Homework 3

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Assignment Date: Monday (06/22/2020)
Collection Date: 06/29/2020 Monday 11:59PM by Email
Grade: Total 20 points
Email Address: sbu.ams326d@gmail.com

Please do both problems and each is worth 10 points.

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Problem 3.1 (10 Points) We set a 2-dimensional Cartesian coordinate system for a section of a river such that the y-axis lies at the river's west bank and points north and the x-axis runs from west to east. The water is assumed to flow strictly northward at all times and its speed, denoted by

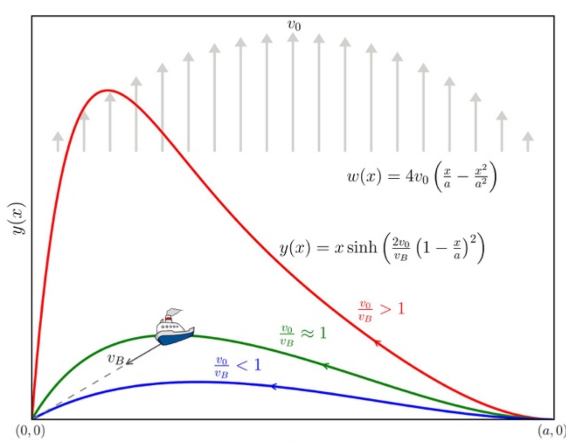
$$w(x) = 4v_0\left(\frac{x}{a} - \frac{x^2}{a^2}\right)$$

varies depending on the x-coordinate of the location. The water flow will drag the boat at full efficiency (meaning the boat flows along with water) and you may ignore the water resistance (Many simplifications were made to make this HW solvable!)

A ferryboat tries to cross the river from a point $(a, 0)$ to dock at $(0,0)$. It can do a lot of good things to cross optimally in time, energy, and safety but it does two simple things: moving at a constant speed v_B relative to water and keeping its head toward the docking point $(0, 0)$ at all times.

Please compute the boat's trajectories for three cases of boat speeds: $v_B = 7,14,21$

In this problem, we assume the river width a = 7777 (any unit) and $v_0 = 14$.



Hints: For your reference, I add some material and hope it's useful.

The boat's velocity components relative to the riverbanks (not to the moving water) are

$$\begin{cases} \dfrac{dx}{dt} = -v_B \cos\alpha = -v_B \dfrac{x}{\sqrt{x^2+y^2}} \\ \dfrac{dy}{dt} = -v_B \sin\alpha + w(x) = -v_B \dfrac{y}{\sqrt{x^2+y^2}} + 4v_0(\dfrac{x}{a} - \dfrac{x^2}{a^2}) \end{cases}$$

which is a system of two DEs with x and y as the dependent variables and t the independent variable. Remember the river flow speed at location x (independent of y) is

$$w(x) = 4v_0(\frac{x}{a} - \frac{x^2}{a^2})$$

Now, numerically, you have two related methods to solve this problem.

Method 1. Directly solve these two DEs as a system of equations, i.e., using Euler or Runge-Kutta methods to solve x as function of t and solve y as function of t.

Method 2. You first eliminate the explicit dependence of t in the two DEs to establish the following initial value problem

$$\begin{cases} \dfrac{dy}{dx} = \dfrac{y}{x} - \dfrac{w(x)}{v_B(\dfrac{x}{\sqrt{x^2+y^2}})} \\ y(x=a) = 0 \end{cases}$$

where, again, the water speed $w(x)$ is not explicitly stated.

Now, you use Euler or Runge-Kutta methods to solve y as function of x and find the trajectory of the boat.

Problem 3.2 (10 Points) The quadrifolium (as shown) is a polar-coordinate equation $r = \sin(2\theta)$. This equation represents a plant with 4 leaves, and never mind what plant it represents (I won't argue with you if you say it's the "lucky clover"). I make a box $(-1,1) \times (-1,1)$ to enclose the quadrifolium. Your test requires you to write a program to throw a needle of length L for 10^6 times to the box and compute the probability of your needle touching the curve (count it one time if your needle touches 2+ spots of the curve, per throw), for needle lengths $L = 0.1, 0.25, 0.5, 1.0$.

Note: throw a needle inside the box means only the coordinate of the center of the needle is inside the box.