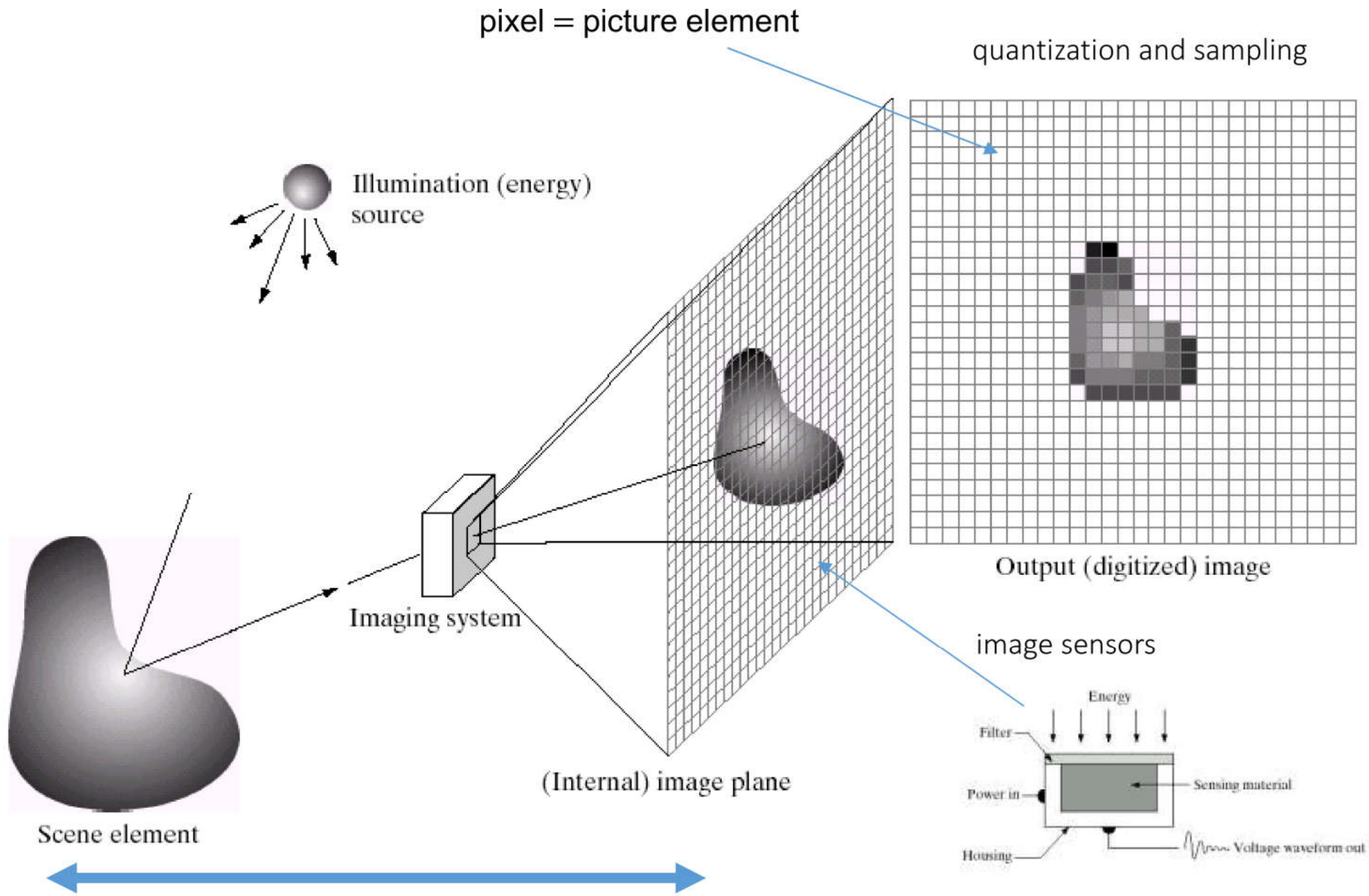
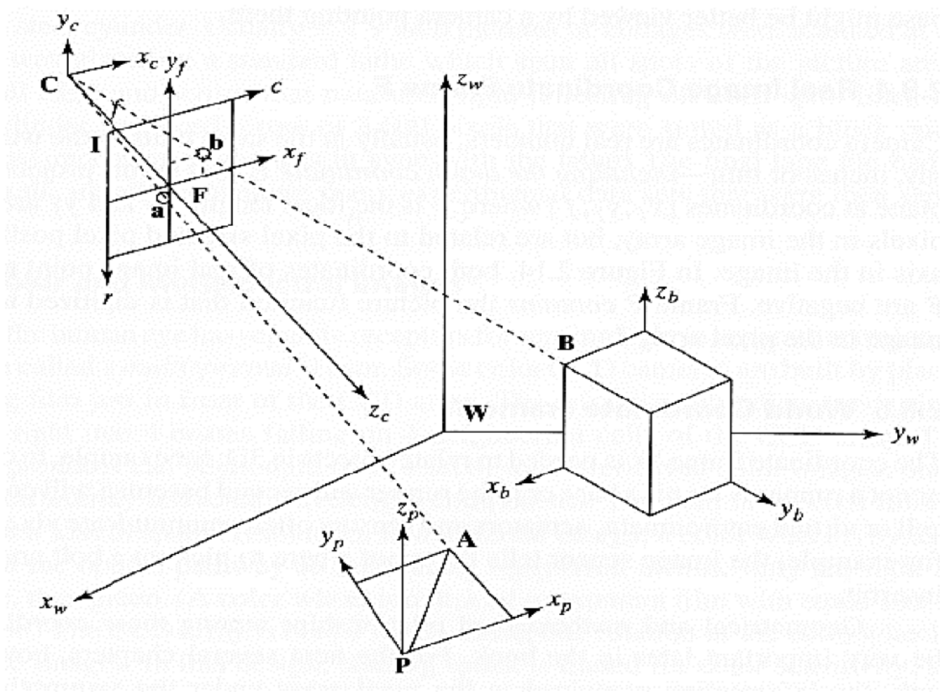
LAST CLASS

Image acquisition: Geometry



Traditional References

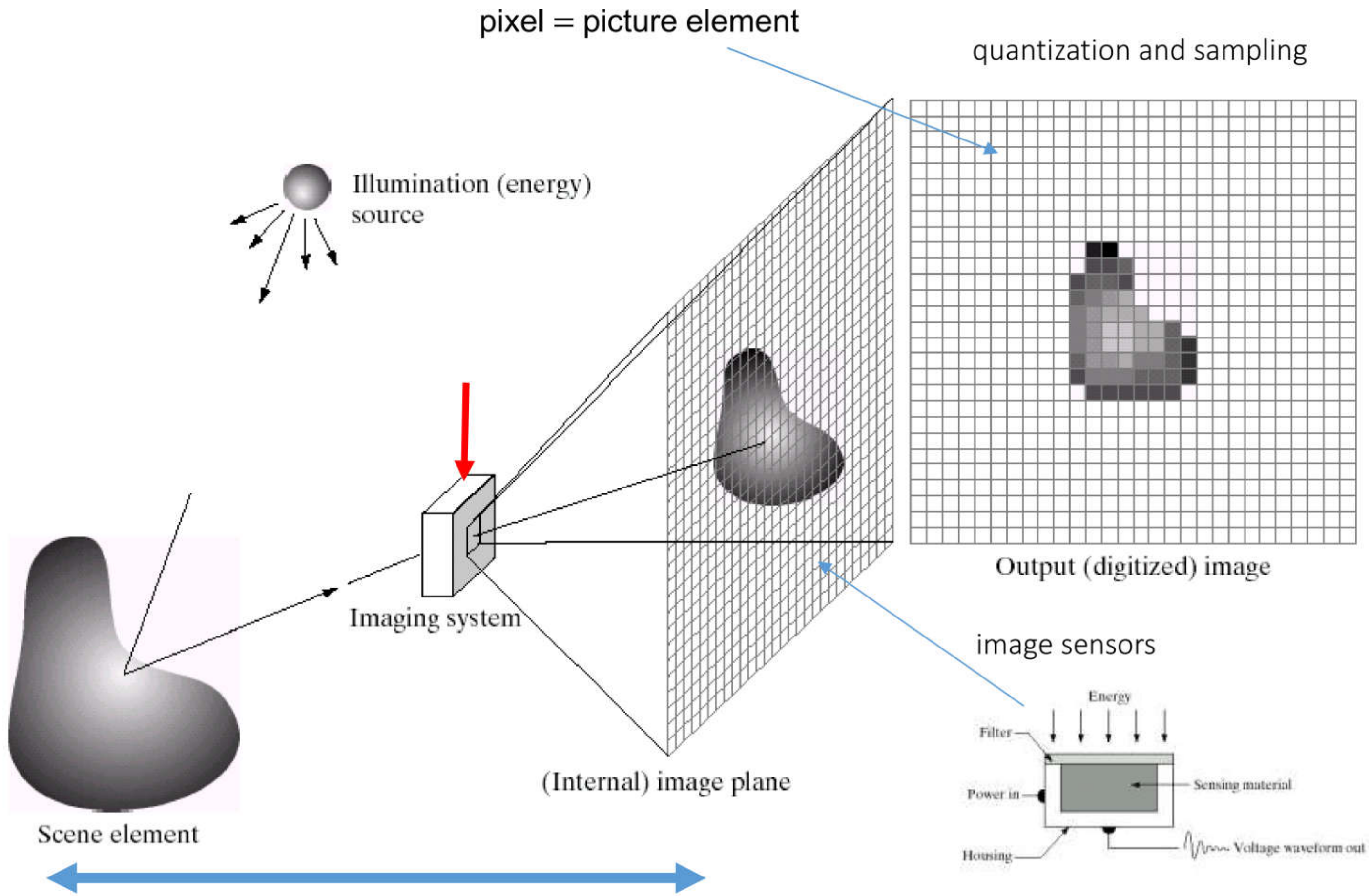
- **Five** reference frames are needed in general for 3D scene analysis.
  - Object
  - World
  - Camera
  - Image
  - Pixel



Geometric Transforms

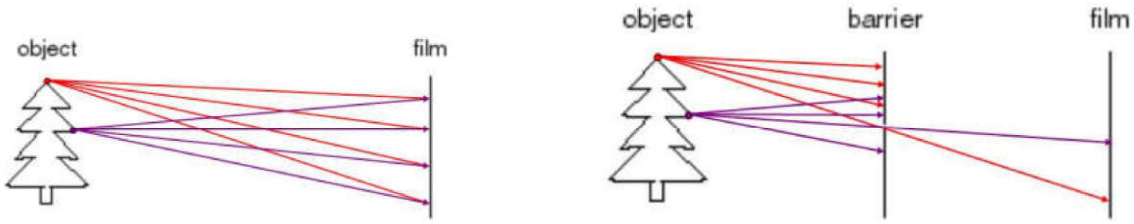
- 2D Translation
- 2D Rotation
- 2D Scaling
- Homogeneous coordinates
- Geometric Transforms using Homogeneous Coordinates
- Composition of transforms
- General form of 2D transformation matrix
- 3D Homogeneous coordinates and 3D geometric transforms

Image acquisition: Geometry

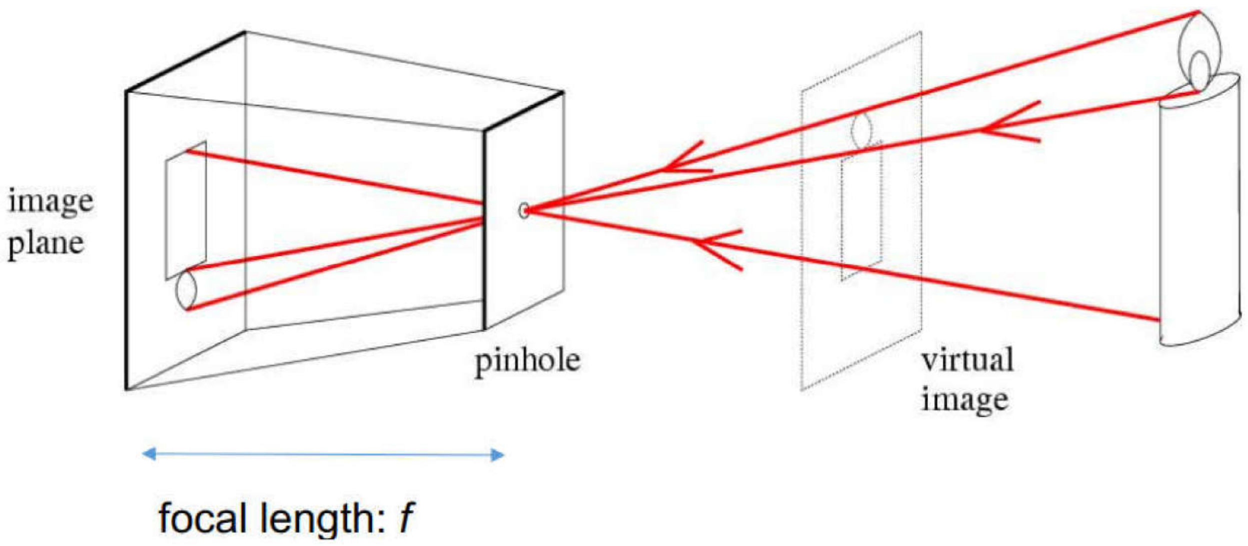


Necessity of an aperture

- Aperture : avoid ambiguity on image plane



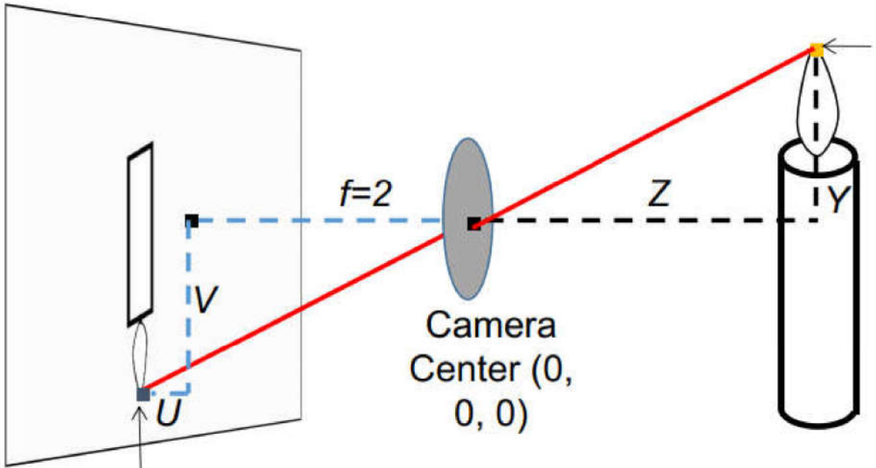
- Pinhole Camera model:



Projection Geometry

- How's a point in the world coordinate P=[X, Y, Z], relates to the image plane coordinates p=[u, v] ?

$$\mathbf{P} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$$



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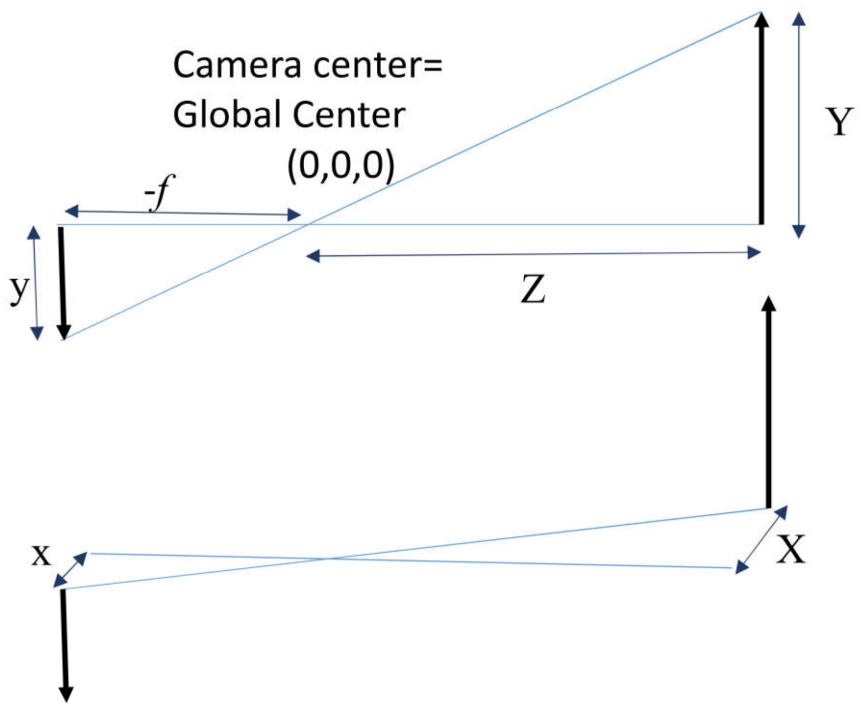
Projection Geometry



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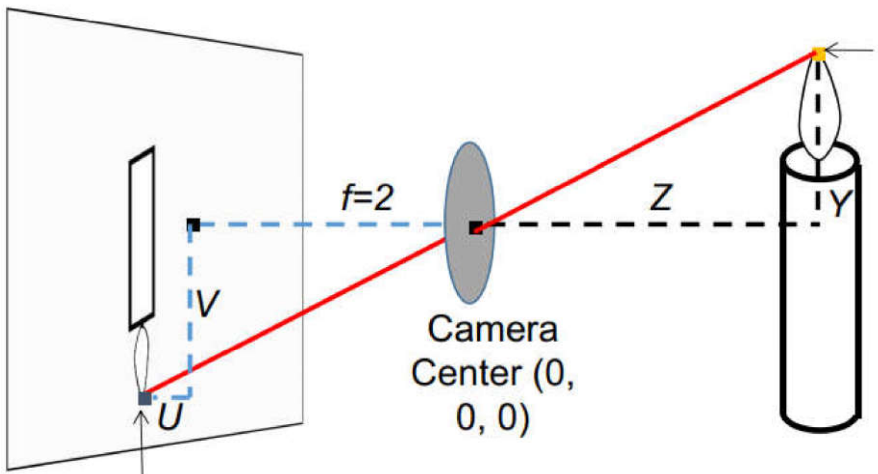
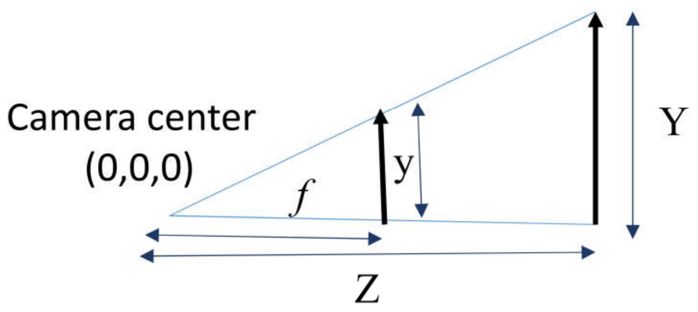


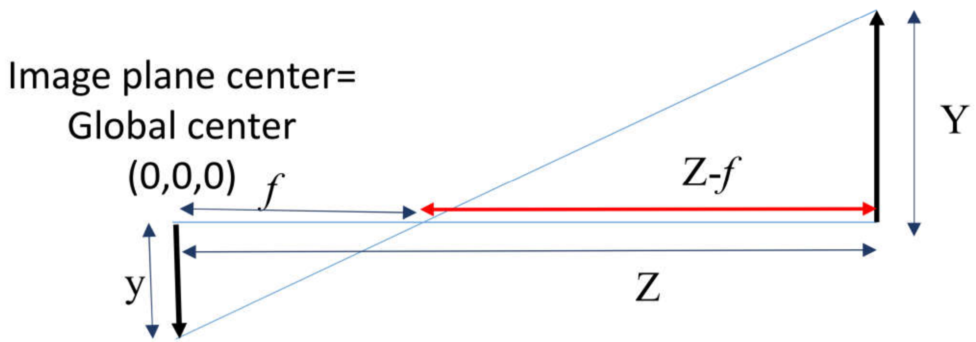
$$\mathbf{p} = \begin{bmatrix} u \\ v \end{bmatrix}$$

$$u = -X * \frac{f}{Z} = -2 * (2/5)$$

$$v = -Y * \frac{f}{Z} = -3 * (2/5)$$

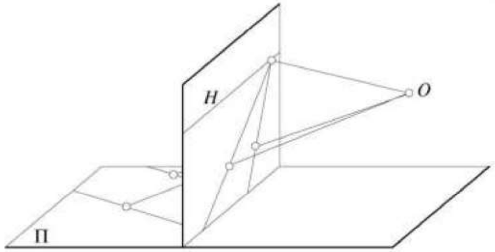
Projection Geometry (other formulations)





Projection Geometry

- Perspective projection is a simplification of real world image formation
  - Lens characteristics are not considered
- Perspective Projection Characteristics:
  - Parallel Lines converge to a vanishing point




  - Depth perception from perspective projection (Julian Beever)


The equations of perspective projection (cont'd)

- Using matrix notation:

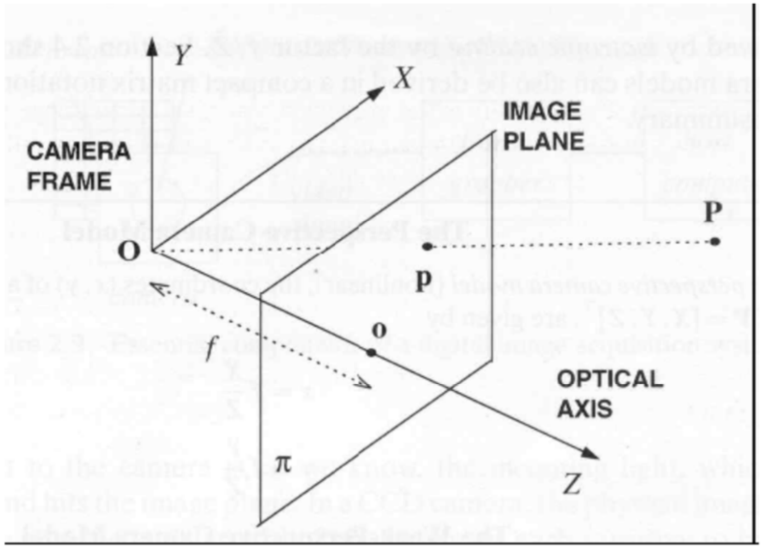
$$\begin{bmatrix} x_h \\ y_h \\ z_h \\ w \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & f & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} x_h \\ y_h \\ z_h \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/f & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

- Verify the correctness of the above matrix
  - homogenize using $w = Z$

$$x = \frac{x_h}{w} = \frac{fX}{Z} \qquad y = \frac{y_h}{w} = \frac{fY}{Z} \qquad z = \frac{z_h}{w} = f$$

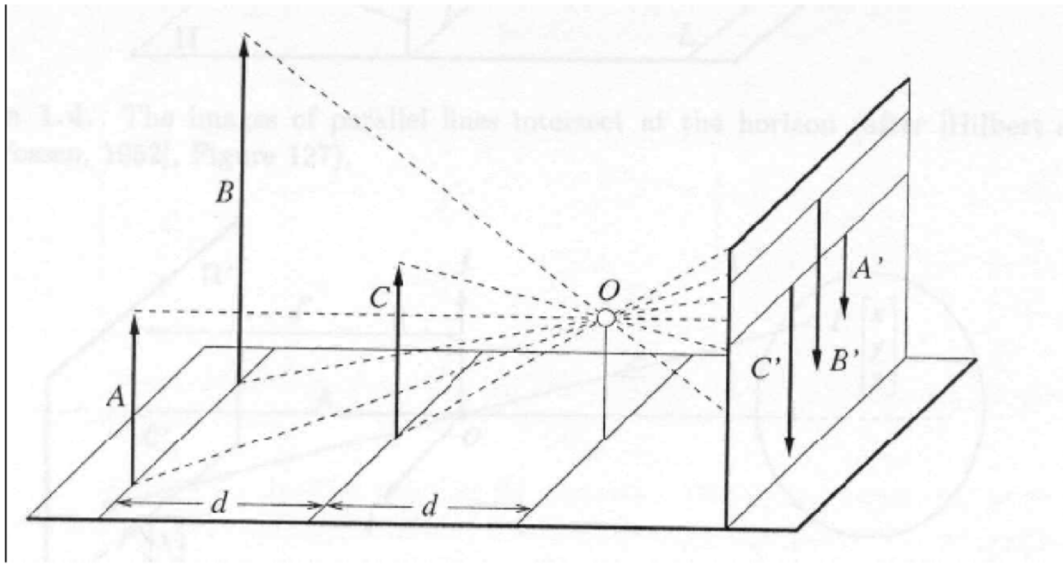
Properties of perspective projection

- Many-to-one mapping
  - The projection of a point is *not* unique
  - Any point on the line *OP* has the same projection



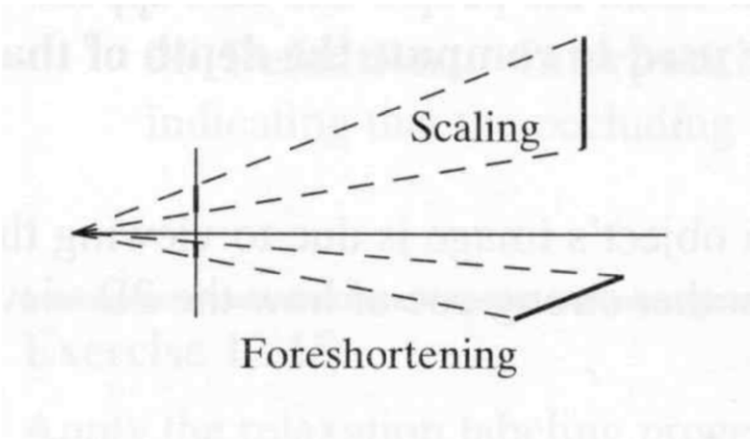
Properties of perspective projection (cont’d)

- Scaling/Foreshortening
  - The distance to an object is inversely proportional to its image size.



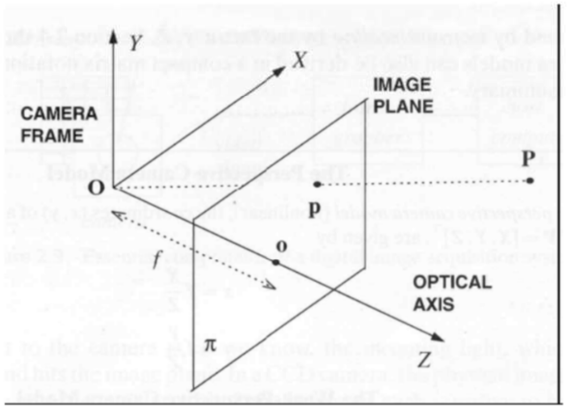
Properties of perspective projection (cont'd)

- When a line (or surface) is parallel to the image plane, the effect of perspective projection is *scaling*.
- When an line (or surface) is not parallel to the image plane, we use the term *foreshortening* to describe the effect of projective distortion



Properties of perspective projection (cont'd)

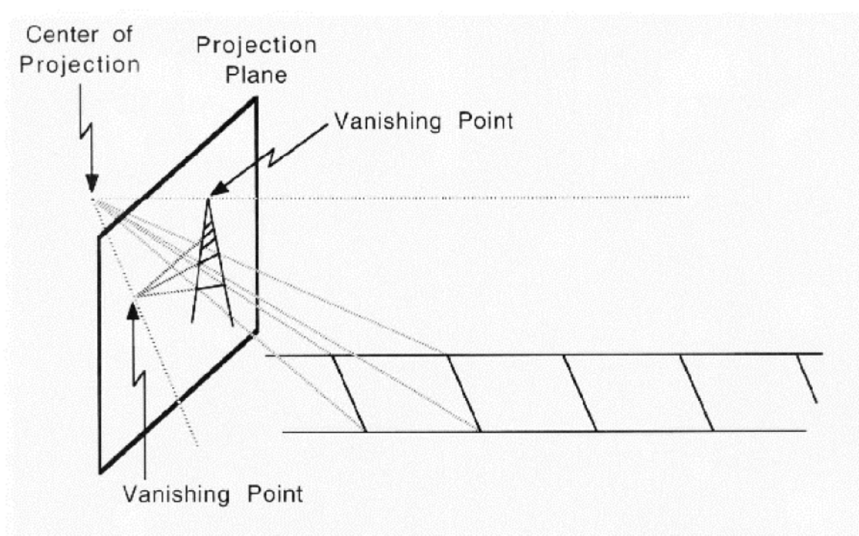
- Effect of focal length
  - As *f* gets smaller, more points project onto the image plane (*wide-angle camera*).
  - As *f* gets larger, the field of view becomes smaller (more *telescopic*).



$$x = \frac{x_h}{w} = \frac{fX}{Z} \qquad y = \frac{y_h}{w} = \frac{fY}{Z} \qquad z = \frac{z_h}{w} = f$$

Properties of perspective projection (cont'd)

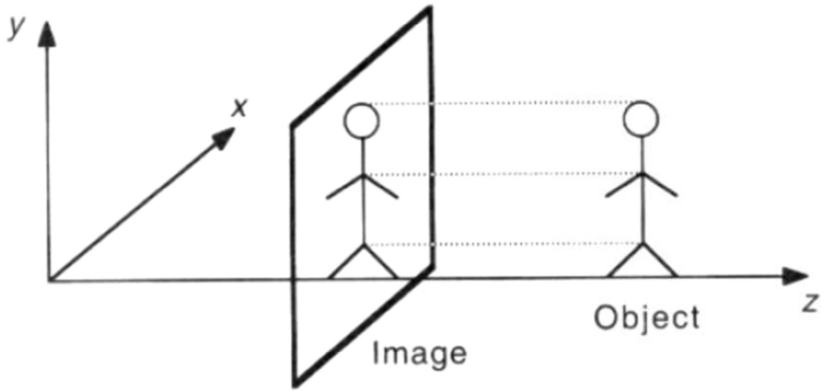
- Vanishing point
  - Parallel lines in space project perspectively onto lines that on extension intersect at a single point in the image plane called *vanishing point* or *point at infinity*.



Warning: vanishing points might lie outside of the image plane!

Orthographic Projection

- The projection of a 3D object onto a plane by a set of parallel rays <u>orthogonal</u> to the image plane.
- It is the limit of perspective projection as $f -> \infty$ (i.e., $f/Z -> 1$)



$$x = \frac{Xf}{Z} \qquad y = \frac{Yf}{Z} \qquad z = f$$

orthographic proj. eqs: $x = X, \quad y = Y \quad$ (drop Z)

Orthographic Projection (cont'd)

- Using matrix notation:

$$\begin{bmatrix} x_h \\ y_h \\ z_h \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

- Verify the correctness of the above matrix (homogenize using *w*=1):

$$x = \frac{x_h}{w} = X \qquad y = \frac{y_h}{w} = Y$$