Math 323 - Assignment 1

Sada Sólomon, Ignacio - Id 260708051

Winter 2020

Problem 1

Suppose a family contains two children of different ages, and we are interested in the gender of these children. Let F denote a female child, and M denote male child, such that the ordered pair FM denotes an older female child and a younger male child without loss of generality. In the set S, we thus have:

$$S = \{FF,\, FM,\, MF,\, MM\}$$

Let A denote the subset of possibilities containing no males, B denote the subset containing two males, and C the subset containing at least one male. List the elements of $A, B, C, A \cap B, A \cup B, A \cap C, A \cup C, B \cap C, B \cup C$, and $C \cap \bar{B}$.

Solution.

$$\begin{aligned}
A &= \{FF\} \\
B &= \{MM\} \\
\bar{B} &= \{FF,\, FM,\, MF\} \\
C &= \{FM,\, MF,\, MM\} \\
A \cap B &= \{\} = \varnothing \\
A \cup B &= \{FF,\, MM\} \\
A \cap C &= \{\} = \varnothing \\
A \cup C &= \{FF,\, FM,\, MF,\, MM\} = S \\
B \cap C &= \{MM\} = B \\
B \cup C &= \{FM,\, MF,\, MM\} = C \\
C \cap \bar{B} &= \{FM,\, MF\}
\end{aligned}$$

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Problem 2

Suppose two dice are tossed, and the numbers on he upper faces are observed. Let S denote the set of all possible pairs that can be observed. [These pairs can be listed, for example, by letting $(2, 3)$ denote that a 2 was observed on the first die, and a 3 on the second.]

a) *Define the following subsets of S:*

$$A: \textit{The number on the second die is even.}$$
$$B: \textit{The sum of the two numbers is even.}$$
$$C: \textit{At least one number in the pair is odd.}$$

b) *List the points in $A, \bar{C}, A \cap B, A \cap \bar{B}, \bar{A} \cup B$, and $\bar{A} \cap C$*

Solution.

a) We begin by defining the a particular subset of the natural numbers $\mathbb{N}_k$. Let k be a positive integer such that:

$$\mathbb{N}_k = \{1, 2, \ldots, k\}$$

The particular subset N_k has the following multiplicative property for $\alpha \in \mathbb{N}$:

$$\mathbb{N}_{\alpha(k)} = \{\alpha(1), \alpha(2), \ldots, \alpha(k)\}$$

We can thus define the set $S = \{(a, b) : a, b \in \mathbb{N}_6\}$ where a denotes the result of the first die, and b denotes the result of the second die. Now, we can use the same notation to define the required subsets:

$$\begin{aligned}
A &= \{(a, b) : b \in \mathbb{N}_{2(3)}\} \\
B &= \{(a, b) : a + b \in \mathbb{N}_{2(6)}\} \\
C &= \{(a, b) : a \notin \mathbb{N}_{2(3)} \quad \nabla \quad b \notin \mathbb{N}_{2(3)}\}
\end{aligned}$$

Note the use of ∇ (XOR), which is the exclusive $\vee$ (OR).

b)

$$
\begin{aligned}
A &= \{(1,2),(1,4),(1,6),(2,2),(2,4),(2,6),(3,2),(3,4),(3,6),(4,2),(4,4),(4,6),\\
&\qquad (5,2),(5,4),(5,6),(6,2),(6,4),(6,6)\}\\
\bar{C} &= \{(2,2),(2,4),(2,6),(4,2),(4,4),(4,6),(6,2),(6,4),(6,6)\}\\
A \cap B &= \{(1,2),(1,4),(1,6),(2,2),(2,4),(2,6),(3,2),(3,4),(3,6),(4,2),(4,4),(4,6),\\
&\qquad (5,2),(5,4),(5,6),(6,2),(6,4),(6,6)\} \cap \{(1,1),(1,3),(1,5),(2,2),\\
&\qquad (2,4),(2,6),(3,1),(3,3),(3,5),(4,2),(4,4),(4,6),(5,1),(5,3),(5,5),\\
&\qquad (6,2),(6,4),(6,6)\}\\
&= \{(2,2),(2,4),(2,6),(4,2),(4,4),(4,6),(6,2),(6,4),(6,6)\} = \bar{C}\\
A \cap \bar{B} &= \{(1,2),(1,4),(1,6),(2,2),(2,4),(2,6),(3,2),(3,4),(3,6),(4,2),(4,4),(4,6),\\
&\qquad (5,2),(5,4),(5,6),(6,2),(6,4),(6,6)\} \cap \{(1,2),(1,4),(1,6),(2,1),\\
&\qquad (2,3),(2,5),(3,2),(3,4),(3,6),(4,1),(4,3),(4,5),(5,2),(5,4),(5,6),\\
&\qquad (6,1),(6,3),(6,5)\}\\
&= \{(1,2),(1,4),(1,6),(3,2),(3,4),(3,6),(5,2),(5,4),(5,6)\}\\
\bar{A} \cup B &= \{(1,1),(1,3),(1,5),(2,1),(2,3),(2,5),(3,1),(3,3),(3,5),(4,1),(4,3),(4,5),\\
&\qquad (5,1),(5,3),(5,5),(6,1),(6,3),(6,5)\} \cup \{(1,1),(1,3),(1,5),(2,2),\\
&\qquad (2,4),(2,6),(3,1),(3,3),(3,5),(4,2),(4,4),(4,6),(5,1),(5,3),(5,5),\\
&\qquad (6,2),(6,4),(6,6)\}\\
&= \{(1,1),(1,3),(1,5),(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),(3,1),(3,3),(3,5),\\
&\qquad (4,1),(4,2),(4,3),(4,4),(4,5),(4,6),(5,1),(5,3),(5,5),(6,1),(6,2),\\
&\qquad (6,3),(6,4),(6,5),(6,6)\}\\
\bar{A} \cap C &= \{(1,1),(1,3),(1,5),(2,1),(2,3),(2,5),(3,1),(3,3),(3,5),(4,1),(4,3),(4,5),\\
&\qquad (5,1),(5,3),(5,5),(6,1),(6,3),(6,5)\} \cap \{(1,1),(1,2),(1,3),(1,4),\\
&\qquad (1,5),(1,6),(2,1),(2,3),(2,5),(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),\\
&\qquad (4,1),(4,3),(4,5),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),(6,1),(6,3),\\
&\qquad (6,5)\}\\
&= \{(1,1),(1,3),(1,5),(2,1),(2,3),(2,5),(3,1),(3,3),(3,5),(4,1),(4,3),(4,5),\\
&\qquad (5,1),(5,3),(5,5),(6,1),(6,3),(6,5)\} = \bar{A}
\end{aligned}
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Problem 3

A survey classified a large number of adults according to whether they were diagnosed as needing eyeglasses to correct their reading vision, and whether they use eyeglasses when reading. The proportions falling into the four resulting categories are given in the following table:

	Uses Eyeglasses for Reading?	
Needs glasses?	Yes	No
Yes	0.44	0.14
No	0.02	0.40

If a single adult is selected from the large group, find the probabilities of the events defined below. The adult

a) needs glasses.

b) needs glasses but does not use them.

c) uses glasses whether the glasses are needed or not.

Solution.

a) An adult from the large group that needs glasses has a probability of $0.44 + 0.14 = 0.60$.

b) An adult from the large group that needs glasses, but does not use them, has a probability of 0.14.

c) An adult from the large group that uses glasses regardless of whether they are needed or not has a probability $0.44 + 0.02 = 0.46$

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