Two phase method :-

$$\text{Min } z = -2x_1 - x_2 \qquad \text{Max } -z = 2x_1 + x_2$$

s.t.

$x_1 + x_2 \geq 2$	$x_1 + x_2 - x_3 = 2$	$x_3 = -2$
$x_1 + x_2 \leq 4$	$x_1 + x_2 + x_4 = 4$	$x_4 = 4$
$x_1, x_2 \geq 0$	$x_i \geq 0 \ \forall i$	

After introducing artificial variable

First phase

$$\text{Min } \omega = a \rightarrow \text{Max } -\omega = -a$$

$x_1 + x_2 - x_3 + a = 2 \Rightarrow a = 2$ ✓

$x_1 + x_2 + x_4 = 4 \Rightarrow x_4 = 4$ ✓

$x_i \geq 0 \quad a \geq 0$

	c_j	0	0	0	0	-1		
	j	x_1	x_2	x_3	x_4	a	b	θ
-1	a	1	1	-1	0	1	2	2 →
0	x_4	1	1	0	1	0	4	4
	z_j	-1	-1	1	0	-1		
	$c_j - z_j$	1 ↑	1	-1	0	0		

Second phase

	Coeff c_j	2	1	0	0		
2	x_1	1	1	-1	0	2	–
$R_2 - R_1$ 0	x_4	0	0	1	1	2	2 →
	z_j	2	2	-2	0	4	
	$c_j - z_j$	0	-1	2 ↑	0		
$R_1 + R_2$ 2	x_1	1	1	0	1	4	
0	x_3	0	0	1	1	2	
	z_j	2	2	0	2	8	
	$c_j - z_j$	0	-1	0	-2		

$(x_1, x_2) = (4, 0)$

$z = 8$

Min $z = x_1 + x_2$	
S.t. $x_1 + 2x_2 \leq 2$	$x_1 + 2x_2 + x_3 = 2$
$3x_1 + 5x_2 \geq 15$	$3x_1 + 5x_2 - x_4 + a = 15$
$x_1, x_2 \geq 0$	$x_i \geq 0 \ \forall i, \ a \geq 0$

Phase-I Min a

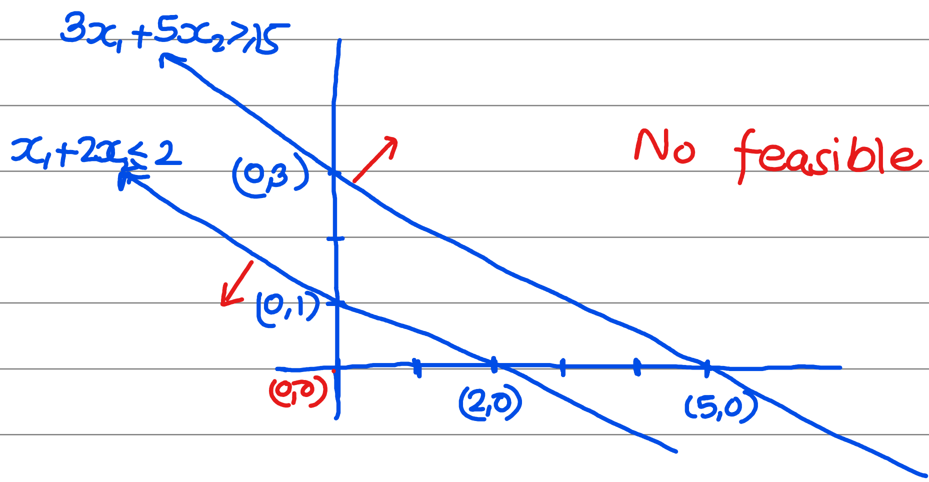
			0	0	0	0	-1		
			x_1	x_2	x_3	x_4	a	b	θ
	0	x_3	1	2	1	0	0	2	1 →
	-1	a	3	5	0	-1	1	15	3
		z_j	-3	-5	0	1	-1	-15	
		$c_j - z_j$	3	5 ↑	0	-1	0		
$\frac{1}{2}R_1$	0	x_2	1/2	1	1/2	0	0	1	2 →
$R_2 - 5R_1$	-1	a	1/2	0	-5/2	-1	1	10	20
		z_j	-1/2	0	5/2	1	-1	-10	
		$c_j - z_j$	1/2 ↑	0	-5/2	-1	0		
$2R_1$	0	x_1	1	2	1	0	0	2	
$R_2 - \frac{1}{2}R_1$	-1	a	0	-1	-3	-1	1	9	
		z_j	0	1	3	1	-1		
		$c_j - z_j$	0	-1	-3	-1	0		

As in the final sol^n of Phase-I artificial variable is still positive $(a = 9)$

$\Rightarrow$ The LPP is infeasible.

Min $z = x_1 + x_2$

$x_1 + 2x_2 \leq 2$	$x_1 + 2x_2 = 2$	$(0,1)\ (2,0)$
$3x_1 + 5x_2 \geq 15$	$3x_1 + 5x_2 = 15$	$(0,3)\ (5,0)$
$x_1, x_2 \geq 0$		



No feasible region/ sol^n

Imp

Min $z = 2 - x_2$

$x_1 - x_2 = 4$	$x_1 - x_2 + a_1 = 4$	$x_1 = 4$
$-x_2 - x_3 = 0$	$x_2 + x_3 + a_2 = 0$	$a_2 = 0$

Min $a_1 + a_2$

Direct second phase.

	c_j	0	1	0	0	0		
		x_1	x_2	x_3	a_1	a_2	b	
0	x_1	1	-1	0	1	0	4	–
0	x_3	0	1	1	0	1	0	0 →
	z_j	0	0	0				
	$c_j - z_j$	0	1 ↑	0				

$R_1 + R_2$

	c_j	0	1	0	
		x_1	x_2	x_3	b
0	x_1	1	0	1	4
1	x_2	0	1	1	0
	z_j	0	1	1	
	$c_j - z_j$	0	0	-1	

$x_2 = 0$

$z = 2$

?

Big-M Charne's M Technique

Min $Z = 4x_1 + 8x_2 + 3x_3 + Ma_1 + Ma_2$

$x_1 + x_2 \geq 2$	$x_1 + x_2 - x_4 + a_1 = 2$
$2x_2 + x_3 \geq 5$	$2x_2 + x_3 - x_5 + a_2 = 5$
$x_i \geq 0 \;\forall i$	$x_i \geq 0 \;\forall i \quad a_i \geq 0$

			-4	-8	-3	0	0	$-M$	$-M$		
			x_1	x_2	x_3	x_4	x_5	a_1	a_2	b	θ
	$-M$	a_1	1	1	0	-1	0	1	0	2	2 $\rightarrow$
	$-M$	a_2	0	2	1	0	-1	0	1	5	$5/2$
		Z_j	$-M$	$-3M$	$-M$	M	M	$-M$	$-M$		
		$C_j - Z_j$	$M-4$	$3M-8$ $\uparrow$	$M-3$	$-M$	$-M$	0	0		
	-8	x_2	1	1	0	-1	0		0	2	$-$
$R_2 - 2R_1$	$-M$	a_2	-2	0	1	2	-1		1	1	1
		Z_j	$2M-8$	-8	$-M$	$8-2M$	M		$-M$		
		$C_j - Z_j$	$4-2M$	0	$M-3$	$2M-8$ $\uparrow$	$-M$		0		

			-4	-8	-3	0	0		
$R_1 + R_2$	-8	x_2	0	1	$1/2$	0	$-1/2$	$5/2$	5
$\frac{1}{2}R_2$	0	x_4	-1	0	$1/2$	1	$-1/2$	$1/2$	1 $\rightarrow$
		Z_j	0	-8	-4	0	4	-20	
		$C_j - Z_j$	-4	0	1 $\uparrow$	0	-4		
$R_1 - \frac{1}{2}R_2$	-8	x_2	1	1	0	-1	0	2	
$2R_2$	-3	x_3	-2	0	1	2	-1	1	
		$C_j - Z_j$	-6	0	0	-2	-3		