Primera capa

$$y^{(1)} = \sigma\left(w^{(1)} \cdot x^{(1)} + b^{(1)}\right)$$

$$z^{(1)} = w^{(1)} \cdot x^{(1)} + b^{(1)} = \sigma\left(\begin{pmatrix} 0,1 & -0,5 \\ -0,3 & -0,9 \\ 0,8 & 0,02 \end{pmatrix} \cdot \begin{pmatrix} 1,8 \\ -3,4 \end{pmatrix} + \begin{pmatrix} 0,1 \\ 0,5 \\ 0,8 \end{pmatrix}\right)$$

$$= \sigma\begin{pmatrix} 0,1 \cdot 1,8 + (0,5) \cdot (-3,4) + 0,1 \\ -0,3 \cdot 1,8 + (-0,9) \cdot (-3,4) + 0,5 \\ 0,8 \cdot 1,8 + 0,02 \cdot (-3,4) + 0,8 \end{pmatrix} = \sigma\begin{pmatrix} 1,98 \\ 3,02 \\ 2,172 \end{pmatrix} = \begin{pmatrix} \sigma(1,98) \\ \sigma(3,021) \\ \sigma(2,172) \end{pmatrix}$$

$$\Rightarrow \boxed{y^{(1)} = \begin{pmatrix} 0,8787 \\ 0,9535 \\ 0,8977 \end{pmatrix} = x^{(2)}}$$

Segunda capa

$$y^{(2)} = \sigma\left(w^{(2)} \cdot x^{(2)} + b^{(2)}\right)$$

$$z^{(2)} = w^{(2)} \cdot x^{(2)} + b^{(2)} = (-0,4 \quad 0,2 \quad -0,5) \cdot \begin{pmatrix} 0,8787 \\ 0,9535 \\ 0,8972 \end{pmatrix} + 0,7$$

$$z^{(2)} = (-0,4 \cdot 0,8787 + 0,2 \cdot 0,9535 - 0,5 \cdot 0,8972) + 0,7$$

$$\boxed{z^{(2)} = 0,09}$$

$$\boxed{y^{(2)} = \sigma(0,09) = 0,5226}$$

Back Propagation

$$\frac{\partial J}{\partial y^{(2)}} = \frac{1}{2} \cdot 2 \left(y^{(2)} - y\right) = 0,5226 - 5 \Rightarrow \boxed{\frac{\partial J}{\partial y^{(2)}} = -4,4774}$$

$$\frac{\partial z^{(2)}}{\partial w^{(2)}} = x^{(2)} = y^{(1)} = \begin{pmatrix} 0,8787 \\ 0,9535 \\ 0,8974 \end{pmatrix}$$

$$\frac{\partial z^{(2)}}{\partial b^{(2)}} = 1$$

$$\cdot \frac{\partial y^{(2)}}{\partial z^{(2)}} = \sigma'(z^{(2)}) = \sigma(z^{(2)})\left(1 - \sigma(z^{(2)})\right) = y^{(2)}\left(1 - y^{(2)}\right)$$

$$\frac{\partial y^{(2)}}{\partial z^{(2)}} = 0,5226\,(1 - 0,5226) \Rightarrow \boxed{\frac{\partial y^{(2)}}{\partial z^{(2)}} = 0,2495}$$

⟶ Para la capa 2:

$$\frac{\partial J}{\partial w^{(2)}} = \frac{\partial J}{\partial y^{(2)}} \cdot \frac{\partial y^{(2)}}{\partial z^{(2)}} \cdot \frac{\partial z^{(2)}}{\partial w^{(2)}} = (-4,4774) \cdot 0,2495 \cdot \begin{pmatrix} 0,3787 \\ 0,9535 \\ 0,8977 \end{pmatrix}$$

$$\Rightarrow \boxed{\frac{\partial J}{\partial w^{(2)}} = \begin{pmatrix} -0,9816 \\ -1,065 \\ -1,002 \end{pmatrix}}$$

$$\frac{\partial J}{\partial b^{(2)}} = \frac{\partial J}{\partial y^{(2)}} \cdot \frac{\partial y^{(2)}}{\partial z^{(2)}} \cdot \frac{\partial z^{(2)}}{\partial b^{(2)}} = (-4,4774) \cdot 0,2495 \cdot 1 \Rightarrow \boxed{\frac{\partial J}{\partial b^{(2)}} = -1,1171}$$

Para la capa 1:

$$\frac{\partial J}{\partial w^{(1)}} = \frac{\partial J}{\partial y^{(2)}} \cdot \frac{\partial y^{(2)}}{\partial z^{(2)}} \cdot \frac{\partial z^{(2)}}{\partial x^{(2)}} \cdot \frac{\partial x^{(2)}}{\partial z^{(1)}} \cdot \frac{\partial z^{(1)}}{\partial w^{(1)}} = (-4,4774) \cdot 0,2495 \cdot (-0,4\ 0,2\ -0,5) \cdot$$

$$\cdot \begin{pmatrix} 0,107 \\ 0,044 \\ 0,092 \end{pmatrix} \cdot \begin{pmatrix} 1,8 \\ -3,4 \end{pmatrix}$$

$$\frac{\partial z^{(2)}}{\partial x^{(2)}} = w^{(2)} = (-0,4\ \ 0,2\ \ -0,5)$$

$$\frac{\partial x^{(2)}}{\partial z^{(1)}} = \sigma'(z^{(1)}) = \sigma(z^{(1)})\left(1 - \sigma(z^{(1)})\right) = y^{(1)}\left(1 - y^{(1)}\right)$$

$$\Rightarrow \boxed{\frac{\partial x^{(2)}}{\partial z^{(1)}} = \begin{pmatrix} 0,107 \\ 0,044 \\ 0,092 \end{pmatrix}}$$

$$\frac{\partial z^{(1)}}{\partial w^{(1)}} = x^{(1)} = \begin{pmatrix} 1,8 \\ -3,4 \end{pmatrix}$$

$$\frac{\partial J}{\partial w^{(1)}} = (-4,4774)(0,2495)(-0,0797)\begin{pmatrix} 1,8 \\ -3,4 \end{pmatrix} \rightarrow \boxed{\frac{\partial J}{\partial w^{(1)}} = \begin{pmatrix} 0,16 \\ -0,30 \end{pmatrix}}$$

$$\frac{\partial J}{\partial b^{(1)}} = \underbrace{\frac{\partial J}{\partial y^{(2)}} \cdot \frac{\partial y^{(2)}}{\partial z^{(2)}} \cdot \frac{\partial z^{(2)}}{\partial x^{(2)}} \cdot \frac{\partial x^{(2)}}{\partial z^{(1)}}}_{=\ 0,089} \cdot \underbrace{\frac{\partial z^{(1)}}{\partial b^{(1)}}}_{=\ 1} \quad \Rightarrow \quad \boxed{\frac{\partial J}{\partial b^{(1)}} = 0,089}$$