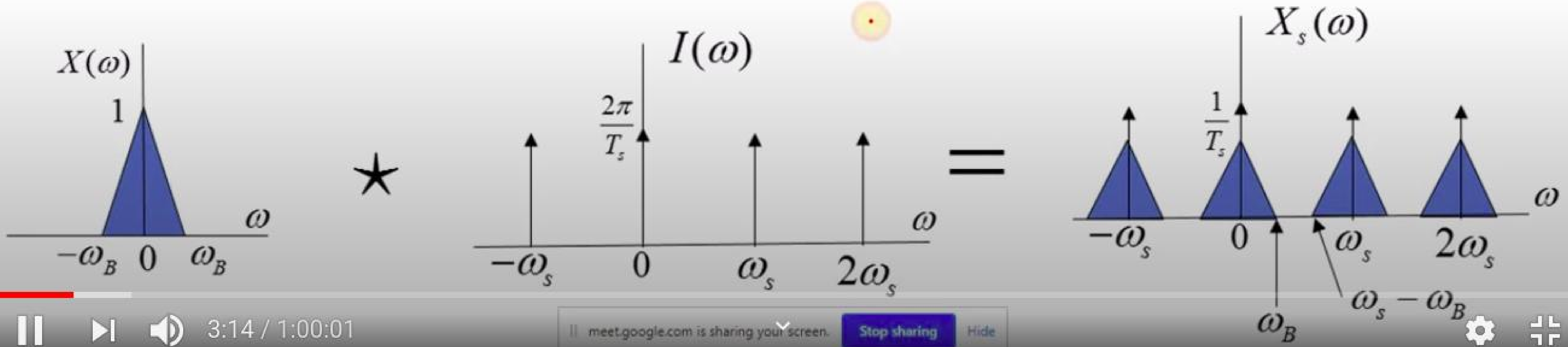
1-D Sampling Theorem

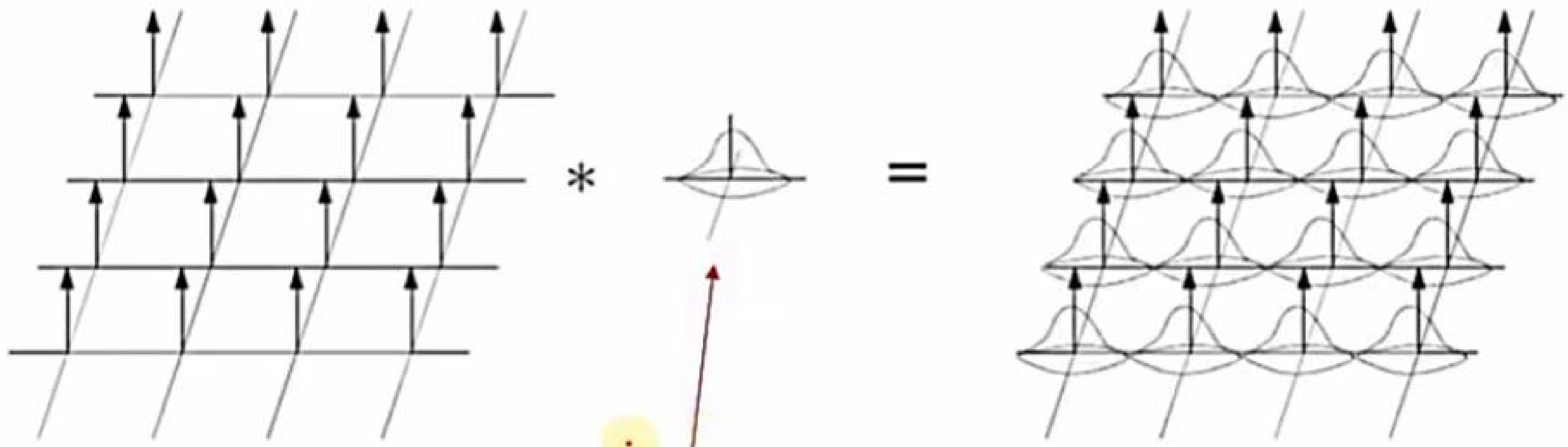
Sampled signal $x_s(t) = x(t)\, it(t) = \sum_{p \in \mathbb{Z}} x(t)\delta(t - pT_s) = \sum_{p \in \mathbb{Z}} x(pT_s)\delta(t - pT_s)$

By the multiplication property of FT,

$$X_s(\omega) = \frac{1}{2\pi} X(\omega) \star I(\omega) = \frac{1}{2\pi} X(\omega) \star [\frac{2\pi}{T_s} \sum_n \delta(\omega - n\omega_s)]$$

$$= \frac{1}{T_s} \sum_n X(\omega - n\omega_s)$$



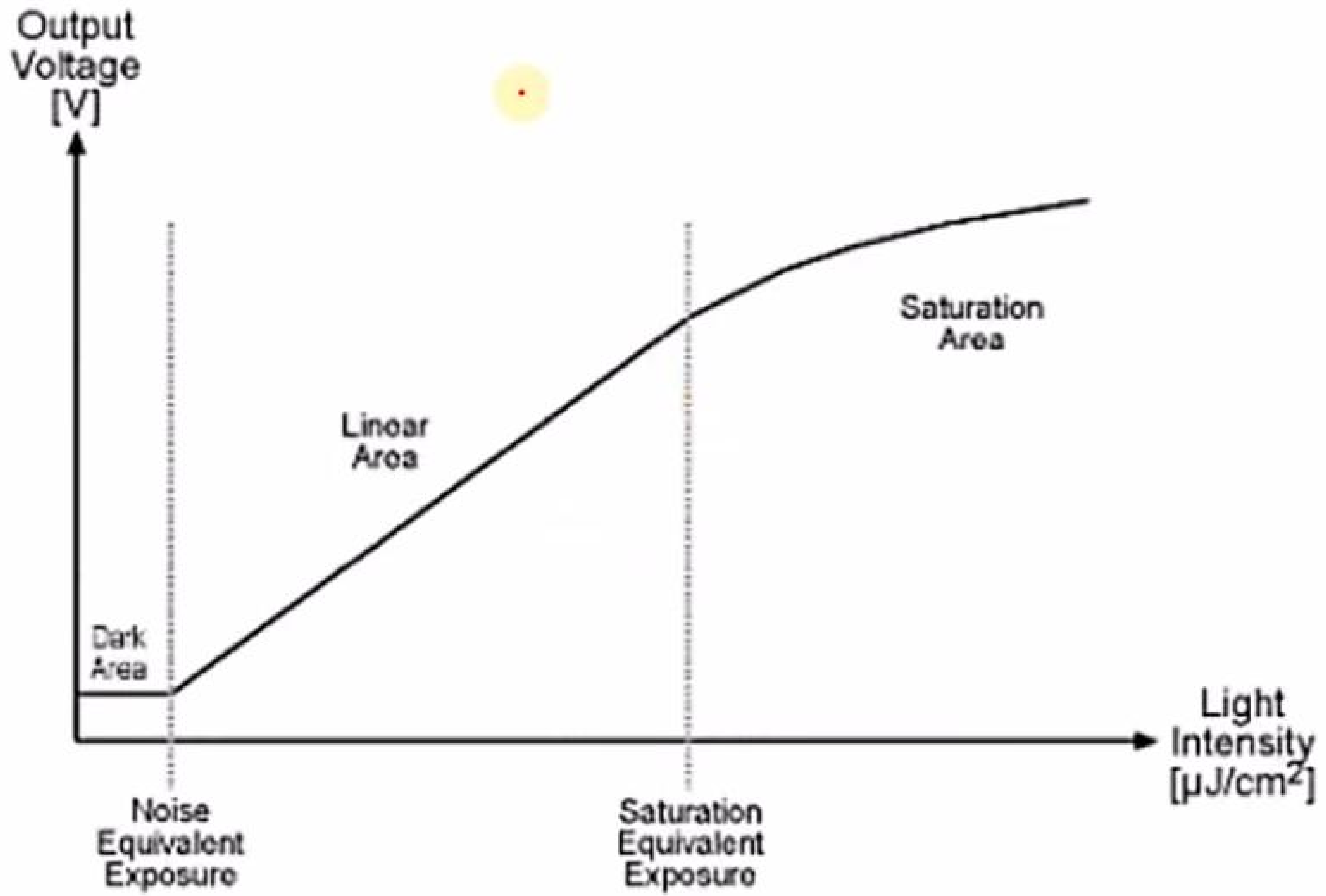


A comb of delta functions

Fourier spectrum of a 2D signal

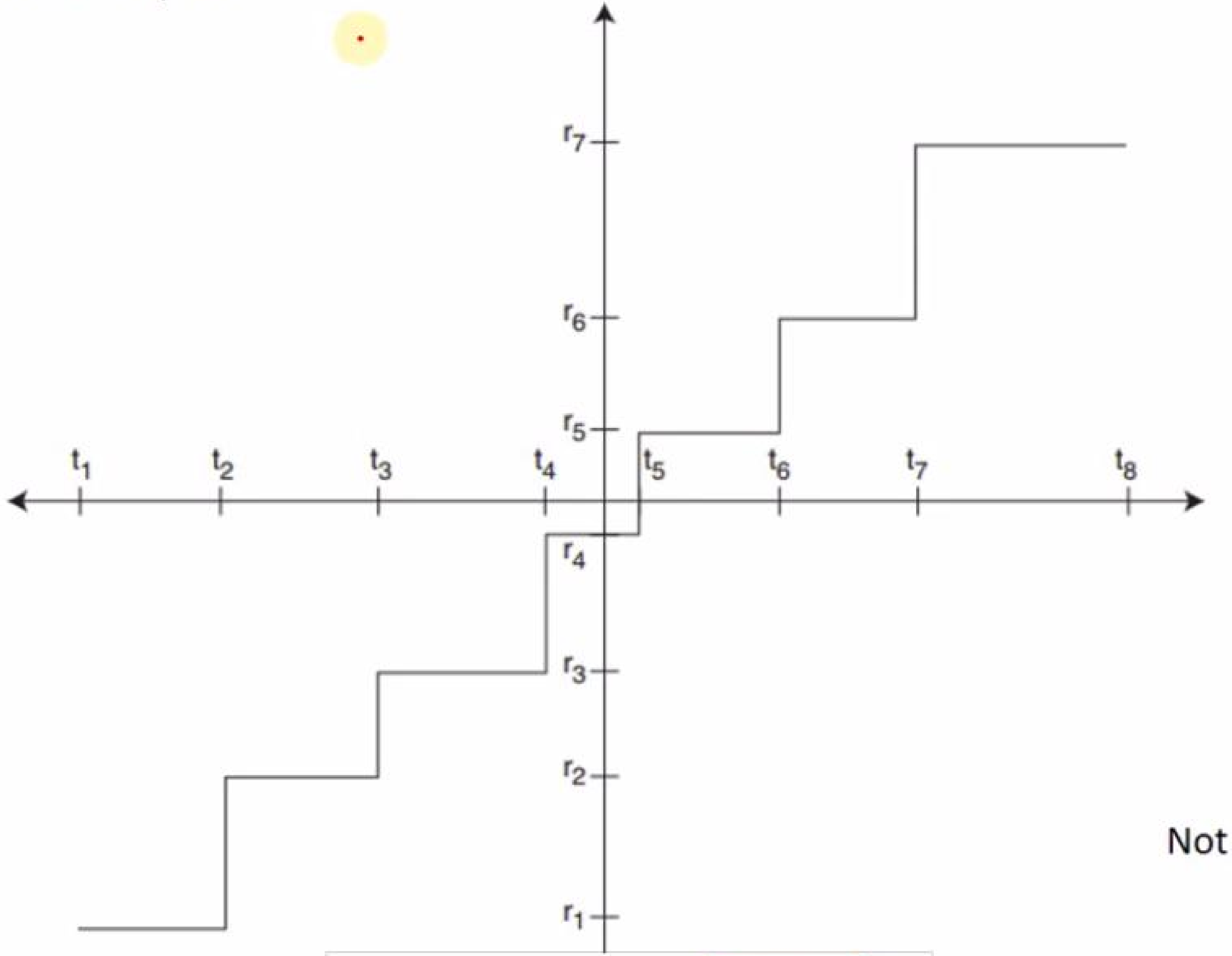
Fourier spectrum of the sampled 2D signal

Nonlinear Response:



Zero-memory Quantizer:

The simplest type of quantizers are called ***zero memory* quantizers** in which quantizing a sample is independent of other samples.



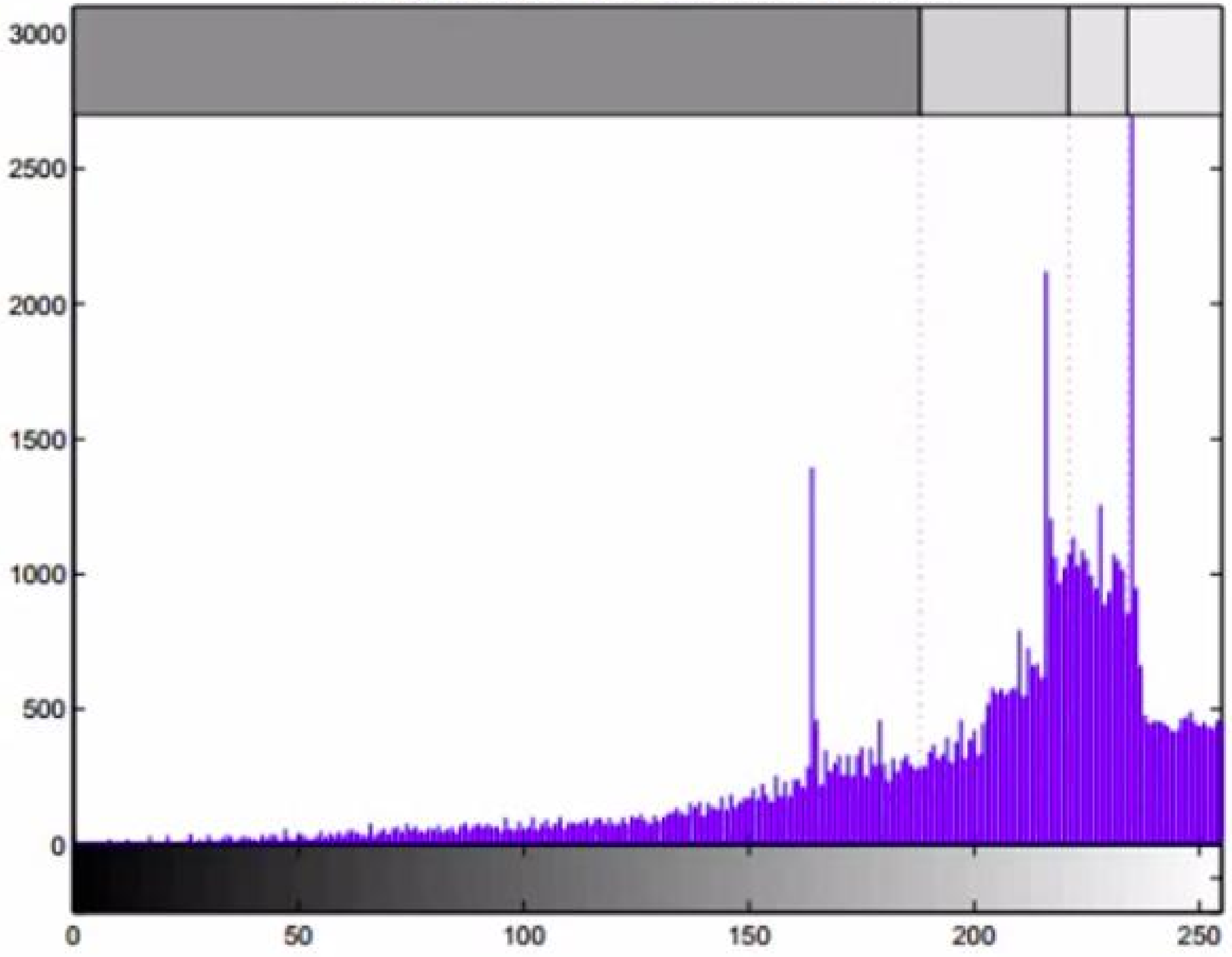
Not necessarily uniform

Nonuniform Quantizer:

If an image never takes on a certain range of gray levels then there is no reason to waste quantization levels in this range of gray levels.

Original image, 256 gray levels

Histogram of original image and quantizer breakpoints



Optimal Quantizer:

$$t_k = \frac{r_k + r_{k-1}}{2} \qquad (1)$$

$$r_k = \frac{\int_{t_k}^{t_{k+1}} x p_X(x) dx}{\int_{t_k}^{t_{k+1}} p_X(x) dx} \qquad (2)$$

- The optimum transition levels lie halfway between the optimum reconstruction levels,
- The optimal reconstruction levels lie at the center of mass of the PDF in between the optimum transition levels.

Basic Relationships Between Pixels

- Neighborhood
- Adjacency
- Connectivity
- Paths
- Regions and boundaries

Neighbors of a Pixel

For an Image I(x,y), $< x, y > \in \mathbb{Z}^2$

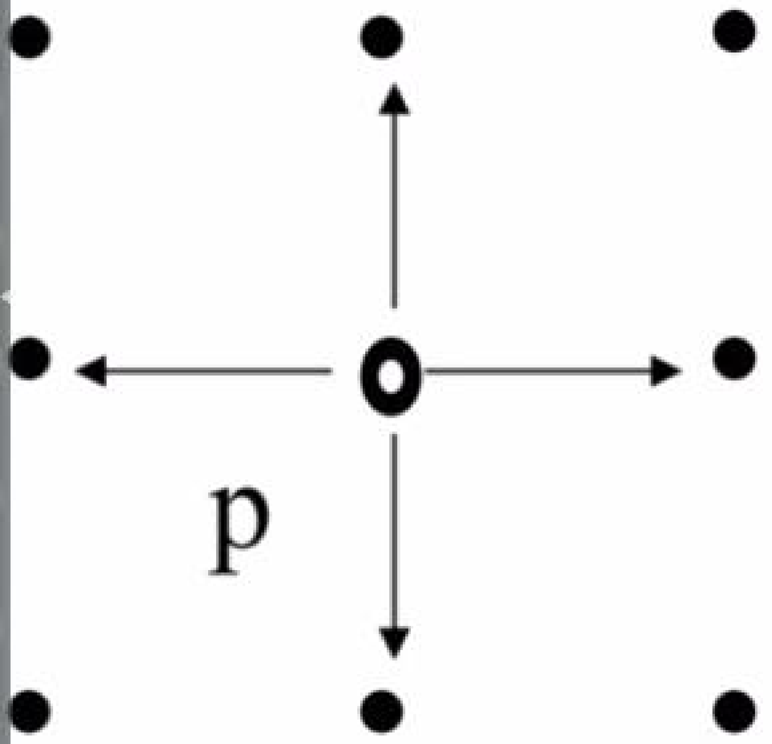
- Any pixel p(x, y) has two vertical and two horizontal neighbors, given by (x+1, y), (x-1, y), (x, y+1), (x, y-1)
- This set of pixels are called the 4-neighbors of p, and is denoted by $N_4(P)$.
- Each of them are at a unit distance from P.

- The four diagonal neighbors of p(x,y) are given by, (x+1, y+1), (x+1, y-1), (x-1, y+1), (x-1 ,y-1)
- This set is denoted by $N_D(P)$.
- Each of them are at Euclidean distance of 1.414 from P.

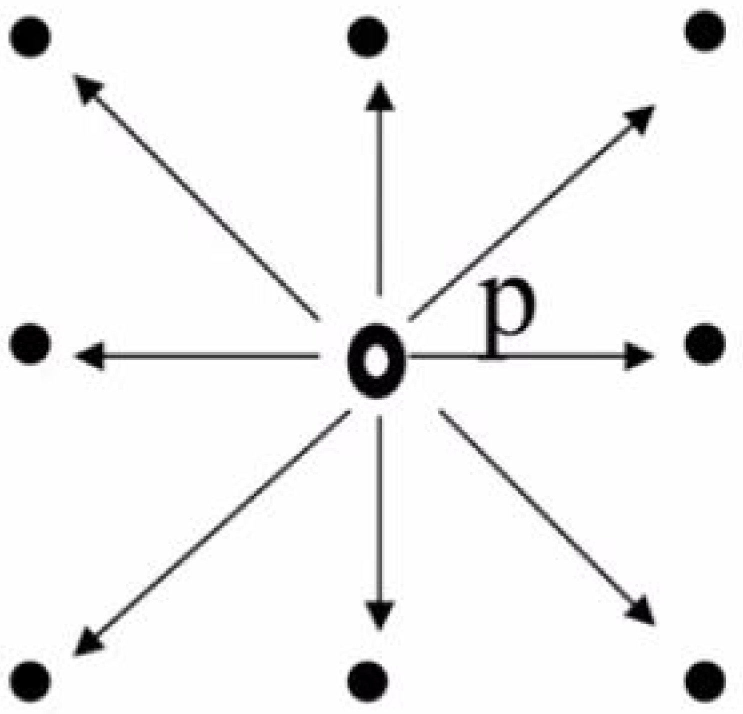
Neighbors of a Pixel

The points $N_D(P)$ and $N_4(P)$ are together known as 8-neighbors of the point P, denoted by $N_8(P)$.

- Some of the points in the $N_4(P)$, $N_D(P)$ and $N_8(P)$ may fall outside a finite resolution image when P lies on the border of image.



$N_4(p)$



$N_8(p)$

Adjacency

- Two pixels are connected if they are neighbors and their gray levels satisfy some specified criterion of similarity.

- For example, in a binary image two pixels are connected if they are 4-neighbors and have same value (0/1).

Let V be set of gray levels values used to define adjacency.

- 4-adjacency: Two pixels p and q with values from V are 4-adjacent if q is in the set $N_4(P)$.
- 8-adjacency: Two pixels p and q with values from V are 8-adjacent if q is in the set $N_8(P)$.
- m-adjacency: Two pixels p and q with values from V are m-adjacent if,

– q is in $N_4(P)$.
– q is in $N_D(P)$ and the set $[N_4(P) \cap N_4(Q)]$ is empty
(has no pixels whose values are from V)

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Adjacency

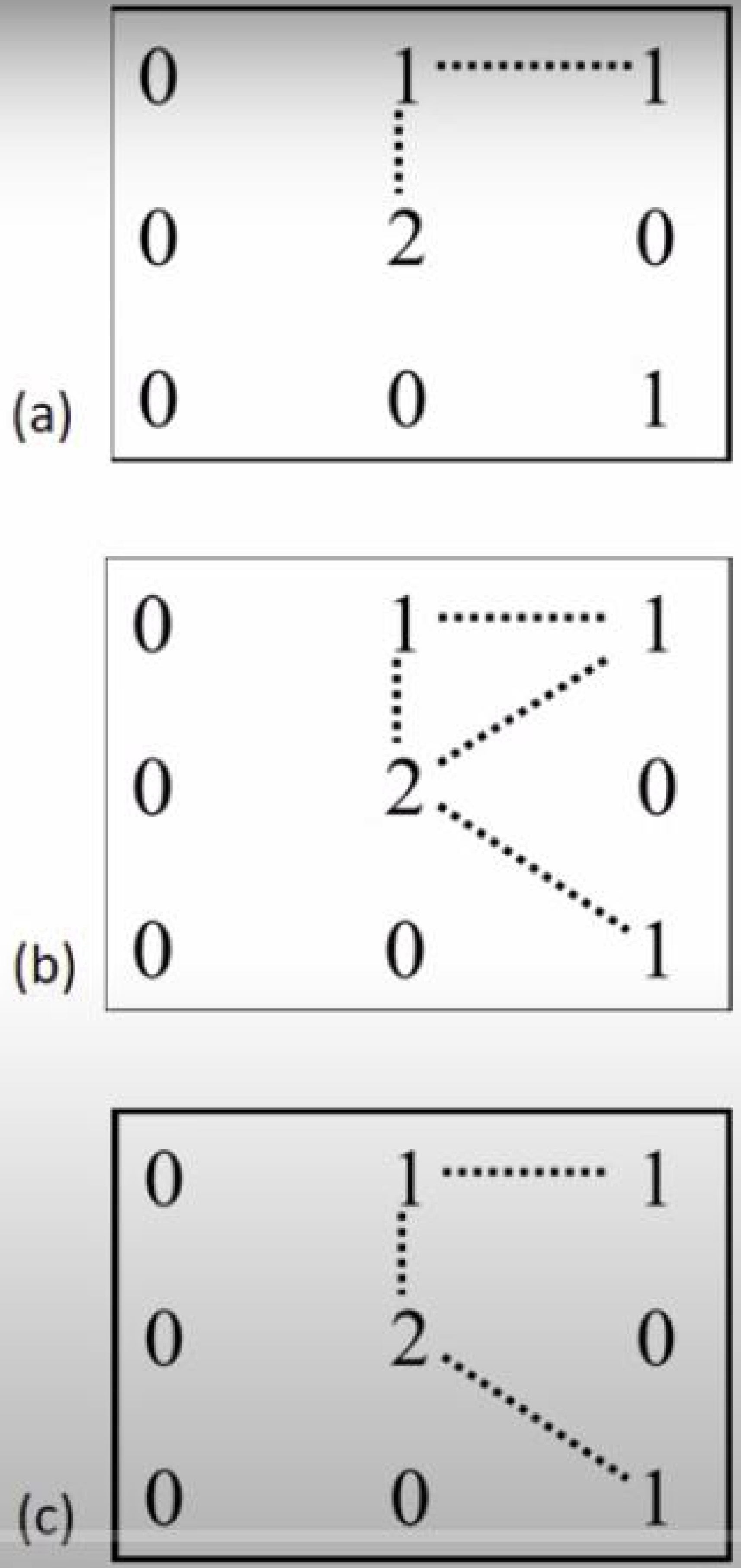
$V = \{1,2\}$

Let V be the set of gray-level values used to define connectivity;
then two pixels p, q that have values from the set V are:

a. 4-connected,if q is in the set $N_4(P)$

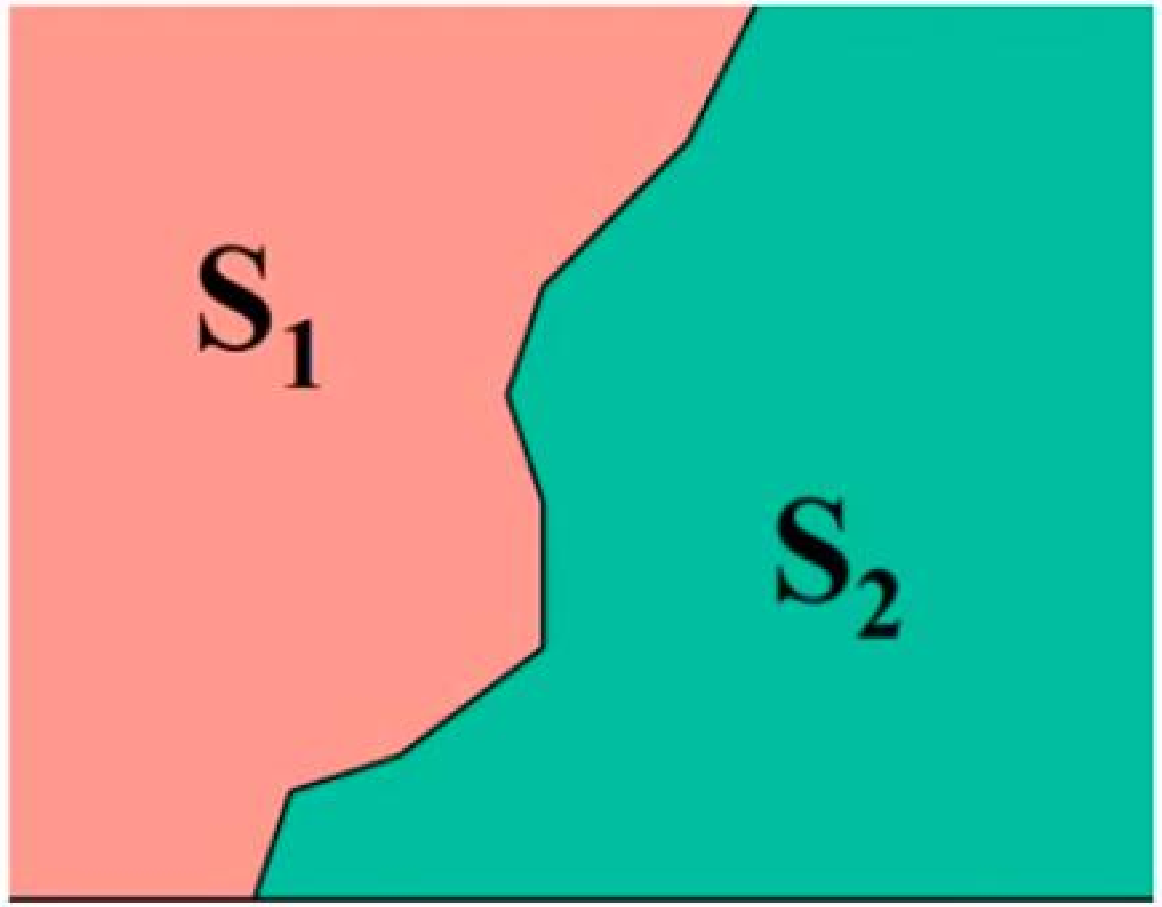
b. 8-connected,if q is in the set $N_8(P)$

c. m-connected, iff
– q is in $N_4(P)$.
– q is in $N_D(P)$ and the set $[N_4(P) \cap N_4(Q)]$ is empty (has no pixels whose values are from V)



Adjacency

- Two image subsets S_1 and S_2 are adjacent if some pixel in S_1 is adjacent to some pixel in S_2



Paths

A path from pixel p with coordinates (x, y) to pixel q with coordinates (s, t) is a sequence of **distinct** pixels with coordinates:

$$(x_0, y_0), (x_1, y_1), (x_2, y_2) \dots (x_{n-1}, y_{n-1})$$

where

- $(x_0, y_0) = (x, y)$
- $(x_{n-1}, y_{n-1}) = (s, t)$
- (x_i, y_i) is **adjacent** to (x_{i-1}, y_{i-1}) $\quad 1 \leq i \leq n-1$

Here **n** is the length of the path.

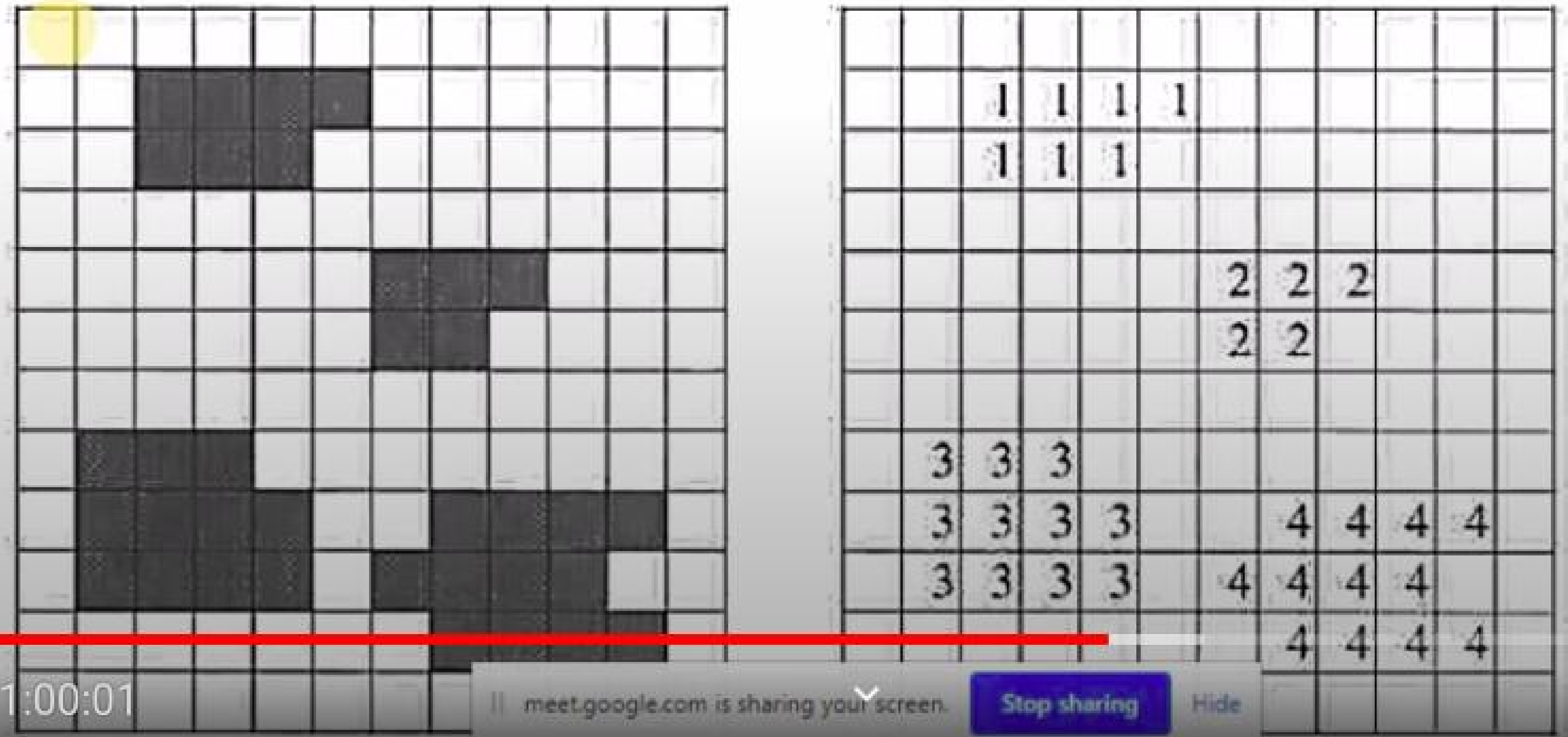
We can define 4-, 8-, and m-paths based on type of adjacency used.

Connected Components

If p and q are pixels of an image subset S then p is connected to q in S if there is a path from p to q consisting entirely of pixels in S.

- For every pixel p in S, the set of pixels in S that are connected to p is called a connected component of S.

- If S has only one connected component then S is called Connected Set.



Connected Components

- **Recursive algorithm**

- Let is assume that region pixels have the value 0 (black) and that background pix-els have the value 255 (white).

(1) Scan the image to find an unlabeled 0 (pixel and assign it a new label L.

(2) Recursively assign a label L to all of its 0 neighbors.

(3) Stop if there are no more unlabeled 0 pixels.

(4) Go to step 1.

• Sequential algorithm

- The sequential algorithm usually requires two passes over the image.

- It works with only two rows of an image at a time.

(1) Scan the image left to right, top to bottom.

(2) If the pixel is 0, then:

(2.1) If only one of its upper and left neighbors has a label, then copy the label.

(2.2) If both have the same label, then copy the label.

(2.3) If both have different labels, then copy the upper's label and enter the labels in the equivalence table as equivalent labels.

(2.4) Otherwise assign a new label to this pixel and enter this label in the equivalence table.

(3) If there are no more pixels to consider, then go to step 2.

(4) Find the lowest label for each equivalent set in the equivalence table.

(5) Scan the image. Replace each label b… et.