

→ 0/1



// Data

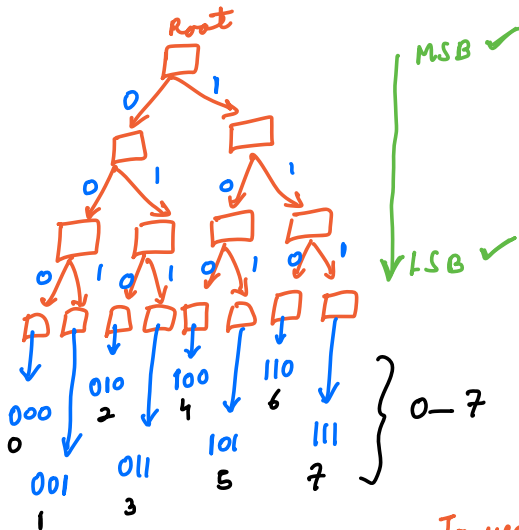
node () of

```
// data
```

3

3

5 0-31 ✓

$$32 \quad 0 - (2^{32} - 1)$$


To use all numbers from 0 to $(2^k - 1)$,
tree of height k is used. ✓

Trie of Bits is a binary tree

Q → Find max value of $A[i] \wedge A[j]$ forall pair (i, j) .

Ex $\rightarrow A = [9, 8, 10, 7]$

Beute $\rightarrow \forall \text{paire}(i,j) \text{ find } A[i] \wedge A[j]$

↳ take max value. $TC = O(N^2)$ ✓

A	B	$A \wedge B$
0	0	0
0	1	0
1	0	0
1	1	1

$$9 \wedge 8 = 1001 \wedge 1000 = 0001 = 1$$

$$9 \wedge_{10} = 1001 \wedge_{10} 1010 = 0011 = 3$$

$$9 \wedge 7 = 1001 \wedge 0111 = 1110 = 14$$

$$8 \wedge_{10} = 2$$

$$8 \wedge 7 = 15 \checkmark$$

$$10^7 = 1010 \wedge 011 = 1101 = 13$$

$$15 = 1111$$

7 $\rightarrow 1111 \wedge 0111 = 1000 = 8$ ✓

$$\Rightarrow 1111 \wedge 1011 = 0100 = 4$$

$11 \rightarrow 1111 \wedge 1101 = 0010 = 2$
 $13 \rightarrow 1111 \wedge 1101 = 0001 = 1$

14 → $1111 \wedge 11110 = 0001 = 1$

15 = 1111

7 = 0111 ✓

$$8 = 1000$$

$$1111 \wedge 1000 = 0111 = \underline{7}$$

Making more significant bits = 2
can be a better choice to get
higher result. ✓

$A = [5, 10, 15, 20, 25, 30]$

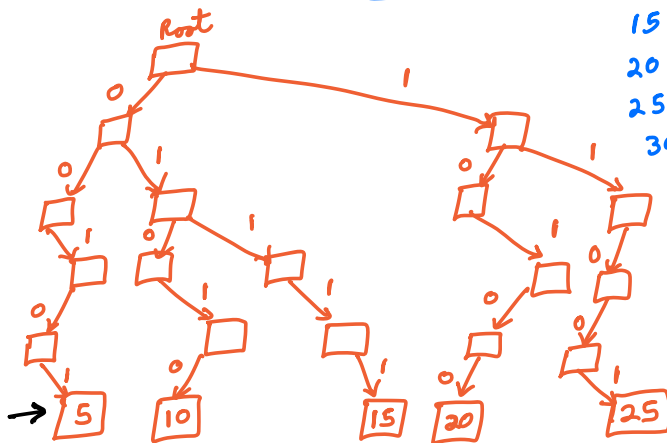
Height = 5 (0-31)

$5 = 00101$
 $10 = 01010$
 $15 = 01111$
 $20 = 10100$
 $25 = 11001$
 $30 = 11110$

$\forall i$ find j which give max $A[i] \wedge A[j]$ ✓

$i > j$

$10 \wedge 5 = 01111 = 15$
 $15 \wedge 5 = 01010 = 10$
 $20 \wedge 10 = 11110 = 30$
 $25 \wedge 5 = 11100 = 28$
 $30 \wedge 5 = 11011 = 27$



$TC = O(H * N)$ ✓

5

$SC = O(H * N)$

1) Query
 2) Insert

Ans = 15 30 ✓

Q → Find max XOR subarray of an array?

eg → $A = [1, 4, 3]$

1) $x \wedge x = 0$
 $x \wedge y \wedge x = y$

2) $x \wedge y \wedge z$
 $(x \wedge y) \wedge z = x \wedge (y \wedge z)$

$a \wedge b \wedge c \wedge d = (a \wedge b) \wedge c \wedge d$
 \downarrow
 $x \wedge d$

$[1] \rightarrow 1$
 $[1, 4] \rightarrow 1 \wedge 4 \rightarrow 5$
 $[1, 4, 3] \rightarrow 1 \wedge 4 \wedge 3 \rightarrow 6$
 $[4] \rightarrow 4$
 $[4, 3] \rightarrow 4 \wedge 3 \rightarrow 7$ ✓ (Ans)
 $[3] \rightarrow 3$

Prefix = $[1, (1 \wedge 4), (1 \wedge 4 \wedge 3)]$

$= [1, 5, 6]$

$P = [0, 1, 5, 6]$ ✓

Bruteforce → $O(N^2)$

\forall subarray find XOR & get max value.

$TC = O(N^3) \rightarrow O(N^2)$

0 1 2 3 4

start	# ends
0	5 (0, 1, 2, 3, 4)
1	4 (1, 2, 3, 4)
2	3 (2, 3, 4)
3	2 (3, 4)
4	1 (4)
$5 + 4 + 3 + 2 + 1 = 15$	
$\leq N$	

Prefix XOR

$A[i] \wedge A[i+1] \wedge A[i+2] \dots \wedge A[j] = X$
 $x \wedge y \wedge y = x$

$X = A[i] \wedge A[i+1] \wedge \dots \wedge A[j]$
 $\wedge A[0] \wedge A[1] \wedge A[2] \dots \wedge A[i-1]$
 $\wedge A[0] \wedge A[1] \wedge A[2] \dots \wedge A[i-1]$

$P[i] = P[0] \wedge P[1] \wedge P[2] \dots \wedge P[i]$

$X = P[j] \wedge P[i-1]$ ✓

$TC = O(H * N)$

$i \geq 0$

$SC = O(H * N)$ ✓

Find the max XOR pair in Prefix XOR array.

$A = [1, 4, 3]$ $H=3$

$p = 0^1 1 = 1^1 4 = 5^1 3 = 6$

1011011
 $= 110 = 6$

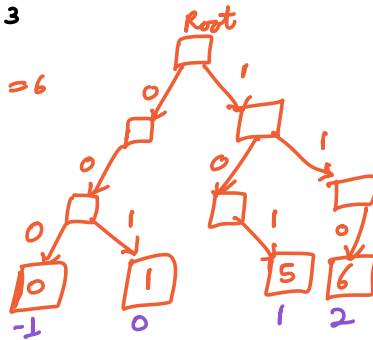
1) Query
 2) Insert

Ans = 1, 5, 7 ✓

$1^1 0 = 1$

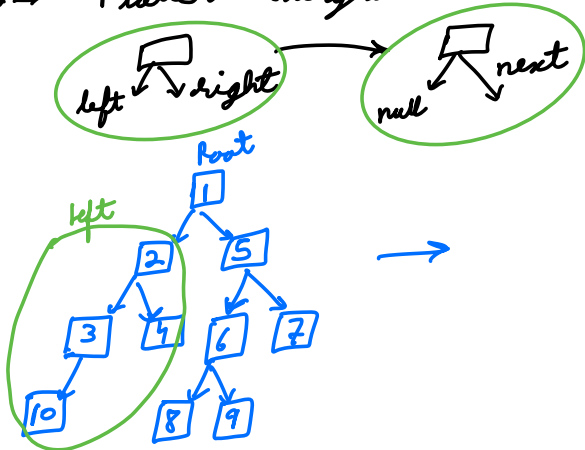
$5^1 0 = 5$

$6^1 1 = 7$



$\leftarrow (i-1) + 1$
 start $(i-j)$ subarray

Q → Flatten Binary tree to LL via preorder traversal. (Epifi)



Preorder = 1, 2, 3, 10, 4, 5, 6, 8, 9, 7

pair <node> flatten(root) of

H, T
 ↓
 Head
 Tail

if (root == null) return {null, null};

→ L = flatten(root.left);

→ R = flatten(root.right);

root.left = null;

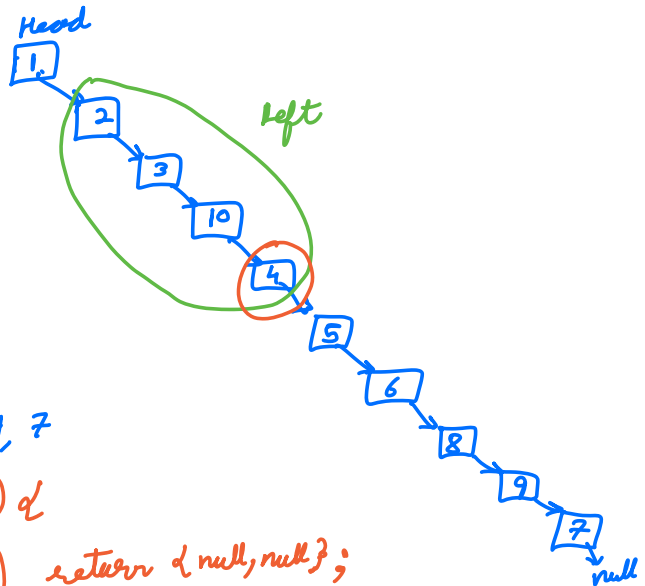
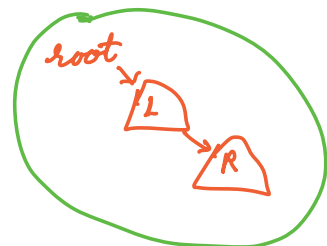
if (L.H != null) {

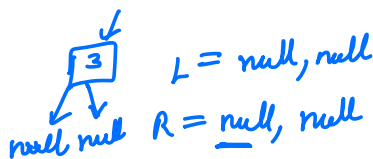
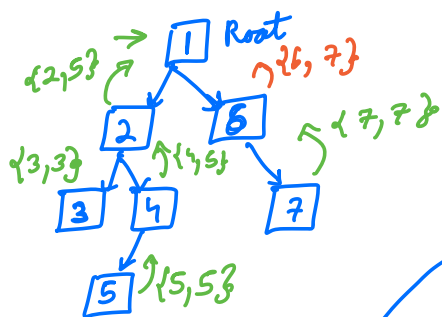
root.right = L.H;

L.T.right = R.H;

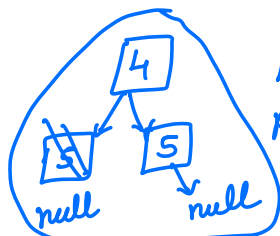
} else { root.right = R.H; }

return {root, (R.T != null ? R.T : (L.T != null ? L.T : root))};



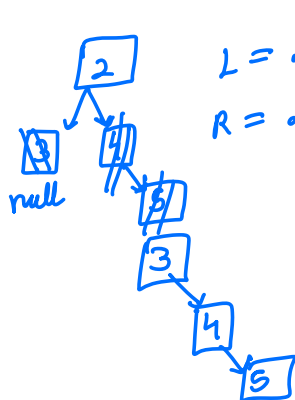


$\{3, 3\}$



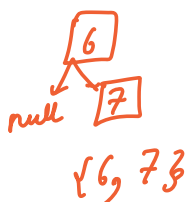
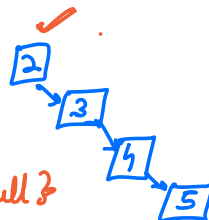
$L = \{5, 5\}$
 $R = \{\text{null}, \text{null}\}$

$\{4, 5\}$ ✓



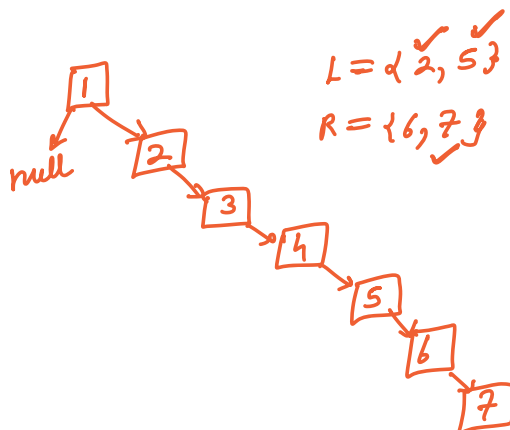
$L = \{3, 3\}$
 $R = \{4, 5\}$
 ↑

$\{2, 5\}$



$L = \{\text{null}, \text{null}\}$
 $R = \{7, 7\}$

$TC = O(N)$
 $SC = O(H)$



$L = \{2, 5\}$
 $R = \{6, 7\}$

$\{1, 7\}$

(Ans)