```
In [ ]:
                                                                                            H
from google.colab import drive
drive.mount('/content/gdrive')
Mounted at /content/gdrive
In [ ]:
                                                                                            M
%cd gdrive/MyDrive/probstat
/content/gdrive/MyDrive/probstat
In [ ]:
                                                                                            M
import pandas as pd
import numpy as np
In [ ]:
def get_daily_counts(x):
  y=np.array([0])
  y=np.append(y,x)
  y=y[:-1]
  return x-y
In [ ]:
                                                                                            M
def outliers_and_data_cleaning(x):
  Q1 = np.percentile(x, 25, interpolation = 'midpoint')
  Q3 = np.percentile(x, 75, interpolation = 'midpoint')
  #tukey's rule outliers are with first_quartile-1.5IQR and third_quartile+1.5IQR
  IQR=Q3-Q1
  left=Q1-1.5*IQR
  right=Q3+1.5*IQR
  mean=np.mean(x)
  print("Lower bound",left)
  print("Upper bound", right)
  #replace with mean if the outliers are found
  for i in range(x.size):
    if(x[i] < left and x[i]! = 0):
      x[i]=mean
    elif(x[i]>right and x[i]!=0):
      x[i]=mean
  return x
```

In the above function, we apply tukey's rule to identify the left and right bounds for the data

In []:

```
def remove_outliers_and_create_new_file():
   dataframe = pd.read_csv('../15.csv')
   ne_confirmed = dataframe['NE confirmed'].to_numpy()
   nd_confirmed = dataframe['ND confirmed'].to_numpy()
   ne_deaths = dataframe['NE deaths'].to_numpy()
   nd deaths = dataframe['ND deaths'].to numpy()
   #get the daily counts from each state and each category
   ne_confirmed_per_day = get_daily_counts(ne_confirmed)
   nd confirmed per day = get daily counts(nd confirmed)
   ne_deaths_per_day = get_daily_counts(ne_deaths)
   nd_deaths_per_day = get_daily_counts(nd_deaths)
   #remove outliers by replacing them with mean and not changing zeros
   print("nd_confirmed_per_day")
   nd_confirmed_per_day = outliers_and_data_cleaning(nd_confirmed_per_day)
   print("ne confirmed per day")
   ne_confirmed_per_day = outliers_and_data_cleaning(ne_confirmed_per_day)
   print("nd_deaths_per_day")
   nd_deaths_per_day = outliers_and_data_cleaning(nd_deaths_per_day)
   print("ne_deaths_per_day")
   ne_deaths_per_day = outliers_and_data_cleaning(ne_deaths_per_day)
   #append to a new file
   dataframe['ne_confirmed_per_day'] = ne_confirmed_per_day
   dataframe['nd_confirmed_per_day'] = nd_confirmed_per_day
   dataframe['ne_deaths_per_day'] = ne_deaths_per_day
   dataframe['nd deaths per day'] = nd deaths per day
   dataframe.to_csv('../15_updated.csv')
```

In the above the function we take each column data, find the outliers, replace with the mean of rhe data and output to a new file

```
In [ ]: ▶
```

```
remove_outliers_and_create_new_file()
```

```
nd_confirmed_per_day
Lower bound -333.75
Upper bound 592.25
ne_confirmed_per_day
Lower bound -536.75
Upper bound 1161.25
nd_deaths_per_day
Lower bound -4.5
Upper bound 7.5
ne_deaths_per_day
Lower bound -7.5
Upper bound 12.5
```

Approach

We used tukey's rule to find the lower and upper bounds where outliers exists

- Since removing all the outliers may cause gaps in the time series data, we replace the outliers with the mean data
- changing the outliers does not cause any changes to the data distribution.

In []:	H

```
In [12]:
                                                                                             H
from google.colab import drive
drive.mount('/content/gdrive')
Drive already mounted at /content/gdrive; to attempt to forcibly remount, ca
11 drive.mount("/content/gdrive", force_remount=True).
                                                                                             H
In [13]:
%cd gdrive/MyDrive/probstat
[Errno 2] No such file or directory: 'gdrive/MyDrive/probstat'
/content/gdrive/MyDrive/probstat
In [ ]:
                                                                                             M
%pwd
Out[22]:
'/content/gdrive/My Drive/probstat'
                                                                                             M
In [14]:
import pandas as pd
import numpy as np
In [25]:
                                                                                             H
def get_beta(train_x, train_y):
    train_x_transpose = np.transpose(train_x)
    #find the transpose of X
    x_x_t = np.matmul(train_x_transpose, train_x)
    #We know B = x^{-1}X*Y
    beta = np.matmul(np.matmul(np.linalg.inv(x_x_t), train_x_transpose), train_y)
    return beta
The above function is used for finding the beta values from the x and y values
In [ ]:
                                                                                             H
```

The above function is used to append 1 to the begining of an array for the constant beta term

def pad one(x):

return y

y=np.array([1])
y=np.append(y,x)

In [17]:

The above function is used to get the time series data for the past p days and the predicted value on the p+1 day

```
In [18]: ▶
```

```
def AR(x, p):
   print("AR("+str(p)+")")
   x three = x[70:91]
   x_{four} = x[91:100]
   n = x_{four.size}
   mape = 0
   mse = 0
   #change the input data based on the value of p.i.e, if p=3 then three consucitive recor
   x_data, y_data = get_regression_data(x[70:100], p)
   for i in range(n - 1):
        #Beta values are calculated by appending 1 to each row and doing the matrix inverse
        #beta is calculated at each stage
        beta = get_beta(x_data[0:21 + i - p, :], y_data[0:21 - p + i])
        #find the predicted array
        pred = beta * x_{data}[21 + i - p + 1, :]
        #find the predicted value
        pred = np.sum(pred)
        # if the denominator is zeo in the MAPE calculation changed the denominator to 1
        if (x four[i] == 0):
            mape = mape + abs((x_four[i] - pred))
        else:
            mape = mape + abs((x_four[i] - pred) / x_four[i])
        mse = mse + ((x_four[i] - pred) * (x_four[i] - pred))
   mape = 100 / n * (mape)
   mse = mse / n
   print("Mean Absolute Percentage Error", mape)
   print("Mean Squared error", mse)
   return mape, mse
```

- The above function is used to find the predicted value using auto regression method.
- The metrics are calculated after the each prediction and the final metrics are printed

```
In [19]:
```

```
def get_predicted_value(x, alpha):
    #x is the data of the last p-1 days
    n = x.size
    rev_alpha = 1 - alpha
    coeff = 1
    pred = 0
    # when we expand the EWMA we get the below coefficients for each term
    for i in range(n):
        pred = pred + (x[n - i - 1] * (coeff))
        coeff = coeff * rev_alpha
    pred = pred * alpha
    #return the predicted value on the pth day
    return pred
```

Get the pth day predicted value based on the alpha and previous p-1 days data

```
In [20]: ▶
```

```
def EWMA(x, alpha):
   print("EWMA("+str(alpha)+")")
   #x three is the data of the first weeks
   x_{three} = x[70:91]
   #x_four is the data of the fourth week
   x_{four} = x[91:100]
   #number of predicted days in the fourth week
   n=x four.size
   mse = 0
   mape = 0
   for i in range(n):
        #get the predicted value on t-day from t-1 days data
        #t-1 days data is passed at each iteration
        pred = get predicted value(x[70:91 + i], alpha)
        #if the denominator is zeo in the MAPE calculation changed the denominator to 1
        if(x_four[i]==0):
            mape = mape + abs((x_four[i] - pred))
        else:
            mape = mape + abs((x_four[i] - pred) / x_four[i])
        mse = mse + ((x_four[i] - pred) * (x_four[i] - pred))
   mape = (100 / n) * (mape)
   mse = mse / n
   print("Mean Absolute Percentage Error", mape)
   print("Mean Squared error", mse)
    return mape, mse
```

Perform EWMA on the data to find the predicted value of the fourth week based on the data of the data before the current day

In [21]:

```
def run_all_models(x):
    #Autoregressive model with p
    AR(x, 3)
    AR(x, 5)

#Exponential weighted moving average with alpha
    EWMA(x, 0.5)
    EWMA(x, 0.8)
```

Run all the models with the parameters for an attribute

In [22]: ▶

```
def q1_a():
    #get the data from the file
    dataframe = pd.read_csv('../15_updated.csv')
    ne_confirmed = dataframe['ne_confirmed_per_day'].to_numpy()
    nd_confirmed = dataframe['nd_confirmed_per_day'].to_numpy()
    ne_deaths = dataframe['ne_deaths_per_day'].to_numpy()
    nd_deaths = dataframe['nd_deaths_per_day'].to_numpy()

    print("NE Confirmed")
    run_all_models(ne_confirmed)
    print("ND Confirmed")
    run_all_models(nd_confirmed)
    print("NE Deaths")
    run_all_models(ne_deaths)
    print("ND Deaths")
    run_all_models(nd_deaths)
```

In [24]:

```
q1_a()
```

NE Confirmed

AR(3)

Mean Absolute Percentage Error 32.96850430025964

Mean Squared error 8349.724692558942

AR(5)

Mean Absolute Percentage Error 52.11422476794114

Mean Squared error 18891.37117302536

EWMA(0.5)

Mean Absolute Percentage Error 36.87492078436932

Mean Squared error 12446.65933430085

EWMA(0.8)

Mean Absolute Percentage Error 32.35788333457912

Mean Squared error 10892.235627411601

ND Confirmed

AR(3)

Mean Absolute Percentage Error 404.9411215660536

Mean Squared error 155572.02981181236

AR(5)

Mean Absolute Percentage Error 938.3191264733968

Mean Squared error 340283.1046553315

EWMA(0.5)

Mean Absolute Percentage Error 39.24418879275243

Mean Squared error 331.4057628966122

EWMA(0.8)

Mean Absolute Percentage Error 28.747277708178856

Mean Squared error 187.76483880995954

NE Deaths

AR(3)

Mean Absolute Percentage Error 47.09115008203833

Mean Squared error 1.7882953488439088

AR(5)

Mean Absolute Percentage Error 58.42903933084258

Mean Squared error 1.8807861237175874

EWMA(0.5)

Mean Absolute Percentage Error 11.121908575296402

Mean Squared error 1.7777744125300556

EWMA(0.8)

Mean Absolute Percentage Error 17.7777785059521

Mean Squared error 1.93777777777219

ND Deaths

AR(3)

Mean Absolute Percentage Error 54.33218017569266

Mean Squared error 0.49843168790177056

AR(5)

Mean Absolute Percentage Error 63.589199494864296

Mean Squared error 0.8226222970787114

EWMA(0.5)

Mean Absolute Percentage Error 55.5695202615526

Mean Squared error 0.5855089373064304

EWMA(0.8)

Mean Absolute Percentage Error 58.56516928220713

Mean Squared error 0.6996864856289055

```
In [22]: from google.colab import drive
drive.mount('/content/gdrive')
```

Drive already mounted at /content/gdrive; to attempt to forcibly remount, call drive.mount("/content/gdrive", force_remount=True).

In [18]: cd gdrive/MyDrive/

/content/gdrive/MyDrive

In [23]: import pandas as pd
import numpy as np
import math

```
In [24]: df = pd.read_csv("15_daily_filtered_mean.csv")
    df_feb = df[(df['Date'] >= '2021-02-01') & (df['Date'] <= '2021-02-28')]
    df_mar = df[(df['Date'] >= '2021-03-01') & (df['Date'] <= '2021-03-31')]
    df_feb = df_feb.loc[:, 'ND confirmed':]
    df_mar = df_mar.loc[:, 'ND confirmed':]
    print(df_mar)</pre>
```

	ND confirmed	NE confirmed	ND deaths	NE deaths
404	108	377	2	2
405	113	276	1	7
406	110	358	0	1
407	94	325	0	20
408	82	337	1	1
409	33	386	0	0
410	29	141	0	0
411	97	195	0	2
412	99	301	3	10
413	108	281	3	2
414	120	265	1	2
415	93	287	2	3
416	34	165	0	0
417	25	114	0	0
418	148	317	1	1
419	133	216	1	1
420	119	276	0	1
421	113	254	1	4
422	0	395	0	2
423	0	137	0	0
424	0	179	0	0
425	0	245	1	2
426	486	206	3	26
427	197	286	1	9
428	183	363	1	1
429	159	531	0	0
430	47	225	0	0
431	55	126	0	0
432	245	350	0	2
433	216	408	0	3
434	205	2319	0	2

One Sample W-Test

In [28]:

```
feb_mean = df_feb.mean()
cols = len(df_feb.columns)
n = len(df mar)
lambda MLE = df_mar.sum().div(n)
se MLE = df mar.var().div(math.sqrt(n))
w = (lambda_MLE - feb_mean).div(se_MLE).abs()
print(w)
# alpha=0.05
reject hypo = (w > 1.96)
print("1-Sample W-Test")
print("H0: mean(mar)=mean(feb), Reject:\n ", reject_hypo)
ND confirmed
            0.020000
NE confirmed
            0.000110
ND deaths
            0.867704
            0.411946
NE deaths
dtype: float64
1-Sample W-Test
H0: mean(mar)=mean(feb), Reject:
 ND confirmed
              False
NE confirmed
            False
ND deaths
            False
NE deaths
            False
dtype: bool
```

Results of Walds 1 sample testing for mean of cases and death

Null hypothesis (H0):

the mean of daily cases and the mean of daily deaths for Feb'21 is different from the corresponding mean of daily values for March'21

Mean of Feb'21 cases/deaths = Mean of March'21 cases/deaths.

Alternate hypothesis(H1):

Mean of Feb'21 cases/deaths **not equal to** mean of March'21 cases/deaths.

Procedure:

We have taken the guess value as mean of daily values from Feb'21 cases/deaths and alpha = 0.05 as given in documentation and the MLE estimator for mean of daily values from Mar'21 cases/deaths becomes the sample mean of daily values from Mar'21 cases/deaths.

Result:

walds 1 sample testing for mean of state ND confirmed cases is w=0.020000

walds 1 sample testing for mean of state NE confirmed cases is w=0.000110

walds 1 sample testing for mean of state ND deaths is w=0.867704

walds 1 sample testing for mean of state ND deaths is w=0.411946

which are less than z_alpha = 1.96 so accept the NULL hypothesis

Type *Markdown* and LaTeX: α^2

One sample Z-Test

```
In [29]: # True Variance
       df_t = df.loc[:, 'ND confirmed':]
       mean = df.loc[:, 'ND confirmed':].mean()
       diff_sum = df_t.sub(mean).sum()
       True Sample Dev = diff_sum.mul(diff_sum).div(len(df_t) - 1) ** (1 / 2)
       print("1-Sample Z-Test")
       z = (df mar.mean() - feb mean).div(True Sample Dev.div(math.sqrt(n))).abs()
       print(z)
       reject hypo = (z > 1.96)
       print("H0: mean(mar)=mean(feb), Reject:\n ", reject hypo)
       1-Sample Z-Test
       ND confirmed 2.334509e+13
       NE confirmed 1.015164e+12
       ND deaths
                    7.242974e+12
       NE deaths
                    4.125351e+13
       dtype: float64
       H0: mean(mar)=mean(feb), Reject:
         ND confirmed
                      True
       NE confirmed
                    True
       ND deaths
                    True
       NE deaths
                    True
       dtype: bool
```

Result of Z testing for mean of cases and death

Null hypothesis (H0):

Mean of Feb'21 cases/deaths = Mean of March'21 cases/deaths.

Alternate hypothesis(H1):

Mean of Feb'21 cases/deaths not equal to mean of March'21 cases/deaths.

Result:

Z-sample testing for mean of state ND confirmed cases is w=2.334509e+13

Z-sample testing for mean of state NE confirmed cases is w=1.015164e+12

Z-sample testing for mean of state ND deaths is w=7.242974e+12

Z-sample testing for mean of state ND deaths is w=4.125351e+13

which are greater than z_alpha = 1.96 so reject the NULL hypothesis

One Sample T-test

```
In [30]: print("1-Sample T-test")
        df t = df mar.loc[:, 'ND confirmed':]
        mean = df_mar.loc[:, 'ND confirmed':].mean()
        diff sum = df t.sub(mean).sum()
        Sample Dev = diff_sum.mul(diff_sum).div(len(df_t) - 1) ** (1 / 2)
        t = (df mar.mean() - feb mean).div(Sample Dev.div(math.sqrt(n))).abs()
        print(t)
        reject hypo = (t > 2.04)
        print("H0: mean(mar)=mean(feb), Reject:\n ", reject hypo)
        ## Comment about applicability of tests
        1-Sample T-test
        ND confirmed 3.594604e+16
                     3.821252e+14
        NE confirmed
        ND deaths
                      9.644333e+14
        NE deaths
                      6.224550e+15
        dtype: float64
        H0: mean(mar)=mean(feb), Reject:
         ND confirmed
                       True
        NE confirmed
                      True
        ND deaths
                      True
        NE deaths
                      True
        dtype: bool
```

Result of T 1 sample testing for mean of cases and death

Null hypothesis (H0):

Mean of Feb'21 cases/deaths = Mean of March'21 cases/deaths.

Alternate hypothesis(H1):

Mean of Feb'21 cases/deaths not equal to Mean of March'21 cases/deaths.

Procedure:

We have taken the alpha = 0.05,n = 30 as we took 30 days of data as given in documentation and calculated the numerator and denominator of t.

Result:

As the calculated values for mean of state

ND confirmed cases = 3.594604e+16

NE confirmed cases = 3.821252e+14

ND deaths = 9.644333e+14

NE deaths = 6.224550e + 15

which are greater than t value 2.04 we are rejecting the NULL hypothesis.

Two Sample Walds Test

```
2-Sample Walds-test
ND confirmed 1.591266
NE confirmed 0.036141
ND deaths
           0.521922
        1.723450
NE deaths
dtype: float64
H0: mean(mar)=mean(feb), Reject:
ND confirmed False
NE confirmed
           False
ND deaths
          False
NE deaths
           False
dtype: bool
```

Result of Walds 2 sample testing for mean of cases and death

Null hypothesis (H0):

Mean of Feb'21 cases/deaths = Mean of March'21 cases/deaths.

Alternate hypothesis(H1):

Mean of Feb'21 cases/deaths not equal to Mean of March'21 cases/deaths.

Procedure:

We have taken the alpha = 0.05 as given in documentation and calculated the numerator and denominator of w.

Result:

As the w value for mean of state

ND confirmed cases 1.591266

NE confirmed cases 0.036141

ND deaths 0.521922

NE deaths 1.723450

which are less than 1.96 we are accepting the NULL hypothesis.

Type *Markdown* and LaTeX: α^2

Two Sample Unpaired T-test

```
In [32]: print("2-Sample T-test")
        Var_feb = ((df_feb - df_feb.mean()) ** 2).sum().div(len(df_feb) - 1)
        Var_mar = ((df_mar - df_mar.mean()) ** 2).sum().div(len(df_mar) - 1)
        pool_std = (Var_feb / len(df_feb) + Var_mar / len(df_mar)) ** 0.5
        t = ((df feb.mean() - df mar.mean()) / pool std).abs()
        print(t)
        reject hypo = (t > 2.0423)
        print("H0: mean(mar)=mean(feb), Reject:\n", reject hypo)
        2-Sample T-test
        ND confirmed
                     1.564428
        NE confirmed
                      0.035535
        ND deaths
                      0.512856
        NE deaths
                      1.693888
        dtype: float64
        H0: mean(mar)=mean(feb), Reject:
         ND confirmed
                       False
        NE confirmed
                      False
        ND deaths
                      False
        NE deaths
                      False
```

Result of T 2 sample unpaired testing for mean of cases and death

Null hypothesis (H0):

dtype: bool

Mean of Feb'21 cases/deaths = Mean of March'21 cases/deaths.

Alternate hypothesis(H1):

Mean of Feb'21 cases/deaths **not equal to** Mean of March'21 cases/deaths.

Procedure:

We have taken the alpha = 0.05 n=27, m=30 as given in documentation and calculated the numerator and denominator of t value.

Result:

As the t value for mean of state

ND confirmed cases 1.564428

NE confirmed cases 0.035535

ND deaths 0.512856

NE deaths 1.693888

which are less than 2.0423 we are accepting the NULL hypothesis.

Applicability Of Tests

Wald's Test

In Wald's we need to have a Asymptomatically Normal estimator. Since N is fairly low, we cannot assume CLT applies in case of sample mean and hence this test is not applicable.

The above reasoning works well for 2 sample test as well, since we need both estimators to be Asymptomatically Normal. Hence this test is also not applicable.

Z-test

In Z-test, we need to have true standard deviation and either the N is large or the X is normally distributed. Since neither of the second requirement is true, we say the test is not applicable.

T-test

In one sample T-test, the requirement is that the data is normally distributed, since this is not the case and N is low as well. We say that the test is not applicable

In unpaired T-test, the required distributions should be independent and need to be normally distributed. In the current scenario, neither is the case so we say that the test is not applicable

In paired T-test, the difference distribution should be normally distributed but we cannot assume the same, hence we say that the test is not applicable In [1]: import pandas as pd
import numpy as np
from scipy import stats
import matplotlib.pyplot as plt

In [2]: df = pd.read_csv("/Users/kodalialekhya/Downloads/15_new.csv")

In [4]: df = df.drop(columns='Unnamed: 0')

In [5]: df.head()

Out [5]:

	Date	ND confirmed	NE confirmed	ND deaths	NE deaths	ne_confirmed_per_day	nd_confirmed_per_day
0	2020- 01-22	0	0	0	0	0	0
1	2020- 01-23	0	0	0	0	0	0
2	2020- 01-24	0	0	0	0	0	0
3	2020- 01-25	0	0	0	0	0	0
4	2020- 01-26	0	0	0	0	0	0

In [6]: data = df[(df.Date>'2020-09-30') & (df.Date<'2021-01-01')]
 data.head()</pre>

Out[6]:

		Date	ND confirmed	NE confirmed	ND deaths	NE deaths	ne_confirmed_per_day	nd_confirmed_per_da
_	253	2020- 10-01	22218	45870	256	494	594	37
	254	2020- 10-02	22694	46647	264	494	777	47
	255	2020- 10-03	23134	47070	271	498	423	44
	256	2020- 10-04	23550	47475	274	500	405	41
	257	2020- 10-05	23862	47910	277	503	435	31

```
In [11]: def get_xy(x):
    n = len(x)
    x = sorted(x)
    x_cdf = []
    y_cdf = []
    y_curr = 0

    x_cdf.append(0)
    y_cdf.append(0)

    for i in x:
        y_curr += 1/n
        y_cdf.append(y_curr)
        x_cdf.append(i)

    return x_cdf,y_cdf
```

```
In [13]: def draw_ecdf(x1, y1, x2, y2, max_diff, max_ind):
    plt.figure(figsize=(20,10))
    plt.step(x1, y1, where="post", label="CDF-D1")
    plt.step(x2, y2, where="post", label="CDF-D2")
    plt.yticks(np.arange(0, 1.1, 1/10))
    plt.title("Empirical CDF")
    plt.xlabel("Sample Points")
    plt.ylabel("Pr[X<x]")
    plt.scatter([max_ind],[0], color='red', marker='x', s=100, label=f
    plt.grid(which="both")
    plt.legend()
    plt.show()</pre>
```

```
In [35]: | def ks_1_sample_test(data1,data2, statement, threshold=0.05):
           x1, v1 = get xv(data1)
           n = len(data2)
           diff=[]
           for i in range(n):
             diff.append( np.absolute( y1[i] - data2[i] ) )
           max_diff = np.max(diff)
           max_ind = np.argmax(diff)
           if max_diff > threshold:
             print(f"Max value = {max_diff} > C: {threshold}, We reject H0: "+s
           else:
             print(f"Max value = {max diff} <= C: {threshold}, We reject H0: "+</pre>
In [36]: mean_nd_confirmed = np.mean(data['nd_confirmed_per_day'].values)
         mean nd deaths = np.mean(data['nd deaths per day'].values)
In [37]: | def calc_poisson(param, x):
           return stats.poisson.cdf(x, param)
In [38]: |x1, y1 = get_xy(data['ne_confirmed_per_day'].values)
         val = calc poisson(mean nd confirmed, x1)
         ks_1_sample_test(data['ne_confirmed_per_day'].values, val, "The cases
         Max value = 0.9130434782608696 > C: 0.05, We reject H0: The cases fol
         low poisson distrubtion
```

Result of 1 sample KS test for last 3 months of 2020 for NE state cases with poisson distribution

Null Hypothesis (H0):

Distribution of last 3 months of 2020 for NE state cases equals poisson distribution

Alternate Hypothesis (H1):

Distribution of last 3 months of 2020 for NE state cases not equals poisson distribution

Procedure:

We have obtained parameters for poisson distribution by using MME on last 3 months for ND state data. We have taken the c = 0.05 and n=92 as given in documentation and calculated the maximum difference of the CDF of the distributions at all the points.

Result:

As the KS test value is 0.913 which is greater than 0.05 we are rejecting the NULL hypothesis.

```
In [18]: x1, y1 = get_xy(data['ne_deaths_per_day'].values)
val = calc_poisson(mean_nd_deaths, x1)
ks_1_sample_test(data['ne_deaths_per_day'].values, val, "The deaths for
```

Max value = 0.4979724320106867 > C: 0.05, We reject H0: The deaths fo llow poisson distrubtion

Result of 1 sample KS test for last 3 months of 2020 for NE state deaths with poisson distribution

Null Hypothesis (H0):

Distribution of last 3 months of 2020 for NE state deaths equals poisson distribution

Alternate Hypothesis (H1):

Distribution of last 3 months of 2020 for NE state deaths not equals poisson distribution

Procedure:

We have obtained parameters for poisson distribution by using MME on last 3 months for ND state data. We have taken the c = 0.05 and n=92 as given in documentation and calculated the maximum difference of the CDF of the distributions at all the points.

Result:

As the KS test value is 0.497 which is greater than 0.05 we are rejecting the NULL hypothesis.

```
In [40]: def calc_geometric(param, x):
    return stats.geom.cdf(x, param)
```

```
In [44]: x1, y1 = get_xy(data['ne_confirmed_per_day'].values)
val = calc_poisson(1/mean_nd_confirmed, x1)
ks_1_sample_test(data['ne_confirmed_per_day'].values, val, "The cases
```

Max value = 0.9961051655953749 > C: 0.05, We reject H0: The cases fol low geometric distrubtion

Result of 1 sample KS test for last 3 months of 2020 for NE state cases with geometric distribution

Null Hypothesis (H0):

Distribution of last 3 months of 2020 for NE state cases equals geometric distribution

Alternate Hypothesis (H1):

Distribution of last 3 months of 2020 for NE state cases not equals geometric distribution

Procedure:

We have obtained parameters for geometric distribution by using MME on last 3 months for ND state data. We have taken the c = 0.05 and n=92 as given in documentation and calculated the maximum difference of the CDF of the distributions at all the points.

Result:

As the KS test value is 0.996 which is greater than 0.05 we are rejecting the NULL hypothesis.

```
In [45]: x1, y1 = get_xy(data['ne_deaths_per_day'].values)
val = calc_poisson(1/mean_nd_deaths, x1)
ks_1_sample_test(data['ne_deaths_per_day'].values, val, "The cases fol
```

Max value = 0.7725453890382081 > C: 0.05, We reject H0: The cases follow geometric distrubtion

Result of 1 sample KS test for last 3 months of 2020 for NE state deaths with geometric distribution

Null Hypothesis (H0):

Distribution of last 3 months of 2020 for NE state deaths equals geometric distribution

Alternate Hypothesis (H1):

Distribution of last 3 months of 2020 for NE state deaths not equals geometric distribution

Procedure:

We have obtained parameters for geometric distribution by using MME on last 3 months for ND state data. We have taken the c = 0.05 and n=92 as given in documentation and calculated the maximum difference of the CDF of the distributions at all the points.

Result:

As the KS test value is 0.772 which is greater than 0.05 we are rejecting the NULL hypothesis.

```
In [46]: def calc_binomial(n, p, x):
    return stats.binom.cdf(x, n, p)
```

```
In [23]: var_nd_confirmed = np.var(data['nd_confirmed_per_day'].values)
    n = (mean_nd_confirmed * mean_nd_confirmed)/(mean_nd_confirmed - var_n
    p = mean_nd_confirmed/n

x1, y1 = get_xy(data['ne_confirmed_per_day'].values)
    val = calc_binomial(n, p, x1)
    ks_1_sample_test(data['ne_confirmed_per_day'].values, val, "The cases
```

Max value = 1.0 > C: 0.05, We reject H0: The cases follow binomial distrubtion

Result of 1 sample KS test for last 3 months of 2020 for NE state cases with binomial distribution

Null Hypothesis (H0):

Distribution of last 3 months of 2020 for NE state cases equals binomial distribution

Alternate Hypothesis (H1):

Distribution of last 3 months of 2020 for NE state cases not equals binomial distribution

Procedure:

We have obtained parameters for binomial distribution by using MME on last 3 months for ND state data. We have taken the c = 0.05 and n=92 as given in documentation and calculated the maximum difference of the CDF of the distributions at all the points.

Result:

As the KS test value is 1 which is greater than 0.05 we are rejecting the NULL hypothesis.

```
In [24]: var_nd_deaths = np.var(data['nd_deaths_per_day'].values)
n = (mean_nd_deaths * mean_nd_deaths)/(mean_nd_deaths - var_nd_deaths)
p = mean_nd_deaths/n

x1, y1 = get_xy(data['ne_deaths_per_day'].values)
val = calc_binomial(n, p, x1)
ks_1_sample_test(data['ne_deaths_per_day'].values, val, "The cases fol
```

Max value = 1.0 > C: 0.05, We reject H0: The cases follow binomial distrubtion

Result of 1 sample KS test for last 3 months of 2020 for NE state deaths with binomial distribution

Null Hypothesis (H0):

Distribution of last 3 months of 2020 for NE state deaths equals binomial distribution

Alternate Hypothesis (H1):

Distribution of last 3 months of 2020 for NE state deaths not equals binomial distribution

Procedure:

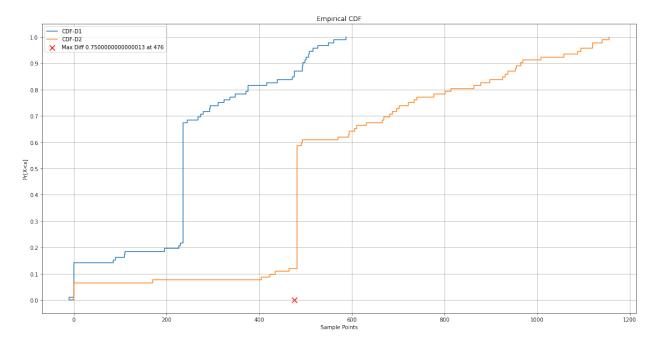
We have obtained parameters for binomial distribution by using MME on last 3 months for ND state data. We have taken the c = 0.05 and n=92 as given in documentation and calculated the maximum difference of the CDF of the distributions at all the points.

Result:

As the KS test value is 1 which is greater than 0.05 we are rejecting the NULL hypothesis.

```
In [62]: | def ks_2_sample_test(data1,data2, threshold=0.05, draw=True):
           x1, y1 = get_xy(data1)
           x2, y2 = get_xy(data2)
           n = int(min([max(x1), max(x2)])) +10
           y1_all = []
           temp=0
           for i in np.arange(n):
             ind = np.where(np.array(x1) == i)[0]
             if len(ind)==0:
               y1_all.append(temp)
             else:
               y1_all.append(y1[ind[-1]])
               temp = y1[ind[-1]]
           y2_all = []
           temp=0
           for i in np.arange(n):
             ind = np.where(np.array(x2) == i)[0]
             if len(ind)==0:
               y2_all.append(temp)
             else:
               y2_all_append(y2[ind[-1]])
               temp = y2[ind[-1]]
           diff=[]
           for i in range(n):
             diff.append( np.absolute( y1_all[i] - y2_all[i] ) )
           max_diff = np.max(diff)
           max_ind = np.argmax(diff)
           if draw:
             draw_ecdf(x1,y1,x2,y2, max_diff, max_ind)
           if max diff > threshold:
             print(f"Max value = {max_diff} > C: {threshold}, We reject H0")
           else:
             print(f"Max value = {max diff} <= C: {threshold}, We reject H0")</pre>
```

In [63]: ks_2_sample_test(data['nd_confirmed_per_day'].values, data['ne_confirm
598



Max value = 0.7500000000000013 > C: 0.05, We reject H0

Result of 2 sample KS test for last 3 months of 2020 for ND state and NE state cases

Null hypothesis (H0):

Distribution of ND state cases equals distribution of NE state cases

Alternate hypothesis(H1):

Distribution of ND state cases not equals to distribution of NE state cases

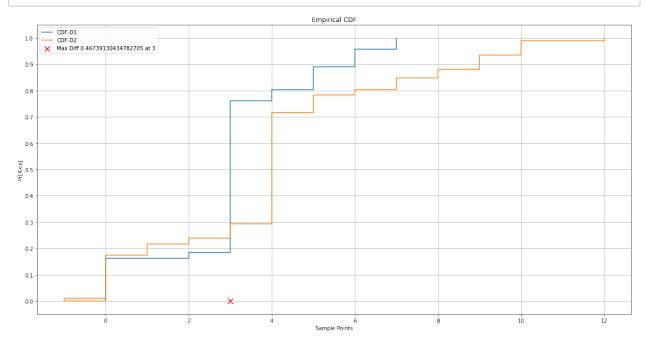
Procedure:

We have taken c = 0.05 as given in documentation and calculated the maximum difference of the CDF of the distributions at all the points.

Result:

As the KS test value is 0.75 which is greater than 0.05 we are rejecting the NULL hypothesis.

In [27]: ks_2_sample_test(data['nd_deaths_per_day'].values, data['ne_deaths_per



Max value = 0.46739130434782705 > C: 0.05, We reject H0

Result of 2 sample KS test for last 3 months of 2020 for ND state and NE state deaths

Null hypothesis (H0):

Distribution of ND state deaths equals distribution of NE state deaths

Alternate hypothesis(H1):

Distribution of ND state deaths not equals to distribution of NE state deaths

Procedure:

We have taken the c=0.05 as given in documentation and calculated the maximum difference of the CDF of the distributions at all the points.

Result:

As the KS test value is 0.467 which is greater than 0.05 we are rejecting the NULL hypothesis.

```
In [60]: def permutation_test(X, Y, n=1000, threshold=0.05):
           T_{obs} = abs(np.mean(X) - np.mean(Y))
           xy = np.append(X,Y)
           p value = 0.0
           T = []
           for i in range(n):
             permutation = np.random.permutation(xy)
             X1 = permutation[:len(X)]
             Y1 = permutation[len(X):]
             Ti = abs(np.mean(X1) - np.mean(Y1))
             T.append(Ti)
           T = np.array(T)
           p_value = np.sum(T>T_obs)/len(T)
           print("The p-value is: ", p_value)
           if (p_value <= threshold):</pre>
             print("p-value less than or equal to the threshold ==> Reject the
             print("p-value greater than threshold ==> Accept the Null Hypothes
```

```
In [61]: x1 = data['nd_confirmed_per_day'].values
    x2 = data['ne_confirmed_per_day'].values

y1=data['nd_deaths_per_day'].values
    y2=data['ne_deaths_per_day'].values

permutation_test(x1,x2)
    permutation_test(y1,y2)
```

The p-value is: 0.0
p-value less than or equal to the threshold ==> Reject the Null Hypot hesis
The p-value is: 0.003
p-value less than or equal to the threshold ==> Reject the Null Hypot hesis

Result of Permutation test for last 3 months of 2020 for ND state and NE state cases

Null hypothesis (H0):

Distribution of ND state cases equals distribution of NE state cases

Alternate hypothesis(H1):

Distribution of ND state cases not equals distribution of NE state cases

Result:

As the Permutation test value is 0.0 which is less than 0.05 we are rejecting the NULL hypothesis.

Result of Permutation test for last 3 months of 2020 for ND state and NE state deaths

Null hypothesis (H0):

Distribution of ND state deaths equals distribution of NE state deaths

Alternate hypothesis(H1):

Distribution of ND state deaths not equals distribution of NE state deaths

Result:

As the Permutation test value is 0.003 which is less than 0.05 we are rejecting the NULL hypothesis.

```
In [2]:
                                                                                            H
from google.colab import drive
drive.mount('/content/gdrive')
Mounted at /content/gdrive
In [3]:
                                                                                            M
%cd gdrive/MyDrive/probstat
/content/gdrive/MyDrive/probstat
In [ ]:
                                                                                            H
%pwd
Out[25]:
'/content/gdrive/My Drive/probstat'
                                                                                            H
In [4]:
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import gamma
In [20]:
                                                                                            M
def plot_gamma_distribution(posteriors):
  weeks=[]
  for i in range(5,9):
      weeks.append("week"+str(i))
  for posterior in posteriors:
    alpha=posterior[0]
    beta=posterior[1]
    x = np.linspace(gamma.ppf(0.01, alpha), gamma.ppf(0.99, alpha), 100)
    y1 = gamma.pdf(x, a=alpha)
    plt.plot(x, y1)
  plt.legend(weeks)
  plt.xlabel("x")
  plt.ylabel("pdf")
  plt.title("Posterior distributions")
  plt.show()
```

Plotting the gamma distributions of posterior

```
In [15]:
```

```
def get_new_params(prior_alpha,prior_beta,x):
    #we get the sum because of the sum of exponents of exponential
    posterior_alpha=prior_alpha+1+np.sum(x)
    # we get the sum because of the sum of exponents of lambda which is the size
    posterior_beta=prior_beta+x.size
    return posterior_alpha,posterior_beta
```

Getting the alpha and beta of the new posterior from the prior data and sample data.

```
In [13]:

def get_posteriors_for_distribution(x):
```

```
In [18]:

def get_MAP(posteriors):
    MAPS=[]
    #differentiating the gamma w.r.t x gives (alpha-1)/beta
    for posterior in posteriors:
        alpha = posterior[0]
        beta = posterior[1]
        map=(alpha-1)/beta
        MAPS.append(map)
    return MAPS
```

Calculate the MAPs for the posterior distribution which is a gamma distribution. MAP=(alpha-1)/beta

In [10]: ▶

```
def q1_d():
   #get data from new file
   dataframe = pd.read_csv('../15_updated.csv')
   #get daily stats and conver to numpy
   ne_deaths = dataframe['ne_deaths_per_day'].to_numpy()
   nd_deaths = dataframe['nd_deaths_per_day'].to_numpy()
   #add the total deaths from both states
   total_deaths = ne_deaths[131:187] + nd_deaths[131:187]
   #get posterior gamma distributions for week 5,6,7,8
   posteriors = get_posteriors_for_distribution(total_deaths)
   print("posteriors", posteriors)
   #get the MAPs by finding the MLE of the gamma posterior
   maps=get_MAP(posteriors)
   print("MAPS", maps)
   #plot the posterior gamma distributions
   plot_gamma_distribution(posteriors)
```

Approach

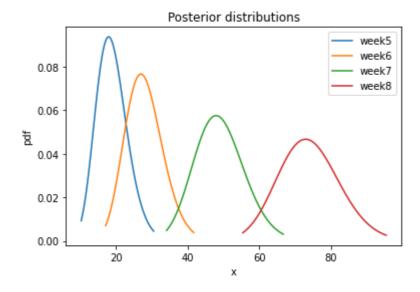
- The exponential prior can also be written as a gamma distribution with alpha=0 and beta=mean of exponential dist=1/lambda MME
- The exponential prior and MLE of the poission gives a gamma distribution with alpha and beta with alpha-1 being the coefficient of lambda and beta being the coefficient of the exponent.
- Taking the posterior of the previous week as the prior of the current week we get the a new gamma distribution in each

In [21]:

q1_d()

posteriors [[19, 7.261682242990654], [28, 14.261682242990654], [49, 21.26168 2242990652], [74, 28.261682242990652]]

MAPS [2.4787644787644787, 1.8931847968545217, 2.2575824175824177, 2.58300264 55026456]



In []:

```
df = pd.read_csv("15_daily_filtered_mean.csv")
df_feb = df[(df['Date'] >= '2021-02-01') & (df['Date'] <= '2021-02-28')]
df_mar = df[(df['Date'] >= '2021-03-01') & (df['Date'] <= '2021-03-31')]
df_feb = df_feb.loc[:, 'ND confirmed':]
df_mar = df_mar.loc[:, 'ND confirmed':]
print(df_mar)</pre>
```

	ND confir	med N	E confirme	d ND	deaths	NE deaths
404		108	37		2	2
405		113	27	6	1	7
406		110	35	8	0	1
407		94	32	.5	0	20
408		82	33	7	1	1
409		33	38	6	0	0
410		29	14	1	0	0
411		97	19	5	0	2
412		99	36	1	3	10
413		108	28	1	3	2
414		120	26	5	1	2
415		93	28	7	2	3
416		34	16	5	0	0
417		25	11	.4	0	0
418		148	31	.7	1	1
419		133	21	.6	1	1
420		119	27	6	0	1
421		113	25	4	1	4
422		0	39		0	2
423		0	13		0	0
424		0	17		0	0
425		0	24		1	2
426		486	26		3	26
427		197	28	6	1	9
428		183	36		1	1
429		159	53		0	0
430		47	22		0	0
431		55	12	.6	0	0
432		245	35		0	2
433		216	46		0	3
434		205	231	.9	0	2

One Sample W-Test

In []:

```
ND confirmed
            0.020000
NE confirmed
            0.000110
ND deaths
            0.867704
NE deaths
            0.411946
dtype: float64
1-Sample W-Test
H0: mean(mar)=mean(feb), Reject:
 ND confirmed
             False
NE confirmed
            False
            False
ND deaths
NE deaths
            False
dtype: bool
```

Results of Walds 1 sample testing for mean of cases and death

Null hypothesis (H0):

the mean of daily cases and the mean of daily deaths for Feb'21 is different from the corresponding mean of daily values for March'21

Mean of Feb'21 cases/deaths = Mean of March'21 cases/deaths.

Alternate hypothesis(H1):

Mean of Feb'21 cases/deaths not equal to mean of March'21 cases/deaths.

Procedure:

We have taken the guess value as mean of daily values from Feb'21 cases/deaths and alpha = 0.05 as given in documentation and the MLE estimator for mean of daily values from Mar'21 cases/deaths becomes the sample mean of daily values from Mar'21 cases/deaths.

Result:

walds 1 sample testing for mean of state ND confirmed cases is w=0.020000

walds 1 sample testing for mean of state NE confirmed cases is w=0.000110

walds 1 sample testing for mean of state ND deaths is w=0.867704

walds 1 sample testing for mean of state ND deaths is w=0.411946

which are less than z alpha = 1.96 so accept the NULL hypothesis

Type *Markdown* and LaTeX: α^2

One sample Z-Test

```
1-Sample Z-Test
ND confirmed
            2.334509e+13
NE confirmed
            1.015164e+12
ND deaths
            7.242974e+12
NE deaths
            4.125351e+13
dtype: float64
H0: mean(mar)=mean(feb), Reject:
 ND confirmed
             True
NE confirmed
            True
ND deaths
            True
NE deaths
            True
dtype: bool
```

Result of Z testing for mean of cases and death

Null hypothesis (H0):

Mean of Feb'21 cases/deaths = Mean of March'21 cases/deaths.

Alternate hypothesis(H1):

Mean of Feb'21 cases/deaths not equal to mean of March'21 cases/deaths.

Result:

Z-sample testing for mean of state ND confirmed cases is w=2.334509e+13

Z-sample testing for mean of state NE confirmed cases is w=1.015164e+12

Z-sample testing for mean of state ND deaths is w=7.242974e+12

Z-sample testing for mean of state ND deaths is w=4.125351e+13

which are greater than z alpha = 1.96 so reject the NULL hypothesis

One Sample T-test

```
1-Sample T-test
ND confirmed
            3.594604e+16
NE confirmed
            3.821252e+14
ND deaths
            9.644333e+14
NE deaths
            6.224550e+15
dtype: float64
H0: mean(mar)=mean(feb), Reject:
 ND confirmed
             True
NE confirmed
            True
ND deaths
            True
NE deaths
            True
dtype: bool
```

Result of T 1 sample testing for mean of cases and death

Null hypothesis (H0):

Mean of Feb'21 cases/deaths = Mean of March'21 cases/deaths.

Comment about applicability of tests

Alternate hypothesis(H1):

Mean of Feb'21 cases/deaths not equal to Mean of March'21 cases/deaths.

Procedure:

We have taken the alpha = 0.05,n = 30 as we took 30 days of data as given in documentation and calculated the numerator and denominator of t.

Result:

As the calculated values for mean of state

ND confirmed cases = 3.594604e+16

NE confirmed cases = 3.821252e+14

ND deaths = 9.644333e+14

NE deaths = 6.224550e+15

which are greater than t value 2.04 we are rejecting the NULL hypothesis.

Two Sample Walds Test

```
In [ ]: ▶
```

```
2-Sample Walds-test
ND confirmed 1.591266
NE confirmed
            0.036141
ND deaths
            0.521922
NE deaths
            1.723450
dtype: float64
H0: mean(mar)=mean(feb), Reject:
ND confirmed
            False
NE confirmed
            False
ND deaths
            False
NE deaths
            False
dtype: bool
```

Result of Walds 2 sample testing for mean of cases and death

Null hypothesis (H0):

Mean of Feb'21 cases/deaths = Mean of March'21 cases/deaths.

Alternate hypothesis(H1):

Mean of Feb'21 cases/deaths not equal to Mean of March'21 cases/deaths.

Procedure:

We have taken the alpha = 0.05 as given in documentation and calculated the numerator and denominator of w.

Result:

As the w value for mean of state

ND confirmed cases 1.591266

NE confirmed cases 0.036141

ND deaths 0.521922

NE deaths 1.723450

which are less than 1.96 we are rejecting the NULL hypothesis.

Type *Markdown* and LaTeX: α^2

Two Sample Unpaired T-test

```
In [ ]: ▶
```

```
2-Sample T-test
ND confirmed
            1.564428
NE confirmed
            0.035535
ND deaths
            0.512856
NE deaths
            1.693888
dtype: float64
H0: mean(mar)=mean(feb), Reject:
ND confirmed
             False
NE confirmed
            False
ND deaths
            False
NE deaths
            False
dtype: bool
```

Result of T 2 sample unpaired testing for mean of cases and death

Null hypothesis (H0):

Alternate hypothesis(H1):

Mean of Feb'21 cases/deaths not equal to Mean of March'21 cases/deaths.

Procedure:

We have taken the alpha = 0.05 n=27, m=30 as given in documentation and calculated the numerator and denominator of t value.

Result:

As the t value for mean of state

ND confirmed cases 1.564428

NE confirmed cases 0.035535

ND deaths 0.512856

NE deaths 1.693888

which are less than 2.0423 we are rejecting the NULL hypothesis.

Applicability Of Tests

Wald's Test

In Wald's we need to have a Asymptomatically Normal estimator. Since N is fairly low, we cannot assume CLT applies in case of sample mean and hence this test is not applicable.

The above reasoning works well for 2 sample test as well, since we need both estimators to be Asymptomatically Normal. Hence this test is also not applicable.

Z-test

In Z-test, we need to have true standard deviation and either the N is large or the X is normally distributed. Since neither of the second requirement is true, we say the test is not applicable.

T-test

In one sample T-test, the requirement is that the data is normally distributed, since this is not the case and N is low as well. We say that the test is not applicable

In unpaired T-test, the required distributions should be independent and need to be normally distributed. In the current scenario, neither is the case so we say that the test is not applicable

In paired T-test, the difference distribution should be normally distributed but we cannot assume the same, hence we say that the test is not applicable