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Homework 1 Solutions

1. Summation using for-loops

b) The values of x_1 - x_4 from the code are as follows:
$x_1 = 1.8848 * 10^{-8}$
$x_2 = 0.0189$
$x_3 = 0$
$x_4 = 0$

c) The calculated value of x_2 is greater than the calculated value of x_1 by a magnitude of roughly 10^6. This is likely because of the difference in the amount of iterations performed to calculate y_1 and y_2; namely, y_2 (and therefore x_2) required 10^3 more iterations of addition to calculate than y_1 (and therefore x_1), and thus any truncation error that occurred was exacerbated by that magnitude.

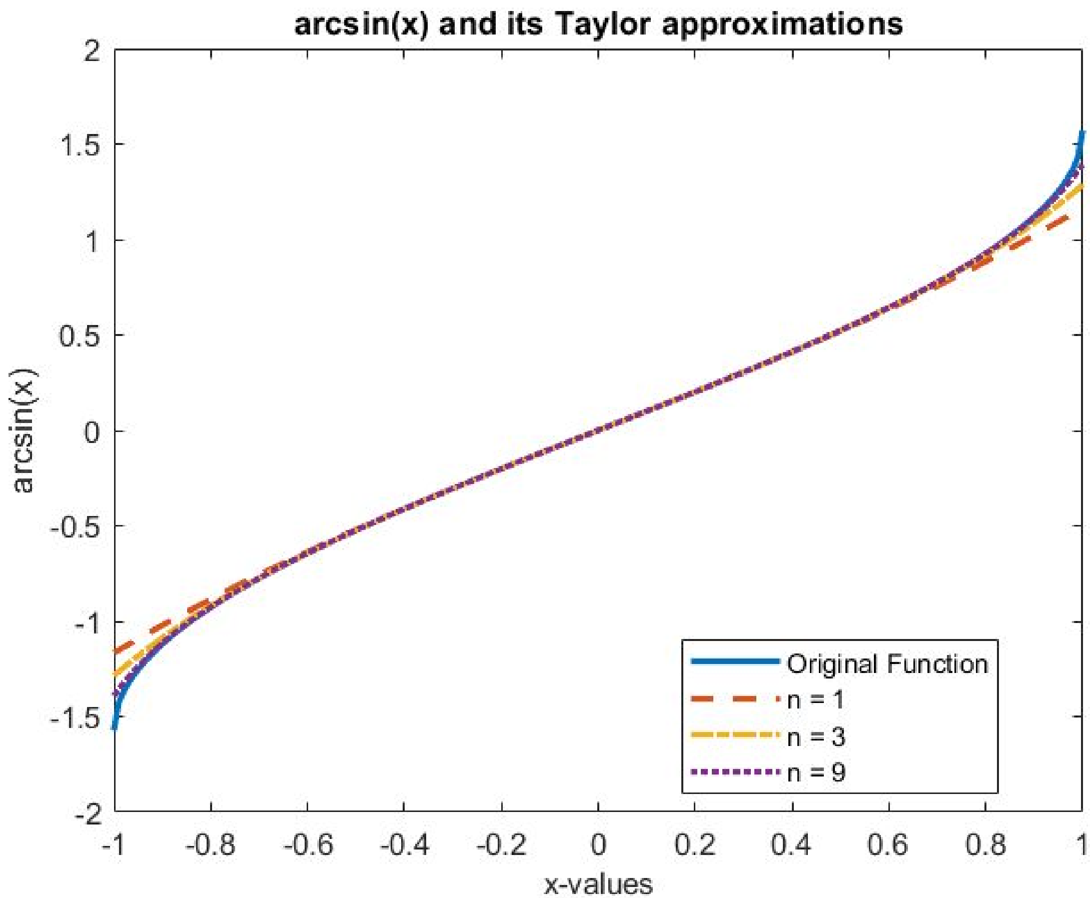
d) Of the calculated values for x_1 - x_4, only x_3 and x_4 are exactly 0. This is likely due to the different increments of addition. In other words, a less accurate result for x_1 and x_2 was likely caused by the fact that the calculation of y_1 and y_2 involved adding 0.1 to each variable for each iteration, whereas the calculation of y_3 and y_4 involved adding a number other than 0.1 for each iteration.

e) The differences in accuracy between x_1, x_2, x_3, and x_4 could be due to the differences in addition increments in the calculations of y_1, y_2, y_3, and y_4. In fraction form, 0.25 is expressed as $\frac{1}{4}$ and 0.5 is expressed as $\frac{1}{2}$, both of which have single-digit denominators. In contrast, 0.1 is expressed as $\frac{1}{10}$ in fraction form, which has a double-digit denominator. If the MATLAB stores each decimal number in fraction form, then it would require far more storage space to perform and store addition in increments of 0.1 than in increments of 0.25 or 0.5, which would exacerbate any truncation errors that may occur during this addition.

Problem 1 Code

```
%% Problem 1
y1 = 0;
for i = 1:10^5
    y1 = y1 + 0.1;
end

y2 = 0;
for i = 1:10^8
    y2 = y2 + 0.1;
end

y3 = 0;
for i = 1:10^8
    y3 = y3 + 0.25;
end

y4 = 0;
for i = 1:10^8
    y4 = y4 + 0.5;
end

x1 = abs((10^4) - y1);
x2 = abs(y2 - (10^7));
x3 = abs((2.5 * 10^7) - y3);
x4 = abs(y4 - (5 * 10^7));
```

2. The code produced the following figure:



<u>Problem 2 Code</u>

```
%% Problem 2
x = -1:10^-2:1;
y1 = asin(x);
y2 = asinTaylor(1, x);
y3 = asinTaylor(3, x);
y4 = asinTaylor(9, x);

plot(x, y1, '-', x, y2, '--', x, y3, '-.', x, y4, ':', 'Linewidth', [2]);
set(gca, 'Fontsize', 10);

legend('Original Function', 'n = 1', 'n = 3', 'n = 9', 'Location', 'Best');
xlabel('x-values', 'Fontsize', [10]);
ylabel('arcsin(x)');
title('arcsin(x) and its Taylor approximations');
```

```
function output = asinTaylor(n, x)
    output = 0;
    for k = 0:1:n
        output = output + ((factorial(2*k))*(x.^(2*k + 1)))/((4^k)*(factorial(k)^2)*(2*k + 1));
    end
end
```