Units & Dimensions

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Conversion **FACTOR**

To find conversion factor between the units of the same physical quantity in two different systems of units

Question Find conversion factor between **SI unit** of Force & **CGS unit** of Force.

Force
SI unit : Newton
CGS unit : Dyne

Solution:

Let 1 Newton = X Dyne

The dimensions of force = $[M^1 L^1 T^{-2}]$

∴ Equation in dimensional form

$$1\left[M_1^1 L_1^1 T_1^{-2}\right] = x\left[M_2^1 L_2^1 T_2^{-2}\right]$$

$$\therefore \mathbf{x} = \frac{\left[M_1^1 L_1^1 T_1^{-2}\right]}{\left[M_2^1 L_2^1 T_2^{-2}\right]} = \left[\frac{M_1}{M_2}\right]^1 \left[\frac{L}{L^1}\right]^1 \left[\frac{T}{T_{12}}\right]^{-2}$$

In SI system, In CGS system,

In SI system,	In CGS system,
$L \longrightarrow m$	$L \longrightarrow cm$
$M \longrightarrow kg$	$M \longrightarrow g$
$T \longrightarrow s$	$T \longrightarrow s$

$$\therefore \quad x = \left(\frac{m}{cm}\right)^{1}\left(\frac{kg}{g}\right)^{1}\left(\frac{s}{s}\right)^{-2}$$

$$= \left(10^{2}\,\frac{cm}{cm}\right)^{1}\left(10^{3}\,\frac{g}{g}\right)^{1}\quad(1)^{-2}$$

$$\therefore \quad x = 10^{5}$$

$\therefore$ 1 Newton = 10^{5} dyne

Question

Find conversion factor between SI unit of Energy and CGS unit of Energy

Force

SI unit : Joule

CGS unit : Erg

Example

The value of acceleration due to gravity is 980 cm/s^2. What will be its value if the unit of length is **kilometer** and that of time is **minute**?

A. 35.3
B. 65.7
C. 85
D. 105

Solution:

The value of **acceleration due to gravity** is 980 cm/s^2 . What will be its value
if the unit of length is **kilometer** and that of time is **minute**?

Any physical quantity can be represented by = magnitude × units

$$\text{So,}\quad 980\,\frac{cm}{s^2} = x\,\frac{km}{min^2}$$

$$\left(\begin{array}{c} 1\,km = 10^3\,m \\ \& \\ 1\,m = 10^2\,cm \end{array}\right)$$

$$\Rightarrow \quad x = 980\left(\frac{cm}{km}\right)\frac{min^2}{s^2}$$

$$= 980\left(\frac{1}{10^5}\right)(60)^2$$

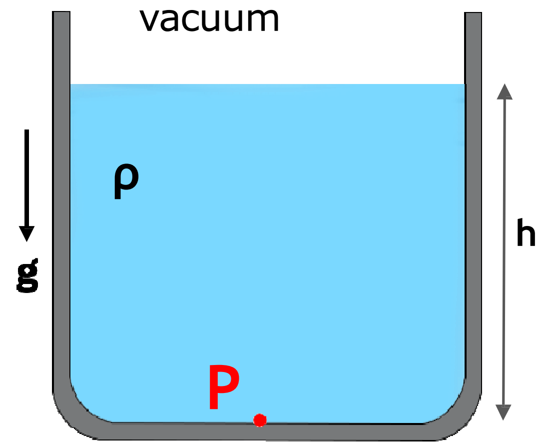
$$\therefore \text{Acceleration due to gravity} = 35.3\,\frac{km}{min^2}$$

$$\boxed{x = 35.3}$$

How is dimensional analysis used?

To establish relationship between related physical quantities.

P is dependent on **h, ρ, g**



How is dimensional analysis used?

To establish the relation $P = h\rho g$

Let us assume that,

$$P = k \; h^x \; \rho^y \; g^z$$

Dimensions of P $= [M^1 L^{-1} T^{-2}]$

Dimensions of h $= [M^0 L^1 T^0]$

Dimensions of ρ $= [M^1 L^{-3} T^0]$

Dimensions of g $= [M^0 L^1 T^{-2}]$

$P \rightarrow$ Hydrostatic Pressure

$h \rightarrow$ Height of liquid volume

$\rho \rightarrow$ Density

$g \rightarrow$ acceleration due to gravity

How is dimensional analysis used?

$\therefore$ our equation becomes,

$$[M^1 L^{-1} T^{-2}] = k[M^0 L^1 T^0]^x \quad [M^1 L^{-3} T^0]^y \quad [M^0 L^1 T^{-2}]^z$$

$$\therefore \quad [M^1 L^{-1} T^{-2}] = k[M^y L^{x-3y+z} T^{-2Z}]$$

Comparing L.H.S and R.H.S,

$x - 3y + z = -1 \quad \Rightarrow \quad y = 1$

$-2z = -2 \quad \therefore \quad z = 1 \quad \therefore \quad x = 1$

Substituting these values,

$P = k \; h^1 \rho^1 g^1$

Here, $k = 1$

$\therefore \quad \mathbf{p = h \rho g}$

Value of 'k' which is a constant cannot be found using dimensional analysis, it can be found out experimentally

Example It has been observed that **velocity of ripple waves** produced in water depends upon their **Wavelength** **(λ)**, **Density of water** **(ρ)** and **Surface Tension** **(S).**

Prove that $v^2 \propto \frac{S}{\lambda\rho}$

Surface tension = $\frac{\text{Force}}{\text{Length}}$

Solution:

According to the problem,

$$v \propto \lambda^a \rho^b S^c$$

$$\Rightarrow \quad v = k\, \lambda^a \rho^b S^c$$

Where k is a dimensionless constant.

$$\Rightarrow \quad LT^{-1} = L^a (ML^{-3})^b (MT^{-2})^c$$

$$\Rightarrow \quad M^0L^1T^{-1} = M^{b+c}\, L^{a-3b}\, T^{-2c}$$

$$\Rightarrow \quad M^0L^1T^{-1} = M^{b+c}\,L^{a-3b}\,T^{-2c}$$

Using the principle of Homogeneity, we get

$b + c = 0$, $a - 3b = 1$, $-2c = -1$

Solving these equations we get,

$$a = -\frac{1}{2}, \quad b = -\frac{1}{2}, \quad c = \frac{1}{2}$$

So, the relation becomes,

$$v = k\,\lambda^{-1/2}\,\rho^{-1/2}\,S^{1/2}$$

$$\Rightarrow \quad v \propto \sqrt{\frac{S}{\lambda\rho}} \quad \Rightarrow \quad v^2 \propto \frac{S}{\lambda\rho}$$

Example

If velocity of **light** **(v)**, **force (F)** and **time (t)** are taken as **fundamental dimensions**. Then dimensions of **mass** in this system will be …….

A. $F^1T^3v^0$
B. $F^1T^1v^{-1}$
C. $F^2T^0v^1$
D. $F^2T^0v^{-1}$

t

Example

If velocity of **light** **(v)**, **force (F)** and **time (t)** are taken as **fundamental dimensions**. Then dimensions of mass in this system will be ....

Solution:

We know,

Dimensions of mass $= [M]$

Dimensions of velocity, $v = [LT^{-1}]$

Dimensions of force, $F = [MLT^{-2}]$

Dimensions of time, $t = [T]$

Let **mass** $= v^a F^b t^c$

$\therefore \quad [M] = [LT^{-1}]^a [MLT^{-2}]^b [T]^c$

$\Rightarrow \quad [M] = [M^b L^{a+b} T^{-a-2b+c}]$

Equating the powers of dimensions on both sides

for M $\longrightarrow$ $\boxed{1 = b}$ $\qquad \Rightarrow \boxed{c = 1}$

for L $\longrightarrow$ $0 = a + b \Rightarrow 0 = a + 1 \Rightarrow \boxed{a = -1}$

for T $\longrightarrow$ $0 = -a - 2b + c \Rightarrow 0 = +1 - 2 + c$

Dimensions of mass in this system [mass] $= v^{-1} F^1 t^1$

Example

If Pressure **(P)**, Length **(L)** and Momentum **(J)** are taken as **fundamental quantities.** Find the dimensions of **Energy** **(E)**

A. $P^1L^3J^0$
B. $P^1L^3J^{-1}$
C. $P^2L^0J^1$
D. $P^2L^0J^{-1}$

Example If Pressure **(P)**, Length **(L)** and Momentum **(J)** are taken as **fundamental quantities.** Find the dimensions of **Energy** **(E)**

Solution: Let $E = k\, P^a L^b J^c$

$$\Rightarrow \quad [ML^2T^{-2}] = [ML^{-1}T^{-2}]^a [L]^b [MLT^{-1}]^c$$

$$\Rightarrow \quad [ML^2T^{-2}] = [M^{a+c}\, L^{-a+b+c}\, T^{-2a-c}]$$

for $M \rightarrow 1 = a + c$ ...(i)

for $L \rightarrow 2 = -a + b + c$ ...(ii)

for $T \rightarrow -2 = -2a - c$ ...(iii)

We know,

Dimensions of pressure, $P = [ML^{-1}T^{-2}]$

Dimensions of Length, $L = [L]$

Dimensions of momentum, $J = [MLT^{-1}]$

Dimensions of energy $E = [ML^2T^{-2}]$

Equating the powers of dimensions an both sides

Example

If Pressure **(P)**, Length **(L)** and Momentum **(J)** are taken as **fundamental quantities.** Find the dimensions of **Energy** **(E)**

Solution:

If pressure (P), length (L) and momentum (J) are taken as fundamental quantities find the dimensions of energy (E)

$1 = a + c \quad \ldots(i)$

$2 = -a + b + c \quad \ldots(ii)$

$-2 = -2a - c \quad \ldots(iii)$

Using, (i) + (iii), we get

$-1 = -a \Rightarrow \boxed{a = 1}$

$\therefore$ From (i) $\boxed{c = 0}$

and from (ii)

$2 = -1 + b + 0 \Rightarrow \boxed{b = 3}$

$\therefore$ Dimensions of energy in new system are

$[E] = P^1L^3J^0$

LIMITATIONS OF DIMENSIONAL ANALYSIS

The method of dimensions has the following limitations:

(i) By this method the value of dimensionless constant cannot be calculated.

(ii) By this method the equation containing trigonometric, exponential and logarithmic terms cannot be analyzed.

(iii) If a physical quantity in mechanics depends on more than three factors, then relation among them cannot be established because we can have only three equations by equalizing the powers of M, L and T.

(iv) It doesn't tell whether the quantity is vector or scalar.

1.5 Advantages of SI system

(i) **SI is a coherent system of units.** All derived units can be obtained by simple multiplication or devision of fundamental units without introducing any numerical factor.

(ii) **SI is a rational system of units.** It uses only one unit for a given physical quantity. For example all forms of energy are measured in joule, heat energy in calories and electrical energy in watt hour.

(iii) **SI is a metric system.** The mulitples and subsultiples of SI units can be expressed as powers of 10.

(iv) **SI is an absolute system of units.** It does not used gravitational units. The use of 'g' is not required.

(v) **SI is an internationally accepted system of units.**