Math 323 - Assignment 1

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Problem 1

Suppose a family contains two children of different ages, and we are interested in the gender of these children. Let F denote a female child, and M denote male child, such that the ordered pair FM denotes an older female child and a younger male child without loss of generality. In the set S, we thus have:

$$S = \{FF,\, FM,\, MF,\, MM\}$$

Let A denote the subset of possibilities containing no males, B denote the subset containing two males, and C the subset containing at least one male. List the elements of $A, B, C, A \cap B, A \cup B, A \cap C, A \cup C, B \cap C, B \cup C$, and $C \cap \bar{B}$.

Solution.

$$\begin{aligned}
A &= \{FF\} \\
B &= \{MM\} \\
\bar{B} &= \{FF,\, FM,\, MF\} \\
C &= \{FM,\, MF,\, MM\} \\
A \cap B &= \{\} = \varnothing \\
A \cup B &= \{FF,\, MM\} \\
A \cap C &= \{\} = \varnothing \\
A \cup C &= \{FF,\, FM,\, MF,\, MM\} = S \\
B \cap C &= \{MM\} = B \\
B \cup C &= \{FM,\, MF,\, MM\} = C \\
C \cap \bar{B} &= \{FM,\, MF\}
\end{aligned}$$

Problem 2

Suppose two dice are tossed, and the numbers on he upper faces are observed. Let S denote the set of all possible pairs that can be observed. [These pairs can be listed, for example, by letting $(2, 3)$ denote that a 2 was observed on the first die, and a 3 on the second.]

a) Define the following subsets of S:

$$A : \textit{The number on the second die is even.}$$

$$B : \textit{The sum of the two numbers is even.}$$

$$C : \textit{At least one number in the pair is odd.}$$

b) List the points in $A, \bar{C}, A \cap B, A \cap \bar{B}, \bar{A} \cup B$, and $\bar{A} \cap C$

Solution.

a) We begin by defining the a particular subset of the natural numbers $\mathbb{N}_k$. Let k be a positive integer such that:

$$\mathbb{N}_k = \{1, 2, \ldots, k\}$$

The particular subset N_k has the following multiplicative property for $\alpha \in \mathbb{N}$:

$$\mathbb{N}_{\alpha(k)} = \{\alpha(1), \alpha(2), \ldots, \alpha(k)\}$$

We can thus define the set $S = \{(a, b) : a, b \in \mathbb{N}_6\}$ where a denotes the result of the first die, and b denotes the result of the second die. Now, we can use the same notation to define the required subsets:

$$A = \{(a, b) : b \in \mathbb{N}_{2(3)}\}$$

$$B = \{(a, b) : a + b \in \mathbb{N}_{2(6)}\}$$

$$C = \{(a, b) : a \notin \mathbb{N}_{2(3)} \quad \nabla \quad b \notin \mathbb{N}_{2(3)}\}$$

Note the use of ∇ (XOR), which is the exclusive $\vee$ (OR).

b)

$$
\begin{aligned}
A = \{&(1,2),(1,4),(1,6),(2,2),(2,4),(2,6),(3,2),(3,4),(3,6),(4,2),(4,4),(4,6),\\
&(5,2),(5,4),(5,6),(6,2),(6,4),(6,6)\}\\
\bar{C} = \{&(2,2),(2,4),(2,6),(4,2),(4,4),(4,6),(6,2),(6,4),(6,6)\}\\
A \cap B = \{&(1,2),(1,4),(1,6),(2,2),(2,4),(2,6),(3,2),(3,4),(3,6),(4,2),(4,4),(4,6),\\
&(5,2),(5,4),(5,6),(6,2),(6,4),(6,6)\} \cap \{(1,1),(1,3),(1,5),(2,2),\\
&(2,4),(2,6),(3,1),(3,3),(3,5),(4,2),(4,4),(4,6),(5,1),(5,3),(5,5),\\
&(6,2),(6,4),(6,6)\}\\
= \{&(2,2),(2,4),(2,6),(4,2),(4,4),(4,6),(6,2),(6,4),(6,6)\} = \bar{C}\\
A \cap \bar{B} = \{&(1,2),(1,4),(1,6),(2,2),(2,4),(2,6),(3,2),(3,4),(3,6),(4,2),(4,4),(4,6),\\
&(5,2),(5,4),(5,6),(6,2),(6,4),(6,6)\} \cap \{(1,2),(1,4),(1,6),(2,1),\\
&(2,3),(2,5),(3,2),(3,4),(3,6),(4,1),(4,3),(4,5),(5,2),(5,4),(5,6),\\
&(6,1),(6,3),(6,5)\}\\
= \{&(1,2),(1,4),(1,6),(3,2),(3,4),(3,6),(5,2),(5,4),(5,6)\}\\
\bar{A} \cup B = \{&(1,1),(1,3),(1,5),(2,1),(2,3),(2,5),(3,1),(3,3),(3,5),(4,1),(4,3),(4,5),\\
&(5,1),(5,3),(5,5),(6,1),(6,3),(6,5)\} \cup \{(1,1),(1,3),(1,5),(2,2),\\
&(2,4),(2,6),(3,1),(3,3),(3,5),(4,2),(4,4),(4,6),(5,1),(5,3),(5,5),\\
&(6,2),(6,4),(6,6)\}\\
= \{&(1,1),(1,3),(1,5),(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),(3,1),(3,3),(3,5),\\
&(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),(5,1),(5,3),(5,5),(6,1),(6,2),\\
&(6,3),(6,4),(6,5),(6,6)\}\\
\bar{A} \cap C = \{&(1,1),(1,3),(1,5),(2,1),(2,3),(2,5),(3,1),(3,3),(3,5),(4,1),(4,3),(4,5),\\
&(5,1),(5,3),(5,5),(6,1),(6,3),(6,5)\} \cap \{(1,1),(1,2),(1,3),(1,4),\\
&(1,5),(1,6),(2,1),(2,3),(2,5),(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),\\
&(4,1),(4,3),(4,5),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),(6,1),(6,3),\\
&(6,5)\}\\
= \{&(1,1),(1,3),(1,5),(2,1),(2,3),(2,5),(3,1),(3,3),(3,5),(4,1),(4,3),(4,5),\\
&(5,1),(5,3),(5,5),(6,1),(6,3),(6,5)\} = \bar{A}
\end{aligned}
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