Fuzzy Logic & Neural Networks (CS-514)

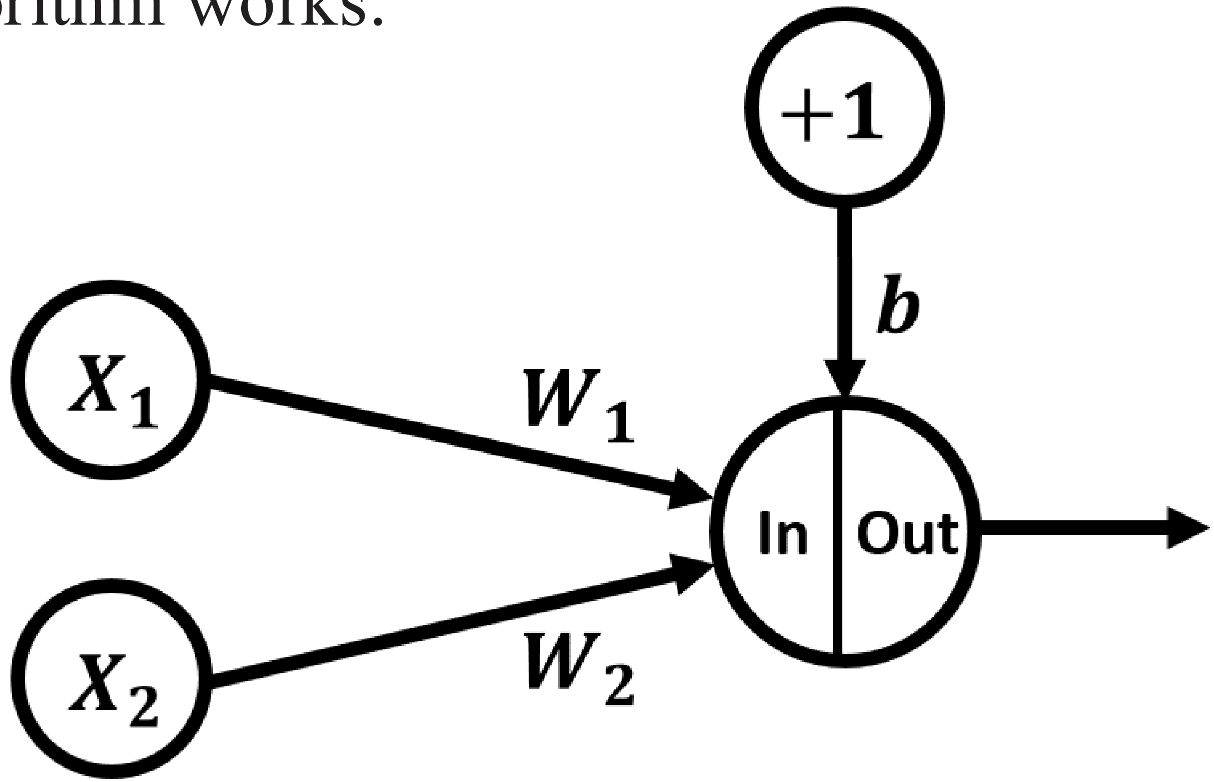
Dr. Sudeep Sharma

IIIT Surat

sudeep.sharma@iiitsurat.ac.in

Backpropagation

- Simple example to learn how the backpropagation algorithm works:

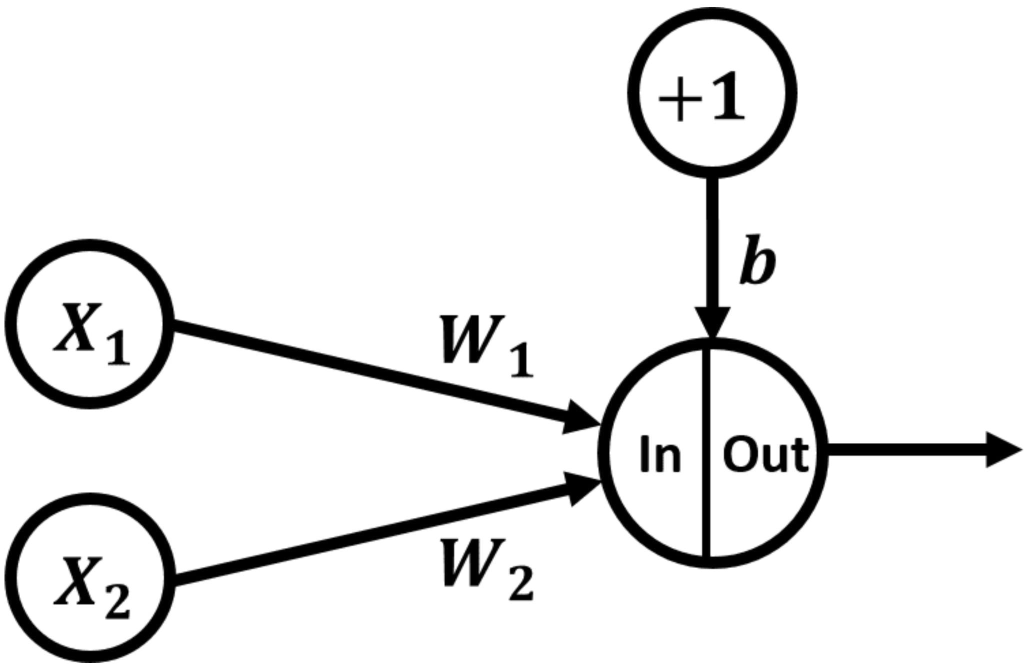


- 2 inputs (X_1 and X_2), Weights W_1 , W_2 and bias b

Source: Internet

Backpropagation

- Assume that output layer uses the sigmoid activation function:



$$\hat{y}_i = \frac{1}{(1 + e^{-s})}$$

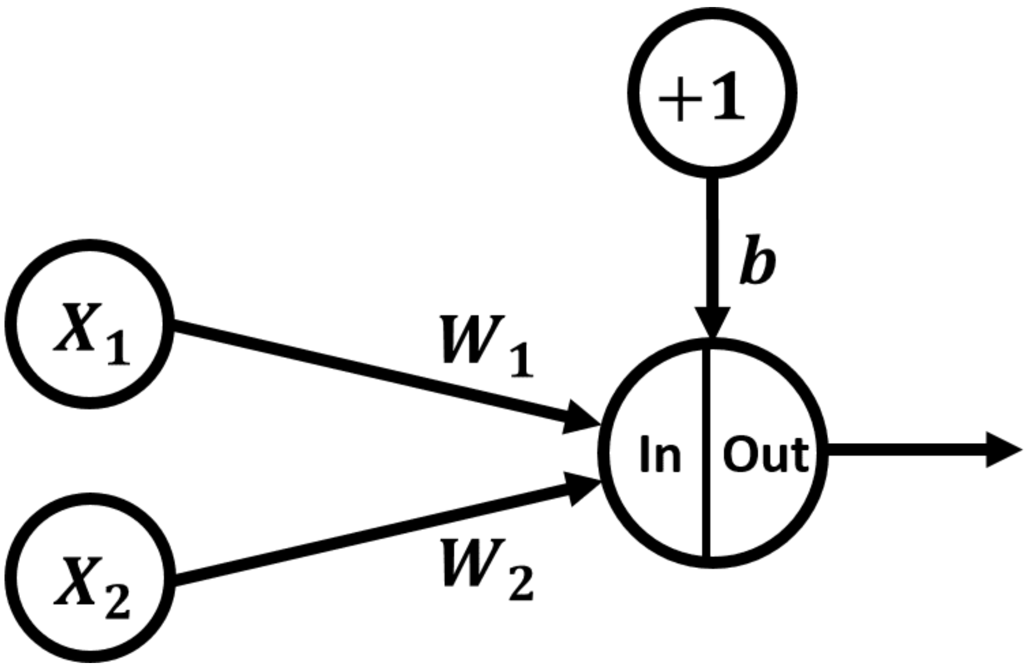
- where

$$s = X_1 * W_1 + X_2 * W_2 + b$$

Source: Internet

Backpropagation

- **Forward Pass:**



$$\hat{y}_i = \frac{1}{(1 + e^{-s})}$$

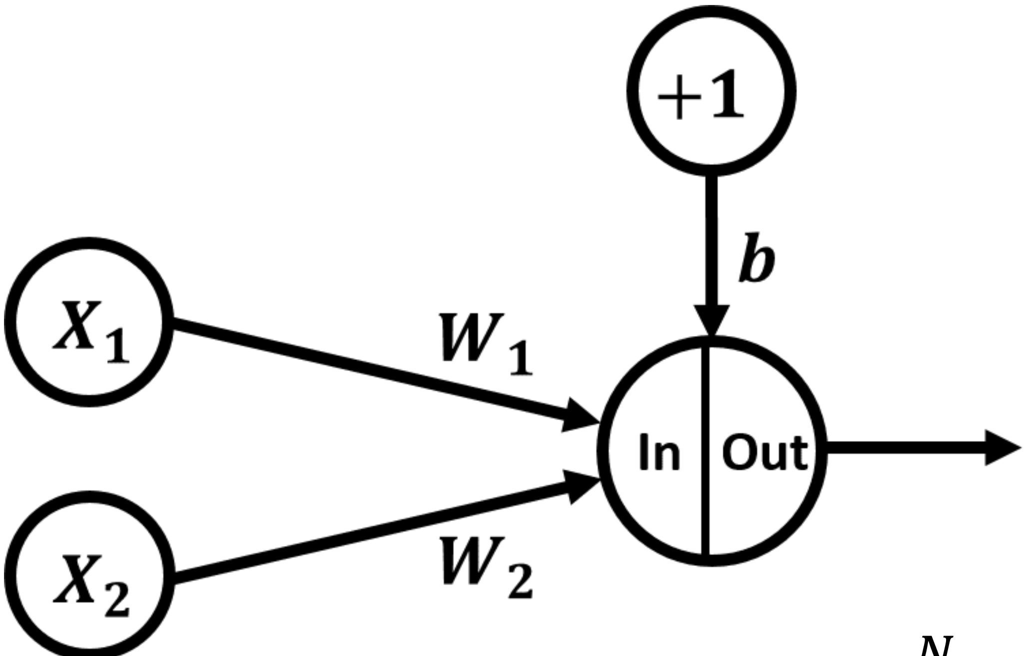
- where

$$s = X_1 * W_1 + X_2 * W_2 + b$$

Source: Internet

Backpropagation

➢ **Loss Calculations:**



s=X₁* W₁+ X₂*W₂+b

$$\hat{y}_i = \frac{1}{(1 + e^{-s})}$$

$$Loss = \frac{1}{2N}\sum_{i=1}^{N}(y_i - \hat{y}_i)^2$$

$$e_i = (y_i - \hat{y}_i)$$

Source: Internet

Backpropagation

➢ **Backward Pass**

Gradient Calculations:

$$Loss = \frac{1}{2N}\sum_{i=1}^{N}(y_i - \hat{y}_i)^2$$

$$\frac{\partial Loss}{\partial \hat{y}_i} = -\frac{(y_i - \hat{y}_i)}{N}$$

Backpropagation

- Gradient Calculations:

$$Loss = \frac{1}{2N}\sum_{i=1}^{N}(y_i - \hat{y}_i)^2$$

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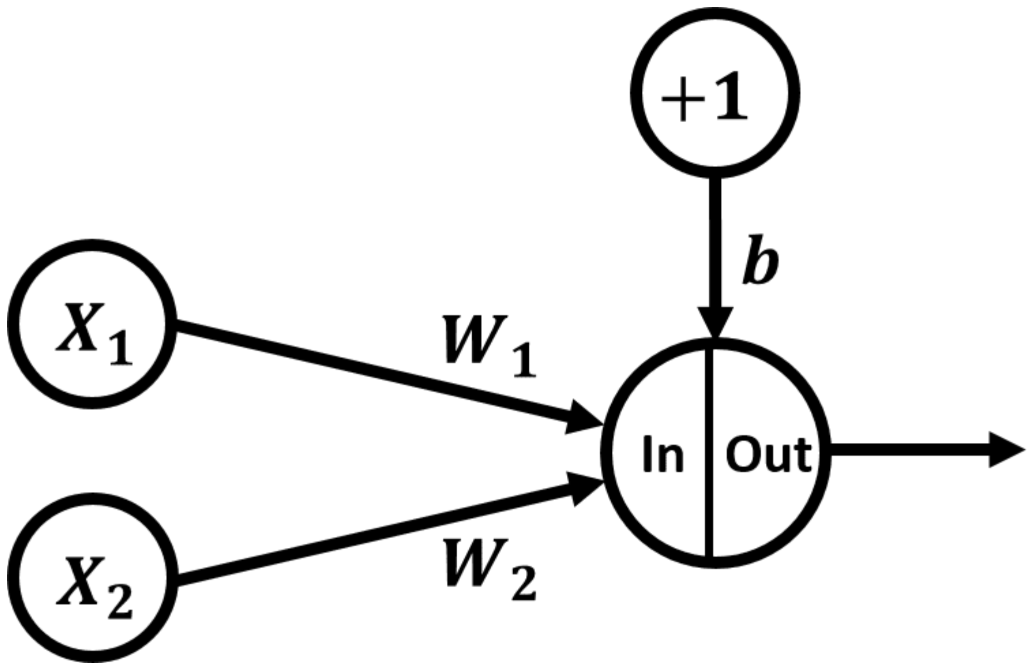

$$\frac{\partial \hat{y}_i}{\partial s} = \frac{\partial}{\partial s}\hat{y}_i = \frac{\partial}{\partial s}\frac{1}{(1+e^{-s})} = \hat{y}_i(1-\hat{y}_i)$$

Source: Internet

Backpropagation

➢ Gradient Calculations:

$$Loss = \frac{1}{2N}\sum_{i=1}^{N}(y_i - \hat{y}_i)^2$$

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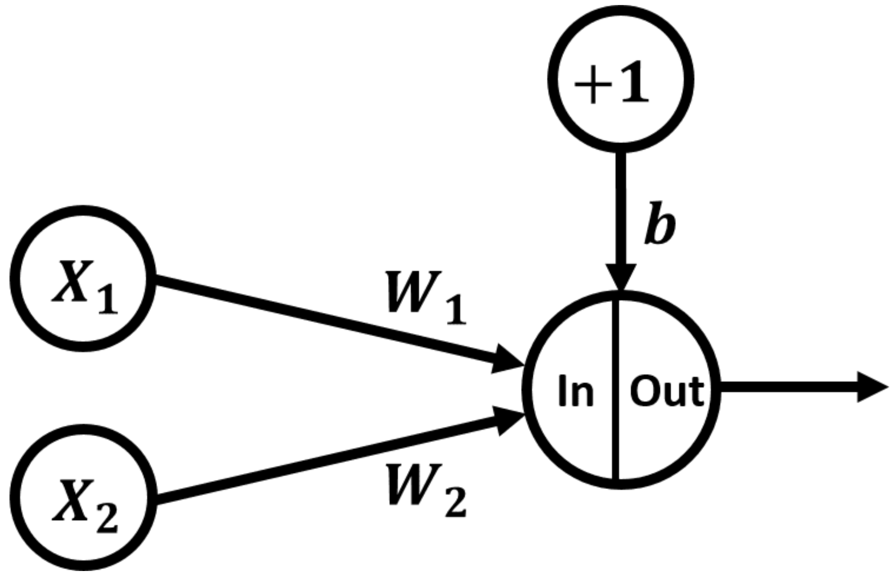
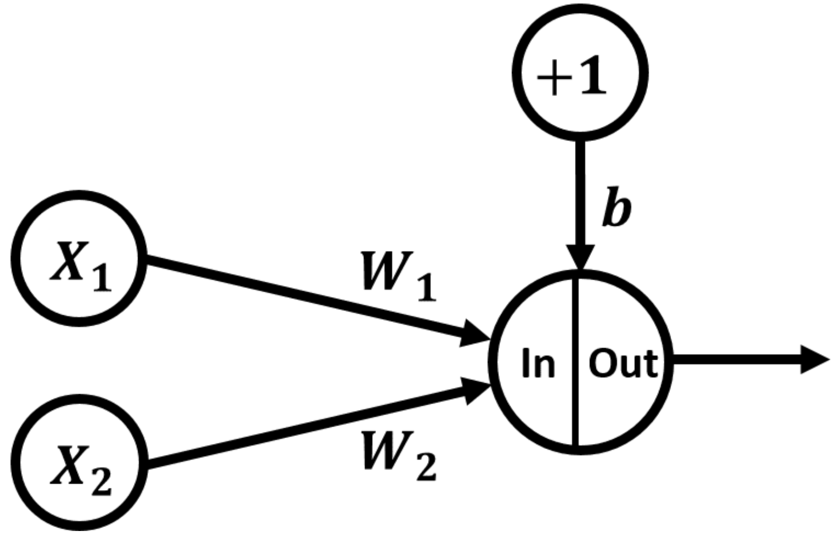


$$\frac{\partial \hat{y}_i}{\partial s} = \frac{\partial}{\partial s}\hat{y}_i = \frac{\partial}{\partial s}\frac{1}{(1+e^{-s})} = \hat{y}_i(1-\hat{y}_i)$$

$$\frac{\partial s}{\partial w_1} = \frac{\partial}{\partial w_1}(X_1 w_1 + X_2 w_2 + b) = X_1$$

Source: Internet

Backpropagation



- Gradient Calculations:

$$\frac{\partial Loss}{\partial \hat{y}_i} = -\frac{(y_i - \hat{y}_i)}{N}$$

$$\frac{\partial \hat{y}_i}{\partial s} = \frac{\partial}{\partial s}\hat{y}_i = \frac{\partial}{\partial s}\frac{1}{(1+e^{-s})} = \hat{y}_i(1-\hat{y}_i)$$

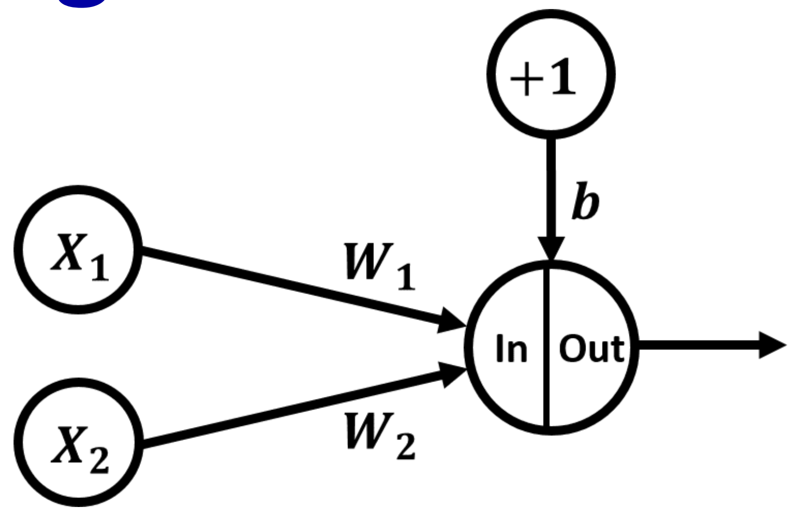
$$\frac{\partial s}{\partial w_1} = \frac{\partial}{\partial w_1}(X_1 w_1 + X_2 w_2 + b) = X_1$$

$$\frac{\partial Loss}{\partial w_1} = \frac{\partial Loss}{\partial \hat{y}_i}\frac{\partial \hat{y}_i}{\partial s}\frac{\partial s}{\partial w_1} = -\frac{(y_i - \hat{y}_i)}{N}\hat{y}_i(1-\hat{y}_i)X_1$$

Source: Internet

Backpropagation

- Gradient Calculations:



$$\frac{\partial Loss}{\partial \hat{y}_i} = -\frac{(y_i - \hat{y}_i)}{N}$$

$$\frac{\partial \hat{y}_i}{\partial s} = \frac{\partial}{\partial s}\hat{y}_i = \frac{\partial}{\partial s}\frac{1}{(1+e^{-s})} = \hat{y}_i(1-\hat{y}_i)$$

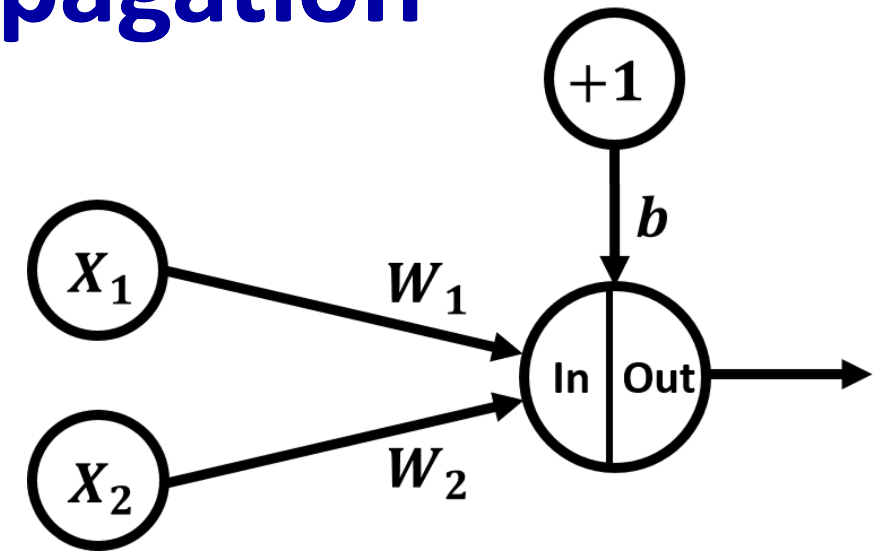
$$\frac{\partial s}{\partial w_2} = \frac{\partial}{\partial w_2}(X_1 w_1 + X_2 w_2 + b) = X_2$$

$$\frac{\partial Loss}{\partial w_2} = \frac{\partial Loss}{\partial \hat{y}_i}\frac{\partial \hat{y}_i}{\partial s}\frac{\partial s}{\partial w_2} = -\frac{(y_i - \hat{y}_i)}{N}\hat{y}_i(1-\hat{y}_i)X_2$$

Source: Internet

Backpropagation

- Gradient Calculations:



$$\frac{\partial Loss}{\partial \hat{y}_i} = -\frac{(y_i - \hat{y}_i)}{N}$$

$$\frac{\partial \hat{y}_i}{\partial s} = \frac{\partial}{\partial s}\hat{y}_i = \frac{\partial}{\partial s}\frac{1}{(1+e^{-s})} = \hat{y}_i(1-\hat{y}_i)$$

$$\frac{\partial s}{\partial b} = \frac{\partial}{\partial b}(X_1 w_1 + X_2 w_2 + b) = 1$$

$$\frac{\partial Loss}{\partial b} = \frac{\partial Loss}{\partial \hat{y}_i}\frac{\partial \hat{y}_i}{\partial s}\frac{\partial s}{\partial b} = -\frac{(y_i - \hat{y}_i)}{N}\hat{y}_i(1-\hat{y}_i)$$

Source: Internet

Backpropagation

- Weights Update Rule:

$$w(new) = w(old) \ - \eta \frac{\partial Loss}{\partial w}$$

η is the learning rate

$$w_1(new) = w_1(old) \ - \eta \frac{\partial Loss}{\partial w_1}$$

$$w_2(new) = w_2(old) \ - \eta \frac{\partial Loss}{\partial w_2}$$

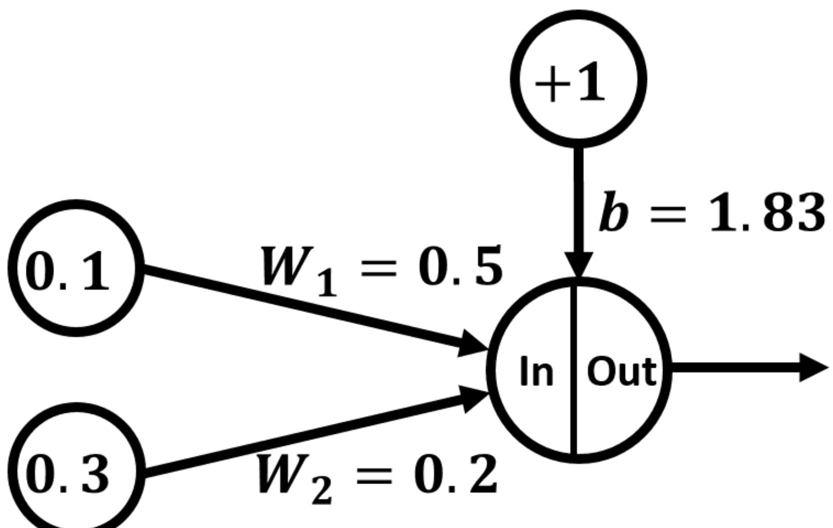
$$b(new) = b(old) \ - \eta \frac{\partial Loss}{\partial b}$$

Backpropagation

- To make things simple, a single training sample is used in this example:

X1	X2	Desired Output
0.1	0.3	0.1

W1	W2	b
0.5	0.2	1.83

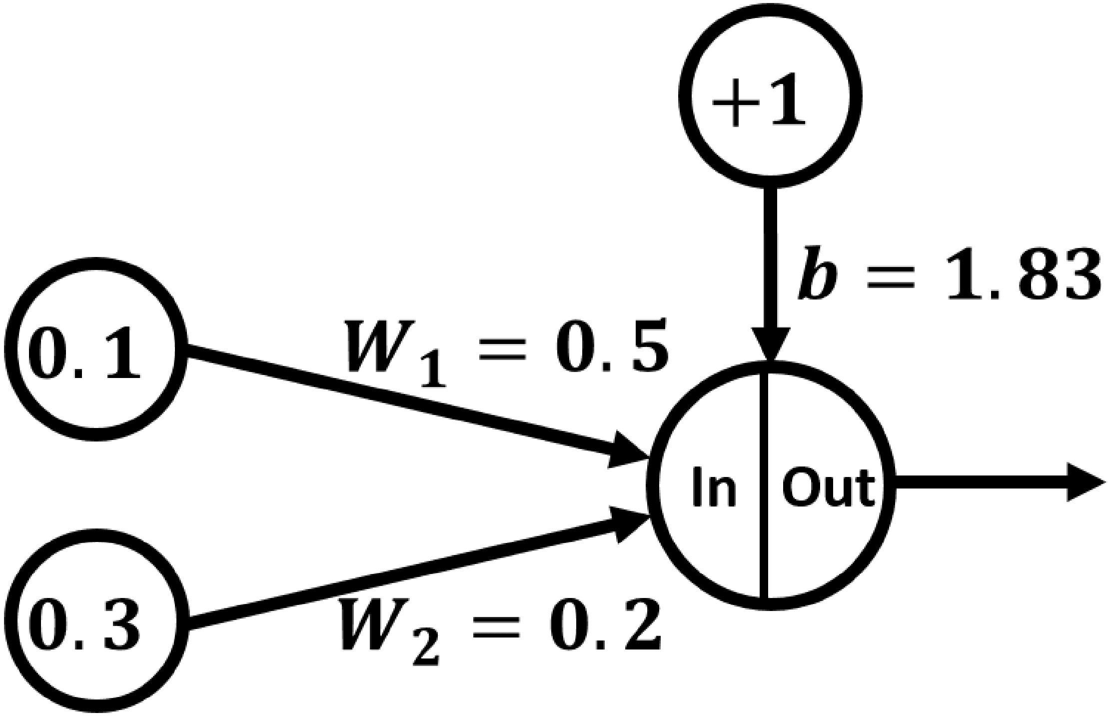


Source: Internet

Backpropagation

Iteration 1

- **Forward Pass** using the given values:



$$\hat{y}_i = \frac{1}{(1 + e^{-s})}$$

$$\hat{y}_i = \frac{1}{(1 + e^{-1.94})}$$

$$\hat{y}_i = 0.874$$

- where

$s = X_1 * W_1 + X_2 * W_2 + b$

$s = 0.1 * 0.5 + 0.3 * 0.2 + 1.83$

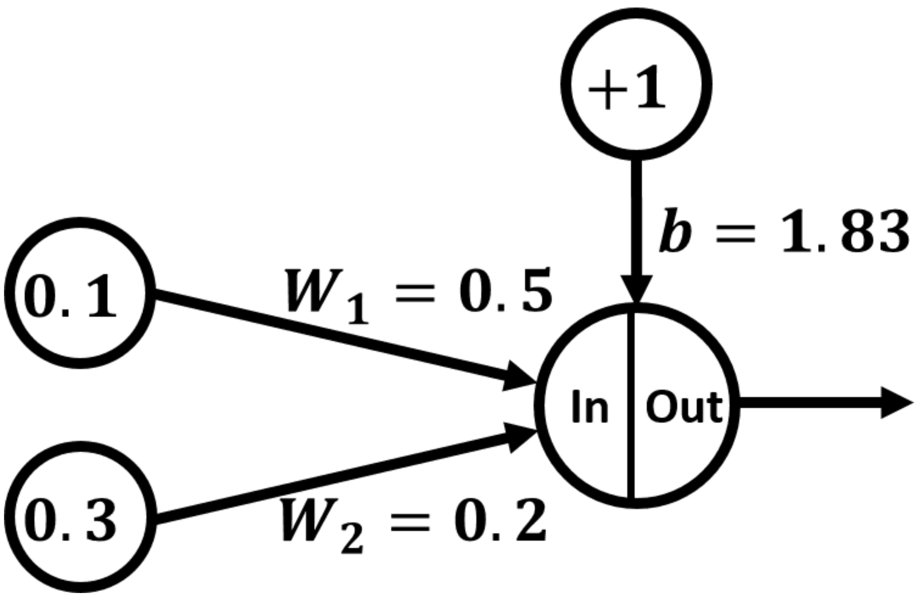
$s = 1.94$

Source: Internet

Backpropagation

Iteration 1

- **Loss Calculations:**



s=X₁* W₁+ X₂*W₂+b

$$s = X_1 * W_1 + X_2 * W_2 + b$$

$$\hat{y}_i = \frac{1}{(1 + e^{-s})}$$

$$\hat{y}_i = 0.874$$

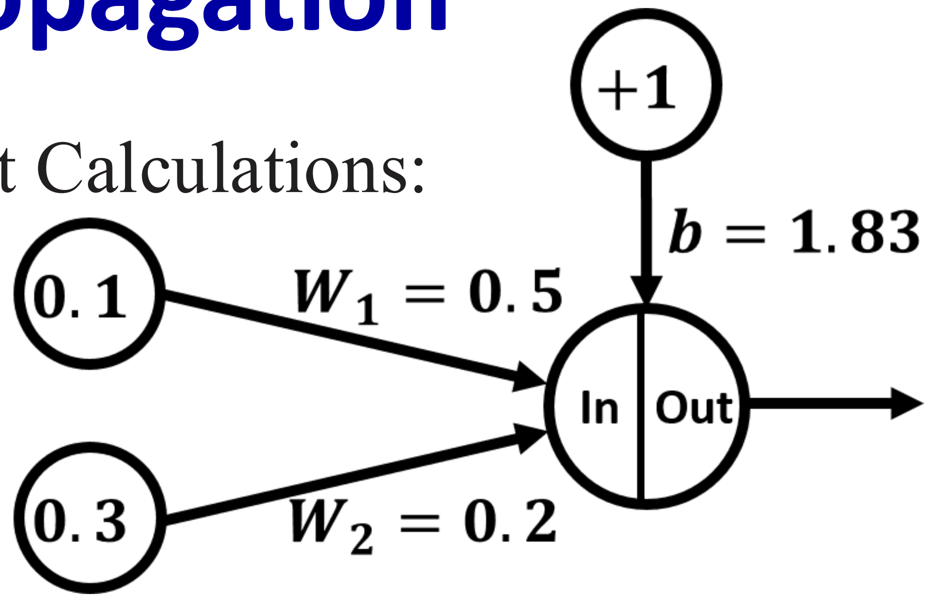
$$Loss = \frac{1}{2N}\sum_{i=1}^{N}(y_i - \hat{y}_i)^2 = 0.299$$

Source: Internet

Backpropagation

Iteration 1

- Backward Pass Gradient Calculations:



$$\frac{\partial Loss}{\partial \hat{y}_i} = -\frac{(y_i - \hat{y}_i)}{N} = 0.774$$

$$\frac{\partial \hat{y}_i}{\partial s} = \frac{\partial}{\partial s}\hat{y}_i = \frac{\partial}{\partial s}\frac{1}{(1+e^{-s})} = \hat{y}_i(1-\hat{y}_i) = 0.11$$

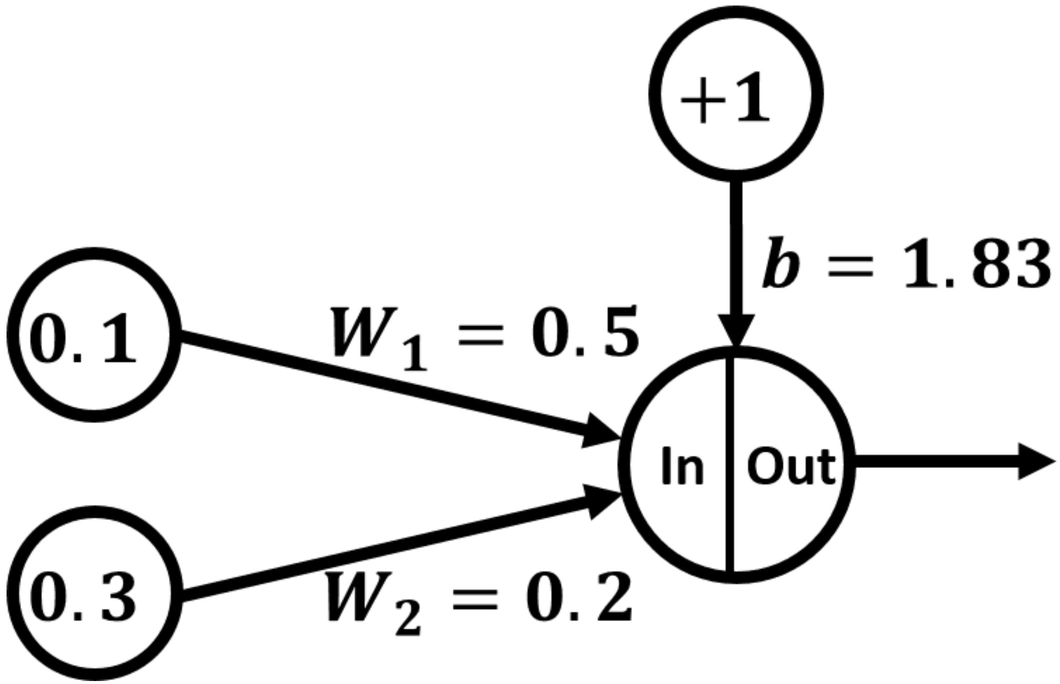
$$\frac{\partial s}{\partial w_1} = \frac{\partial}{\partial w_1}(X_1 w_1 + X_2 w_2 + b) = X_1 = 0.1$$

Source: Internet

Backpropagation

Iteration 1

- Gradient Calculations:



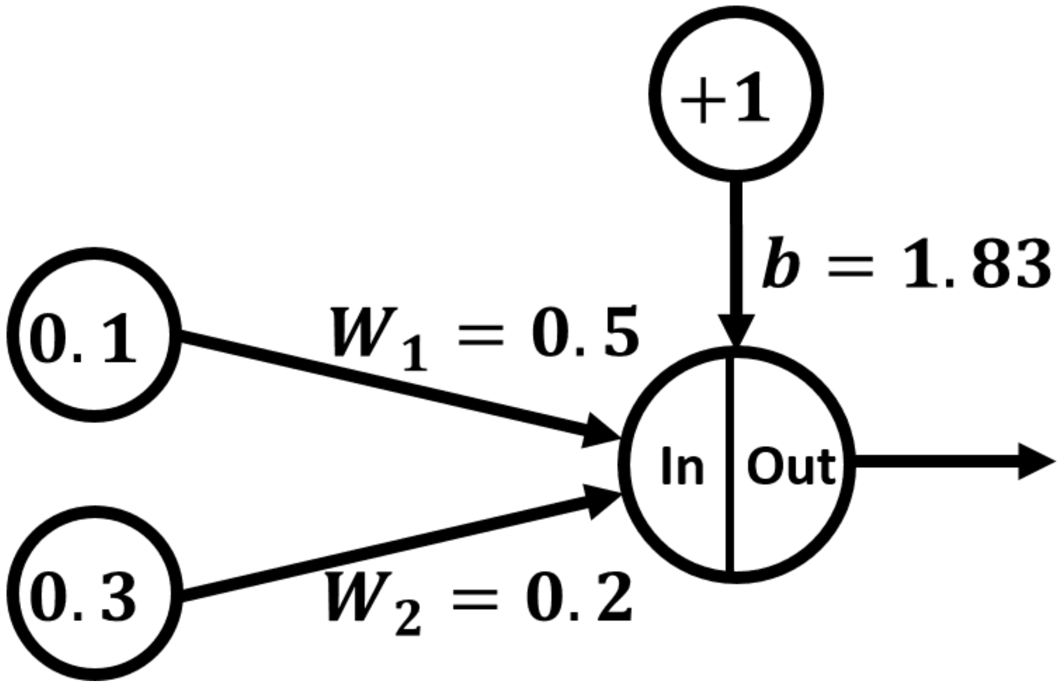
$$\frac{\partial Loss}{\partial w_1} = \frac{\partial Loss}{\partial \hat{y}_i}\frac{\partial \hat{y}_i}{\partial s}\frac{\partial s}{\partial w_1} = -\frac{(y_i - \hat{y}_i)}{N}\hat{y}_i(1-\hat{y}_i)X_1 = 0.0085$$

Source: Internet

Backpropagation

Iteration 1

- Gradient Calculations:



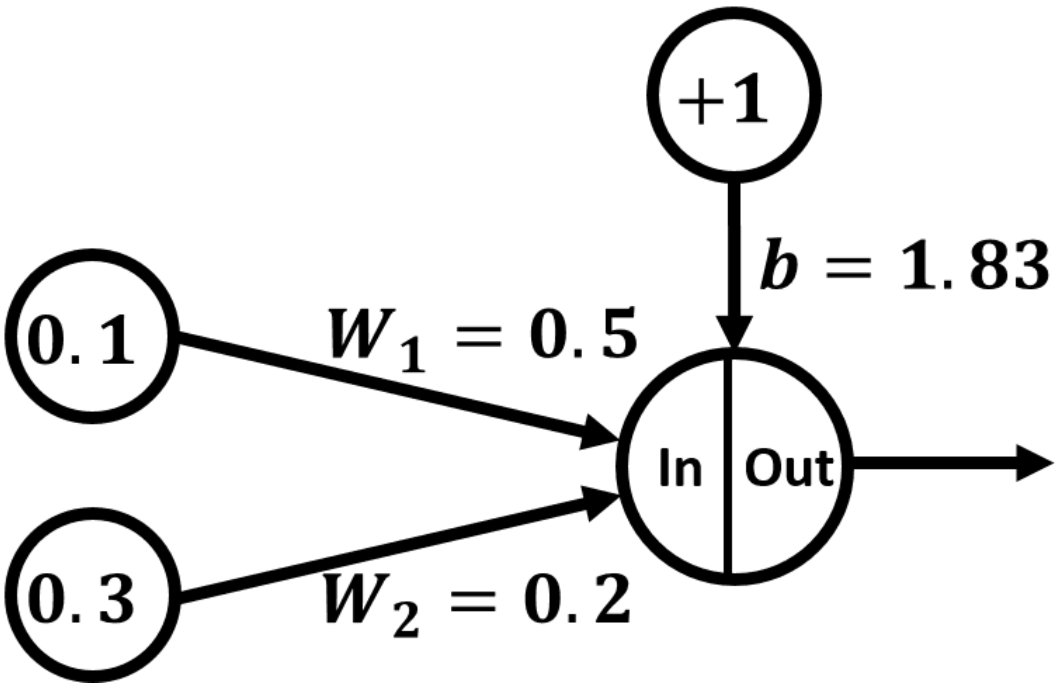
$$\frac{\partial Loss}{\partial w_2} = \frac{\partial Loss}{\partial \hat{y}_i}\frac{\partial \hat{y}_i}{\partial s}\frac{\partial s}{\partial w_2} = -\frac{(y_i - \hat{y}_i)}{N}\hat{y}_i(1-\hat{y}_i)X_2 = 0.0255$$

Source: Internet

Backpropagation

Iteration 1

- Gradient Calculations:



$$\frac{\partial Loss}{\partial b} = \frac{\partial Loss}{\partial \hat{y}_i}\frac{\partial \hat{y}_i}{\partial s}\frac{\partial s}{\partial b} = -\frac{(y_i - \hat{y}_i)}{N}\,\hat{y}_i(1-\hat{y}_i) = 0.085$$

Source: Internet

Backpropagation

Iteration 1

- Weights Update: Take learning rate as 0.5

$$w_1(new) = w_1(old) - \eta \frac{\partial Loss}{\partial w_1} = 0.5 - 0.5(0.0085) = 0.49575$$

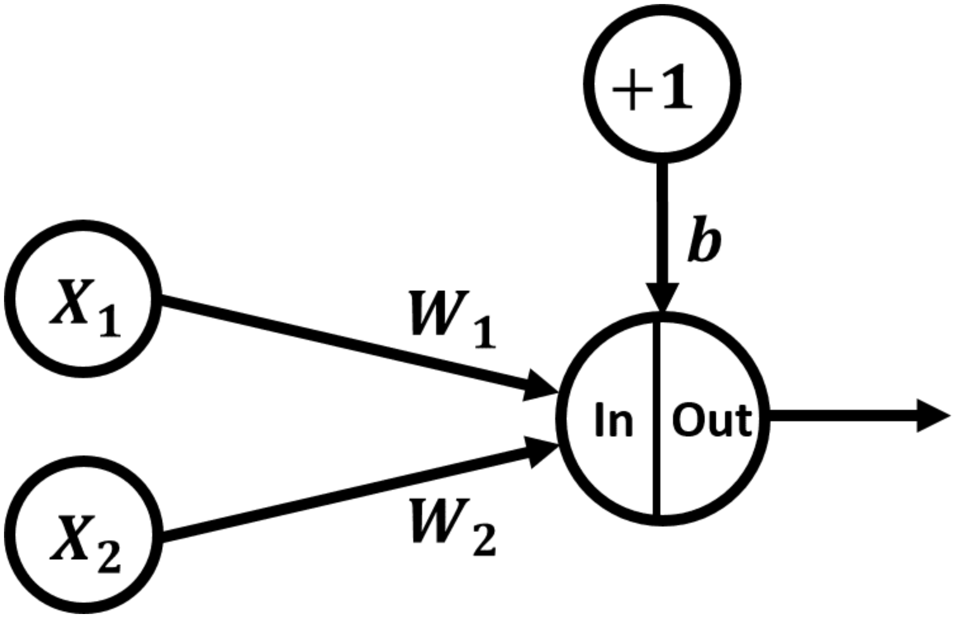
$$w_2(new) = w_2(old) - \eta \frac{\partial Loss}{\partial w_2} = 0.2 - 0.5(0.0255) = 0.18725$$

$$b(new) = b(old) - \eta \frac{\partial Loss}{\partial b} = 1.83 - 0.5(0.085) = 1.7875$$

Backpropagation

Iteration 2

- **Forward Pass** using the given values:



$$\hat{y}_i = \frac{1}{(1+e^{-s})}$$

$$\hat{y}_i = \frac{1}{(1+e^{-1.894})}$$

$$\hat{y}_i = 0.869$$

- where

$$s = X_1 * W_1 + X_2 * W_2 + b$$

$$s = 0.1 * 0.496 + 0.3 * 0.187 + 1.788$$

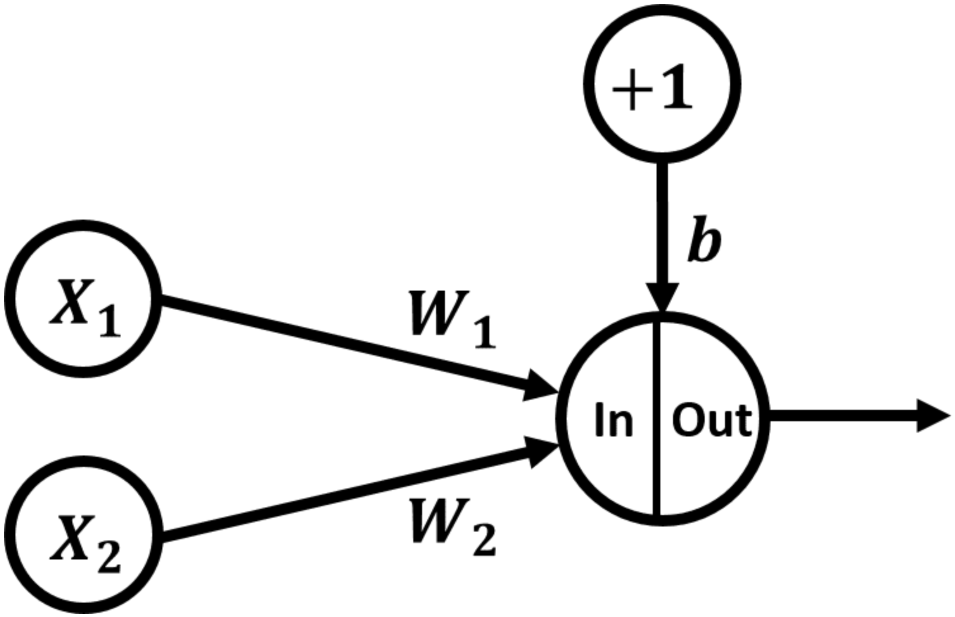
$$s = 1.894$$

Source: Internet

Backpropagation

Iteration 2

- **Loss Calculations:**



s=X₁* W₁+ X₂*W₂+b

$$\hat{y}_i = \frac{1}{(1+e^{-s})}$$

$$\hat{y}_i = 0.869$$

$$Loss = \frac{1}{2N}\sum_{i=1}^{N}(y_i - \hat{y}_i)^2 = 0.296$$

Source: Internet

Backpropagation

Iteration 2

- Backward Pass:

$$\frac{\partial Loss}{\partial w_1} = \frac{\partial Loss}{\partial \hat{y}_i}\frac{\partial \hat{y}_i}{\partial s}\frac{\partial s}{\partial w_1} = -\frac{(y_i - \hat{y}_i)}{N}\hat{y}_i(1 - \hat{y}_i)X_1 = 0.0088$$

$$\frac{\partial Loss}{\partial w_2} = \frac{\partial Loss}{\partial \hat{y}_i}\frac{\partial \hat{y}_i}{\partial s}\frac{\partial s}{\partial w_2} = -\frac{(y_i - \hat{y}_i)}{N}\hat{y}_i(1 - \hat{y}_i)X_2 = 0.026$$

$$\frac{\partial Loss}{\partial b} = \frac{\partial Loss}{\partial \hat{y}_i}\frac{\partial \hat{y}_i}{\partial s}\frac{\partial s}{\partial b} = -\frac{(y_i - \hat{y}_i)}{N}\hat{y}_i(1 - \hat{y}_i) = 0.088$$

Backpropagation

Iteration 2

- Weights Update: Take learning rate as 0.5

$$w_1(new) = w_1(old) - \eta \frac{\partial Loss}{\partial w_1} = 0.496 - 0.5(0.0088) = 0.492$$

$$w_2(new) = w_2(old) - \eta \frac{\partial Loss}{\partial w_2} = 0.187 - 0.5(0.026) = 0.174$$

$$b(new) = b(old) - \eta \frac{\partial Loss}{\partial b} = 1.79 - 0.5(0.088) = 1.746$$