Computer Lab III: Nonlinear Dynamics

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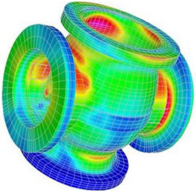
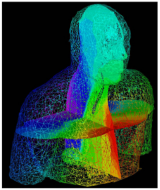


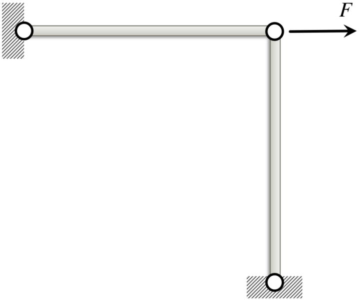


Outline

Learning goals

- Familiarize yourselves with the solution to nonlinear dynamic problems
- Test the dynamic behavior of nonlinear structures under different conditions
- Understand the nonlinear Newmark method
- Explore different parameters of the algorithm and understand their impact on the solution

Case study A



- Perform a dynamic analysis of the 2-bar truss which is subjected to the horizontal load
- The material behavior is assumed to be elastic perfectly plastic
- The code structure is identical to the one used in the previous Computer Lab

Case study A

Task 1

Complete the provided Newmark algorithm

$\ddot{\mathbf{u}}_{k+1} \leftarrow 0$

$\mathbf{u}_{k+1} \leftarrow \mathbf{u}_k + \dot{\mathbf{u}}_k \Delta t + \ddot{\mathbf{u}}_k \left(\frac{1}{2} - \beta\right) \Delta t^2 + \ddot{\mathbf{u}}_{k+1} \beta \Delta t^2$

$\dot{\mathbf{u}}_{k+1} \leftarrow \dot{\mathbf{u}}_k + \ddot{\mathbf{u}}_k \left(1 - \gamma\right) \Delta t + \ddot{\mathbf{u}}_{k+1} \gamma \Delta t$

$\varepsilon \leftarrow \mathbf{f}\left(t_{k+1}\right) - \mathbf{r}^*\left(\mathbf{u}_{k+1}, \dot{\mathbf{u}}_{k+1}\right) - \mathbf{M}\ddot{\mathbf{u}}_{k+1}$

while $\|\varepsilon\| >= Tol$ **do**

    $\Delta\ddot{\mathbf{u}}_{k+1} \leftarrow \left(\mathbf{M} + \mathbf{C}\gamma\Delta t + \mathbf{K}\beta\Delta t^2\right)^{-1} \varepsilon$

    $\ddot{\mathbf{u}}_{k+1} \leftarrow \ddot{\mathbf{u}}_{k+1} + \Delta\ddot{\mathbf{u}}_{k+1}$

    $\dot{\mathbf{u}}_{k+1} \leftarrow \dot{\mathbf{u}}_{k+1} + \Delta\ddot{\mathbf{u}}_{k+1} \gamma \Delta t$

    $\mathbf{u}_{k+1} \leftarrow \mathbf{u}_{k+1} + \Delta\ddot{\mathbf{u}}_{k+1} \beta \Delta t^2$

    $\varepsilon \leftarrow \mathbf{f}\left(t_{k+1}\right) - \mathbf{r}\left(\mathbf{u}_{k+1}, \dot{\mathbf{u}}_{k+1}\right) - \mathbf{M}\ddot{\mathbf{u}}_{k+1}$

end while

*Notation: $\mathbf{r}\left(\mathbf{u}_{k+1}, \dot{\mathbf{u}}_{k+1}\right) =$ Damping Forces + Elastic Forces

Task 2

Set the Rayleigh damping parameters such that the damping ratio for the first two natural frequencies of the system is 0.01 and 0.1 respectively.

The first two natural frequencies of the system are:

$$\omega_1 = 1.2247 \text{ rad/s}$$

$$\omega_2 = 1.2247 \text{ rad/s}$$

Task 2

Rayleigh damping

$$\mathbf{C} = \alpha \mathbf{K} + \beta \mathbf{M}$$

$$\mathbf{\Phi}^T \mathbf{C} \mathbf{\Phi} = \mathbf{\Phi}^T \left(\alpha \mathbf{K} + \beta \mathbf{M} \right) \mathbf{\Phi}$$

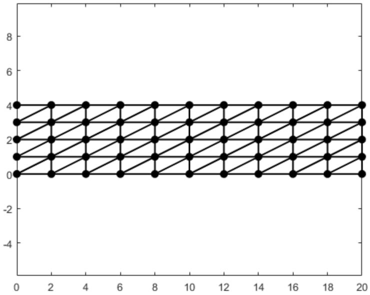
$$\tilde{\mathbf{C}} = \alpha \mathbf{\Omega}^2 + \beta \mathbf{I}$$

where

$$\tilde{\mathbf{C}} = \begin{bmatrix} 2\,\zeta_1\,\omega_1 & 0 \\ 0 & 2\,\zeta_2\,\omega_2 \end{bmatrix}$$

$$\begin{bmatrix} \omega_1^2 & 1 \\ \omega_2^2 & 1 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} 2\,\zeta_1\,\omega_1 \\ 2\,\zeta_2\,\omega_2 \end{bmatrix} \rightarrow \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \omega_1^2 & 1 \\ \omega_2^2 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 2\,\zeta_1\,\omega_1 \\ 2\,\zeta_2\,\omega_2 \end{bmatrix}$$

Case study B



- A vertical load is applied to all nodes of the free end
- The material behavior is assumed to be elastic perfectly plastic
- The first two natural frequencies of the system are $\omega_1 = 0.4703$ rad/s and $\omega_2 = 1.9118$ rad/s

Task 3

Apply a linearly increasing load of magnitude 3 which replicates a quasi-static pushover analysis.

Hints:

- Make sure that the load is applied at a rate that minimizes dynamic phenomena
- Use damping to further reduce dynamic response
- Set the time step such that the time of analysis is reduced

Task 4

Apply harmonic loads with frequencies 0.07484 Hz and 0.30426 Hz and magnitudes 0.87 and 2.56 respectively. Then do the same using the natural frequencies of the structure.

Hints:

- Set the damping parameters such that the damping ratio for the first two natural frequencies of the system is 3%
- Set the time step such that the response in the frequency range of the excitation can be accurately captured