Additional Assignments

Saturday, 22 September 2018 5:09 PM

Problem Statement 1: [100 marks]
A company manufactures LED bulbs with a faulty rate of 30%. If I randomly select 6 chosen LEDs, what is the probability of having 2 faulty LEDs in my sample? Calculate the average value of this process. Also evaluate the standard deviation associated with it.

Faulty rate = 30%

$P(\text{Faulty LED}) = 0.3 \Rightarrow p$

$P(\text{Good LED}) = 0.7 \Rightarrow q$

Total LED chosen $\Rightarrow n = 6$

Since the probabity and the total no. of outcomes are given, we would use Binomial distribution

A) probability of having 2 faulty LEDs

$$P(X=2) = {}^6C_2 \, p^2 q^4$$
$$= \frac{6!}{2! \, 4!} \cdot (0.3)^2 \cdot (0.7)^4$$
$$= 15 \times 0.09 \times 0.49 \times 0.49$$
$$= 0.325$$

B) average value of this process

$$\mu = np$$
$$= 6 \times 0.3 = 1.8$$

C) evaluate the standard deviation

$$\sigma = \sqrt{npq}$$
$$= \sqrt{6 \times 0.3 \times 0.7}$$
$$= \sqrt{1.26} = 1.12$$

Problem Statement 2: [100 marks]
Gaurav and Barakha are both preparing for entrance exams. Gaurav attempts to solve 8 questions per day with a correction rate of 75%, while Barakha averages around 12 questions per day with a correction rate of 45%. What is the probability that each of them will solve 5 questions correctly? What happens in cases of 4 and 6 correct solutions? What do you infer from it? What are the two main governing factors affecting their ability to solve questions correctly? Give a pictorial representation of the same to validate your answer.

Since the number of questions and the correction rate is given, we would use Binomial distribution

Gaurav

Total questions per day

$n = 8$

Correction rate $= 75\%$

$P(\text{Correct eval}) = 0.75 = p$

$P(\text{Wrong eval}) = 0.25 = q$

$$P(X=5) = {}^{8}C_{5}\, p^5 q^3$$
$$= 56 \times (0.75)^5 \times (0.25)^3$$
$$= 56 \times 0.237 \times 0.015$$
$$= 0.207$$

$$P(X=4) = {}^{8}C_{4}\, p^4 q^4$$
$$= 70 \times (0.75)^4 \times (0.25)^4$$
$$= 70 \times 0.316 \times 0.004$$
$$= 0.08$$

$$P(X=6) = {}^{8}C_{6}\, p^6 q^2$$
$$= 28 \times (0.75)^6 \times (0.25)^2$$
$$= 0.311$$

Barakha

$n = 12$

$= 45\%$

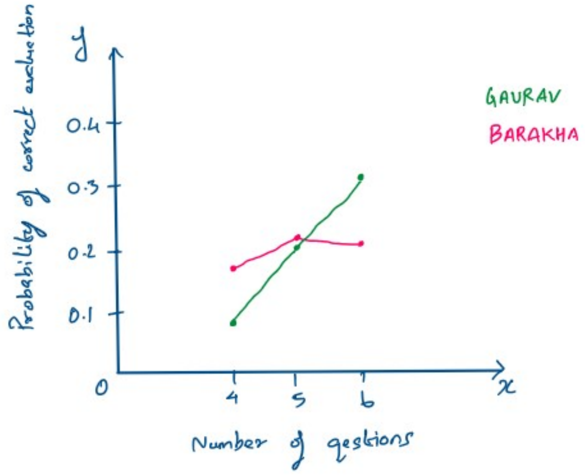
$p = 0.45$

$q = 0.55$

$$P(X=5) = {}^{12}C_{5}\, p^5 q^7$$
$$= 792 \times (0.45)^5 \times (0.55)^7$$
$$= 792 \times 0.018 \times 0.0152$$
$$= 0.222$$

$$P(X=4) = {}^{12}C_{4}\, p^4 q^8$$
$$= 495 \times (0.45)^4 \times (0.55)^8$$
$$= 495 \times 0.041 \times 0.00837$$
$$= 0.169$$

$$P(X=6) = {}^{12}C_{6}\, p^6 q^6$$
$$= 924 \times (0.45)^6 \times (0.55)^6$$
$$= 0.212$$



Observations :-

Gaurav shows a steady increse of correct evaluation

Barakha's correct evaluation starts declining after certain no. of questions

Problem Statement 3: [100 marks]
Customers arrive at a rate of 72 per hour to my shop. What is the probability of k customers arriving in 4 minutes? a) 5 customers, b) not more than 3 customers, c) more than 3 customers. Give a pictorial representation of the same to validate your answer.

We have only mean and no outcomes, hence we would use Poisson distribution

$$\text{Customer arrival rate} = \mu = 72/hr = 72/60\ min = 1.2\ /min$$

$$\text{new mean for 4mins} \Rightarrow \mu = 4 \times 1.2\ /min = 4.8\ /min$$

a) 5 customers

$$P(X=x) = \frac{e^{-\mu} \cdot \mu^x}{x!}$$

$$P(X=5) = \frac{e^{-4.8} \times (4.8)^5}{5!}$$

$$= 0.174$$

b) not more than 3 customers

$$P(X<=3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$$

$$= \frac{e^{-4.8}(4.8)^0}{0!} + \frac{e^{-4.8}(4.8)^1}{1!} + \frac{e^{-4.8}(4.8)^2}{2!} + \frac{e^{-4.8}(4.8)^3}{3!}$$

$$= e^{-4.8}\left[\frac{(4.8)^0}{0!} + \frac{(4.8)^1}{1!} + \frac{(4.8)^2}{2!} + \frac{(4.8)^3}{3!}\right]$$

$$= 0.00822\left[1 + 4.8 + \frac{(4.8)^2}{2!} + \frac{(4.8)^3}{3!}\right]$$

$$= 0.445$$

c) more than 3 customers.

$$P(X>3) = 1 - P(X<=3)$$

$$= 1 - 0.445$$

$$= 0.554$$

Problem Statement 4: [100 marks]
I work as a data analyst in Aeon Learning Pvt. Ltd. After analyzing data, I make reports, where I have the efficiency of entering 77 words per minute with 6 errors per hour. What is the probability that I will commit 2 errors in a 455-word financial report?
What happens when the no. of words increases (in case of 1000 words) or decreases (255 words)? How is the λ affected? How does it influence the PMF? Give a pictorial representation of the same to validate your answer.

We have only mean and no outcomes, hence we would use Poisson distribution

Typing speed = 77 words/min

No. of Errors = 6/hr
= 6/60 min
= 0.1 / min

A) 2 errors in a 455-word

Total words in report = 455
Total time to type report = 455/77
= 5.9 mins

Mean for 455 words ⇒ $\mu = 5.9 \times 0.1$ /min
$= 0.59$ /min

$$P(X=2) = \frac{e^{-0.59}(0.59)^2}{2!}$$

$$= \frac{0.55 \times 0.348}{2 \times 1}$$

$$= 0.0957$$

Total words in report = 1000
Total time to type report = 1000/77
= 129 mins

Mean for 455 words ⇒ $\mu = 12.9 \times 0.1$ /min
$= 1.29$ /min

$$P(X=2) = \frac{e^{-1.29} \times (1.29)^2}{2!}$$

$$= 0.22$$

C) 2 errors in a 255-word
Total words in report = 255
Total time to type report = 255/77
= 3.3 mins

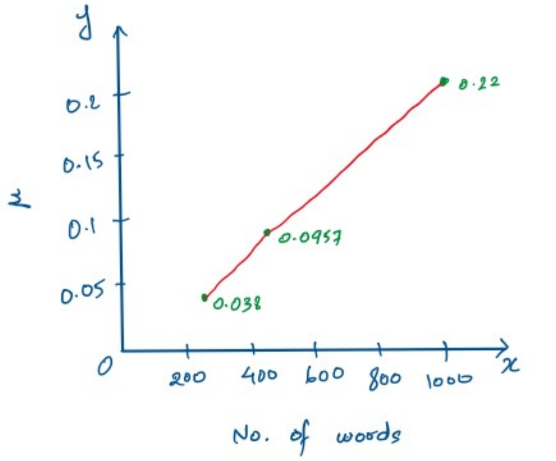
Mean for 455 words ⇒ $\mu = 3.3 \times 0.1$ /min
$= 0.33$ /min

$$P(X=2) = \frac{e^{-0.33} \times (0.33)^2}{2!}$$

$$= 0.038$$

Observations :-

As the number of words increases the μ increases as well



Problem Statement 5: [100 marks]
The current measured in a copper wire is modelled by a continuous random variable X. X is in milliamperes. Assume that the range of X is [0, 20 mA]. The probability density function is given by, $f(x) = 0.05$ for $0 \le x \le 20$. What is the probability that a current measurement is less than 10 milliamperes? Draw the PDF and the CDF diagrams as well.

The outcomes are uncertain hence it is Continuous

As the probability density function is same for all the range of outcome this is a uniform distribution

$x = [0, 20]$

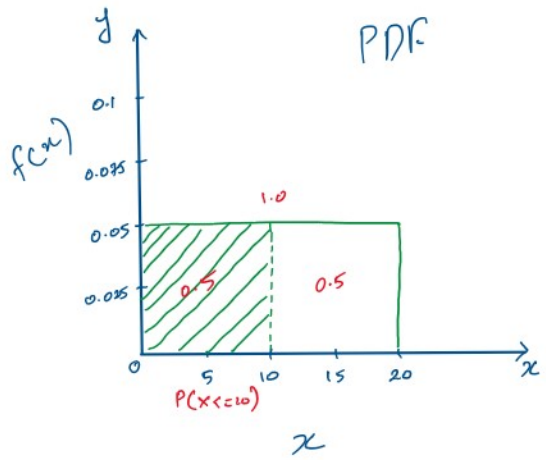
probability density function $f(x) = \frac{1}{b-a} = \frac{1}{20-0} = \frac{1}{20} = 0.05$

$$P(X <= 10) = \int_0^{10} f(x)\, dx$$

$$= \int_0^{10} 0.05\, dx$$

$$= [10 \times 0.05 - 0 \times 0.05]$$

$$= 0.5 - 0 = 0.5$$



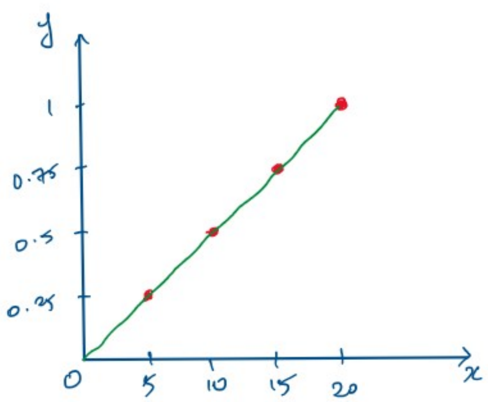
CDF

$$F_X(0 <= x <= 5) = \int_0^5 f(x)\, dx$$

$$= \int_0^5 0.05\, dx$$

$$= [5 \times 0.05 - 0 \times 0.05]$$

$$= 0.25$$



$$F_X(0 <= x <= 10) = \int_0^{10} f(x)\, dx$$

$$= 10 \times 0.05$$

$$= 0.5$$

$$f_X(0 <= x <= 15) = \int_0^{15} f(x)\, dx$$

$$= 15 \times 0.05$$

$$= 0.75$$

$$f_X(0 <= x <= 20) = \int_0^{20} f(x)\, dx$$

$$= 20 \times 0.05$$

$$= 1$$