating Variable Dynamics in Different Regimes

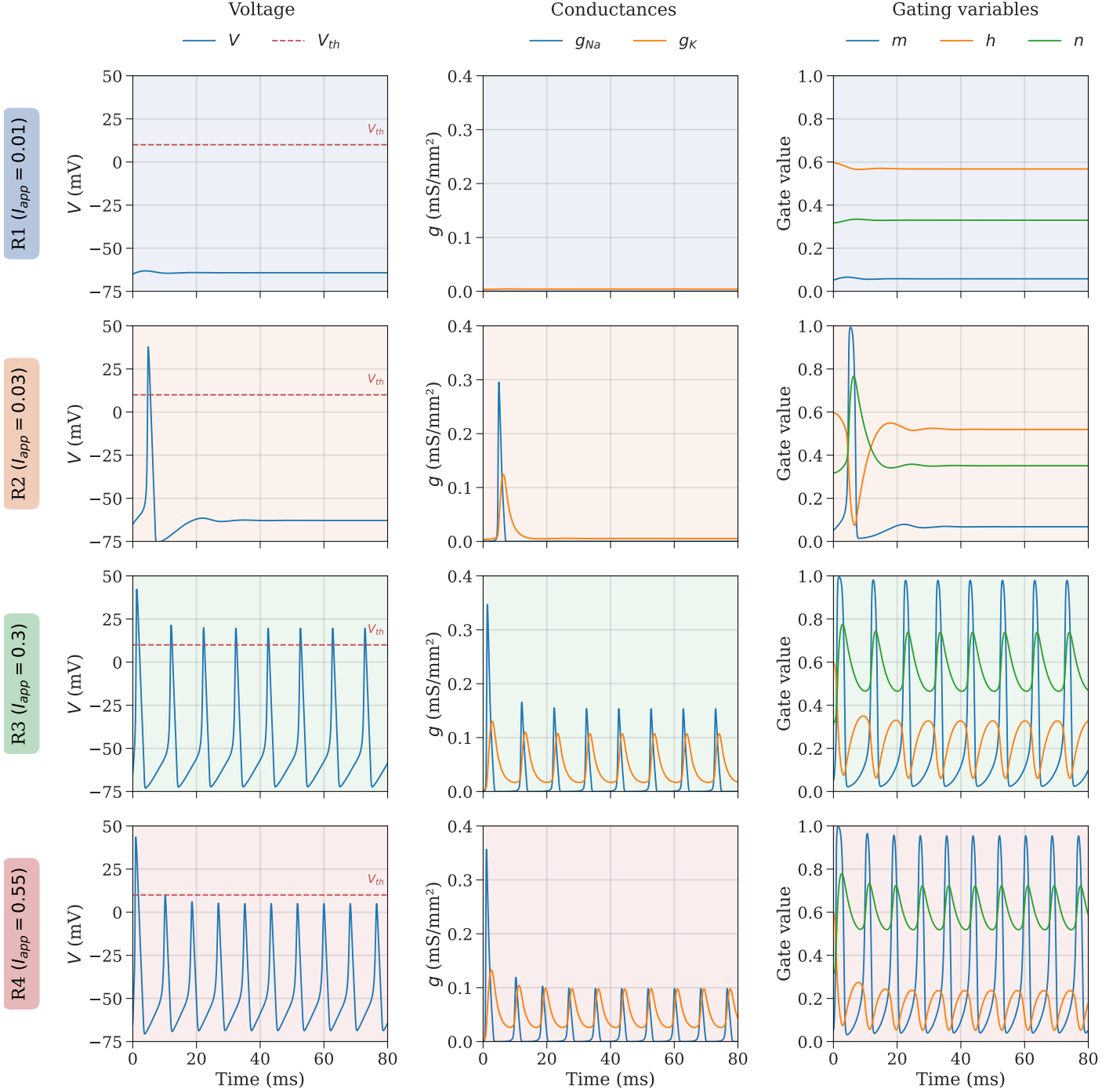


FIG. 2. Representative voltage, conductance, and gating variable dynamics of a Hodgkin-Huxley neuron across four dynamical regimes (R1-R4) under increasing applied current I_{app} . Each row corresponds to one regime. Columns show membrane potential V , sodium/potassium conductances (g_{Na} , g_K), and gating variables (m , h , n). The horizontal dashed line in the voltage plots denotes the spike threshold $V_{th} = 10$ mV. Regimes transition from no spiking (R1) to finite spiking (R2), continuous periodic spiking (R3), and limited initial spikes (R4).

C. Phase Plane Plots

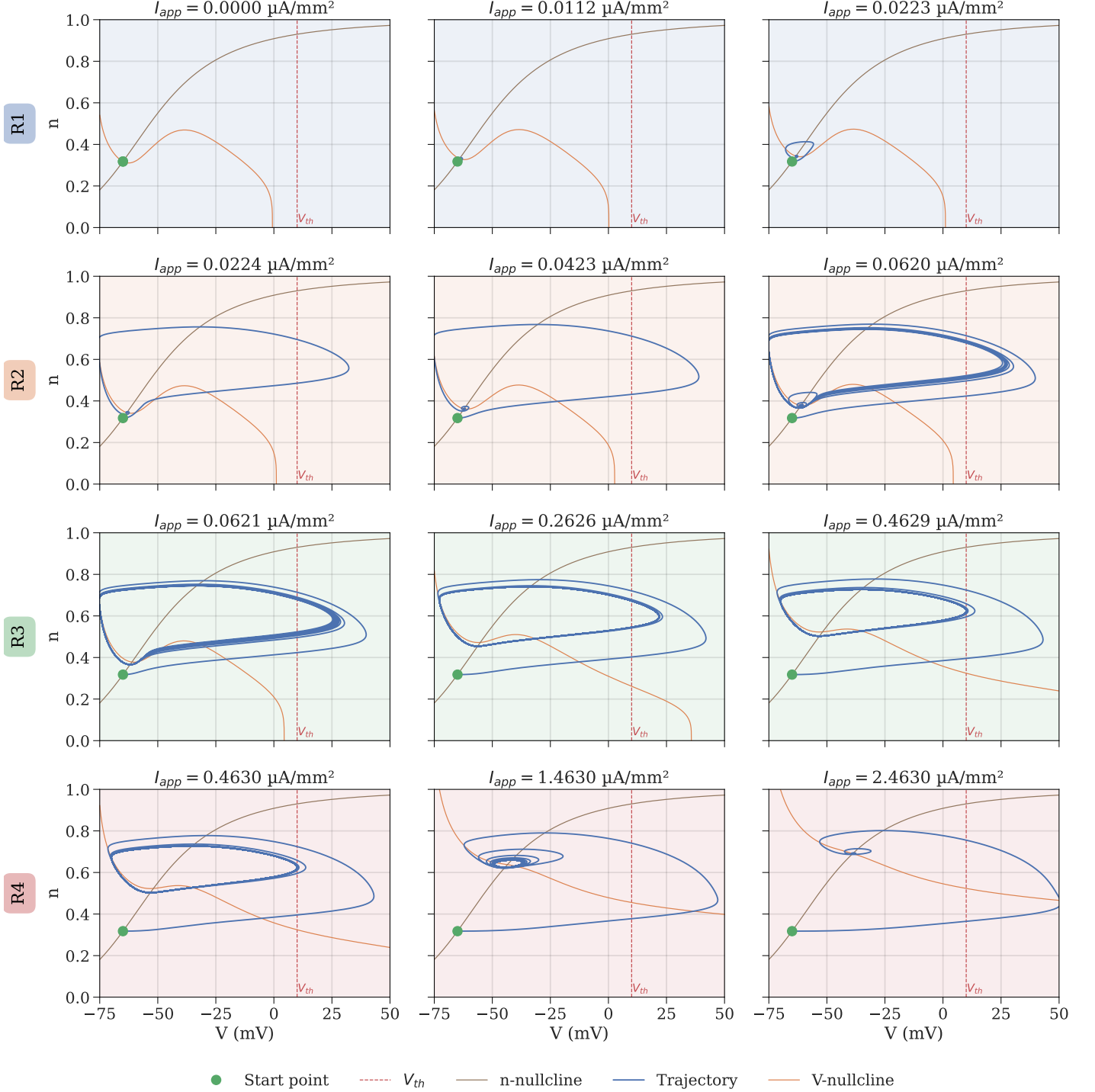


FIG. 3. Phase-plane analysis of the Hodgkin-Huxley neuron. Each panel shows the membrane potential V versus the potassium gating variable n . The **trajectory** begins at the **start point** and evolves under the applied current indicated in each plot. The **V-nullcline** and **n-nullcline** are overlaid to highlight fixed points. The threshold voltage (V_{th}) is marked for reference. Within each row, the first panel corresponds to the lower current threshold of the regime, and the last panel to the upper current threshold. Comparing the last panel of a row with the first panel of the next row illustrates how the dynamics change between regimes.

In **Regime R1** ($I_{app} < I_1 = 0.0224 \mu\text{A}/\text{mm}^2$), the trajectory remains close to the initial state and no action

potentials (APs) are generated. Immediately after crossing I_1 , the trajectory crosses the threshold potential V_{th} , producing APs.

In **Regime R2** ($I_1 < I_{\text{app}} < I_2$), trajectories repeatedly cross V_{th} and generate APs, but eventually return to the stable fixed point. For **Regime R3** ($I_2 < I_{\text{app}} < I_3$), the trajectory converges to a stable limit cycle, resulting

in periodic APs. The maximum voltage of the limit cycle decreases as I_{app} increases.

Finally, in **Regime R4** ($I_{\text{app}} > I_3$), the limit cycle lies entirely below V_{th} , so no APs are observed. With further increase in I_{app} , the limit cycle gradually shrinks towards the fixed point.