Homework 1

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Assignment Date: Wednesday (05/27/2020)

Collection Date: 06/01/2020 Monday 11:59PM by Email

Grade: Total 20 points

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Please do both problems and each is worth 10 points.

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Problem 1.1 (10 Points) Write a computer program to find the root of the following equation (you may use the provided mathematical functions such as exp(), sin(), etc.)

$$f(x) = 2.020^{-x^3} - x^4 \sin(x^3) - 1.949$$

by going through the following steps. (Your results must have 4 digits of accuracy.)

(a) Plot a figure for the function $f(x)$ above in interval $x \in [-1, 2]$ by using any graphing program of your choice.

(b) From the above figure, please use eyeballs to guess the value (labeled as "x_1") of one of the roots. Report x_1.

(c) Since the "root x_1" you obtained above is an approximation, you design a small interval around the root $[x_1 - \delta, x_1 + \delta]$ where "δ" is small enough that within this interval there is one and only one root. Report "δ".

(d) Use the Bisection method, and the interval you selected in (c), to find the root x_0. Please report the number of iterations you need to get this solution. Report the solution.

(e) Repeat the above steps (b)-(d) to find all roots in interval $x \in [-1, 2]$

Note: If you like to use the Newton's method, you may skip (c). You may use any one or two or all three methods to find the roots. For a single root, if you use Bisection a few iterations and Newton's a few more, I won't blame you and you get full grades for correct roots.

Problem 1.2 (10 Points)

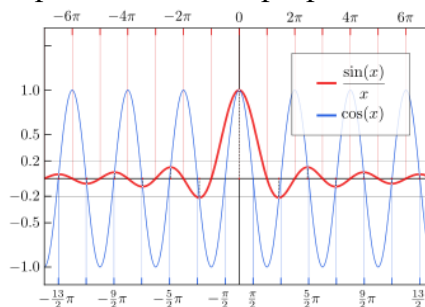T1-2: The sinc function $\text{sinc}(x) = \frac{\sin x}{x}$ as shown by the curve below.
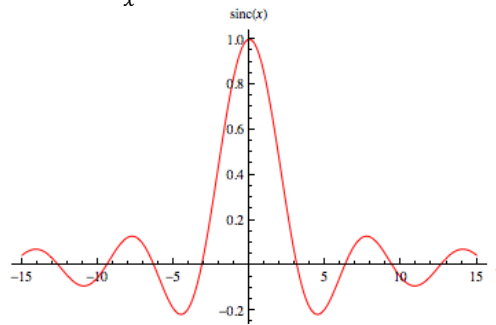
It has infinitely many roots at $x = \pm\pi, \pm2\pi, \pm3\pi, \dots$ which are easy to find. Now, we take its ($\text{sinc}(x)$) derivative to get a new function,

$$\text{sinc}(x) = \frac{d}{dx}\left(\frac{\sin x}{x}\right) = \frac{\cos x}{x} - \frac{\sin x}{x^2} = \frac{1}{x}(\cos(x) - \text{sinc}(x))$$

Please use the root-finding algorithms including the bisection method and Newton's method, Secant method, etc to write a program to find the first five roots of $f(x) \; \forall \, x > 0$. You may use the same method, or two different methods, to find them.

Your roots should have four significant digits. Please report the numbers of iterations you performed to achieve these roots of this accuracy.

The following figure may help understand the properties of the roots:

Source of figure: Wiki.