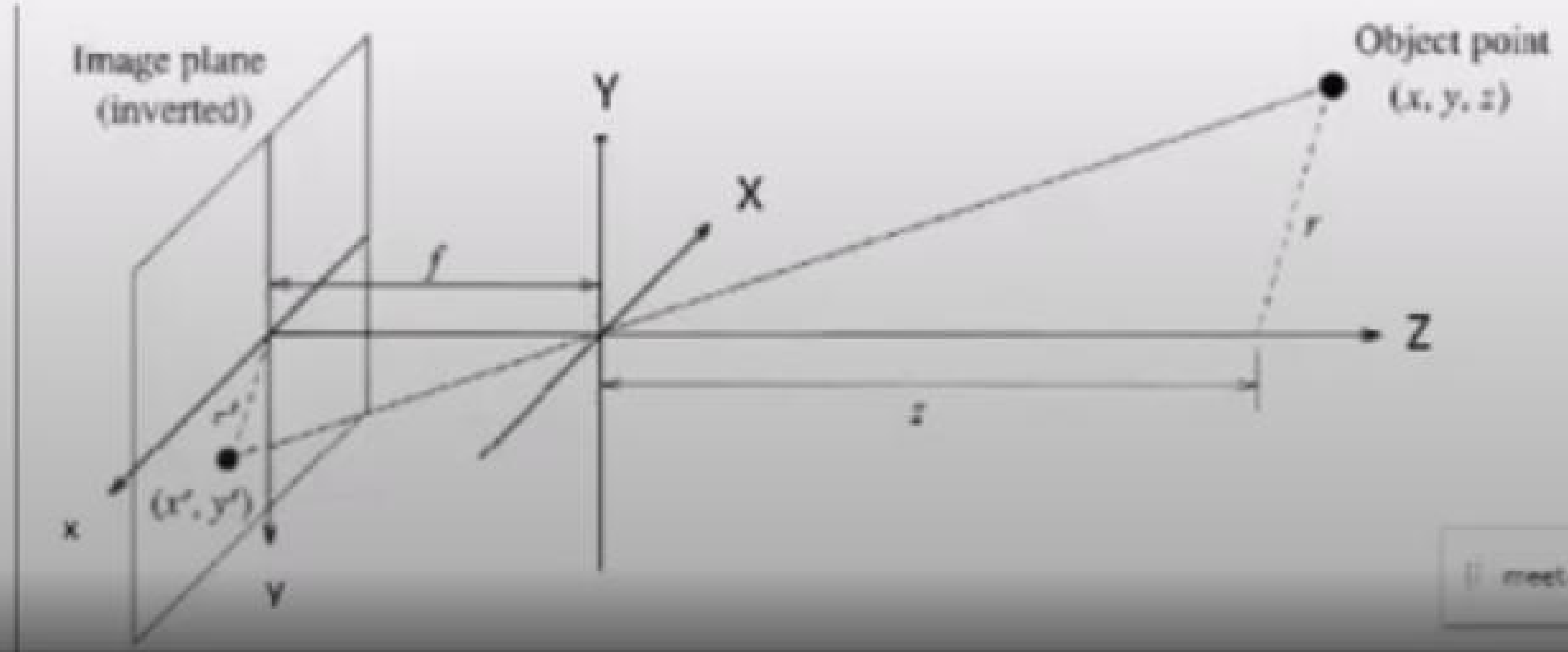
World and Camera coordinate systems (cont'd)

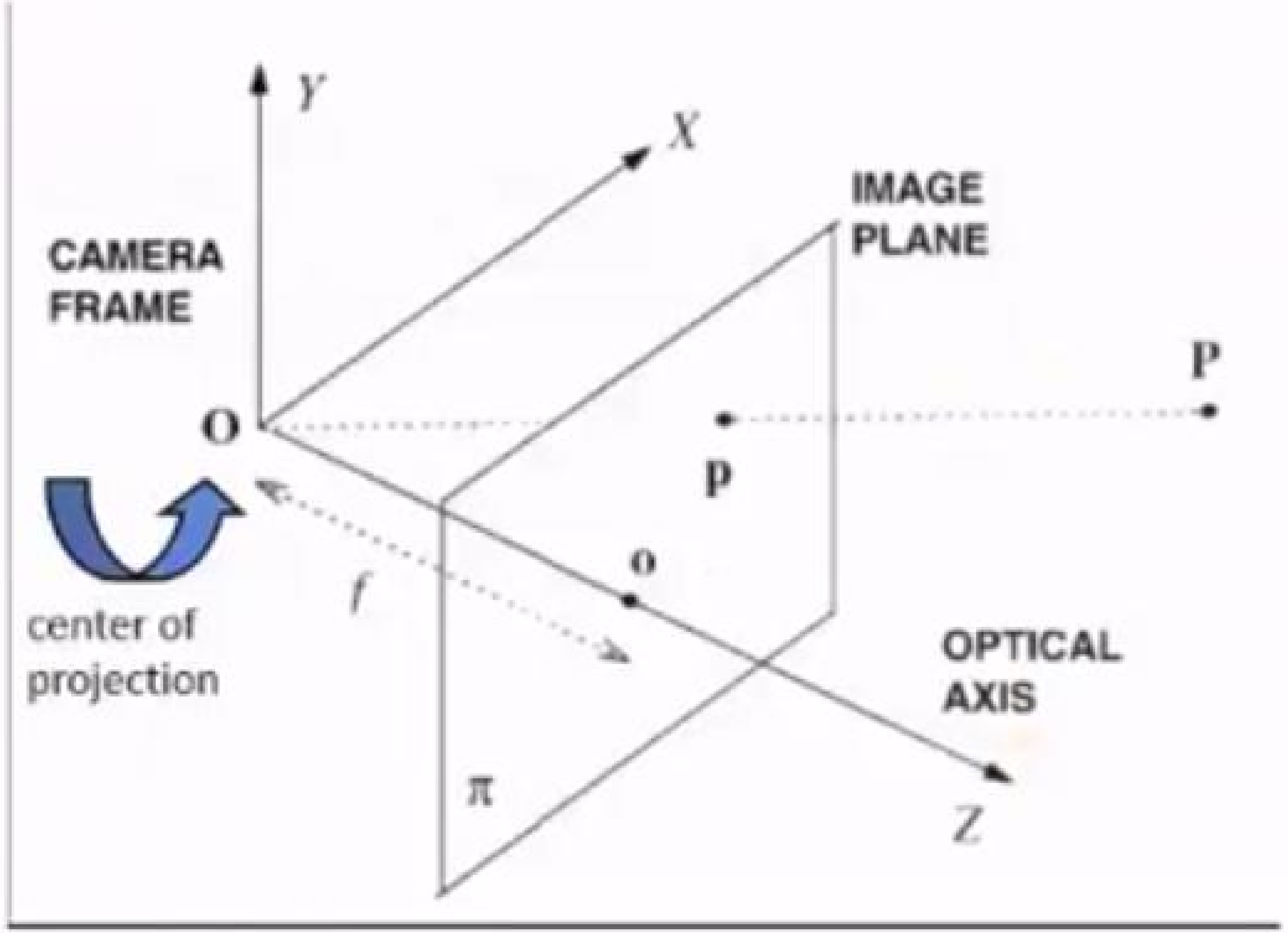
- To simplify the derivation of the perspective projection equations, we will make the following assumptions:
  (1) the center of projection coincides with the origin of the world coordinate system.
  (2) the camera axis (i.e., optical axis) is aligned with the world's z-axis.



World and Camera coordinate systems (cont'd)

(3) avoid image inversion by assuming that the image plane is in front of the center of projection.

(4) the origin of the image plane is the principal point.



2D Translation using homogeneous coordinates

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & dx \\ 0 & 1 & dy \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

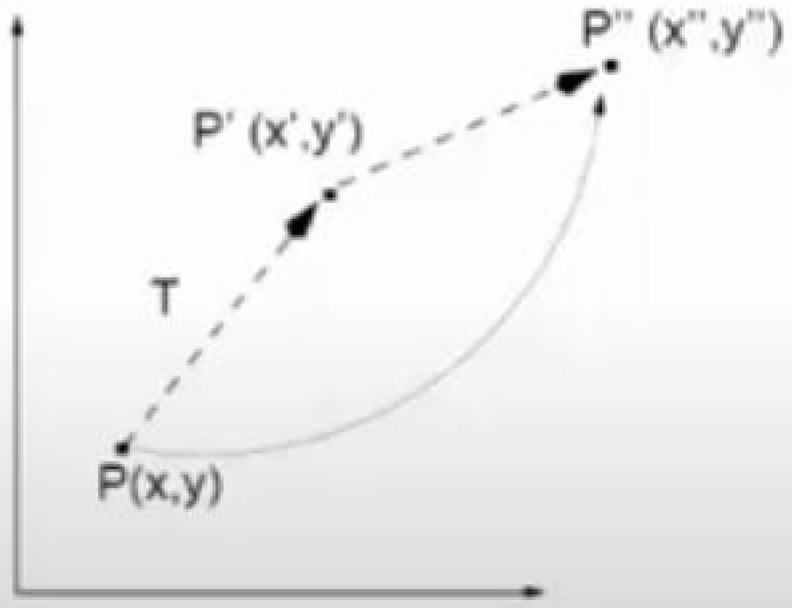
w=1

$$x' = x + dx, \quad y' = y + dy$$

$$P' = T(dx, dy)\, P$$

2D Translation using homogeneous coordinates

- Successive translations:



$$P' = T(dx_1, dy_1)\, P, \quad P'' = T(dx_2, dy_2)\, P'$$

$$P'' = T(dx_2, dy_2)T(dx_1, dy_1)\, P = T(dx_1 + dx_2, dy_1 + dy_2)\, P$$

$$\begin{bmatrix} 1 & 0 & dx_2 \\ 0 & 1 & dy_2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & dx_1 \\ 0 & 1 & dy_1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & dx_1 + dx_2 \\ 0 & 1 & dy_1 + dy_2 \\ 0 & 0 & 1 \end{bmatrix}$$

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2D Scaling using homogeneous coordinates

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

w=1

$$x' = x\, s_x,\ y' = y\, s_y$$

$$\underline{P' = S(s_x, s_y)\, P}$$

2D Scaling using homogeneous coordinates

- Successive scalings:

$$P' = S(s_{x_1}, s_{y_1})\ P, \quad P'' = S(s_{x_2}, s_{y_2})\ P'$$

$$P'' = S(s_{x_2}, s_{y_2}) S(s_{x_1}, s_{y_1})\ P = S(s_{x_1} s_{x_2}, s_{y_1} s_{y_2})\ P$$

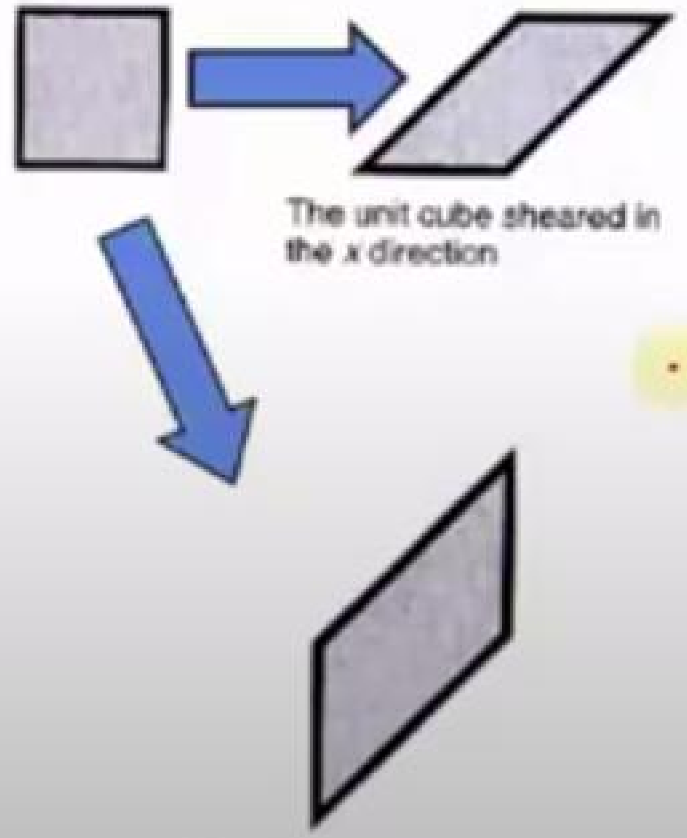
$$\begin{bmatrix} s_{x_2} & 0 & 0 \\ 0 & s_{y_2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_{x_1} & 0 & 0 \\ 0 & s_{y_1} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} s_{x_2} s_{x_1} & 0 & 0 \\ 0 & s_{y_2} s_{y_1} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2D shear transformation

- Shearing along x-axis:

changes object shape!

$$x' = x + ay,\ y' = y \qquad SH_x = \begin{bmatrix} 1 & a & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



The unit cube sheared in the x direction

- Shearing along y-axis

$$x' = x,\ y' = bx + y \qquad SH_y = \begin{bmatrix} 1 & 0 & 0 \\ b & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The unit cube sheared in the y direction

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2D Rotation using homogeneous coordinates

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} cos(\theta) & -sin(\theta) & 0 \\ sin(\theta) & cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

w=1

$$x' = xcos(\theta) - ysin(\theta), \quad y' = xsin(\theta) + ycos(\theta)$$

$$\underline{P' = R(\theta)\ P}$$

General form of transformation matrix

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Representing a sequence of transformations as a single transformation matrix is more efficient!

$$x' = a_{11}x + a_{12}y + a_{13}$$

$$y' = a_{21}x + a_{22}y + a_{23}$$

(only 4 multiplications and 4 additions)

General form of transformation matrix

$$
\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} =
\begin{array}{c}
\begin{array}{cc} \text{rotation, scale} & \text{translation} \end{array} \\
\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix}
\end{array}
\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}
$$

- Representing a sequence of transformations as a single transformation matrix is more efficient!

$$x' = a_{11}x + a_{12}y + a_{13}$$

$$y' = a_{21}x + a_{22}y + a_{23}$$

(only 4 multiplications and 4 additions)

3D Homogeneous coordinates

- Add one more coordinate: $(x,y,z) \rightarrow (x_h, y_h, z_h, w)$
- Recover (x,y,z) by homogenizing (x_h, y_h, z_h, w):

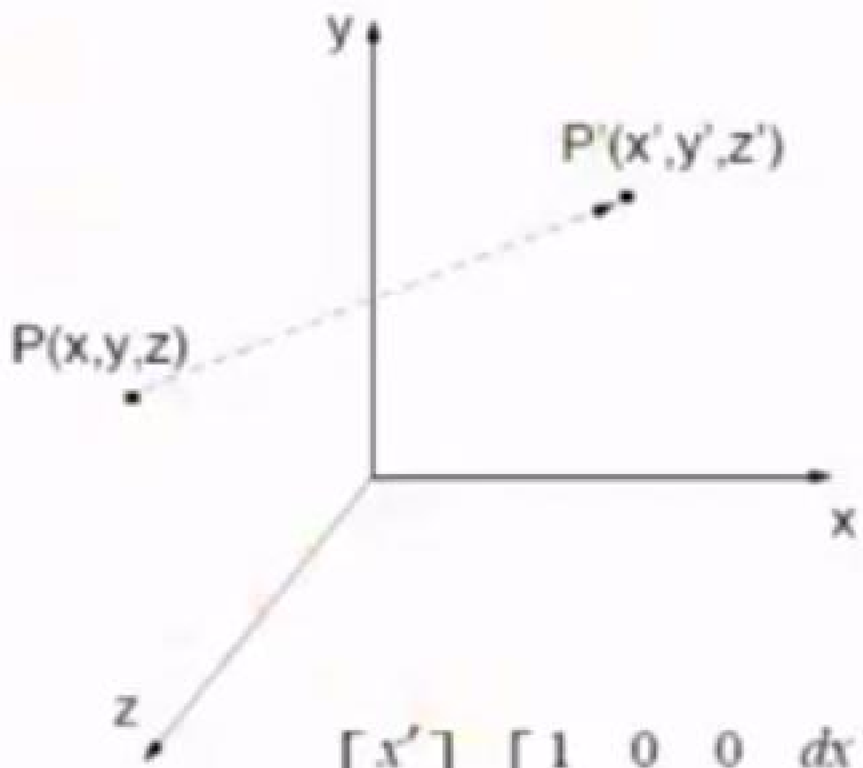
$$x = \frac{x_h}{w}, \; y = \frac{y_h}{w}, \; z = \frac{z_h}{w}, \; w \neq 0$$

- In general, $x_h = xw$, $y_h = yw$, $z_h = zw$

$$(x, y, z) \rightarrow (xw, yw, zw, w) \qquad (w \neq 0)$$

- Each point (x, y, z) corresponds to a line in the 4D-space of homogeneous coordinates.

3D Translation



$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & dx \\ 0 & 1 & 0 & dy \\ 0 & 0 & 1 & dz \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\underline{P' = T(dx, dy, dz)\ P}$$

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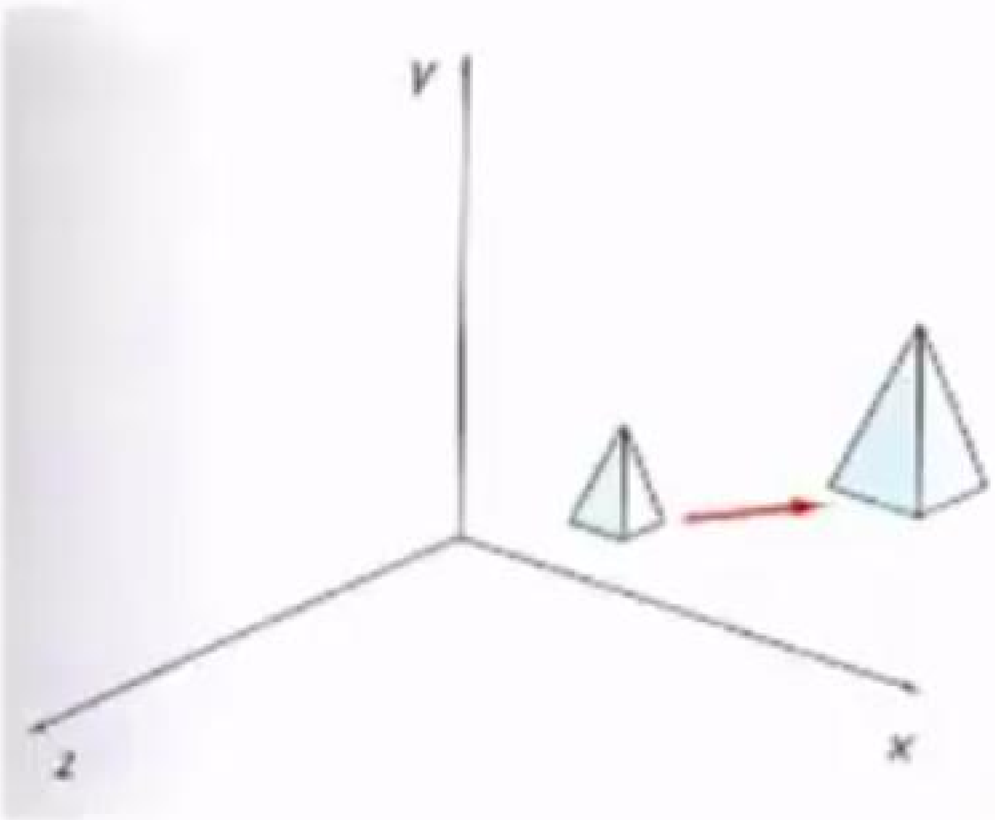
3D Scaling



$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\underline{P' = S(s_x, s_y, s_z)\ P}$$

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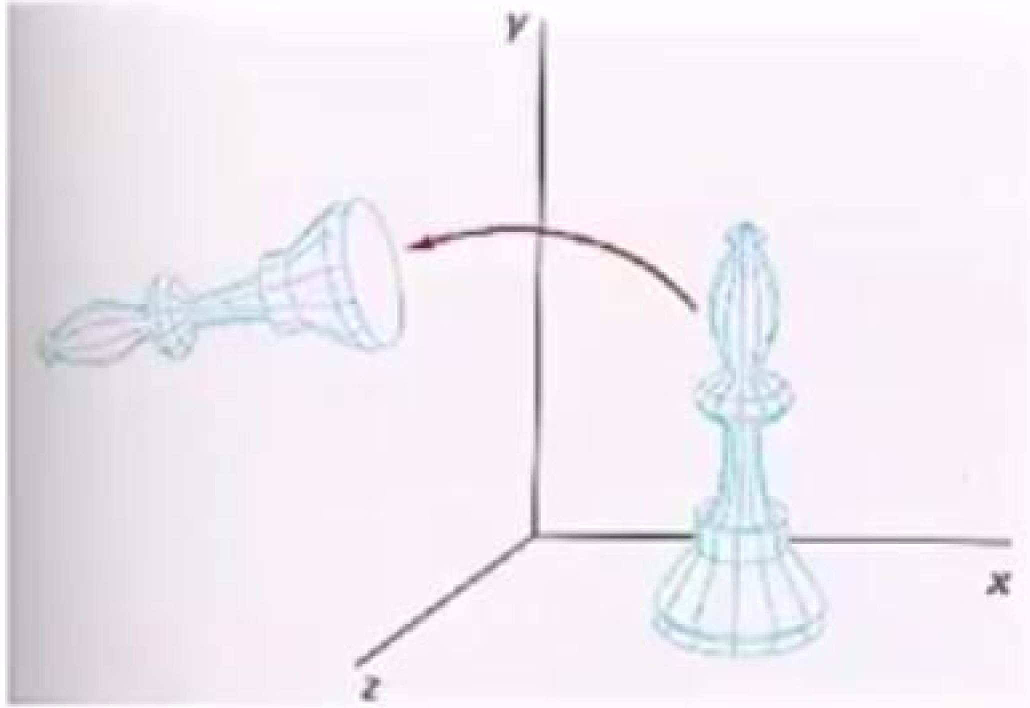
3D Rotation

- Rotation about the *z*-axis:



$$x' = x\cos(\theta) - y\sin(\theta)$$

$$y' = x\sin(\theta) + y\cos(\theta)$$

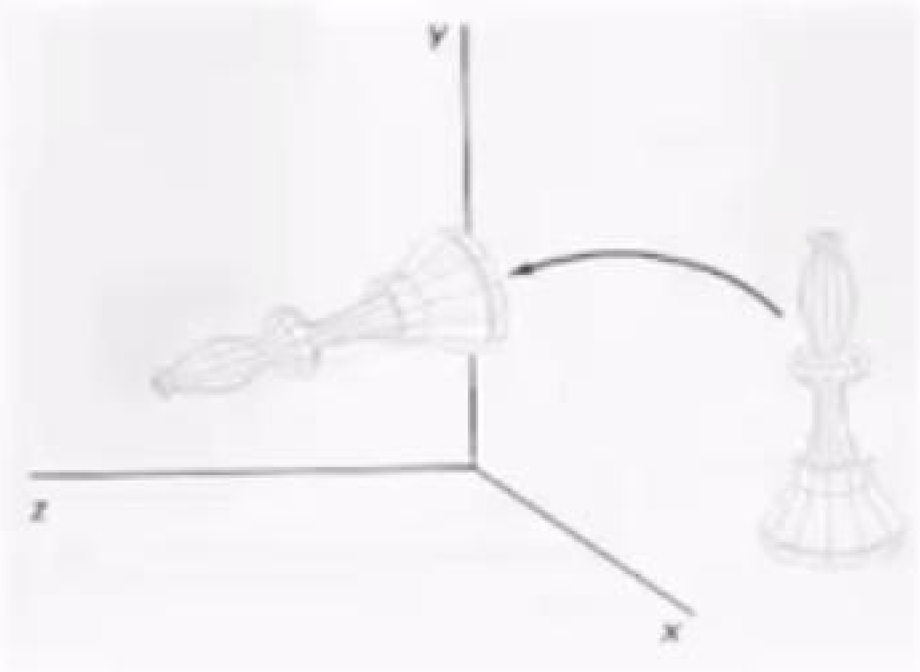
$$z' = z$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\underline{P' = R_z(\theta)\ P}$$

3D Rotation

- Rotation about the *x*-axis:



$$x' = x$$

$$y' = y\cos(\theta) - z\sin(\theta)$$

$$z' = y\sin(\theta) + z\cos(\theta)$$

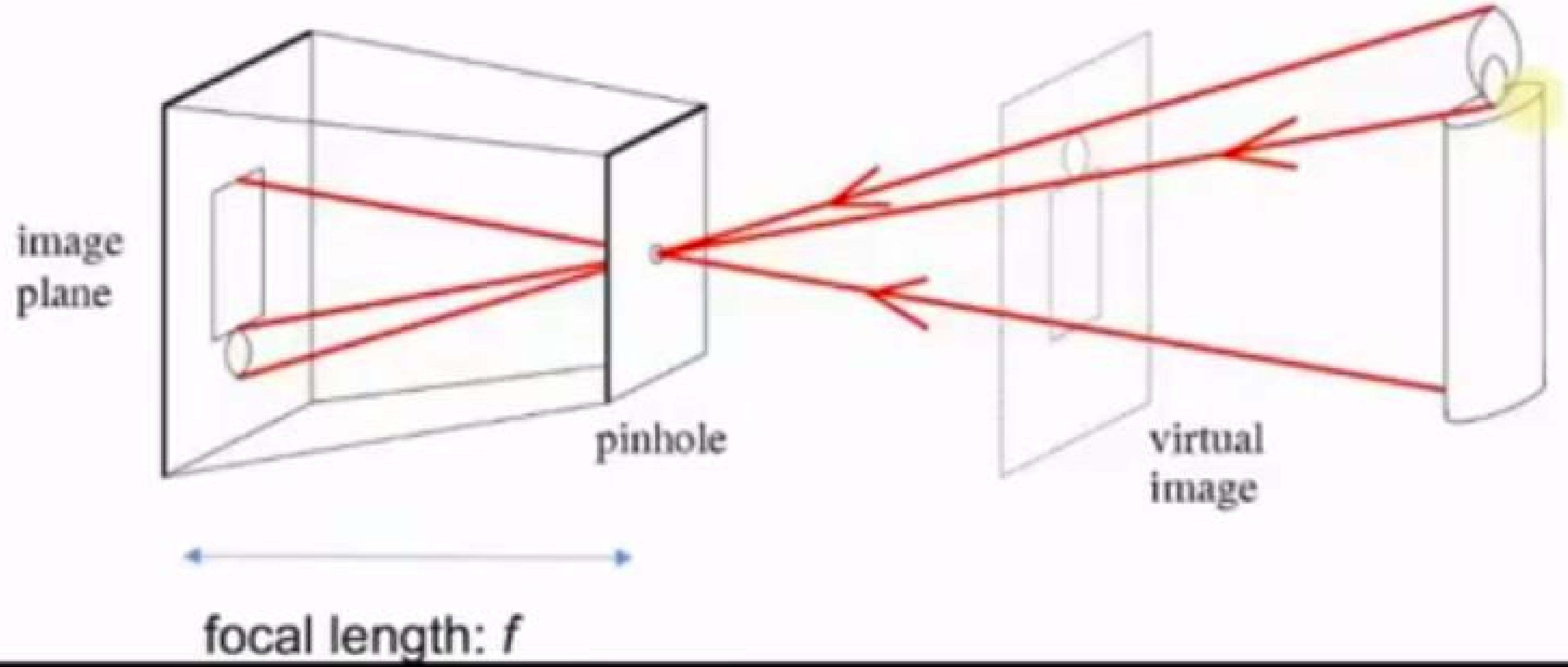
$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) & 0 \\ 0 & \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\underline{P' = R_x(\theta)\ P}$$

Necessity of an aperture

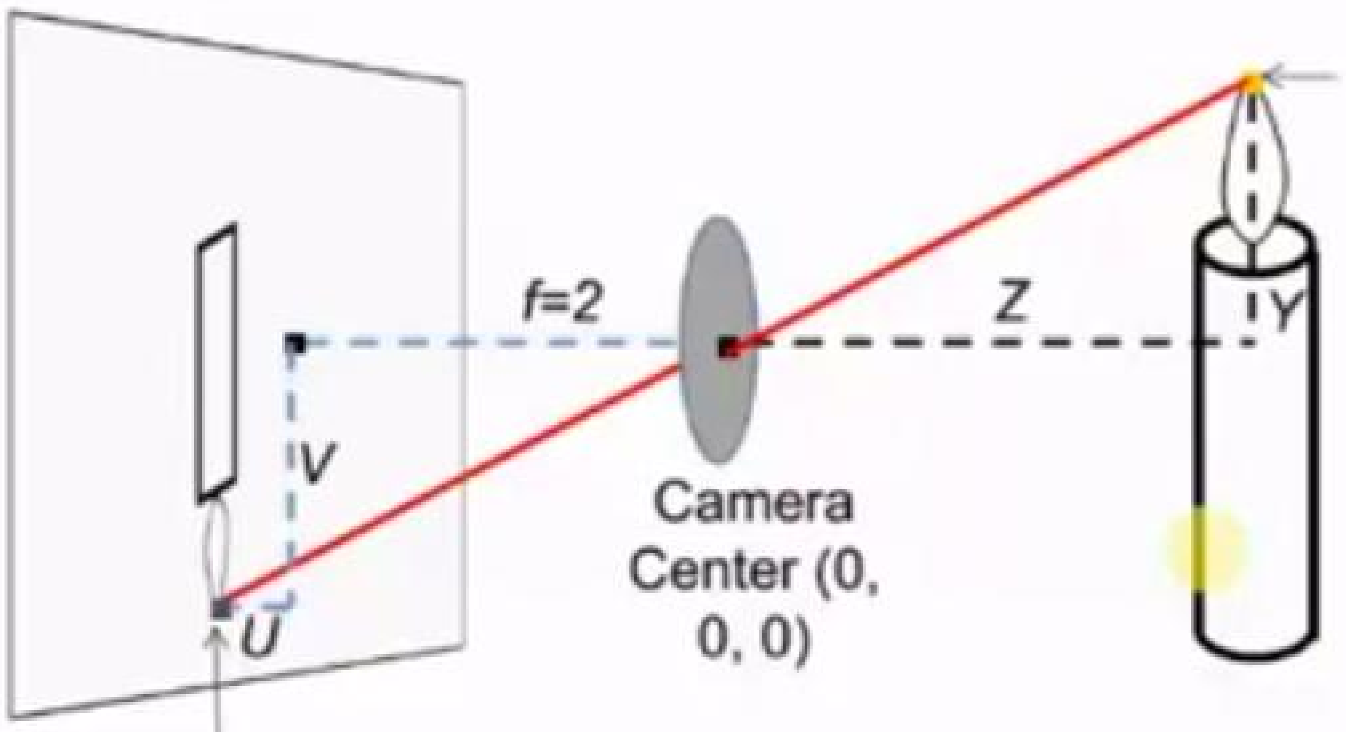
- Aperture : avoid ambiguity on image plane

- Pinhole Camera model:



Projection Geometry

- How's a point in the world coordinate P=[X, Y, Z], relates to the image plane coordinates p=[u, v] ?



$$\mathbf{P} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$$

$$\mathbf{p} = \begin{bmatrix} u \\ v \end{bmatrix}$$