

## Quick Navigation

★★★★★ Average Rating: 4.95 (56 votes)

## Solution

### Overview

Finding the  $k$  closest points to the origin will require us to first be able to calculate the distance of a given point to the origin before we can start to evaluate the relative closeness of any two points.

In order to evaluate the distance from the origin to a given point, we must use the **Euclidean distance** equation. This equation starts with the **Pythagorean theorem**:  $(a^2 + b^2 = c^2)$  which calculates the distance of the hypotenuse ( $c$ ) of a right triangle when the length of the other two sides ( $a, b$ ) is known.

Given two Euclidean points, we can determine the values for  $a$  and  $b$  by taking the difference of the two  $x$  coordinates ( $a = x_1 - x_2$ ) and the two  $y$  coordinates ( $b = y_1 - y_2$ ). Plugging these values into the Pythagorean theorem and solving for the length of  $c$ , we get the Euclidean distance equation ( $dist = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ ).

In this problem, with one of the two Euclidean coordinates being the origin ( $0, 0$ ), this simplifies the Euclidean distance equation back to the original Pythagorean theorem ( $dist = \sqrt{(x - 0)^2 + (y - 0)^2} = \sqrt{x^2 + y^2}$ ).

We can also simplify the process of comparing two points by using the squared Euclidean distance instead of the precise Euclidean distance, as both will yield the same result. This allows us to remove the square root from each side of the equation ( $\sqrt{x_1^2 + y_1^2} < \sqrt{x_2^2 + y_2^2} \Leftrightarrow (x_1^2 + y_1^2) < (x_2^2 + y_2^2)$ ) which will significantly reduce the overall computational time for each comparison made.

### Approach 1: Sort with Custom Comparator

#### Intuition

We can reframe the problem as finding  $k$  points with the smallest **squared Euclidean distance** from the origin. When seeking the smallest elements in a list, an intuitive first step is to sort the list, as this will bring the smallest elements to the front.

Therefore, in this problem, we can sort the entire `points` array using a **custom comparator** function that applies the squared Euclidean distance equation. After the sorting process is completed, we just return the first  $k$  elements of the sorted array.

This solution is trivial, and while it gets the job done, it should not be considered an ideal candidate for an interview response. As we will see, there are more efficient options from which to choose.

#### Algorithm

- Sort the array with a **custom comparator** function.  
The custom comparator function will use the **squared Euclidean distance** equation to compare two points.

- Return the first  $k$  elements of the array.

C++ Java JavaScript Python3

```
C++ Solution {
public int[][] kClosest(int[][] points, int k) {
    // Sort the array with a custom lambda comparator function
    Arrays.sort(points, (a, b) -> squaredDistance(a) - squaredDistance(b));
    // Return the first k elements of the sorted array
    return Arrays.copyOf(points, k);
}

private int squaredDistance(int[] point) {
    // Calculate and return the squared Euclidean distance
    return point[0] * point[0] + point[1] * point[1];
}
}

Java Solution {
public int[][] kClosest(int[][] points, int k) {
    // Sort the array with a custom lambda comparator function
    Arrays.sort(points, (a, b) -> squaredDistance(a) - squaredDistance(b));
    // Return the first k elements of the sorted array
    return Arrays.copyOf(points, k);
}

private int squaredDistance(int[] point) {
    // Calculate and return the squared Euclidean distance
    return point[0] * point[0] + point[1] * point[1];
}
}

JavaScript Solution {
function squaredDistance(point) {
    return point[0] * point[0] + point[1] * point[1];
}

function kClosest(points, k) {
    // Sort the array with a custom lambda comparator function
    points.sort((a, b) -> squaredDistance(a) - squaredDistance(b));
    // Return the first k elements of the sorted array
    return points.slice(0, k);
}
}

Python3 Solution {
def squaredDistance(point):
    return point[0] * point[0] + point[1] * point[1]

def kClosest(points, k):
    # Sort the array with a custom lambda comparator function
    points.sort(key=squaredDistance)
    # Return the first k elements of the sorted array
    return points[:k]
}
}
```

#### Complexity Analysis

Here  $N$  refers to the length of the given array `points`.

- Time complexity:  $O(N \cdot \log N)$  for the sorting of `points`.  
While sorting methods vary between different languages, most have a worst-case or average time complexity of  $O(N \cdot \log N)$ .
- Space complexity:  $O(\log N)$  to  $O(N)$  for the extra space required by the sorting process.

As with the time complexity, the space complexity of the sorting method used can vary from language to language. C++'s STL, for example, uses QuickSort most of the time but will switch to either HeapSort or InsertionSort depending on the nature of the data. Java uses a variant of QuickSort with dual pivots when dealing with arrays of primitive values. The implementation of both C++'s and Java's sort methods will require an average of  $O(\log N)$  extra space. Python, on the other hand, uses TimSort, which is a hybrid of MergeSort and InsertionSort and requires  $O(N)$  extra space. Unlike most other languages, Javascript's sort method will actually vary from browser to browser. Since the adoption of ECMAScript 2019, however, the sort method is required to be stable, which generally means MergeSort or TimSort and a space complexity of  $O(N)$ .

### Approach 2: Max Heap or Max Priority Queue

#### Intuition

While we must iterate over all elements in the `points` array, we only need to keep track of the  $k$  closest points encountered so far. We could therefore choose to store them in a separate data structure. In order to keep this data structure capped at  $k$  elements, we will need to keep track of the point that is farthest away from the origin and thus the next point to be removed when a closer point is found.

The ideal data structure for this purpose is a **max heap** or **max priority queue**. These data structures allow access to the max value in constant time and perform replacements in logarithmic time.

**Note:** We can simulate max heap functionality in a min heap data structure by inserting  $-dist$  instead of  $dist$ , if necessary.

At the start of our iteration through `points`, we will insert the first  $k$  elements into our heap. Once the heap is "full", we can then compare each new point to the farthest point stored in the heap. If the new point is closer, then we should remove the farthest point from the heap and insert the new point.

After the entire `points` array has been processed, we can create an array from the points stored in the heap and then return the answer.

#### Algorithm

- Use a **max heap** (or **max priority queue**) to store points by distance.  
Store the first  $k$  elements in the heap.  
Then only add new elements that are closer than the top point in the heap while removing the top point to keep the heap at  $k$  elements.

- Return an array of the  $k$  points stored in the heap.

C++ Java JavaScript Python3

```
C++ Solution {
public int[][] kClosest(int[][] points, int k) {
    int[] entry = {squaredDistance(points[0]), 0};
    if (maxQ.size() < k) {
        // Fill the max PQ up to k points
        maxQ.add(entry);
    } else if (entry[0] < maxQ.peek()[0]) {
        // If the max PQ is full and a closer point is found,
        // discard the farthest point and add this one
        maxQ.poll();
        maxQ.add(entry);
    }
}

Java Solution {
public int[][] kClosest(int[][] points, int k) {
    int[] entry = {squaredDistance(points[0]), 0};
    if (maxQ.size() < k) {
        // Fill the max PQ up to k points
        maxQ.add(entry);
    } else if (entry[0] < maxQ.peek()[0]) {
        // If the max PQ is full and a closer point is found,
        // discard the farthest point and add this one
        maxQ.poll();
        maxQ.add(entry);
    }
}

JavaScript Solution {
function squaredDistance(point) {
    return point[0] * point[0] + point[1] * point[1];
}

function kClosest(points, k) {
    let maxQ = [];
    let entry = [squaredDistance(points[0]), 0];
    if (maxQ.length < k) {
        // Fill the max PQ up to k points
        maxQ.push(entry);
    } else if (entry[0] < maxQ[0][0]) {
        // If the max PQ is full and a closer point is found,
        // discard the farthest point and add this one
        maxQ.shift();
        maxQ.push(entry);
    }
}

Python3 Solution {
def squaredDistance(point):
    return point[0] * point[0] + point[1] * point[1]

def kClosest(points, k):
    maxQ = []
    entry = [squaredDistance(points[0]), 0]
    if len(maxQ) < k:
        # Fill the max PQ up to k points
        maxQ.append(entry)
    else if entry[0] < maxQ[0][0]:
        # If the max PQ is full and a closer point is found,
        # discard the farthest point and add this one
        maxQ.pop(0)
        maxQ.append(entry)
}
}
}
```

## LeetCode Challenge + GIVEAWAY!

Run Code Result

i C#

```
SortedDictionary<double, List<(int, int)> dict = new SortedDictionary<double, List<(int, int)>>();
List<(int, int)> points;
var x1 = 0;
var y1 = 0;
foreach(var point in points) {
    var x2 = point[0];
    var y2 = point[1];
    var dist = Math.Sqrt((double)(Math.Pow(x1 - x2, 2) + Math.Pow(y1 - y2, 2)));
    if(dict.ContainsKey(dist)) {
        dict[dist].Add((x2, y2));
    } else {
        dict.Add(dist, new List<(int, int)>{ (x2, y2) });
    }
}
var result = new int[k];
var j = 0;
foreach(var item in dict) {
    if(j == k) break;
    foreach(var point in item.Value) {
        result[j] = point.x;
        j++;
    }
}
}

Testcase Run Code Result
```

Accepted Runtime: 132 ms

Your input [[1,3],[-2,2]]

Output [[-2,2]] Diff

Expected [[-2,2]]

Console + Use Example Testcases

```
SortedDictionary<double, List<(int, int)> dict = new SortedDictionary<double, List<(int, int)>>();
List<(int, int)> points;
var x1 = 0;
var y1 = 0;
foreach(var point in points) {
    var x2 = point[0];
    var y2 = point[1];
    var dist = Math.Sqrt((double)(Math.Pow(x1 - x2, 2) + Math.Pow(y1 - y2, 2)));
    if(dict.ContainsKey(dist)) {
        dict[dist].Add((x2, y2));
    } else {
        dict.Add(dist, new List<(int, int)>{ (x2, y2) });
    }
}
var result = new int[k];
var j = 0;
foreach(var item in dict) {
    if(j == k) break;
    foreach(var point in item.Value) {
        result[j] = point.x;
        j++;
    }
}
}

Testcase Run Code Result
```

Accepted Runtime: 132 ms

Your input [[1,3],[-2,2]]

Output [[-2,2]] Diff

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Console + Use Example Testcases

```
SortedDictionary<double, List<(int, int)> dict = new SortedDictionary<double, List<(int, int)>>();
List<(int, int)> points;
var x1 = 0;
var y1 = 0;
foreach(var point in points) {
    var x2 = point[0];
    var y2 = point[1];
    var dist = Math.Sqrt((double)(Math.Pow(x1 - x2, 2) + Math.Pow(y1 - y2, 2)));
    if(dict.ContainsKey(dist)) {
        dict[dist].Add((x2, y2));
    } else {
        dict.Add(dist, new List<(int, int)>{ (x2, y2) });
    }
}
var result = new int[k];
var j = 0;
foreach(var item in dict) {
    if(j == k) break;
    foreach(var point in item.Value) {
        result[j] = point.x;
        j++;
    }
}
}

Testcase Run Code Result
```

Accepted Runtime: 132 ms

Your input [[1,3],[-2,2]]

Output [[-2,2]] Diff

Expected [[-2,2]]

```

14     maxQ.push();
15 }
16 }
17
18 // Return all points stored in the max PQ
19 int[][] answer = new int[k][2];
20 for (int i = 0; i < k; i++) {
21     int entryIndex = maxQ.pop()[1];
22     answer[i] = points[entryIndex];
23 }
24 return answer;
25 }
26
27 private int squaredDistance(int[] point) {
28     // Calculate and return the squared Euclidean distance
29     return point[0] * point[0] + point[1] * point[1];
30 }
31

```

### Complexity Analysis

Here  $N$  refers to the length of the given array `points`.

- Time complexity:  $O(N \cdot \log k)$

Adding to/removing from the heap (or priority queue) only takes  $O(\log k)$  time when the size of the heap is capped at  $k$  elements.

- Space complexity:  $O(k)$

The heap (or priority queue) will contain at most  $k$  elements.

## Approach 3: Binary Search

### Intuition

Since this problem is asking us to identify the first  $k$  sorted points, another approach that may come to mind is a **binary search**. In a standard binary search approach, we would have a sorted array of data, a defined target condition, and a set range of values to attempt. The binary search process involves picking a midpoint of the range and then figuring out on which side of the midpoint the target lies in  $O(1)$  time. By repeating this process, we can isolate the target condition in only  $O(\log N)$  time.

Without a sorted `points` array, applying a binary search technique to the current problem would require us to modify the standard method. For this modified approach we would first choose a target distance, then we would iterate through every point during each binary search loop to check if our target distance contains exactly  $k$  points. If it contains less than  $k$  points, we will increase our target distance, and vice versa, until we find a target distance that contains exactly  $k$  points. This would result in an average time complexity of  $O(N \cdot \log N)$ , which is no better than the standard sorting method.

In this case, however, we can improve upon the time complexity of this modified binary search by eliminating one set of points at the end of each iteration. If the target distance yields fewer than  $k$  closer points, then we know that each of those points belongs in our answer and can then be ignored in later iterations. If the target distance yields more than  $k$  closer points, on the other hand, we know that we can discard the points that fall outside the target distance.

By roughly halving the remaining points in each iteration of the binary search, we reduce the total number of processes to  $N + \frac{N}{2} + \frac{N}{4} + \frac{N}{8} + \dots + \frac{N}{N} = 2N$ . This results in an average time complexity of  $O(N)$ .

Since we're going to be using the midpoint of the range of distances for each iteration of our binary search, we should calculate the actual Euclidean distance for each point, rather than using the squared distance as in the other approaches. An even distribution of the points in the input array will yield an even distribution of distances, but an uneven distribution of squared distances.

As the efficiency of this solution relies on averaging as close to a middle split of the points as possible on each iteration of the binary search, the use of Euclidean distances will be more efficient than the use of squared Euclidean distances. We can precompute these distances in a separate array prior to performing the binary search, however, to lessen the overall processing required. This will also allow us to use an array of reference indices in our binary search, rather than having to create and modify more complex arrays during each iteration.

During each iteration of the binary search, we will split the points into two arrays, `closer`, which contains all of the points that are closer than or equal to the current target distance, and `farther`, which contains all of the points that are farther than the target distance. If the `closer` array contains fewer than  $k$  points, we can add those points to our answer array (`closest`) and adjust  $k$  to reflect the number of points still left to be found. Then we can focus on the remaining points in the `farther` array for the next round. If the `closer` array contains more than  $k$  points, we can discard the `farther` array. In either case, we will need to update our range to match the array we keep.

Once the answer array is complete, we can build and return an array of the  $k$  closest points.

### Algorithm

1. Precompute the Euclidean distances of each point.
2. Define the initial binary search range by identifying the farthest computed distance.
3. Perform a binary search from low to high using the reference distances.
  - Calculate the midpoint of the remaining range as the target distance.
  - Split the remaining points into those closer and those farther than the target distance.
  - If the `closer` array has fewer than  $k$  points, add them to the `closest` array and adjust the value of  $k$ .
  - Keep only the appropriate remaining array for the next iteration and update the binary search range.
4. Once  $k$  elements have been added to the `closest` array, return the  $k$  closest points.

C++ Java JavaScript Python3

```

class Solution {
2 public int[][] kClosest(int[][] points, int k) {
3     // Precompute the Euclidean distance for each point,
4     // define the initial binary search range,
5     // and create a reference list of point indices
6     double[] distances = new double[points.length];
7     double low = 0, high = 0;
8     List<Integer> remaining = new ArrayList<>();
9     for (int i = 0; i < points.length; i++) {
10         distances[i] = euclideanDistance(points[i]);
11         high = Math.max(high, distances[i]);
12         remaining.add(i);
13     }
14
15     // Perform a binary search of the distances
16     // to find the k closest points
17     List<Integer> closest = new ArrayList<>();
18     while (k > 0) {
19         double mid = low + (high - low) / 2;
20         List<List<Integer>> result = splitDistances(remaining, distances, mid);
21         List<Integer> closer = result.get(0), farther = result.get(1);
22         if (closer.size() > k) {
23             // If more than k points are in the closer distances
24             // then discard the farther points and continue
25             remaining = closer;
26             high = mid;
27         } else {
28             closest.addAll(closer);
29             k -= closer.size();
30         }
31     }
32 }

```

### Complexity Analysis

Here  $N$  refers to the length of the given array `points`.

- Time complexity:  $O(N)$

While this binary search variant has a worst-case time complexity of  $O(N^2)$ , it has an average time complexity of  $O(N)$ . It achieves this by halving (on average) the remaining elements needing to be processed at each iteration, which results in  $N + \frac{N}{2} + \frac{N}{4} + \frac{N}{8} + \dots + \frac{N}{N} = 2N$  total processes. This yields an average time complexity of  $O(N)$ .

- Space complexity:  $O(N)$

An extra  $O(N)$  space is required for the arrays containing distances and reference indices.

## Approach 4: QuickSelect

### Intuition

While the previous approach was successful in reducing the time complexity to only  $O(N)$ , it did so at the expense of pushing the space complexity to  $O(N)$ . But what if we could use an in-place approach and modify the `points` array directly? Bringing the  $k$  closest points forward to the beginning of the array would effectively result in a partial sort of `points`. This method is called the **QuickSelect algorithm**.

In fact, anytime we are tasked with finding the  $k$  (or  $k^{th}$ ) [smallest/largest/etc] element(s), we should always consider whether the QuickSelect algorithm can be applied. To understand why this is the case, we will briefly introduce the QuickSelect

```

3     SortedDictionary<double, List<(int, int)>> dict = new
4     SortedDictionary<double, List<(int, int)>>();
5     var x1 = 0;
6     foreach(var point in
7     points) {
8         var x2 = point[0];
9         var y2 = point[1];
10        var dist =
11            Math.Sqrt((double)(Math.Pow(x1-
12            x2, 2) + Math.Pow(y1-y2, 2)));
13
14        if(dict.ContainsKey(dist)) {
15            dict[dist].Add((x2, y2));
16        } else{
17            dict.Add(dist,
18            new List<(int, int)>((x2, y2)));
19        }
20    }
21    var result = new int[k];
22    for(j = 0; foreach(var item in
23    dict) {
24
25    }

```

Testcase Run Code Result

Accepted Runtime: 132 ms

Your input `[[1,3],[-2,2]]`  
1

Output `[-2,2]`  
 Diff

Expected `[-2,2]`

```

3     SortedDictionary<double, List<(int, int)>> dict = new
4     SortedDictionary<double, List<(int, int)>>();
5     var x1 = 0;
6     foreach(var point in
7     points) {
8         var x2 = point[0];
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11            Math.Sqrt((double)(Math.Pow(x1-
12            x2, 2) + Math.Pow(y1-y2, 2)));
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18            new List<(int, int)>((x2, y2)));
19        }
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21    var result = new int[k];
22    for(j = 0; foreach(var item in
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25    }

```

Testcase Run Code Result

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Your input `[[1,3],[-2,2]]`  
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Output `[-2,2]`  
 Diff

Expected `[-2,2]`

```

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13        if(dict.ContainsKey(dist)) {
14            dict[dist].Add((x2, y2));
15        } else{
16            dict.Add(dist,
17            new List<(int, int)>((x2, y2)));
18        }
19    }
20    var result = new int[k];
21    for(j = 0; foreach(var item in
22    dict) {
23
24    }

```

Testcase Run Code Result

Accepted Runtime: 132 ms

Your input `[[1,3],[-2,2]]`  
1

Output `[-2,2]`  
 Diff

Expected `[-2,2]`

```

3     SortedDictionary<double, List<(int, int)>> dict = new
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15        } else{
16            dict.Add(dist,
17            new List<(int, int)>((x2, y2)));
18        }
19    }
20    var result = new int[k];
21    for(j = 0; foreach(var item in
22    dict) {
23
24    }

```

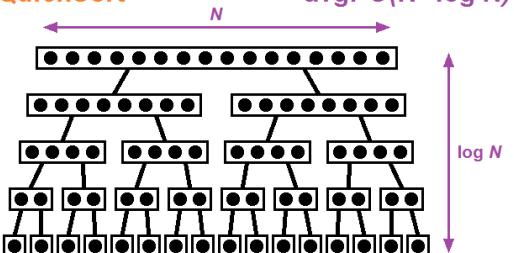
algorithm before diving into how it can be applied to this problem.

The QuickSelect algorithm is essentially a partial application of one of the most common sorting methods: the **QuickSort algorithm**. Both the QuickSort algorithm and its derivative, the QuickSelect algorithm, were invented by Tony Hoare between 1959 and 1961. In order to more easily understand the QuickSelect algorithm, we should first examine how the QuickSort algorithm works.

The QuickSort algorithm operates by **recursively** performing a partial sort of a given range of values. First, a **pivot** value is chosen from the values in the range. Then the QuickSort function uses two pointers, which start from opposite ends of the range and move inward, to swap values in the range. These values are swapped as necessary to ensure that all values lower than the pivot are on one side, and the remaining higher values are on the other. Once the values are thus partitioned, the QuickSort function can be recursively called on each **partition** with progressively smaller ranges until the array is completely sorted.

Since the partition size roughly halves with each recursion, the total recursion stack averages a depth of  $\log N$ , and as each layer of recursion includes all  $N$  values in total, this leads to an overall time complexity for the QuickSort algorithm of  $O(N \cdot \log N)$ . Due to the **recursion stack** necessary for the QuickSort process, it also requires  $O(\log N)$  extra space.

## QuickSort



But if we don't care about fully sorting the array of values and instead only want to make sure that we select the first  $k$  values, we can simplify this process. At each recursive branching of the QuickSort function, we can ignore the partition which does not include the  $k^{\text{th}}$  value. This is the basis for the QuickSelect algorithm.

An immediate benefit of being able to ignore one of the two resulting partitions at each step is that we no longer need to use recursion to branch the process. We can instead convert the function to a more space-friendly iterative solution that uses only a constant amount of space.

A typical QuickSelect function (`quickSelect()`) starts with two pointers (`left`, `right`) that define the entire range of indices in the given array. The function will iteratively apply a partitioning helper function (`partition()`) which will return the index of the borderline between the two subsequent partitions.

Inside the partition helper function, the first step is to find a suitable pivot value. For this, we can call on another helper function (`choosePivot()`). The efficiency of the QuickSelect algorithm relies heavily upon picking a good pivot candidate; the closer the pivot is to the median value, the more likely each successive partitioning is to suitably narrow the range of values.

Common methods for selecting a pivot candidate include picking the first, last, or middle index of the range, or picking the median value between those three elements. Other, more complex methods for selecting a pivot value exist, but their suitability depends upon the nature of the array in question.

If the range is already sorted or nearly sorted, for example picking the first or last index can potentially lead to the worst-case time complexity of  $O(N^2)$  for the QuickSelect process. With no information about the nature of the order of the elements in `points`, we'll simply choose the element at the middle index of the range for this solution, using the simple median formula ( $a + \text{floor}((b - a) / 2)$ ).

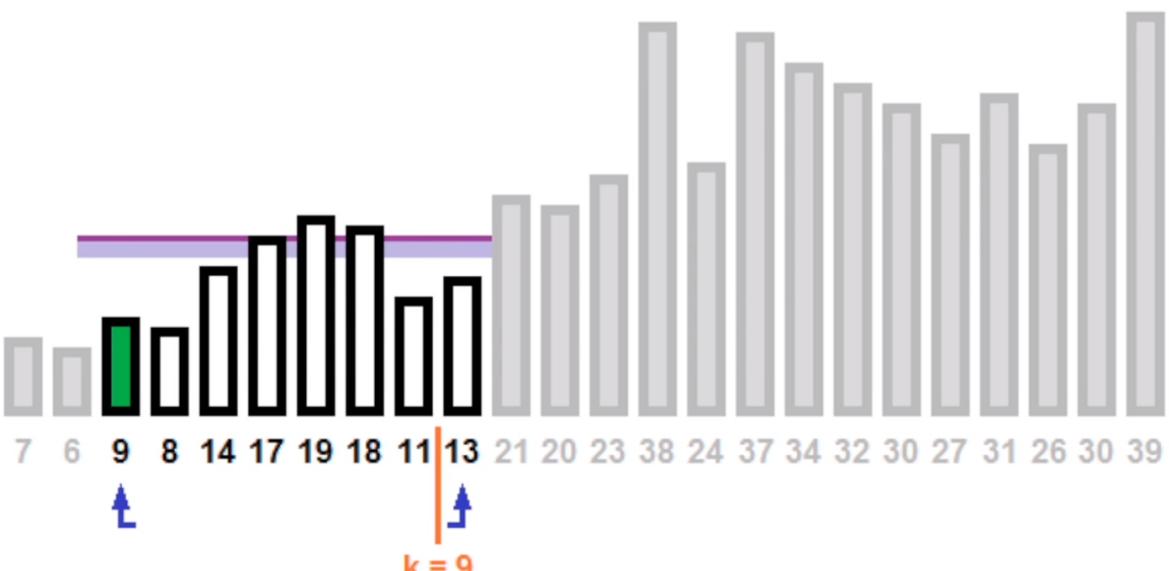
**Note:** Since we're choosing a pivot distance from among the remaining points, we should use the squared Euclidean distance rather than the actual Euclidean distance to save processing time. Unlike the binary search solution, where we used the midpoint of the range of distances, using the distance of a random choice of the remaining points as our pivot distance will not result in an unbalanced split, on average.

After choosing a pivot value, the partition function will swap the values of the elements in the range until it is partitioned into two sides with values less than the pivot value on one side and the remaining values on the other. Like finding the pivot, there are multiple methods available to accomplish this, but we'll use a basic version in which we start with pointers at either end of its range (`left`, `right`) and move inward, swapping elements with values larger than the pivot value to the right side.

Once the two pointers meet, we'll need to make sure the left pointer has completely moved past the end of the left side partition, then we can return it back to the QuickSelect function as the `pivotIndex` representing the left-most edge of the right partition.

If `pivotIndex` is equal to  $k$ , then we know that the first  $k$  values in the array will be the ones we want to select. Since the order of the elements in the output array does not matter, an array containing those  $k$  values can immediately be returned as the solution. Otherwise, the QuickSelect function should adjust its range pointers appropriately, keeping only the partition which includes the  $k^{\text{th}}$  value. This process will continue to narrow the range until a match is found and the solution is returned.

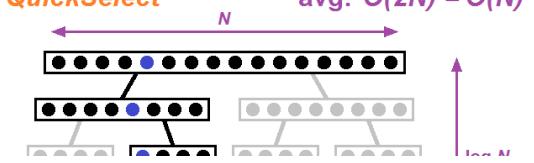
partition: 4  
pivot value: 17



left value: good

Unlike the QuickSort algorithm, the QuickSelect algorithm roughly halves the remaining elements needed to process at each iteration, so the total number of processes will average at  $N + \frac{N}{2} + \frac{N}{4} + \dots + \frac{N}{N} = 2N$ , which results in an average time complexity of  $O(N)$ , down from the  $O(N \cdot \log N)$  of the QuickSort algorithm.

## QuickSelect



```

7 *   {
8 *     var x2 = point[0];
9 *     var y2 = point[1];
10 *    var dist =
11 *      Math.Sqrt((double)(Math.Pow(x1-
12 *      x2, 2) + Math.Pow(y1-y2, 2)));
13 *
14 *    if(dict.ContainsKey(key))
15 *    {
16 *      dict[key].Add((x2, y2));
17 *    }
18 *    else{
19 *      dict.Add(key,
20 *        new List<int, int>{ (x2, y2)} );
21 *    }
22 *  }
23 *  var result = new int[k];
24 *  for(var item in
25 *    dict)
26 *  {
27 *    Run Code Result
  
```

Accepted Runtime: 132 ms

Your input `[[1,3],[-2,2]]`

Output `[-2,2]`

Expected `[-2,2]`

```

3 *  SortedDictionary<double,
4 *  List<int, int>> dict = new
5 *  SortedDictionary<double,
6 *  List<int, int>>();
7 *  var x1 = 0;
8 *  var y1 = 0;
9 *  foreach(var point in
10 *  points)
11 *  {
12 *    var x2 = point[0];
13 *    var y2 = point[1];
14 *    var dist =
15 *      Math.Sqrt((double)(Math.Pow(x1-
16 *      x2, 2) + Math.Pow(y1-y2, 2)));
17 *    if(dict.ContainsKey(key))
18 *    {
19 *      dict[key].Add((x2, y2));
20 *    }
21 *    else{
22 *      dict.Add(key,
23 *        new List<int, int>{ (x2, y2)} );
24 *    }
25 *  }
26 *  var result = new int[k];
27 *  for(var item in
28 *    dict)
29 *  {
30 *    Run Code Result
  
```

Accepted Runtime: 132 ms

Your input `[[1,3],[-2,2]]`

Output `[-2,2]`

Expected `[-2,2]`

```

3 *  SortedDictionary<double,
4 *  List<int, int>> dict = new
5 *  SortedDictionary<double,
6 *  List<int, int>>();
7 *  var x1 = 0;
8 *  var y1 = 0;
9 *  foreach(var point in
10 *  points)
11 *  {
12 *    var x2 = point[0];
13 *    var y2 = point[1];
14 *    var dist =
15 *      Math.Sqrt((double)(Math.Pow(x1-
16 *      x2, 2) + Math.Pow(y1-y2, 2)));
17 *    if(dict.ContainsKey(key))
18 *    {
19 *      dict[key].Add((x2, y2));
20 *    }
21 *    else{
22 *      dict.Add(key,
23 *        new List<int, int>{ (x2, y2)} );
24 *    }
25 *  }
26 *  var result = new int[k];
27 *  for(var item in
28 *    dict)
29 *  {
30 *    Run Code Result
  
```

Accepted Runtime: 132 ms

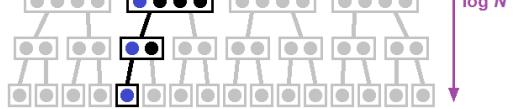
Your input `[[1,3],[-2,2]]`

Output `[-2,2]`

Expected `[-2,2]`

```

3 *  SortedDictionary<double,
4 *  List<int, int>> dict = new
5 *  SortedDictionary<double,
6 *  List<int, int>>();
7 *  var x1 = 0;
8 *  var y1 = 0;
9 *  foreach(var point in
10 *  points)
11 *  {
12 *    var x2 = point[0];
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14 *    var dist =
15 *      Math.Sqrt((double)(Math.Pow(x1-
16 *      x2, 2) + Math.Pow(y1-y2, 2)));
17 *    if(dict.ContainsKey(key))
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24 *    }
25 *  }
26 *  var result = new int[k];
27 *  for(var item in
28 *    dict)
29 *  {
30 *    Run Code Result
  
```



## Algorithm

1. Return the result of a **QuickSelect algorithm** on the `points` array to  $k$  elements.
2. In the QuickSelect function:
  - o Repeatedly **partition** a range of elements in the given array while homing in on the  $k^{th}$  element.
3. In the partition function:
  - o Choose a **pivot** element. The pivot value will be squared Euclidean distance from the origin to the pivot element and will be compared to the squared Euclidean distance of all other points in the partition.
  - o Start with pointers at the left and right ends of the partition, then while the two pointers have not yet met:
    - If the value of the element at the left pointer is smaller than the pivot value, increment the left pointer.
    - Otherwise, swap the elements at the two pointers and decrement the right pointer.
  - o Make sure the left pointer is past the last element whose value is lower than the pivot value.
  - o Return the value of the left pointer as the new pivot index.
4. Return the first  $k$  elements of the array.

```
C++ Java JavaScript Python3
 31 int[] temp = points[left];
 32 points[left] = points[right];
 33 points[right] = temp;
 34 right--;
 35 } else {
 36   left++;
 37 }
 38 }
 39
 40 // Ensure the left pointer is just past the end of
 41 // the left range then return it as the new pivotIndex
 42 if (squaredDistance(points[left]) < pivotDist)
 43   left++;
 44 return left;
45 }
46
47 private int[] choosePivot(int[][] points, int left, int right) {
48   // Choose a pivot element of the array
49   return points[left + (right - left) / 2];
50 }
51
52 private int squaredDistance(int[] point) {
53   // Calculate and return the squared Euclidean distance
54   return point[0] * point[0] + point[1] * point[1];
55 }
56
```

```
15 *
16   else{
17     dict.Add(dist,
18       new List<(int, int)>{<(x2, y2)>});
19   }
20 }
21 var result = new int[k]
22 [];
23 var j =0;
24 foreach(var item in
25 dict)
26   j
```

Testcase Run Code Result

Accepted Runtime: 132 ms

Your input `[[1,3],[-2,2]]`

Output `[-2,2]`

Expected `[-2,2]`

Console Use Example Testcases

```
3 SortedDictionary<double,
List<(int, int)>> dict = new
SortedDictionary<double,
List<(int, int)>>();
4 List<(int, int)>();
5 var x1 = 0;
6 var y1 = 0;
7 foreach(var point in
8 points)
9   {
10     var x2 = point[0];
11     var y2 = point[1];
12     var dist =
13     Math.Sqrt((double)(Math.Pow(x1-
14 x2, 2) + Math.Pow(y1-y2, 2)));
15
16     if(dict.ContainsKey(key))
17       {
18         dict[dist].Add(<x2, y2>);
19       }
20     else{
21       dict.Add(dist,
22         new List<(int, int)>{<x2, y2>});
23     }
24   }
25 var result = new int[k]
26 [];
27 var j =0;
28 foreach(var item in
29 dict)
30   j
```

Testcase Run Code Result

Accepted Runtime: 132 ms

Your input `[[1,3],[-2,2]]`

Output `[-2,2]`

Expected `[-2,2]`

Console Use Example Testcases

```
3 SortedDictionary<double,
List<(int, int)>> dict = new
SortedDictionary<double,
List<(int, int)>>();
4 List<(int, int)>();
5 var x1 = 0;
6 var y1 = 0;
7 foreach(var point in
8 points)
9   {
10     var x2 = point[0];
11     var y2 = point[1];
12     var dist =
13     Math.Sqrt((double)(Math.Pow(x1-
14 x2, 2) + Math.Pow(y1-y2, 2)));
15
16     if(dict.ContainsKey(key))
17       {
18         dict[dist].Add(<x2, y2>);
19       }
20     else{
21       dict.Add(dist,
22         new List<(int, int)>{<x2, y2>});
23     }
24   }
25 var result = new int[k]
26 [];
27 var j =0;
28 foreach(var item in
29 dict)
30   j
```

Testcase Run Code Result

Accepted Runtime: 132 ms

Your input `[[1,3],[-2,2]]`

Output `[-2,2]`

Expected `[-2,2]`

Console Use Example Testcases

```
3 SortedDictionary<double,
List<(int, int)>> dict = new
SortedDictionary<double,
List<(int, int)>>();
4 List<(int, int)>();
5 var x1 = 0;
6 var y1 = 0;
7 foreach(var point in
8 points)
9   {
10     var x2 = point[0];
11     var y2 = point[1];
12     var dist =
13     Math.Sqrt((double)(Math.Pow(x1-
14 x2, 2) + Math.Pow(y1-y2, 2)));
15
16     if(dict.ContainsKey(key))
17       {
18         dict[dist].Add(<x2, y2>);
19       }
20     else{
21       dict.Add(dist,
22         new List<(int, int)>{<x2, y2>});
23     }
24   }
25 var result = new int[k]
26 [];
27 var j =0;
28 foreach(var item in
29 dict)
30   j
```

Testcase Run Code Result

## Complexity Analysis

Here  $N$  refers to the length of the given array `points`.

- Time complexity:  $O(N)$ .

Similar to the earlier binary search solution, the QuickSelect solution has a worst-case time complexity of  $O(N^2)$  if the worst pivot is chosen each time. On average, however, it has a time complexity of  $O(N)$  because it halves (roughly) the remaining elements needing to be processed at each iteration. This results in  $N + \frac{N}{2} + \frac{N}{4} + \frac{N}{8} + \dots + \frac{N}{N} = 2N$  total processes, yielding an average time complexity of  $O(N)$ .

- Space complexity:  $O(1)$ .

The QuickSelect algorithm conducts the partial sort of `points` in place with no recursion, so only constant extra space is required.

## Report Article Issue

Comments: 30 ▾

Type comment here... (Markdown is supported)

[Post](#)

TheLesterKing ★ 221 December 27, 2021 12:15 AM

I don't understand why it's required to use the euclidean distance in binary search instead of the squared distance as in the other approaches.  
Could someone clarify?

▲ 1 ▾ Show 2 replies ▾ Reply

epeian 🇺🇸 ★ 17 Last Edit: December 26, 2021 9:04 PM

One line python, though the binary search one and quick selection one definitely worth learning and practicing.

```
class Solution:
    def kClosest(self, points: List[List[int]], k: int) -> List[List[int]]:
        return heapq.nsmallest(k, points, key = lambda x: x[0]** 2 + x[1]** 2)
```

▲ 1 ▾ ▾ Reply

sxg646 ★ 7 Last Edit: December 14, 2021 9:45 AM

In the QuickSelect algo, choose\_pivot function, isn't it simply  $(left+right)/2$ . I think " $left + (right-left)/2$ " is just complicating it.

▲ 1 ▾ Show 6 replies ▾ Reply

AlphaMonkey9 ★ 54 20 hours ago

Is the first 2 methods enough for an interview in Meta?

▲ 0 ▾ ▾ Reply

dkwan ★ 1 2 days ago

Referring to Solution 4's animated explanation of quick select. I don't believe this partitioning algorithm works if the pivot value happens to be the minimum value of the array.

Since the left pointer will never be less than the pivotValue, you'd always decrement the right pointer until you reach the pivotValue (minimum number), where you eventually swap it to the leftmost index of the array. However, you'd immediately swap it out again in the next iteration, decrement the right pointer and never have a chance to swap it back to its proper spot.

▲ 0 ▾ ▾ Share ▾ Report

yilmazali ★ 49 January 16, 2022 8:17 AM

python solution 3 TLEs

▲ 0 ▾ ▾ Reply

hexoo992 ★ 3 January 10, 2022 7:26 AM

Got LTE using quickselect, but the same code passed the tests before :(

▲ 0 ▾ ▾ Reply

The C++ code from author for binary search has some typos (syntax/logic). I'd request the author to fix it. Great problem & solutions + explanation overall!! Here is my c++ version for binary search:

```
class Solution {
public:
    vector<vector<int>> kClosest(vector<vector<int>>& points, int k) {
        vector<vector<int>> result;
        vector<pair<int,int>> dist;

        // compute distance array and track position of point, for copying points later
        for(int i = 0; i < points.size(); i++) {
            dist.push_back({i, calculateDistance(points[i])});
        }
    }
```

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Dadagda ★ 57 Last Edit: December 28, 2021 12:36 PM

Time complexity of approach 3 (binary search) doesn't seem correct. The one in the article assumes the points are roughly evenly distributed. When it's not the case, doing binary search based on distance would lead to unbounded time. consider an extreme case where all N-1 points have distance 0.00000001 (or infinitely small number) but the last point has distance 1.000.000.000.000 (or infinitely large number). The time complexity doesn't scale with N!

▲ 0 □ Show 1 reply ▲ Reply

Shaun\_Kung ★ 0 Last Edit: December 27, 2021 5:33 PM

I have one question for approach 2 with priority\_queue where in the implementation the instantiated priority\_queue maxPQ only specifies the element type pair<int, int> and the third template parameter compare is a default one std::less (see the definition in [https://en.cppreference.com/w/cpp/container/priority\\_queue](https://en.cppreference.com/w/cpp/container/priority_queue)).

If I understand it correctly this leads to the default relational operators of pair (<https://www.cplusplus.com/reference/utility/pair/operators/>) to be used in heap insertion. However according to the implementation of operator <, the comparing does not only check the first element but also the second. This behavior is different from our expectation here: only checking the first element -> the squared distance.

While I also tried to test the implementation from approach 2 and I do not see any wrong answers.

Does anyone know how it works or where I misunderstood anything from above.

Thanks.

▲ 0 □ Show 2 replies ▲ Reply

◀ 1 2 3 ▶

```
Testcase 1 Run Code Result
SortedDictionary<double, List<int, int>> dict = new SortedDictionary<double, List<int, int>>();
List<int, int>[] points;
var xl = 0;
var yl = 0;
foreach(var point in points)
{
    var x2 = point[0];
    var y2 = point[1];
    var dist =
        Math.Sqrt((double)(Math.Pow(x1-x2, 2) + Math.Pow(y1-y2, 2)));
    if(dict.ContainsKey(dist))
    {
        dict[dist].Add(x2, y2);
    }
    else{
        new List<int, int>{ {x2, y2} };
        dict.Add(dist,
        new List<int, int>{ {x2, y2} });
    }
}
var result = new int[k];
for(j = 0; foreach(var item in dict)
{
    result[j] = item.Value[0];
    j++;
}
```

Testcase 2 Run Code Result

Accepted Runtime: 132 ms

Your input [[1,3],[-2,2]]  
1

Output [[-2,2]] Diff

Expected [[-2,2]]

Console ▾ Use Example Testcases