Image (Layman): what we see and/or feel is an image

An image is a representation, likeness or imitation of an object or thing, a vivid or graphic discription of something introduced to represent something else

object → image
visible ^ non visible

Digital image:

An image is 2D function f(x,y) where x and y are spatial cordinate and the amplitude of f at any point of coordinates (x,y) is called the intensity or grey level of the image at that point

Digital image is composed of a finite number of elements which has a particular location and values referred as a picture elements image elements, pels or pixels

Fractal image

problem domain → image Acquisition → image enhancement → image Restroration → morphological processing → Segmentation → object Recognisation → Representation and discription

Image acquisition preprocessing

ex. An image as a resolution of 100×50 pixels and uses 8-bit grey scale
(a) No of pixels
5000
(b) highest value of pixels
= 256−1 = 255
(c) File size in bytes
= 5000 bytes

Ex a setalite captures 12 bit grey scale image 4096 x 2160 calculate uncompressed file size in mb if converted to 8-bit jpeg then what is the new file size

Sol^n

old size = 4096 x 2160 x 12 bit

$$= \frac{4096 \times 2160 \times 12}{8 \times 1024 / \cancel{1024} \times 1024} = \cancel{4852160} \text{ mB}$$

new size = 4096 x 2160 x 8

Ex A 0200 x 0200 RGB color image uses 8-bits per chanel calculate uncompressed file size for images

Sol' 0200 x 0200 x 3

Ratina: consists of receptors
- Cones: highly sensitive to colors. photopic
- rods: give overall picture with reduced detail scotopik or dim light vision

~~Cones are concentrated~~ ~~around~~

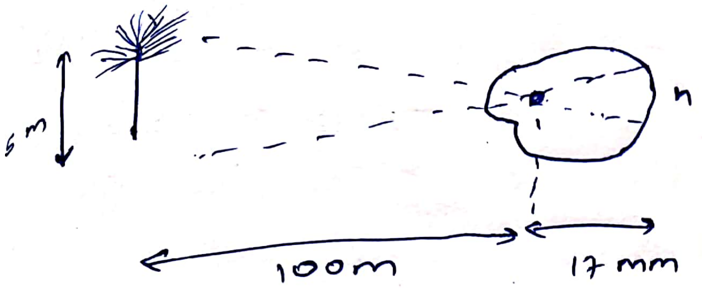
lens: foces light from object onto the retina

cones (6-7 millon) and rods (75 - 150 million)

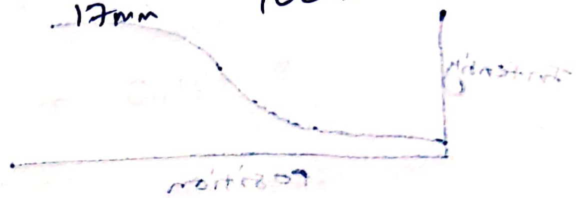
Cones are concentrated around the fovea and are very sensitive to color

Rods are more spread out and are sensitive to light

classical optical theory



$$\frac{h}{17mm} = \frac{15\,m}{100\,m}$$



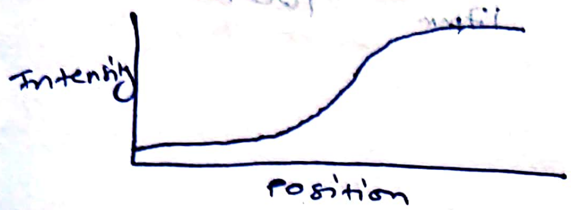
weber ratio

Light source having intensity I can be varied with the increment of illumination ΔI.

- If ΔI is not bright enough then no perceivable change
- As ΔI get stronger, there there is a perceived change
- weber ratio $= \Delta I_c / I$
- where ΔI_c is the increment of illumination discriminable by 50% of the time with background I
- A small value weber ratio means that a small change in intensity
- small weber ratio is good brightness discrimination

Match band effect

In area A, brightness preceived is darker while area B is brighter



simple image model

$$f(x,y) = i(x,y) \cdot r(x,y)$$

i → Incident light

r → reflected light

The amount of source light incident on the scene being viewed $i(x,y)$

the amount of light reflected by the object in the scene $r(x,y)$

$0 < i(x,y) < \infty$ $\quad$ $0 \leq r(x,y) < 1$

$0 < f(x,y) < \infty$

$l = f(x_0, y_0)$ $\quad$ $L_{min} \leq l \leq L_{max}$

$L_{min} = i_{min} \times r_{min}$ $\quad$ $L_{max} = i_{max} \times r_{max}$

$[L_{min}, L_{max}]$ is called grey scale

Common practice, the interval is $[0, L-1]$ where $l = 0$ is considered black and $l = L-1$ is considered white

① Binary image
② Grey scale image
③ RGB image
④ indexed image

① RGB to grey

Ⓐ Luminosity method

$$I = 0.299\,R + 0.587\,G + 0.114\,B$$

Ⓑ Average method

$$I = \frac{R+G+B}{3}$$

Ex-1. (100, 50, 200) → Pixel

Ⓐ luminosity method

$0.299 \times 100 + 0.587 \times 50 + 0.114 \times 200$

$= 29.9 + 5.87 \times 5 + 11.4 \times 2$

$= 82.05 = 82$

Ⓑ Average method

$$= \frac{100+50+200}{3} = 117$$

② Grey to RGB

① Direct mapping, $I = 120$

$(R, G, B) = (120, 120, 120)$

② False color mapping

$I > 150 \rightarrow$ Red

$50 < I \leq 150 \rightarrow$ green

$I \leq 50 \rightarrow$ Blue

Ex $I = 200 \Rightarrow RGB = (255, 0, 0)$

$I = 120 \Rightarrow RGB = (0, 255, 0)$

Scanned with OKEN Scanner

Image sampling and quantization

→ Image is a continuous signal light energy over a plane

→ Continous in x, y and amplitude

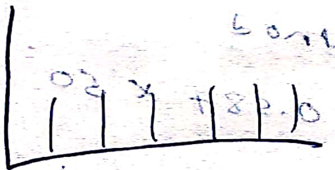
→ Sample both co-ordinate and amplitude

→ Digitizing cordinates is sampling / spatial resolution

→ digitizing amplitude is quantization (intensity resolution)

low { sampling → chess board effect
quantization → false quantorsing }

high contrast



low contrast

(*) Interpolation: using known data to estimate unknown data

zooming may be viewed as over sampling

shrinking may be viewed as under sampling

→ Nearest neighbour interpolation

→ Bilenear → 4 neighbour

→ Bicubic interpolation → 16 neighbour average

Ques

10	20
30	40

produce 4x4 image using bilinear interpolation

sol

10	1	20	6
4	3	5	7
30	2	40	8
12	11	10	9

(0, 0) = 10

(0, 1) = 20

(1, 0) = 30

(1, 1) = 40

(0, 1)

① Horizontal interpolation

$10(1-0.5) + 20 \times 0.5 = 15$

② Horizontal interpolation

$30(1-0.5) + 40(0.5) = 15 + 20 = 35$

③ Vertical interpolation

$15(1-0.5) + 35 \times 0.5 = 25$

④ 20 ⑤ 25

Edge cases can be kept as it is

⑥ = 20 ⑦ = 30 ⑧ = 40 ⑨ = 40 ⑩ = 40

⑪ = 35 ⑫ = 30

EX

10	20	30
40		60
70	80	90

Bilinear

$$\frac{20 + 80 + 60 + 40}{4} = 50$$

nearest also, when too much neighbour take average.

④

(x-1, y-1)	(x, y-1)	(x+1, y-1)
(x-1, y)	(x, y)	(x+1, y)
x-1, y+1	(x, y+1)	(x+1, y+1)

$$N_4(P) = \begin{cases} (x-1, y) \\ (x+1, y) \\ (x, y-1) \\ (x, y+1) \end{cases} \Rightarrow \text{4 neighbour of } P$$

	(x, y-1)	
(x-1, y)	P	(x+1, y)
	(x, y+1)	

8 neighbour of P

x-1, y-1	x, y-1	x+1, y-1
x-1, y	P	x+1, y
x-1, y+1	x, y+1	x+1, y+1

$$N_8(P) = \begin{cases} (x+1, y-1) \\ x, y-1 \\ x+1, y-1 \\ x-1, y-1 \\ x+1, y \\ x-1, y+1 \\ x, y+1 \\ x+1, y+1 \end{cases}$$

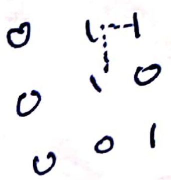
N_D = Diagonal neighbour

① Connectivity: two pixels are connected if they are in the same class (i.e., same color or intensity) and are neighbour

4-connectivity $p, q \Rightarrow q \in N_4(p)$

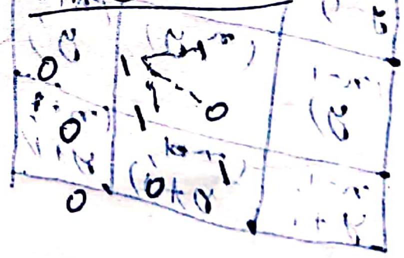
8-connectivity $p, q \Rightarrow q \in N_8(p)$

② 4-connectivity



8-connectivity

m-connectivity



Ex

0	1	0
1	1	0
0	0	0

find connectivity bet^n p and q

(1,2) → 0,2 ⇒ outbound
1,1 = 0
1,3 = 0
2,2 = 1

Ex

p (1) 0 (1) q (1,3)
0 0 0
1 0 1

not 4-connected

Ex

0	1	0
1	1	1
0	1	0

4 connectivity

8 connectivity (should take shortest path)

m-connectivity

Ex

1	0	0
0	1	0
0	0	1
0		

8 connectivity

m connectivity

Adjacency

3	2	1	q
2	2	0	2
1	2	1	
p 1	0	1	2

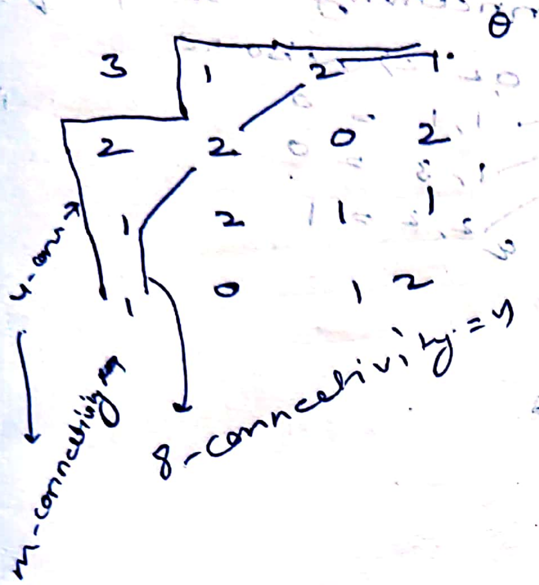
8-connectivity = 4
path length

		1	
	2	0	2
	2	1	
	0	1	2
1			

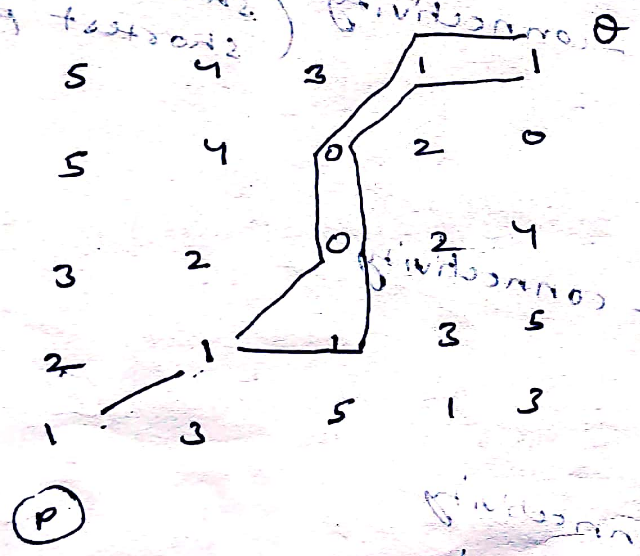
m connectivity

① only take daigonal if no vertical and horizontal
② only take daigonal if $N_4(P) \cup N_4(q) = \phi$

Ques $V = (1, 2)$



Ex : find 8-path and m-path for the following 2-D section with
(a) $V = \{0, 1\}$
(b) $V = \{1, 2\}$ betn P and Q



$V = \{0, 1\} \Rightarrow$
8-connectivity = 5
m-connectivity = 6

$V = \{1, 2\}$
8-connectivity = 5
m-connec → 7

① Euclidean distance

$$D_e(P, Q) = \sqrt{(x-s)^2 + (y-t)^2}$$

② D_4 distance (city block distence) taxi-cab distence, manhatten distence

$$D_4(P, Q) = |x - s| + |y - t|$$

③ D_8 distence (chessboard distence) Chebyshev

$$D_8(P, Q) = \max(|x - s|, |y - t|)$$

Ex Let P and q be the pixels at co-ordinates P(10, 12) and Q(15, 20) find out which distance measure gives minimum distance b/w pixels

sol $D_e(P,Q) = \sqrt{25+64} = \sqrt{89} = 9.$

$D_4(P,Q) = 5 + 8 = 13$

$D_8(P,Q) = 8$

Arithmetic operation

original → Error

Let 10 → 8, 7, 12, 11

$\frac{8+7+12+11}{4} = 9.5$

20 marks

Chapter-2

Ex find digital negative of the given image

4	3	5	2
3	6	4	6
2	2	6	5
7	6	4	1

$s = L - 1 - r$

maximum value $7 = 2^n$

$n = 3$

$L = 2^3 = 8$

$s = 8 - 1 - r \Rightarrow s = 7 - r$

3	4	2	5
4	1	3	1
5	5	1	2
0	1	3	6

(ii) Log transformation

$s = c \log(1 + r)$ c = constant

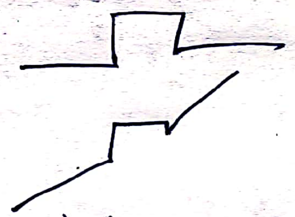
(iii) exponential tranformation

$s = c \exp(r)$

(iv) Gray level slicing

→ with background →

→ without background →



opposite Diagram

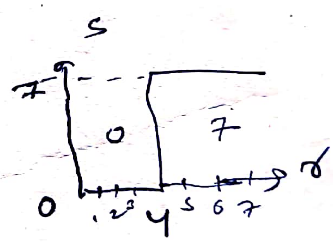
clipping

Ex Threshold $T = 4$

4	3	5	2
3	6	4	6
2	2	6	5
7	6	4	1

Thresholding ⇒

7	0	7	0
0	7	7	7
0	0	7	7
7	7	7	0



$L = 2^3$

$L - 1 = 7 \Rightarrow \max$

$r = 0, 1, 2, 3 \Rightarrow s = 0$

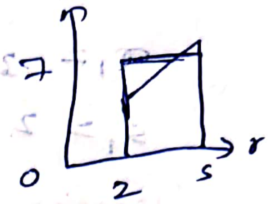
$r = 4, 5, 6, 7 \Rightarrow s = 7$

Scanned with OKEN Scanner

Ex Clipping with $r_1 = 2$, $r_2 = 5$

0	7	7	7
7	0	7	0
7	7	0	7
0	0		

4	3	5	2
3	0	4	0
2	2	0	5
0	6	4	0



$$s = \begin{cases} 7 & 2 \le r \le 5 \\ 0 & \text{otherwise} \end{cases}$$

7	7	7	7
7	0	7	0
7	7	0	7
0	0	4	0

Ex Intensity level = slicing $r_1 = 3$ and $r_2 = 5$

(a) without background

(b) with background

sol (a)

7	7	7	2
7	6	7	6
2	2	6	7
7	6	0	1

(a) without background

$r = 0, 1, 2, 6, 7 \Rightarrow s = 0$

$r = 3, 4, 5 \Rightarrow s = 7$

7	7	7	0
7	0	7	0
0	0	0	7
0	0	7	0

opposite of

(b) with background

7	7	7	2
7	6	7	6
2	2	6	7
7	6	7	1

Ex Bit plane slicing

0	1	1	0
1	0	0	0
0	0	0	1
1	0	0	1

~~1st bit~~

lsb

0	1	0	0
1	1	0	1
1	1	1	0
1	1	0	0

~~2nd Bit~~

mid

1	0	1	0
0	1	1	1
0	0	1	1
1	1	1	0

~~3rd bit~~

MSB

Ex contrast stracching

$r_1 = 3$ $r_2 = 5$

$s_1 = 2$ $s_2 = 6$

$$s = \frac{5 - r}{2} \times 4 + 2$$

stracching - picewise linear

Ex perform contrast method

$$I = \begin{bmatrix} 10 & 20 & 30 & 40 \\ 50 & 60 & 70 & 80 \\ 90 & 100 & 110 & 120 \\ 130 & 140 & 150 & 160 \end{bmatrix}$$

$r_1 = 30$ $r_2 = 120$

$s_1 = 10$ $s_2 = 180$

sol

$s = \alpha r$, $0 \le r < 30$

$s = \beta(r - r_1) + s_1$; $30 \le r < 120$

$s = \gamma(r - r_2) + s_2$, $120 \le r \le 160$

$$\alpha = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{10 - 0}{30 - 0} = 0.333$$

$$\beta = \frac{180 - 10}{120 - 30} = 1.889$$

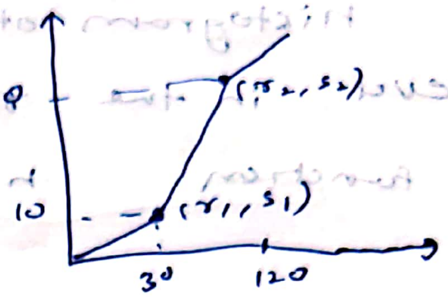
$$r = \frac{255 - 180}{255 - 120} \Rightarrow \frac{75}{135} \Rightarrow \frac{5}{9} \Rightarrow 0.56$$

$s = 0.333\, r$, $0 \le r < 30$

$s = 1.889(r - 30) + 10$, $30 \le r < 120$

$s = 0.56(r - 120) + 180$, $120 \le r < 255$

$$I = \begin{bmatrix} 2.33 & 6.66 & 10 & 28.8 \\ 46 & 64 & 82 & 100 \\ 118 & 136 & 154 & \cancel{172}\ 180 \\ 185 & 190 & 195 & 200 \end{bmatrix}$$



mid poin criterion

$$I = \begin{bmatrix} 3 & 7 & 10 & 29 \\ 46 & 64 & 82 & 100 \\ 118 & 136 & 154 & 180 \\ 185 & 190 & 195 & 200 \end{bmatrix}$$

Perform contrast streching (point processing) (when r_1, s_1, r_2, s_2 are not defined)

$$s = \left(\frac{r - r_{min}}{r_{max} - r_{min}} \right) \times 255 \Rightarrow \frac{r - 10}{190} \times \cancel{255}\ 17$$

$$\Rightarrow \frac{r \times 17 - 17}{10}$$

$$\Rightarrow r \times 1.7 - 17$$

$$I = \begin{bmatrix} 0 & 17 & 34 & 51 \\ 68 & 85 & 102 & 119 \\ 136 & 153 & 170 & 187 \\ 204 & 221 & 238 & 255 \end{bmatrix}$$

Ex apply log transformation

$$I = \begin{bmatrix} 1 & 25 \\ 100 & 200 \end{bmatrix}$$

$$255 = c \log(1 + r)$$

$$255 = c \log(1 + 255)$$

$$s = c \log(1 + r)$$

$$\Rightarrow c = \frac{255}{\log(1 + 255)} = 105.86$$

If $\begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} \longrightarrow c = \frac{3}{\log(1+3)}$

$$I = 105.86 \log_{10}(1 + r)$$

$$I' = \begin{bmatrix} 32 & 150 \\ 212 & 244 \end{bmatrix}$$

Histogram:

Histogram of a digital image with gray level in the range $[0, L-1]$ is a discrete function $h(r_k) = n_k$

Histogram $h(r_k) = n_k$

r_k is the k^{th} intensity values in the image

n_k is the number of pixels with intensity r_k

Normalized histogram $p(r_k) = \frac{n_k}{MN}$

PDF: probability density function

probability of each element

CDF: Cumulative Distribution function

summation of probability of elements <= this element.

(I) Transformation function:-

$T(r) = s$

Inverse transformation function

$r = T^{-1}(s)$

(II) Histogram Equalization

$$P_s(s) = P_r(r) \left| \frac{dr}{ds} \right|$$

$$s = T(r) = \int_0^r p_r(\omega)\, d\omega$$

$$\frac{ds}{dr} = \frac{d(T(r))}{dr} \Rightarrow \left[\frac{d}{dr} \left(\int_0^r P_r(\omega) d\omega \right) \right.$$

$$\Rightarrow P_r(r) = \frac{ds}{dr}$$

$p_k(r_k) = \frac{n_k}{n}$, $k = 0, 1, \ldots L-1$

$s_k = T(r_k) = \sum_{j=0}^{k} p_r(r_j)$

$= \sum_{j=0}^{k} \frac{n_j}{n}$, $k = 0, 1, 2 \ldots L-1$

Ex perform histogram equalization for the following image

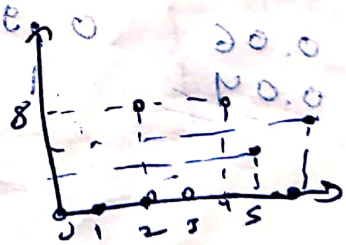
$$f(x,y) = \begin{bmatrix} 1 & 2 & 1 & 1 & 1 \\ 2 & 5 & 3 & 5 & 2 \\ 2 & 5 & 5 & 5 & 2 \\ 2 & 5 & 3 & 5 & 2 \\ 1 & 1 & 1 & 2 & 1 \end{bmatrix}$$

$0 \to 0$, $1 \to 8$, $2 \to 7$, $3 \to 2$, $4 \to 0$, $5 \to 6$, $6 \to 0$, $7 \to 0$

Pixel value	No of pixel	pdf	S_k CDF	$S_k * (L-1)$	mid point approx Histogram equalization
0	0	0	0	0	0
1	8	0.32	0.32	2.24	2
2	8	0.32	0.64	4.48	4
3	2	0.08	0.72	5.04	5
4	0	0	0.72	5.04	5
5	7	0.28	1	7	7
6	0	0	1	7	7
7	0	0	1	7	7

Compare No. of Pixel column and Histogram equalization with Histogram equalization as index of new column

Gray level	No. of pixel
0	0
1	0
2	8
3	0
4	8
5	2
6	0
7	7



Ex

4	4	4	4	4
3	4	5	4	3
3	5	5	5	3
3	4	5	4	3
4	4	4	4	4

		PdF	Sk	Sk×7	
0	0	0	0	0	0
1	0	0	0	0	0
2	0	0	0	0	0
3	6	0.24	0.24	1.68	2
4	14	0.56	0.80	5.6	6
5	5	0.20	1	7	7
6	0	0	0	7	7
7	0	0	0	7	7

0	0
2	6
3	0
4	0
5	0
6	14
7	5

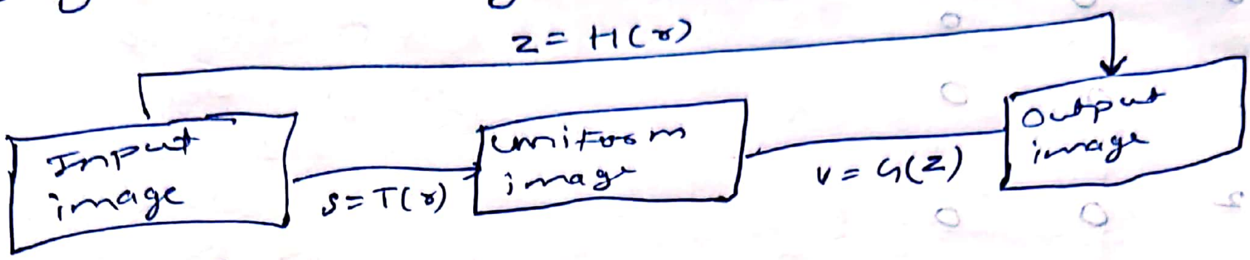
Ex

2	3	3	2
4	2	4	3
3	2	3	5
2	4	2	4

Gray levels	No of Pixels
0	9
1	8
2	11
3	4
5	10
6	15
7	4
8	3

PdF 64	SECdF	CdF×7
0.14	0.14	
0.125	0.26	
0.17	0.43	
0.062	0.49	
0.156	0.653	
0.23	0.88	
0.06	0.94	
0.04		

(II) Histogram matching



$$z = H(r) = G^{-1}(v = s = T(r))$$

$$s = T(r) = \int_0^r p_{in}(w)\,dw \qquad 0 \le r \le 1$$

$v = \cancel{G(z)} =$

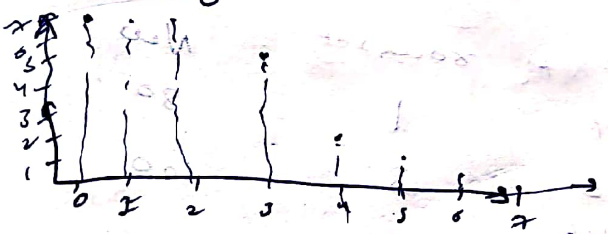
Ques

(a)

r	0	1	2	3	4	5	6	7
n	790	1023	850	656	329	245	122	81

(b)

z	0	1	2	3	4	5	6	7
n	0	0	0	614	819	1230	819	614



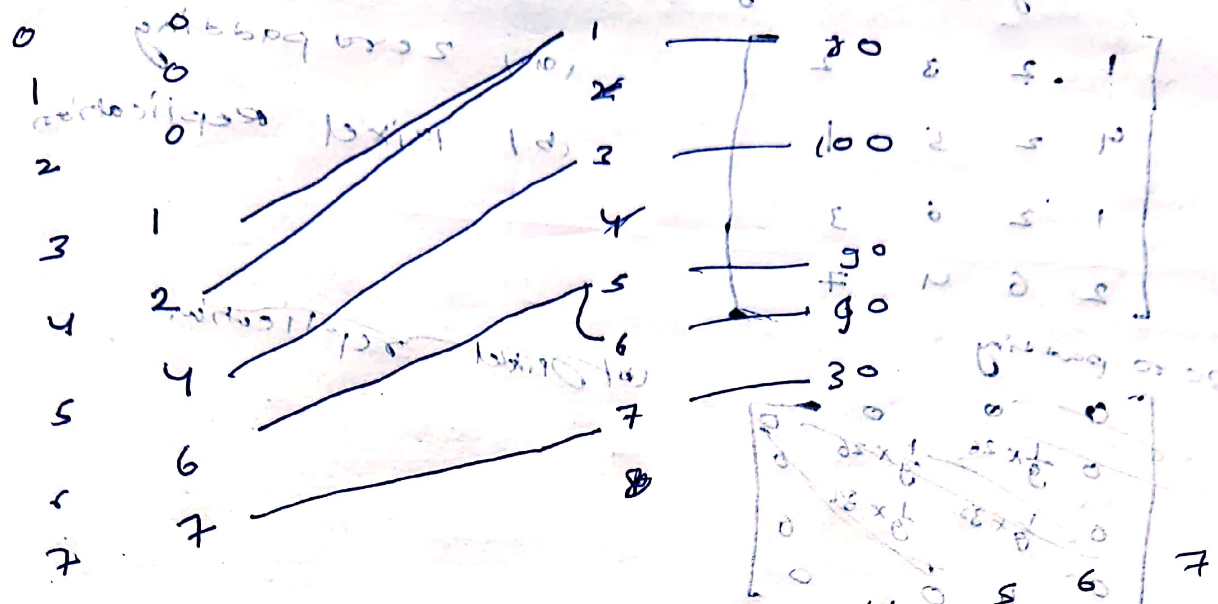
r	n	pdf	cdf	cdf × ~~4095~~ 7	round off	new freq
0	790	0.19	0.19	~~7886~~ 1.3	1	790
1	1023	0.25	0.44	3.09	3	1023
2	850	0.20	0.669	4.55	5	850
3	656	0.16	0.809	5.6	6	985
4	329	0.08	0.889	6.2	6	
5	245	0.06	0.949	6.6	7	448
6	122	0.03	0.979	6.8	7	
7	81	0.02	1.000	7	7	
	4096					

		Pdf	Cdf
0	0	0	
1	0	0	
2	0	0	
3	614	0.149	
4	819	0.199	
5	1230	0.300	
6	819	0.189	
7	614	0.15	
	4096		

First o/p then I/p

grey	freq	Pdf	cdf	cdf × 7	round off	New
0	80	0.20	0.20	1.4	1	80
1	100	0.25	0.45	3.15	3	100
2	90	0.23	0.78	5.46	5	90
3	60	0.15	0.93	6.51	6	80
4	30	0.07	0.97	6.79	6	0
5	20	0.05	0.99	6.93	7	30
6	10	0.025	1	7	7	
7	0	0				
	390					

				×7	
0	0	0	0	0	0
1	0	0	0	0	0
2	0	0	0	0	60
3	60	0.153	0.153	1	80
4	80	0.205	0.358	2	100
5	100	0.256	0.6614	4	80
6	80	0.205	0.819	6	70
7	70	0.179	1	7	
	390				



gray level	0	1	2	3	4	5	6	7
No. of pixel	0	0	0	80	80	100	90	30

⊕ spatial Filtering

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t)\, f(x+s, y+t)$$

neighbouring average

$$\frac{1}{9} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \qquad \frac{1}{16} \times \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

average   weighted average

Ex Calculate the central pixel value after smoothing with a 3x3 box filter ones(3,3)

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 4 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} & & \\ & 0 & \\ & & \end{bmatrix}$$

$$\frac{1}{9} \times (1+2+3+2+4+5+3+4+5)$$

$$= \frac{1}{9} \times 27 = 3$$

3×3 neighbour average

Ques

$$\begin{bmatrix} 1 & 2 & 3 & 2 \\ 4 & 2 & 5 & 1 \\ 1 & 2 & 6 & 3 \\ 2 & 6 & 4 & 7 \end{bmatrix}$$

(a) zero padding

(b) pixel Replication

(a) zero padding

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{9}\times 26 & \frac{1}{9}\times 26 & 0 \\ 0 & \frac{1}{9}\times 32 & \frac{1}{9}\times 81 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(b) ~~pixel replication~~

0	0	0	0	0	0
0	1	2	3	2	0
0	4	2	5	1	0
0	1	2	6	3	0
0	2	6	4	7	0
0	0	0	0	0	0

$$= \begin{bmatrix} 1 & 2 & 2 & 1 \\ 3 & 3 & 3 & 2 \\ 2 & 4 & 4 & 3 \\ 1 & 2 & 3 & 2 \end{bmatrix}$$

(b) pixel replication

1	1	2	3	2	2
1	1	2	3	2	
4	4	2	5	1	1
1	1	2	6	3	3
2	2	6	4	7	7
2	2	6	4	7	

$\times \frac{1}{9}$

$$= \begin{bmatrix} 2 & 2 & 2 & 2 \\ 2 & 3 & 3 & 3 \\ 3 & & 4 & 4 \\ 3 & 4 & 5 & 5 \end{bmatrix}$$

Ex Compute the median value of the center pixel

$$\begin{bmatrix} 1 & 5 & 7 \\ 2 & 4 & 6 \\ 3 & 2 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 5 & 7 \\ 2 & 3 & 6 \\ 3 & 2 & 1 \end{bmatrix}$$

1, 1, 2, 2, 3, 4, 5, 6, 7

mid value

Ex Compute the median value of the marked pixels

$$\begin{bmatrix} 18 & 22 & 33 & 25 & 32 & 24 \\ 34 & 128 & 24 & 172 & 26 & 23 \\ 22 & 19 & 32 & 31 & 28 & 26 \end{bmatrix}$$

18 19 22 24 24 32 32 33 128

$$\begin{bmatrix} 18 & 22 & 33 & 25 & 32 & 24 \\ 24 & 24 & 31 & 31 & 26 & 23 \\ 3 & 19 & 32 & 31 & 28 & 26 \end{bmatrix}$$

Ex

$$\begin{bmatrix} 24 & 22 & 33 & 25 & 32 & 24 \\ 34 & 255 & 24 & 0 & 26 & 23 \\ 23 & 21 & 32 & 31 & 28 & 26 \end{bmatrix}$$

Justify the statement → "Median filter is an effective tool to minimize salt and paper

salt and paper → 255, 0

Soln

255 → 24

0 → 28

~~why~~ weighted ~~why~~ box : ~~weighted~~ box smooths content of image

mask size ↑ blur ↑

sharpning - spatial differentiation :

defination for first ~~differenti~~ derivative
must be zero - in flat segment
must be non zero - at on set of a gray level, step or ramp
must be nonzero ~~along step~~ ramp

defination of second derivative
musts be zero - in flat segments
must be non zero - at on set and end of gray level step or ramp
The shortest distance over which change can occur is between

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

$$\frac{\partial^2 f}{\partial x^2} = f\cancel{(x+1) - f(x)}\ f(x+1) + f(x-1) - 2f(x)$$

slide server

Que

5 5 4 3 2 1 0 0 0 6 0 0 0 0 1 3 1 0 0
for -1 -1 -1 -1 -1 0 0 6 -6 0 0 0 1 2 -2 -1 0 0
-1 0 0 0 0 1 0 6 -12 6 0 0 1 1 -4 1 1 0
-1 0 0 0 0 1 -8 1