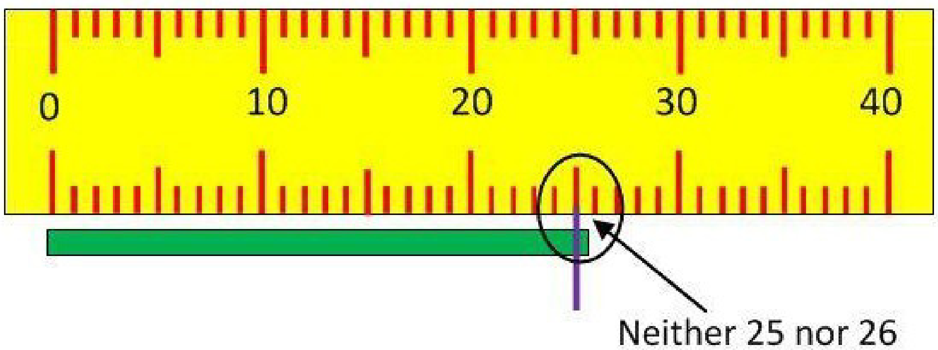
# Units & Dimensions 3

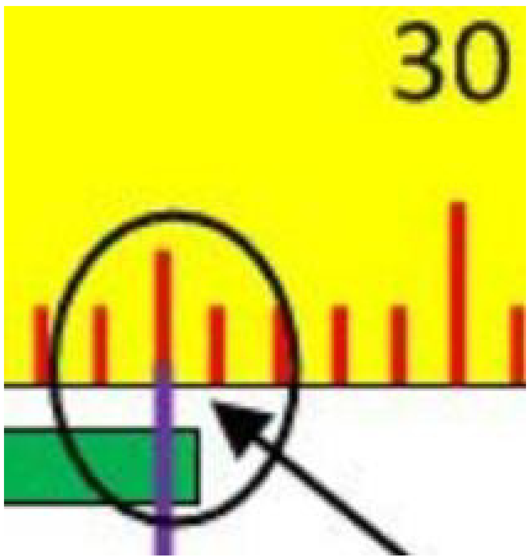
# How to find ERRORS inMeasurements?

# ERROR

Error is amount of uncertainty in measurement of the magnitude of a physical quantity.

Error = Measured Value - True Value.

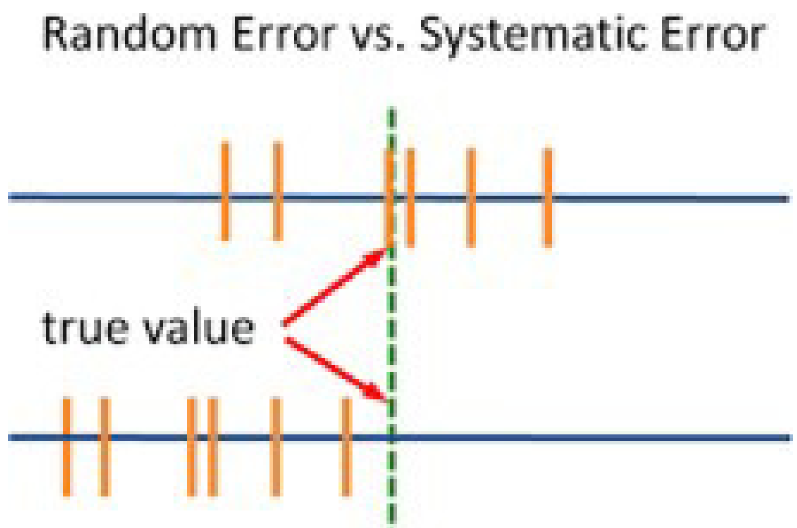


# ERROR

The Types of Errors

- Instrumental errors
- Systematic errors
- Random errors
- Personal errors

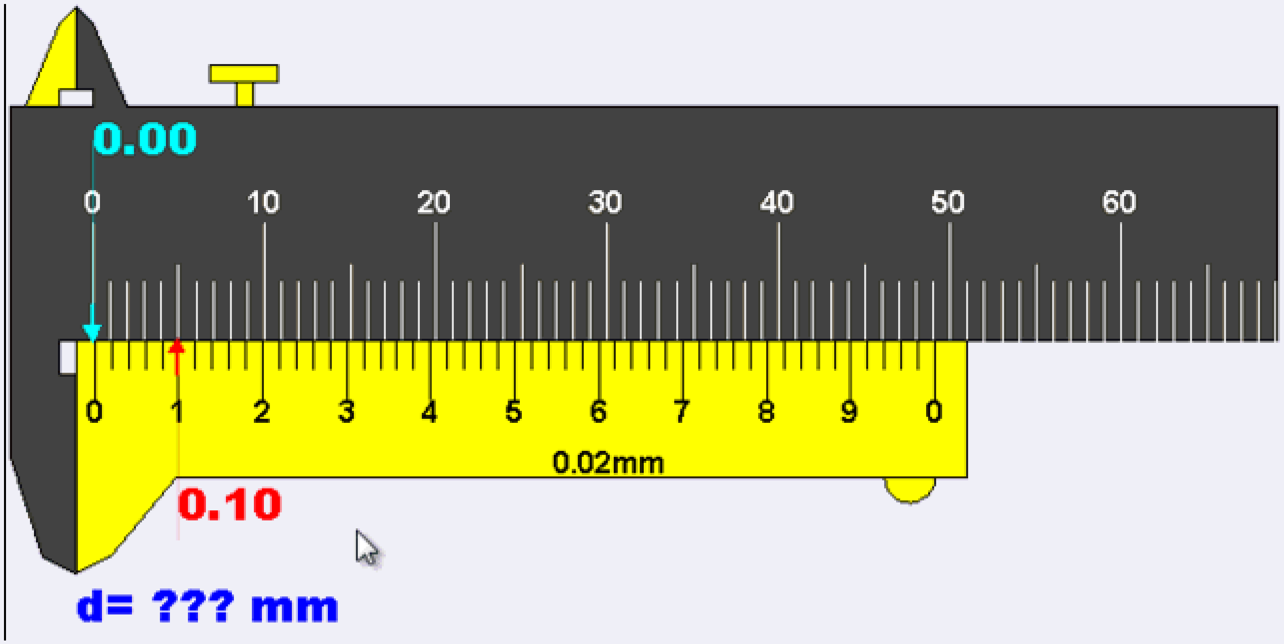


**Instrumental Errors -** These errors arise due to use of faulty instrument.

**Systematic Errors -** These errors arise due to definite cause and takes place according to some rule.

**Random Errors -** These errors are random and occur irregularly. It can be due to changes in the ambient conditions like temperature, pressure, etc. or due to limitations of instrument.

**Personal Errors -** The errors which occur due to limitation of the human senses, judgment, carelessness and responses


0.00
0 10 20 30 40 50 60
0 1 2 3 4 5 6 7 8 9 0
0.02mm
0.10
d= ??? mm

# Calculation Of Errors

**Most Probable Value : $a_{mean}$**

$$= \frac{a_1 + a_2 + \ldots\ldots + a_n}{n} = \frac{1}{n} \sum_{a=1}^{n} a_i$$

**Absolute Error:**

$$= |\Delta a| = |a_{mean} - a|$$

## Calculation Of Errors

**Relative error :**

$$= \frac{\text{Absolute error}}{\text{Most probable value}} = \frac{|a_{mean} - a|}{a_{mean}}$$

**Percentag error :**

$$= \text{Relative error} \times 100\%$$

$$= \frac{|a_{mean} - a|}{a_{mean}} \times 100\,\%$$

Example

Length of the plate measured using Vernier caliper of least count 0.01 cm. The measurements made were 3.11 cm, 3.13 cm, 3.14 cm & 3.14 cm. Find mean length

A. 3.15 cm
B. 3.13 cm
C. 3.14 cm
D. 3.12 cm

**Example** The length of the plate measured using Vernier caliper of least count 0.01 cm. The measurements made were 3.11 cm, 3.13cm, 3.14 cm and 3.14 cm. Find mean length, mean absolute error and % error

**Solution:**

**Step: 1** Data: $a_1 = 3.11$ cm, $a_2 = 3.13$ cm, $a_3 = 3.14$ cm, $a_4 = 3.14$ cm,

**Step: 2** Mean Length $a_m = \dfrac{a_1+a_2+a_3+a_4}{4}$

$$= \frac{3.11+3.13+3.14+3.14}{4}$$

**Step: 3** Mean Length $a_m = \dfrac{12.53}{4} = 3.13$ cm

Example

**Length of the plate measured using Vernier caliper of least count 0.01 cm. The measurements made were 3.11 cm, 3.13 cm, 3.14 cm & 3.14 cm.**
**Find mean absolute error**

A. **0.05 cm**
B. **0.03 cm**
C. **0.02 cm**
D. **0.01 cm**

Example

The length of the plate measured using Vernier caliper of least count 0.01 cm. The measurements made were 3.11 cm, 3.13cm, 3.14 cm and 3.14 cm. Find mean absolute error

## Solution:

Step: 4

$$\text{Mean abs error} = \frac{|a_1-a_m| + |a_2-a_m| + |a_3-a_m| + |a_4-a_m|}{4}$$

Step: 5

$$= \frac{|3.11-3.13| + |3.13-3.13| + |3.14-3.13| + |3.15-3.13|}{4} = 0.01 \text{ cm}$$

Step: 6

Percentag error = $0.01/3.13 \times 100 = 0.319\%$

**Example**

**The length of the plate measured using Vernier caliper of least count 0.01 cm. The measurements made were 3.11 cm, 3.13cm, 3.14 cm and 3.14 cm. Find <u>% error</u>**

A. **3.19 %**
B. **0.319 %**
C. **31.9 %**
D. **3.3 %**

Example

The length of the plate measured using Vernier caliper of least count 0.01 cm. The measurements made were 3.11 cm, 3.13cm, 3.14 cm and 3.14 cm. Find % error

**Solution:**

Step: 4

$$\text{Mean abs error} = \frac{|a_1-a_m|+|a_2-a_m|+|a_3-a_m|+|a_4-a_m|}{4}$$

Step: 5

$$= \frac{|3.11-3.13| + |3.13-3.13|+|3.14-3.13|+|3.15-3.13|}{4} = 0.01\ \text{cm}$$

Step: 6

Percentage error = 0.01/3.13 × 100 = 0.319%

**Example**

The size of the object measured by means of a Vernier calipers is 3.52 cm. If the least count is 0.01 cm, estimate the percentag error in the measurement.

A. 0.28 %
B. 0.31 %
C. 2.81 %
D. 4.12 %

**Example**

The size of the object measured by means of a Vernier calipers is 3.52 cm. If the least count is 0.01 cm, estimate the percentage error in the measurement.

**Solution:**

**Step: 1**

Data : $y = 3.52\ \text{cm}$ and $\Delta y = 0.01\ \text{cm}$

**Step: 2**

$$\%\ \text{error} = \frac{\Delta y}{y} \times 100 = \frac{0.01}{3.52} \times 100$$

**Step: 3**

$\%\ \text{error} = 0.28\%$

Example

An object was weighed by a physical balance and following readings were obtained: 5.04 g, 5.06 g, 4.97 g, 5.00 g & 4.93 g. Find (i) the mean value

A. 5.00 g
B. 5.01 g
C. 5.02 g
D. 5.03 g

**Example** An object was weighed by a physical balance and following readings were obtained :5.04 g 5.06 g, 4.97 g, 5.00 g and 4.93 g. Find (i) the mean value

**Solution:**

**Step: 1** Data: $n = 5$, $m_1 = 5.04$ g, $m_2 = 5.06$ g, $m_3 = 4.97$ g, $m_4 = 5.00$ g $m_5 = 4.93$ g

**Step: 2** Mean value $\bar{m} = \dfrac{m_1+m_2+m_3+m_4+m_5}{n}$

$$= \frac{(5.04 + 5.06 + 4.97 + 5.00 + 4.93)\ g}{5}$$

$$= \frac{25.00}{5} = 5.00\ g$$

Example

An object was weighed by a physical balance and following readings were obtained : 5.04 g, 5.06 g, 4.97 g, 5.00 g & 4.93 g.
Find (ii) the mean absolute error

A. 0.04 g
B. 0.01 g
C. 0.02 g
D. 0.03

Example

An object was weighed by a physical balance and following readings were obtained : 5.04 g 5.06 g, 4.97 g, 5.00 g and 4.93 g.
Find (ii) <u>the mean absolute error</u>

**Solution:**

Step: 3

**Mean absolute error ,**

$$\Delta m = \frac{|m_1 - \bar{m}| + |m_2 - \bar{m}| + |m_3 - \bar{m}| + |m_4 - \bar{m}| + |m_5 - \bar{m}|}{n}$$

$$= \frac{(0.04 + 0.06 + 0.03 + 0.00 + 0.07)\ g}{5} = \frac{0.20}{5} = 0.04\ g$$

Example

An object was weighed by a physical balance and following readings were obtained : 5.04 g, 5.06 g, 4.97 g, 5.00 g & 4.93 g.
Find (iii) the percentage error.

A. 0.5
B. 0.7
C. 0.75
D. 0.8

**Example**

An object was weighed by a physical balance and following readings were obtained :5.04 g 5.06 g, 4.97 g, 5.00 g and 4.93 g.
Find (iii) the percentage error.

**Solution:**

**Step: 3**

Mean absolute error ,

$$\Delta m = \frac{|m_1 - \overline{m}| + |m_2 - \overline{m}| + |m_3 - \overline{m}| + |m_4 - \overline{m}| + |m_5 - \overline{m}|}{n}$$

$$= \frac{(0.04 + 0.06 + 0.03 + 0.00 + 0.07)\ g}{5} = \frac{0.20}{5} = 0.04\ g$$

**Step: 4**

$$\text{Percentage error} = \frac{\Delta m}{m} \times 100\ \%$$

$$= \frac{0.04\ g}{5.00\ g} \times 100\ \% = 0.8\ \%$$

# ERROR

# in
# CALCULATIONS

# How to predict errors in calculations?

1. Addition
2. Subtraction
3. Multiplication
4. Division
5. Powers

## Error Trick


MURPHY'S
LAW
IF IT CAN GO WRONG,
IT WILL!

## Calculating Errors

### 1. ADDITION

$$R = x + y$$

$$dR = dx + dy$$

$$\Delta R = \Delta x + \Delta y$$

Error in R

Error in x

Error in y

Note that errors are always <u>added up</u>

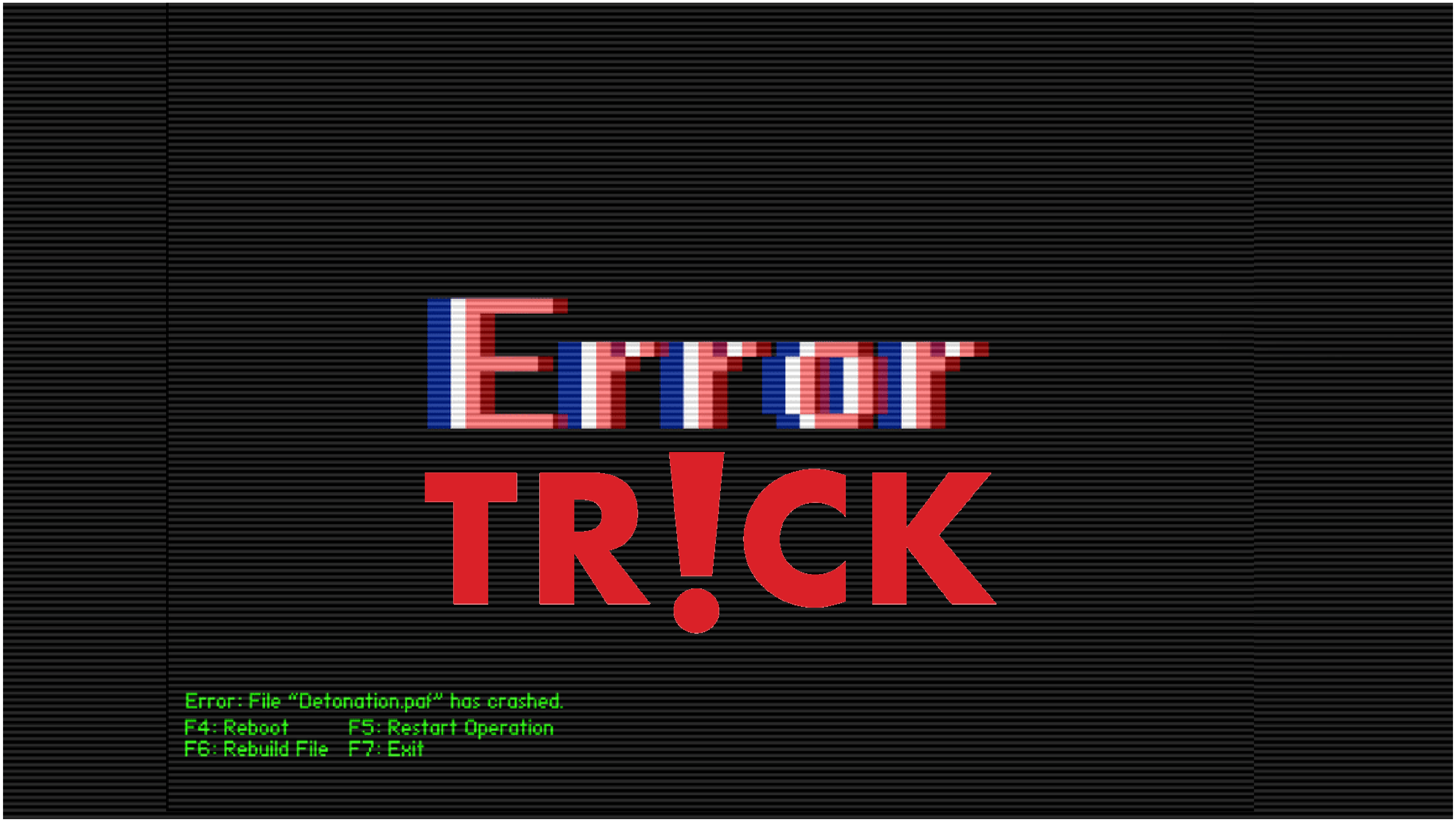
# Calculating Errors

## 2. SUBTRACTION

$R = x - y$

$\Delta R = \Delta x + \Delta y$

Note that errors are always **added up**


Error
TR!CK
Error: File “Detonation.paf” has crashed.
F4: Reboot F5: Restart Operation
F6: Rebuild File F7: Exit

# Error Trick

$R = x^2y^3$

$R = \frac{x^2y^3}{\sqrt{z}}$

# Calculating Errors

## 3. MULTIPLICATION

$R = xy$

$dR = xdy + ydx$

$\Delta R = x\Delta y + y\Delta x$

Error in R

$R = xy$

$dR = xdy + ydx$

$\Delta R = x\Delta y + y\Delta x$

Error in R

**Fractional error = ΔR/R**

$$\frac{\Delta R}{R} = \frac{\cancel{x}\Delta y}{\cancel{x}y} + \frac{y\Delta \cancel{x}}{\cancel{x}y}$$

$$\frac{\Delta R}{R} = \frac{\Delta y}{y} + \frac{\Delta x}{x}$$

## Calculating Errors

### 4. DIVISION

$R = x/y$

$R = xy^{-1}$

$dR = x(-1y^{-2}dy) + (dx)y^{-1}$

**Fractional error = ΔR/R**

$$\frac{\Delta R}{R} = \left|\frac{\cancel{x}(-1y^{-2}\Delta y)}{\cancel{x}y^{-1}}\right| + \left|\frac{(dx)\cancel{y^{-1}}}{x\cancel{y^{-1}}}\right|$$

$$\frac{\Delta R}{R} = \frac{\Delta y}{y} + \frac{\Delta x}{x}$$

# Calculating Errors

## 5. POWERS

$$R = x^2 y^3$$

$$dR = x^2(3y^2 dy) + (2x dx) y^3$$

$$\frac{d}{RR} = \frac{3d}{yy} + \frac{2d}{xx}$$

$$\frac{\Delta R}{R} = \frac{2\Delta x}{x} + \frac{3\Delta y}{y}$$

We can directly write the final expression by looking at the power coefficient

Even if the coefficient is negative, we have to add the errors

**Example**

**Calculate fractional error for**

1. $R = x^2 y$

2. $R = x^{-1} y^4 z$

**Example**

Calculate fractional error for

1. $R = x^2y$
2. $R = x^{-1}y^4z$

**Solution:**

1. $R = x^2y$

$$\frac{\Delta R}{R} = \frac{2\Delta x}{x} + \frac{\Delta y}{y}$$

2. $R = x^{-1}y^4z$

$$\frac{\Delta R}{R} = \frac{\Delta x}{x} + \frac{4\Delta y}{y} + \frac{\Delta z}{z}$$

**Example**

**Calculate fractional error for** $R = \sqrt{x}/y^2$

**A.** $\frac{\Delta R}{R} = \frac{\Delta x}{x} + \frac{-2\Delta y}{y}$

**B.** $\frac{\Delta R}{R} = \frac{-\frac{1}{2}\Delta x}{x} + \frac{-2\Delta y}{y}$

**C.** $\frac{\Delta R}{R} = \frac{\frac{1}{2}\Delta x}{x} + \frac{2\Delta y}{y}$

**D.** $\frac{\Delta R}{R} = \frac{\Delta x}{x} + \frac{2\Delta y}{y}$

Example

Calculate fractional error for $R = \sqrt{x}/y^2$



**Solution:**

$R = x^{1/2}y^{-2}$

$$\frac{\Delta R}{R} = \frac{\frac{1}{2}\Delta x}{x} + \frac{2\Delta y}{y}$$

**When the expressions are not simple, just differentiate and get the results**

Example

The heat dissipated Q in a resistance(R) can be obtained by the measurement of resistance, current(I) and time(t). If the maximum errors in the measurement of these quantities are 2%, 1% and 1% respectively, then what is the maximum error in the measurement of dissipated heat, $Q = [\, I^2Rt/4.2 \,]$ cal?

A. 3 %
B. 4 %
C. 5 %
D. 6 %

Example

The heat dissipated Q in a resistance(R) can be obtained by the measurement of resistance, current(I) and time(t). If the maximum errors in the measurement of these quantities are 2%, 1% and 1% respectively, then what is the maximum error in the measurement of dissipated heat, Q = [ $I^2Rt/4.2$ ] cal?

Solution:

$$Q = \left(\frac{i^2Rt}{4.2}\right)\text{cal}$$

$$\frac{\Delta Q}{Q} = \frac{2\Delta i}{i} + \frac{\Delta R}{R} + \frac{\Delta t}{t}$$

$$\frac{\Delta Q}{Q}(\%) = 2(1\%) + (2\%) + (1\%)$$

$$\frac{\Delta Q}{Q}(\text{in }\%) = 5$$

Example

Two resistors have resistances $x = (24 \pm 0.5)\ \Omega$ and $y = (8 \pm 0.3)\Omega$ . Calculate the absolute error and the percentage relative error in calculating the combination of two resistances when they are in

(a) Series

A. 0.8, 2.0 %
B. 0.8 , 2.5%
C. 1.2, 2.0 %
D. 1.2, 2.5 %

**Example** Two resistors have resistances $x = (24 \pm 0.5)\,\Omega$ and $y = (8 \pm 0.3)\,\Omega$. Calculate the absolute error and the percentage relative error in calculating the combination of two resistances when they are in (a) Series (b) Parallel

**Solution:**

$$R_{eq} = R_1 + R_2$$

$$\Delta R_{eq} = \Delta R_1 + \Delta R_2$$

$$= 0.5 + 0.3$$

$$\boxed{\Delta R_{eq} = 0.8}$$

Absolute error

$$R_{eq} = 24 + 8$$

$$\boxed{R_{eq} = 32}$$

$$\% \text{ error} = \frac{\Delta Req}{R} \times 100$$

$$= \frac{0.8}{32} \times 100 \boxed{= 2.5\%}$$

$$\boxed{32 \pm 2.5\%}$$

**Example**

Two resistors have resistances $x = (24 \pm 0.5)\ \Omega$ and $y = (8 \pm 0.3)\Omega$ . Calculate the absolute error and the percentage relative error in calculating the combination of two resistances when they are in

(b) Parallel

A. 0.2, 3.33 %
B. 0.3 , 6.66 %
C. 0.2, 0.66 %
D. 0.3, 3.33 %

$x = (24 \pm 0.5)\,\Omega$

$y = (8 \pm 0.3)\,\Omega$

**Example** Two resistors have resistances x = (24 ± 0.5) Ω and y = (8 ± 0.3)Ω . Calculate the absolute error and the percentage relative error in calculating the combination of two resistances when they are in
(b) Parallel

**Solution:**

**6 ± 3.33%**

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \; ; \; R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{24 \times 8}{32} = 6$$

Differentiating

$$-\frac{\Delta R_{eq}}{R_{eq}^{\;2}} = \frac{-1\Delta R_1}{R_1^{\;2}} - \frac{1\Delta R_2}{R_2^{\;2}}$$

$$\frac{\Delta R_{eq}}{R_{eq}^{\;2}} = \frac{0.5}{24^2} + \frac{0.3}{8^2}$$

$$\Delta R_{eq} = 6^2\left(\frac{0.5}{24^2}\right) + 6^2\left(\frac{0.3}{8^2}\right)$$

$$\Delta R_{eq} = \frac{0.5}{4^2} + \frac{9}{16}(0.3) = \boxed{0.2}$$

$$\frac{\Delta R_{eq}}{R} \times 100 = \frac{0.2}{6} 100 = \boxed{3.33}$$

Example

Calculate fractional error for $R = \dfrac{x}{x+y}$

A. $\dfrac{\Delta R}{R} = \dfrac{\Delta y}{x-y} - \dfrac{y\Delta x}{x(x+y)}$

B. $\dfrac{\Delta R}{R} = \dfrac{\Delta y}{x+y} + \dfrac{y\Delta x}{x(x+y)}$

C. $\dfrac{\Delta R}{R} = \dfrac{\Delta y}{x+y} - \dfrac{y\Delta x}{x(x-y)}$

D. $\dfrac{\Delta R}{R} = \dfrac{\Delta y}{x-y} + \dfrac{y\Delta x}{x(x+y)}$

Example

Calculate fractional error for $R = \frac{x}{x + y}$



**Solution:**

$$\frac{1}{R} = \frac{x + y}{x} = 1 + \frac{y}{x}$$

$$\frac{-dR}{R^2} = \frac{x(dy) - (dx)y}{x^2}$$

$$\left|\frac{\Delta R}{R^2}\right| = \left|\frac{\Delta y}{x}\right| + \left|\frac{y\Delta x}{x^2}\right|$$

$$\frac{\Delta R}{R} = \frac{\Delta y}{x+y} + \frac{y\Delta x}{x(x+y)} \checkmark$$