Final Exam

Wednesday (July 1st, 2020) 2:00pm-5:00pm EDT Anywhere with Internet

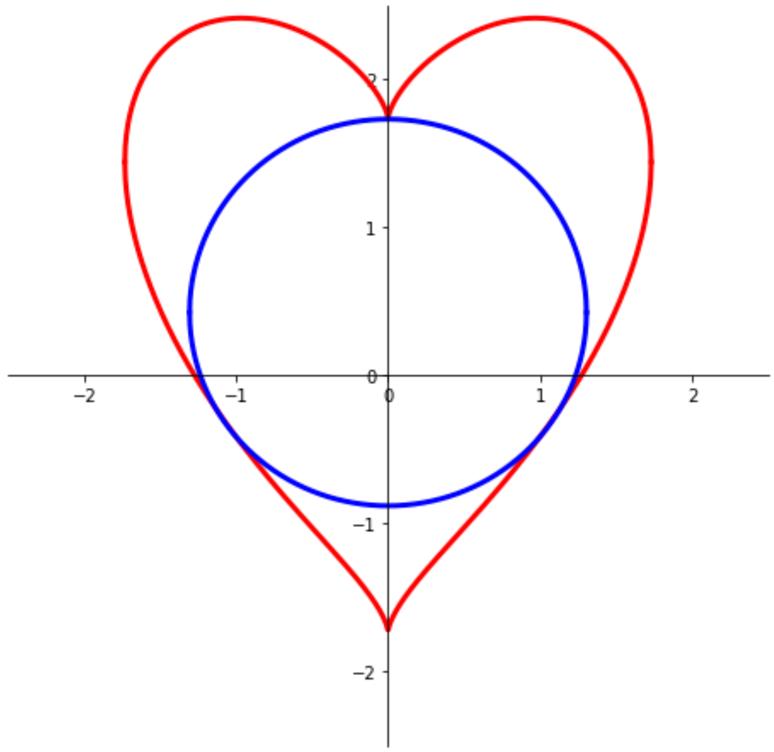
Notes:

1. No communication with a live human, except the proctors, is allowed. Every student must turn on video to allow sufficient view area including the student during the test.
2. Test papers will be examined for abnormality and ~5% students may be interviewed.
3. No late papers are accepted for any reason(s). Same time (EDT) for all regardless of locations.
4. Earn 3x8.5=25.5 points for doing any three of the four problems correctly online;
5. Compose a self-contained report for each problem, the same format as you did for HW sets;
6. Use any language, e.g., C, C++, Fortran, Java, MATLAB, Python, etc;
7. Use any sources for programs as long as you quote the source;
8. Use any computer systems as long as you can e-submit your solutions.
9. Email: sbu.ams326d@gmail.com.

Please read this paragraph before starting the exam:

Academic integrity is expected of all students at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare that I shall not give, use, or receive unauthorized aid in this examination. I have been warned that any suspected instance of academic dishonesty will be reported to the appropriate office and that I will be subjected to the maximum possible penalty permitted under University guidelines.

Problem F-1 (8.5 Points) The heart equation $x^2 + \left(y - \sqrt{|x|}\right)^2 = 3$ can be graphed as



I dig a disc of max area disc the heart. Please use one numerical integration algorithm including rectangle rule, trapezoidal rule, and Monte Carlo methods, etc to compute the area of the remaining heart (for an accuracy of up to four significant digits.). Also, estimate the number of floating-point operations needed for the method you choose.

Problem F-2 (8.5 Points) The US Dow index experienced tremendous dynamics during the past year (shown).

Let's assume Dow index on Monday (5/18/2020) as Dow(0) = 24,000.

During the following 60 trading days (summer session), the Dow index's daily change rate follows a normal distribution $x \sim \mathcal{N}(\mu_1, \sigma_1^2)$ where x is the Dow index's change rate, μ = 0.005 and σ = 0.02, i.e., the next day's Dow is Dow(t + 1) = Dow(t)(1 + x). The Dow index's daily change is about 0.5% fluctuating at ±2.0%.

After the summer session, the next 190 days (which should bring us to May 2021 when the Coronavirus's scourge should end), the Dow index's change will follow another normal distribution $x \sim \mathcal{N}(\mu_2, \sigma_2^2)$ with μ = 0.010 and σ = 0.015.

Please
(1) Compute, and present in a table or figure, the Dow index's daily values (4.0 pts) of the 60+190 days;
(2) Select evenly 6 data points from the summer session (first 60 days) and fit them (4.5 pts) to $Dow(t) = D_0 exp\,(\beta t)$

Problem F-3 (8.5 Points) "My" backyard is a squared 100x100 section of the Amazon forest where 9 trees uniformly distributed at spots whose coordinates are drawn from $(x_i, y_i) \sim \mathcal{U}(0, 100)$ where $\mathcal{U}(0, 100)$ means uniform random numbers $\in (0, 100)$. Their heights are drawn from a normal distribution $z_i \sim \mathcal{N}(12, 1.7^2)$ where the height mean μ = 12 and σ = 1.7 is the standard deviation (and σ^2 is the variance). With such info, generate the 3d coordinates of all trees.

A monkey hops through all trees' tops for the shortest total travel distance (lazy/smart monkey!), starting from the tree whose coordinates (x_1, y_1, z_1) satisfy $x_1 + y_1 + z_1$ is max. Design a path for the monkey (who does not return to its starting tree).

You need to report:
(1) The 3d coordinates of all treetops (2 Pts);

(2) Optimal-distance path (5 Pts) like 1-9-8-4-...;

(3) The optimal distance (1.5 Pts).

Problem F-4 (8.5 Points)

(1) Write a program (3.5 pts) to generate a matrix of the dimension $2^{12} \times 2^{12}$. The matrix's elements $x_{ij} \sim \mathcal{N}(0,1)$ are random numbers distributing, normally, with $\mu = 0$ and $\sigma = 1$.

(2) Write a program (3.0 pts) to compute the dominant e-value and e-vector of the matrix.

(3) Make a histogram (2.0 pts) of the $2^{12} \times 2^{12}$ elements, i.e., counting numbers of elements are in the following intervals: $(-\infty, -1)$, $(-1, -0.5)$, $(-0.5, -0.25)$, $(-0.25, +0.25)$, $(0.25, 0.5)$, $(0.5, 1)$, $(1, \infty)$.