Test 2

Wednesday (June 17, 2020) 1:30pm-3:30pm EDT Anywhere with Internet

Notes:

1. No communication with a live human, except the proctors, is allowed. Every student must turn on video to allow sufficient view area including the student during the test.
2. Test papers will be examined for abnormality and ~5% students may be interviewed.
3. No late papers are accepted for any reason(s). Same time (EDT) for all regardless of locations.
4. Earn 15 points for doing any one of the two problems correctly online;
5. Compose a self-contained report for each problem, the same format as you did for HW sets;
6. Use any language, e.g., C, C++, Fortran, Java, MATLAB, Python, etc;
7. Use any sources for programs as long as you quote the source;
8. Use any computer systems as long as you can e-submit your solutions.
9. Email: sbu.ams326d@gmail.com.

Please read this paragraph before starting the exam:

Academic integrity is expected of all students at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare that I shall not give, use, or receive unauthorized aid in this examination. I have been warned that any suspected instance of academic dishonesty will be reported to the appropriate office and that I will be subjected to the maximum possible penalty permitted under University guidelines.

Problem T2-1 (15 Points) Hypothetically, the number of the coronavirus patients is 10 on 12/1/2019 (considered as Day-1). The daily percentage change $x(t_n)$ of the number of living patients who have the virus, subsequently, follows the following normal distribution

$$\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

where μ is the mean and σ is the standard deviation. You may round up the patient counts to integers. You CANNOT use built-in functions to directly generate random normal distribution numbers, while you CAN use one-line functions to generate uniformly distribution numbers.

(1) During the 1st 45 days, μ = 16% = 0.16 and σ = 6% = 0.06, please compute, and graph, the daily number of infected patients, $y(t_n + 1) = y(t_n)(1 + x(t_n))$, where $t_n = 1, 2, 3 \ldots, 45, 46, \ldots, 90$. In this case, $y(t_1) = 10 =$ number of patients on day1, i.e., 12/1/2019 and $x(t_1)$ is the a normally distributed random number chosen (randomly) from a pool of 90 numbers.

(2) During the 2nd 45 days, do the same as above if μ = −22% = −0.22 and σ = 4% = 0.04, using the 45th day's patient count as the starting number. The number $y(t)$ counts the number of living patients who have the virus, after recoveries and deaths.

(3) Fit the results of the 2nd 45 days (t = 46, 47, ... , 90) in $y(t) = \alpha \exp(\beta t)$. Plot your results. (90 data points and fitted curve)

Problem T2-2 (15 Points) This problem has three parts: (1) Solve the following initial value problem using any method with step size h = Δx = 0.01 to get y (0.00), y(0.01), y(0.02), ... , y(0.50)

$$\begin{cases} \dfrac{dy}{dx} = x + y + xy \\ \quad y(0) = 1 \end{cases}$$

(2) Plot your result x vs. y.

(3) Select x = 0.1, 0.2, 0.3, 0.4, 0.5 with their corresponding $y(x)$ values you found in (1) to interpolate them in a polynomial $y(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4$.