Math 323 - Assignment 1

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Problem 1

Suppose a family contains two children of different ages, and we are interested in the gender of these children. Let F denote a female child, and M denote male child, such that the ordered pair FM denotes an older female child and a younger male child without loss of generality. In the set S, we thus have:

$$S = \{FF,\ FM,\ MF,\ MM\}$$

Let A denote the subset of possibilities containing no males, B denote the subset containing two males, and C the subset containing at least one male. List the elements of $A, B, C, A \cap B, A \cup B, A \cap C, A \cup C, B \cap C, B \cup C,$ and $C \cap \bar{B}$.

Solution.

$$A = \{FF\}$$
$$B = \{MM\}$$
$$\bar{B} = \{FF,\ FM,\ MF\}$$
$$C = \{FM,\ MF,\ MM\}$$
$$A \cap B = \{\} = \varnothing$$
$$A \cup B = \{FF,\ MM\}$$
$$A \cap C = \{\} = \varnothing$$
$$A \cup C = \{FF,\ FM,\ MF,\ MM\} = S$$
$$B \cap C = \{MM\} = B$$
$$B \cup C = \{FM,\ MF,\ MM\} = C$$
$$C \cap \bar{B} = \{FM,\ MF\}$$

Problem 2

Suppose two dice are tossed, and the numbers on he upper faces are observed. Let S denote the set of all possible pairs that can be observed. [These pairs can be listed, for example, by letting $(2,3)$ denote that a 2 was observed on the first die, and a 3 on the second.]

   a) Define the following subsets of S:

$$A : \textit{The number on the second die is even.}$$
$$B : \textit{The sum of the two numbers is even.}$$
$$C : \textit{At least one number in the pair is odd.}$$

   b) List the points in $A, \bar{C}, A \cap B, A \cap \bar{B}, \bar{A} \cup B$, and $\bar{A} \cap C$

Solution.

   a) We begin by defining the a particular subset of the natural numbers $\mathbb{N}_k$. Let k be a positive integer such that:

$$\mathbb{N}_k = \{1, 2, \ldots, k\}$$

The particular subset N_k has the following multiplicative property for $\alpha \in \mathbb{N}$:

$$\mathbb{N}_{\alpha(k)} = \{\alpha(1), \alpha(2), \ldots, \alpha(k)\}$$

We can thus define the set $S = \{(a, b) : a, b \in \mathbb{N}_6\}$ where a denotes the result of the first die, and b denotes the result of the second die. Now, we can use the same notation to define the required subsets:

$$A = \{(a, b) : a \in \mathbb{N}_6,\ b \in \mathbb{N}_{2(3)}\}$$
$$B = \{(a, b) : a + b \in \mathbb{N}_{2(6)}\}$$
$$C = \{(a, b) : a \notin \mathbb{N}_{2(3)} \quad \nabla \quad b \notin \mathbb{N}_{2(3)}\}$$

Note the use of ∇ (XOR), which is the exclusive $\vee$ (OR).

b)

$$A = \{(1,2),(1,4),(1,6),(2,2),(2,4),(2,6),(3,2),(3,4),(3,6),(4,2),(4,4),(4,6),$$
$$(5,2),(5,4),(5,6),(6,2),(6,4),(6,6)\}$$