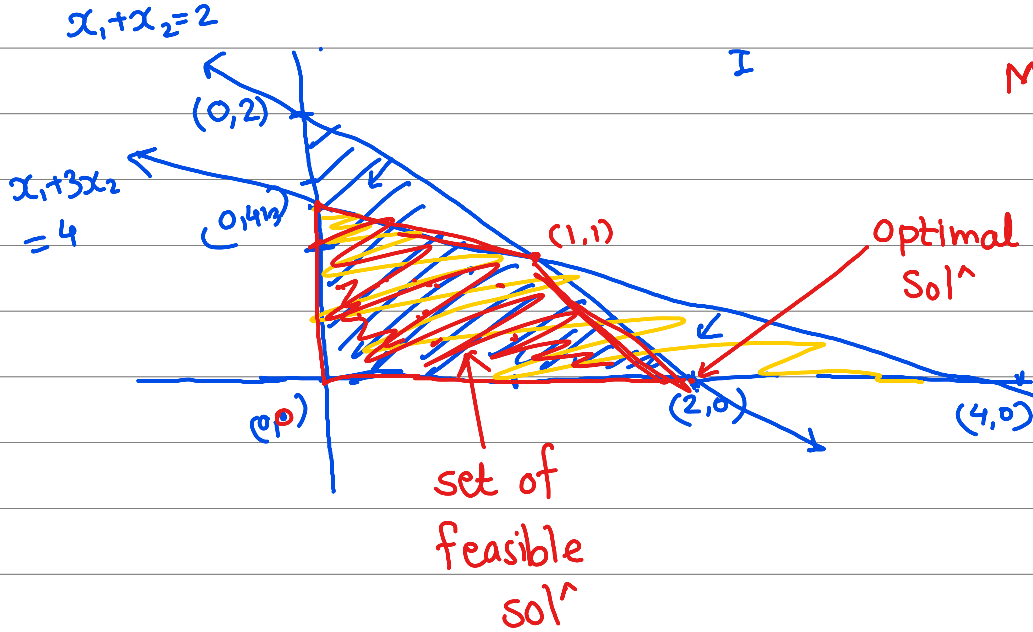


Max $2x_1+x_2$

$x_1+x_2 \le 2$

$x_1+3x_2 \le 4$

$x_1, x_2 \geq 0$

$x_1+x_2=2$

(0,2) (2,0)

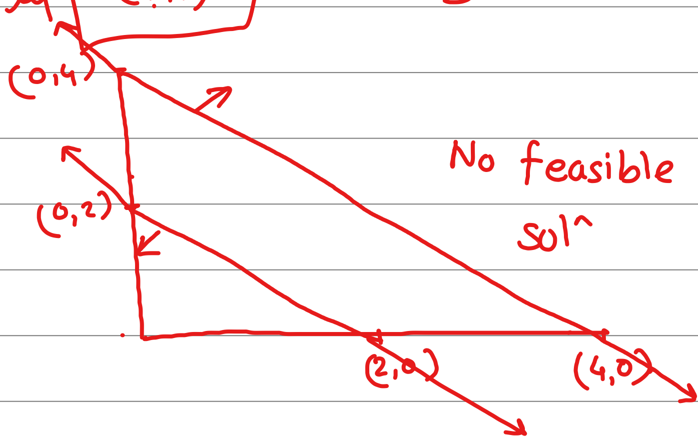
$x_1+3x_2=4$

(0,4/3) (4,0)

$Z=2x_1+x_2$

x_1	(0,0)	0
x_2	(0,4/3)	4/3
x_3	(2,0)	4
x_4	(1,1)	3

$Z_0=\min\{\underline{c}'\underline{x}_i,\ i=1:k\}$



$x_1+x_2 \le 2$

$x_1+x_2 \geq 4$

Let Max $\underline{Z}=\underline{c}'\underline{x}$

$A\underline{x} \le b$

$T=\{\underline{x} \mid A\underline{x} \le b\}$ constraint set.

let $\underline{x}^1$ & $\underline{x}^2$ are to optimum solⁿ to LP

$$\underset{\underline{x} \in T}{\text{Max}}\ Z = \text{Max}\ \underline{c}'\underline{x} = \underline{c}'\underline{x}^1 = \underline{c}'\underline{x}^2 = Z_0 \text{ (say)}$$ optimum value

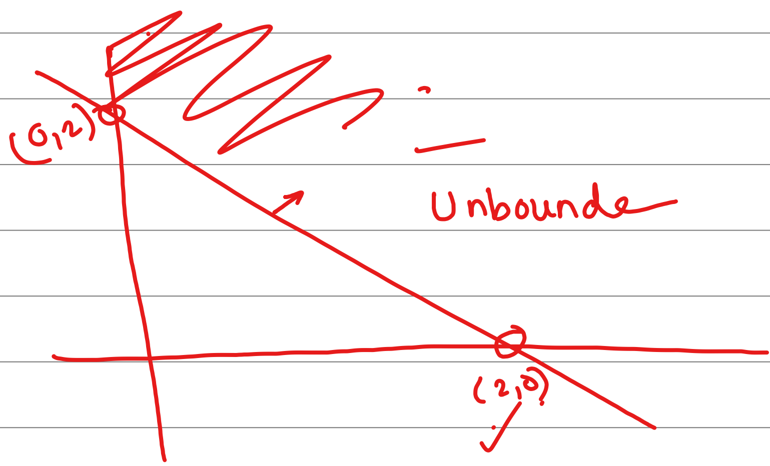
$\underline{x}^0 = \alpha \underline{x}^1 + (1-\alpha)\underline{x}^2$

To show $\underline{x}^0$ is also optimum solⁿ

$$\begin{aligned}\underline{c}'\underline{x}^0 &= c'(\alpha \underline{x}^1 + (1-\alpha)\underline{x}^2) \\ &= \alpha \underline{c}'\underline{x}^1 + (1-\alpha)\underline{c}'\underline{x}^2 \\ &= \alpha Z_0 + (1-\alpha) Z_0 \\ &= Z_0\end{aligned}$$

$\Rightarrow$ $\underline{x}^0$ is also optimum solⁿ

$\Rightarrow$ Set of optimum solⁿ is also convex set.



Min $Z = 2x_1 + 3x_2$

$x_1 + x_2 \geq 2$

$Z_0 = \min\{\underline{c}'\underline{x}_i,\ i = 1:K\}$

$\underline{x} \in T \Rightarrow \quad \underline{x} = \sum_{i=1}^{K} \alpha_i x_i \quad . \quad \alpha_i \geq 0 \quad \sum \alpha_i = 1$

$$\begin{aligned}\underline{c}'\underline{x} &= \sum \alpha_i \underline{c}'\underline{x}_i \\ &\geq \sum \alpha_i Z_0 = Z_0 \quad \Rightarrow\end{aligned}$$

$\underline{c}'\underline{x} \geq z_0 \qquad \forall\ \underline{x} \in T$

$\min_{\underline{x}\in T} \underline{c}'\underline{x} \geq z_0 = \min \{ \underline{c}_i'\underline{x}_i, \quad i = 1{:}k \}$

max $5x + 8y$

subject to

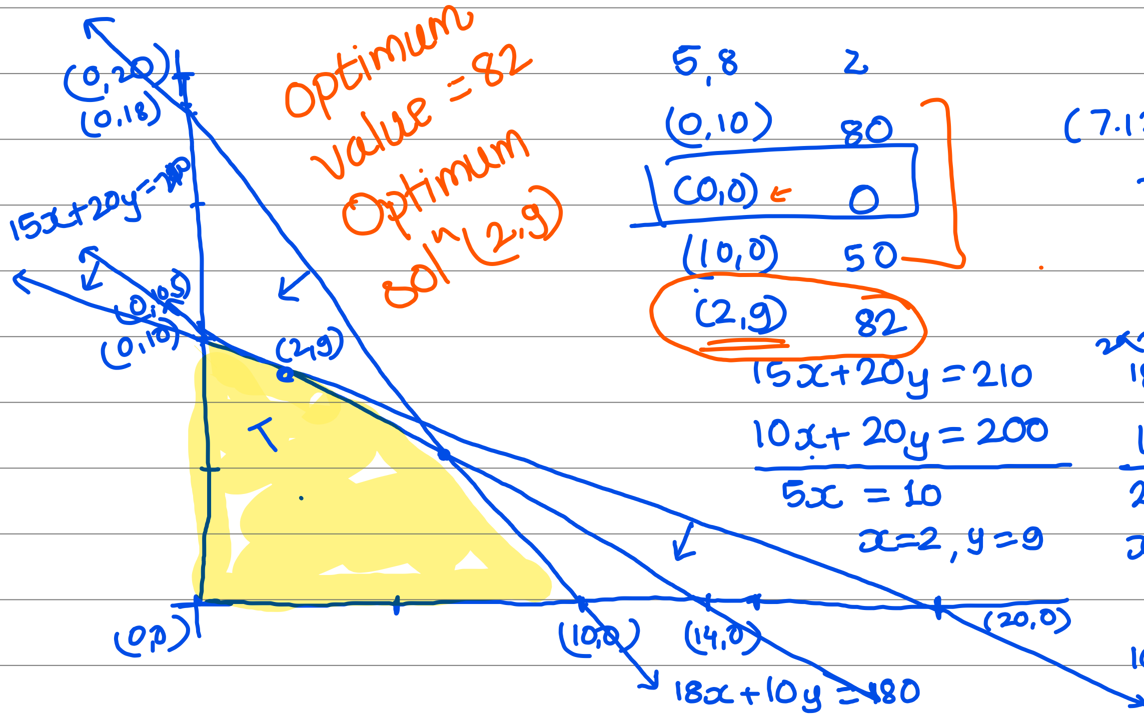
$18x + 10y \leq 180$

$10x + 20y \leq 200$

$15x + 20y \leq 210$

$\underline{\underline{x, y \geq 0}}$

$18x + 10y = 180$	$(0,18)$	$(10,0)$
$10x + 20y = 200$	$(0,10)$	$(20,0)$
$15x + 20y = 210$	$(0,10.5)$	$(14,0)$
$x, y \geq 0$		



5,8	z
(0,10)	80
$\boxed{(0,0)}$	0
(10,0)	50
(2,9)	82

$15x + 20y = 210$

$10x + 20y = 200$

$5x = 10$

$x = 2,\ y = 9$

(7.12, 5.14)

76.85

$2 \times 36 \quad 20 \quad 360$

$18x + 10y = 180$

$15x + 20y = 210$

$21x = 150$

$x = 7.12$

$y = 5.14$

$$2x_1+3x_2 \leq 10$$
$$3x_1+2x_2 \geq 5$$
$$3x_1+5x_2 \leq 15$$
$$x_1,x_2 \geq 0$$

$$2x_1+3x_2+x_3 = 10$$
$$3x_1+2x_2-x_4 = 5$$
$$3x_1+5x_2+x_5 = 15$$

$$2x_1+3x_2 \leq 10 \quad \Rightarrow \quad 2x_1-3x_2' \leq 10$$
$$x_1+5x_2 \leq 15 \qquad\qquad x_1-5x_2' \leq 15$$
$$x_1 \geq 0, \ \boxed{x_2 \leq 0} \qquad\qquad x_1 \geq 0 \quad x_2' \geq 0$$
$$x_2' = -x_2$$

$$x_B = [x_1\, x_2\, x_3]$$
Basic

$$\begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array}$$
$$\Big[\underbrace{\begin{matrix} 2 & 3 & 1 \\ 3 & 2 & 0 \\ 3 & 5 & 0 \end{matrix}}_{B} \ \underbrace{\begin{matrix} 0 & 0 \\ -1 & 0 \\ 0 & 1 \end{matrix}}_{R}\Big] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \\ 15 \end{bmatrix}$$

Basis

$$A\underline{x} = \underline{b} \qquad m=3, \ n=5$$
$m \times n$
3×5

$$\rho(A) \leq \min(m,n)$$

$$A = [B \ \ R] \qquad \rho(A) = \rho(B)$$

$$A\underline{x} = \underline{b}$$
$$\underset{m\times m}{[B} \ \ \underset{m\times(n-m)}{R]} \begin{bmatrix} \underline{x}_B \\ \underline{x}_{NB} \end{bmatrix} = \underline{b}$$
$$B\underline{x}_B + R\underline{x}_{NB} = \underline{b}$$
$$B^{-1}B\underline{x}_B + B^{-1}R\underline{x}_{NB} = B^{-1}\underline{b}$$

$$\underline{x}_B = B^{-1}\underline{b} - B^{-1}R\underline{x}_{NB}$$

Basic Sol^n $\rightarrow \underline{x}_{NB} = \underline{0}$, $\underline{x}_B = B^{-1}\underline{b}$

Max $3x_1+5x_2$

$x_1+2x_2 \le 6$ $\quad$ $x_1+2x_2+x_3 = 6$

$3x_1+x_2 \le 5$ $\quad$ $3x_1+x_2+x_4=5$

$x_1,x_2 \ge 0$ $\quad$ $x_1,x_2,x_3,x_4 \ge 0$

$$A=\begin{bmatrix}1&2&1&0\\3&1&0&1\end{bmatrix} \quad \underline{x}=\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} \quad \underline{b}=\begin{bmatrix}6\\5\end{bmatrix}$$

$m=2,\quad n=4$

$\rho(A)=\underline{m}=2$

$x_1+x_2 \le 2$

$\boxed{\cancel{2x_1+2x_2 \le 4}}$

(1) $B=\begin{bmatrix}1&0\\0&1\end{bmatrix}$ $\quad R=\begin{bmatrix}1&2\\3&1\end{bmatrix}$ $\quad \underline{x}_B=\begin{bmatrix}x_3\\x_4\end{bmatrix}$ $\quad x_{NB}=\begin{bmatrix}x_1\\x_2\end{bmatrix}$

$B^{-1}\underline{b}=\begin{bmatrix}1&0\\0&1\end{bmatrix}\begin{bmatrix}6\\5\end{bmatrix}=\begin{bmatrix}6\\5\end{bmatrix}$ $\quad x_B=[6,5]$

$x_{NB}=[0,0]$

$$\underline{x}=\begin{array}{c}x_1\ x_2\ x_3\ x_4\\(0,0,6,5)\end{array}$$ Basic Solⁿ

(2) $B=\begin{bmatrix}1&2\\3&1\end{bmatrix}$ $\quad \underline{x}_B=[x_1,x_2]$ $\quad \underline{x}_{NB}=[x_3\ x_4]$

$B^{-1}=-\frac{1}{5}\begin{bmatrix}1&-2\\-3&1\end{bmatrix}$ $\quad B^{-1}\underline{b}=-\frac{1}{5}\begin{bmatrix}1&-2\\-3&1\end{bmatrix}\begin{bmatrix}6\\5\end{bmatrix}=-\frac{1}{5}\begin{bmatrix}-4\\-13\end{bmatrix}=\begin{bmatrix}4/5\\13/5\end{bmatrix}$

Basic Solⁿ $\quad \underline{x}=[4/5,13/5,0,0]$

$a_1 \; a_2 \dots a_p$ $\qquad \sum_{i=1}^{p} \alpha_i a_i = 0 \Rightarrow \Rightarrow \alpha_i = 0$ l.i.

for some i $\alpha_i \neq 0$

↳ linearly dependent

$$\underline{x} = (\underbrace{x_1 x_2 \dots x_p}_{>0}, \underbrace{x_{p+1} \dots x_n}_{=\underline{0}})$$

$$\underset{m\times n}{A}\underline{x} = \underline{b}_{m\times 1}$$

$$x_1\underline{a}_1 + x_2\underline{a}_2 + \dots + x_n\underline{a}_n = \underline{b}_{m\times 1}$$

$$\Rightarrow \quad x_1\underline{a}_1 + x_2\underline{a}_2 + \dots + x_p\underline{a}_p = \underline{b}_{m\times 1}$$

① If p vectors are linearly independent

$p \leq m$ ✓

↳ i) $p = m$, $\underline{x}$ is f.s. is also b.f.s.

ii) $p < m$

we can add $m-p$ linearly independent columns

$$x_1\underline{a}_1 + x_2\underline{a}_2 + \dots + x_p\underline{a}_p + 0\,\underline{a}_{p+1} + 0\cdot\underline{a}_{p+2} + \dots + 0\cdot\underline{a}_m = \underline{b}$$

$\Rightarrow$ we get degenerate b.f.s.

② If p ~~vectors~~ columns corresponding to feasible sol^n are linearly dependent.

$$\sum_{i=1}^{p} \alpha_i a_i = 0 \Rightarrow \text{for some } \alpha_j \neq 0$$

Assume $\alpha_j \neq 0$

$$\frac{x_j}{\alpha_j} \sum_{i=1}^{p} \alpha_i a_i = 0.$$

$$\sum_{i=1}^{p} x_i \underline{a}_i = \underline{b}. \qquad (A\underline{x} = \underline{b}$$

$$\sum_{i=1}^{p} x_i a_i - \frac{x_j}{\alpha_j} \sum_{i=1}^{p} \alpha_i a_i = \underline{b} - \underline{0}$$

$$\underbrace{\sum_{\substack{i=1 \\ i \neq j}}^{p} x_i a_i} + \underline{x_j a_j - x_j a_j} = \underline{b}$$

$\underline{x} = [1, 1, 1]$

$\sum \underline{\alpha}_i \underline{a}_i = 0 \Rightarrow \cancel{\alpha_1}\begin{bmatrix}1\\2\end{bmatrix} + \cancel{\alpha_2}\begin{bmatrix}1\\-1\end{bmatrix} + \cancel{\alpha_3}\begin{bmatrix}2\\1\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}.$

choosing $\alpha_1 = 1, \ \alpha_2 = 1, \ \alpha_3 = -1$

$\cancel{x_1}\begin{bmatrix}1\\2\end{bmatrix} + \cancel{x_2}\begin{bmatrix}1\\-1\end{bmatrix} + \cancel{x_3}\begin{bmatrix}2\\1\end{bmatrix} = \begin{bmatrix}4\\2\end{bmatrix}.$

$2\begin{bmatrix}1\\2\end{bmatrix} + 2\begin{bmatrix}1\\-1\end{bmatrix} + 0\begin{bmatrix}2\\1\end{bmatrix} = \begin{bmatrix}4\\2\end{bmatrix}$

$A\underline{x} = b$

$\underline{x} = [1, 1, 1]$

$[2, 2, 0]$

$\underline{x}$ extreme

$x_1 \underline{a}_1 + x_2 \underline{a}_2 + \dots + x_K \underline{a}_K = \underline{b}.$ $\qquad 0 < z < \min \frac{x_i}{|\alpha_i|}$

~~depend~~ $\pm z\left[\alpha_1 \underline{a}_1 + \alpha_2 \underline{a}_2 + \dots + \alpha_K \underline{a}_K = \underline{0}\right]$ $\qquad$ for some $\alpha_j \neq 0$

$z > 0$

$x_i - z\alpha_i \geq 0$

$\frac{1}{2}(\underline{x} + z\underline{\alpha}) + \frac{1}{2}(\underline{x} - z\underline{\alpha}) = \underline{x}$ $\qquad \min\left(\frac{x_i}{\alpha_i}\right) \geq z \quad \forall i$

$(x_1 + z\alpha_1)\underline{a}_1 + \dots + (x_K + z\alpha_K)\underline{a}_K = \underline{b}$

$$\text{Max } z = 6x_1 + 5x_2$$
$$\text{s.t. } x_1 + x_2 \le 5$$
$$3x_1 + 2x_2 \le 12$$
$$x_1, x_2 \ge 0$$

$$x_1 + x_2 + x_3 = 5$$
$$3x_1 + 2x_2 + x_4 = 12$$
$$\Rightarrow x_i \ge 0$$

$$x_3 = 5 - x_1 - x_2 \quad \text{—(1)}$$
$$x_4 = 12 - 3x_1 - 2x_2 \quad \text{—(2)}$$

0, 0 , 6, 5

$(5, 12, 0, 0)$
(x_3, x_4, x_1, x_2)
Basic Nonbasic

$$z = 0$$

$\min(5, 4)$

Basic Nonbasic
(x_1, x_3, x_2, x_4)
6 0 5 0
$(4, 1, 0, 0)$

$$z = 24$$

$$3x_1 = 12 - 2x_2 - x_4$$
$$x_1 = 4 - \frac{2}{3}x_2 - \frac{1}{3}x_4 \quad \text{—(3)}$$
$$x_3 = 5 - 4 + \frac{2}{3}x_2 + \frac{1}{3}x_4 - x_2$$
$$\rightarrow x_3 = 1 - \frac{1}{3}x_2 + \frac{1}{3}x_4 \quad \text{—(4)}$$

$$z = 6x_1 + 5x_2$$
$$= 6\cdot\left(4 - \frac{2}{3}x_2 - \frac{1}{3}x_4\right) + 5x_2$$
$$z = 24 + x_2 - 2x_4$$

$\min(6, 3)$

$$\frac{1}{3}x_2 = 1 - x_3 + \frac{1}{3}x_4 \Rightarrow x_2 = \boxed{3 - 3x_3 + x_4}$$

$$x_1 = 4 - \frac{2}{3}(3 - 3x_3 + x_4) - \frac{1}{3}x_4$$

Basic NonB
$x_1\ x_2 \quad x_3\ x_4$
2 3 0 0
6 5 0 0

$$z = 27$$

$$x_1 = 2 + 2x_3 - x_4$$

$$z = 6x$$

$$z = 24 + x_2 - 2x_4$$
$$= 24 + 3 - 3x_3 + x_4 - 2x_4$$
$$z = 27 - 3x_3 - x_4$$

$(x_1\ x_2\ x_3\ x_4) = (2\ \ 3\ \ 0\ \ 0)$ $z = 27$

Simplex Method.

	C_j	6	5	0	0		
C_j	Basic	x_1	x_2	x_3	x_4	b	θ
0	x_3	1	1	1	0	5	5/1 min
0	x_4	$\boxed{3}$	2	0	1	12	12/3 = 4 → leaving
	Z_j	0	0	0	0	0	
	$C_j - Z_j$	6 ↑ most +ve entering	5	0	0		
0	x_3	0	$\boxed{1/3}$	1	$-1/3$	1	3 min $R_1 - R_2'$
6	x_1	1	2/3	0	1/3	4	6 $\frac{1}{3}R_2$
	Z_j	6	4	0	2	24	
	$C_j - Z_j$	0	↑ 1 most +ve	0	-2		
$3R_1$ 5	x_2	0	1	3	-1	3	3
$R_2 - \frac{2}{3}R_1'$ 6	x_1	1	0	-2	1	2	
	Z_j	6	5	3	1	27	
	$C_j - Z_j$	0	0	-3	-1		

Min $z = -x_1 - x_2$

s.t. $x_1 + x_2 \leq 2$

$x_1 - x_2 \leq 1$

$x_2 \leq 1$

$x_1, x_2 \geq 0$

Max $w = x_1 + x_2$

s.t. $x_1 + x_2 + x_3 = 2.$

$x_1 - x_2 + x_4 = 1$

$x_2 + x_5 = 1$

$x_i \geq 0 \ \forall \ i = 1:5$

Cost	Basic	x_1	x_2	x_3	x_4	x_5	b	θ
	Cost	1	1	0	0	0		
0	x_3	1	1	1	0	0	2	2 min θ
0	x_4	$\boxed{1}$	-1	0	1	0	1	1 → leaving variable
0	x_5	0	1	0	0	1	1	
	z_j	0	0	0	0	0	0	
	$c_j - z_j$	1 max +ve	1	0	0	0		

entering variable

	Cost	Basic	x_1	x_2	x_3	x_4	x_5	b	θ
			1	1	0	0	0		
$R_1 - R_2$	0	x_3	0	$\boxed{2}$	1	-1	0	1	1/2 → leaving
	1	x_1	1	-1	0	1	0	1	–
	0	x_5	0	1	0	0	1	1	1
		z_j	1	-1	0	1	0	1	
		$c_j - z_j$	0	2	0	-1	0		

entering

	Cost	Basic	x_1	x_2	x_3	x_4	x_5	b	θ
			1	1	0	0	0		
$\frac{1}{2}R_1$	1	x_2	0	1	1/2	-1/2	0	1/2	–
$R_2 + R_1$	1	x_1	1	0	1/2	1/2	0	3/2	3
$R_3 - R_1$	0	x_5	0	0	-1/2	$\boxed{1/2}$	1	1/2	1 →
		z_j	1	1	1	0	0	2	
		$c_j - z_j$	0	0	-1	0	0		

Optimal Sol^n $(3/2, 1/2, 0, 0, 1/2)$ ✓

Optimal Value $z = 2$

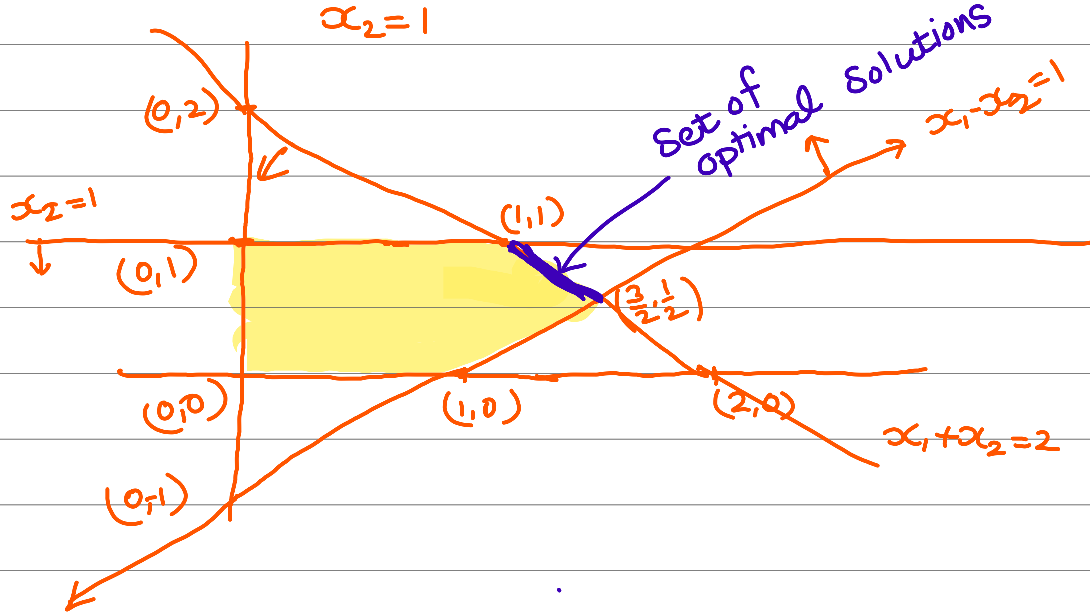
			1	1	0	0	0		θ
$R_1+\frac{1}{2}R_3$	1	x_2	0	1	0	0	1	1	
$R_2-\frac{1}{2}R_3$	1	x_1	1	0	1	0	−1	1	
$2R_3$	0	x_4	0	0	−1	1	2	1	
		z_j	1	1	1	0	0	2	
		c_j-z_j	0	0	−1	0	0		

Alternate Optimal Sol^n :- (1, 1, 0, 1, 0) ✓

Optimal Value :- 2

Min $z = -x_1 - x_2$

$x_1 + x_2 \leq 2$	$x_1 + x_2 = 2$	(0,2), (2,0)
$x_1 - x_2 \leq 1$	$x_1 - x_2 = 1$	(0,−1), (1,0)
$x_2 \leq 1$	$x_2 = 1$	
$x_1, x_2 \geq 0$		



Extreme pts.	z value. min $z = -x_1 - x_2$	
(0,0)	0	
(0,1)	−1	
(1,1)	−2	min
$(\frac{3}{2},\frac{1}{2})$	−2	