Informed Kinodynamic Planning via Markov Chain Monte Carlo Sampling.

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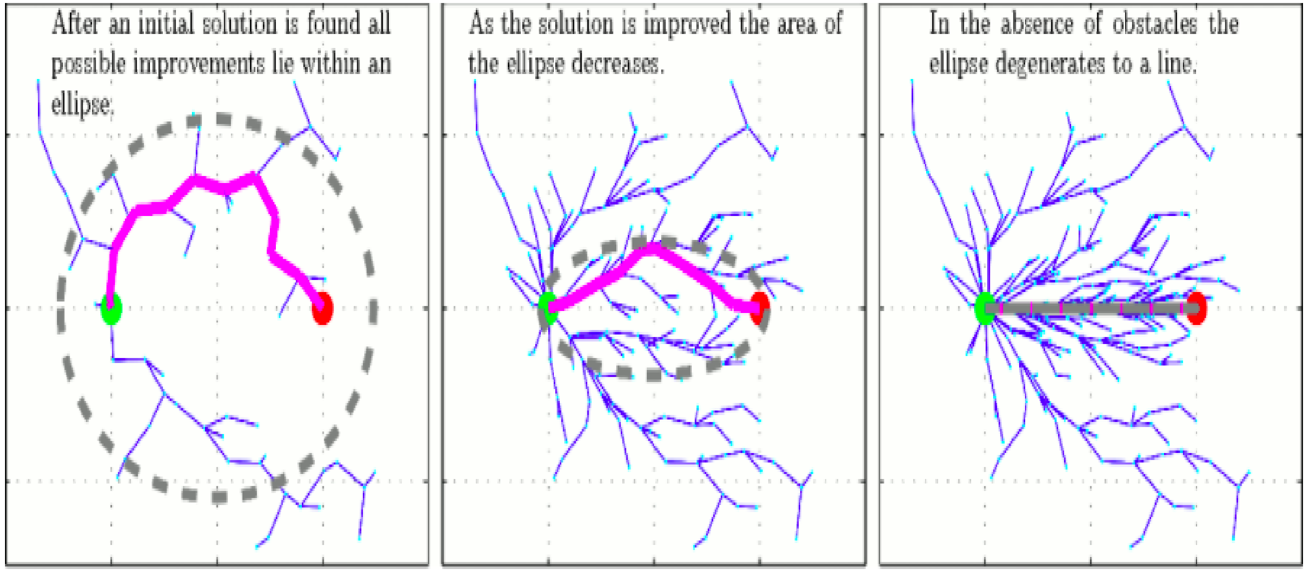
Introduction

1) In algorithms like informed RRT*, once an initial solution is found, the performance can be dramatically improved by restricting subsequent samples to regions of the state space that can potentially improve the current solution.

2) When the planning problem lies in a Euclidean space, this region X_inf, called the informed set, can be sampled directly. But in kinodynamic planning it's not possible.



59 iterations, $c_{best} = 148.24$
(a)

175 iterations, $c_{best} = 107.12$
(b)

1142 iterations, $c_{best} = 100$
(c)

1) However, when planning with differential constraints in non-Euclidean state spaces, no analytic solutions exists to sampling X_inf directly.

2) The main insight is to recast this problem as one of sampling uniformly within the sub-level-set of an implicit non-convex function. This recasting enables us to apply Monte Carlo sampling methods to solve our problem. This sampler can accelerate the convergence rate to high-quality solutions in high-dimensional problems.

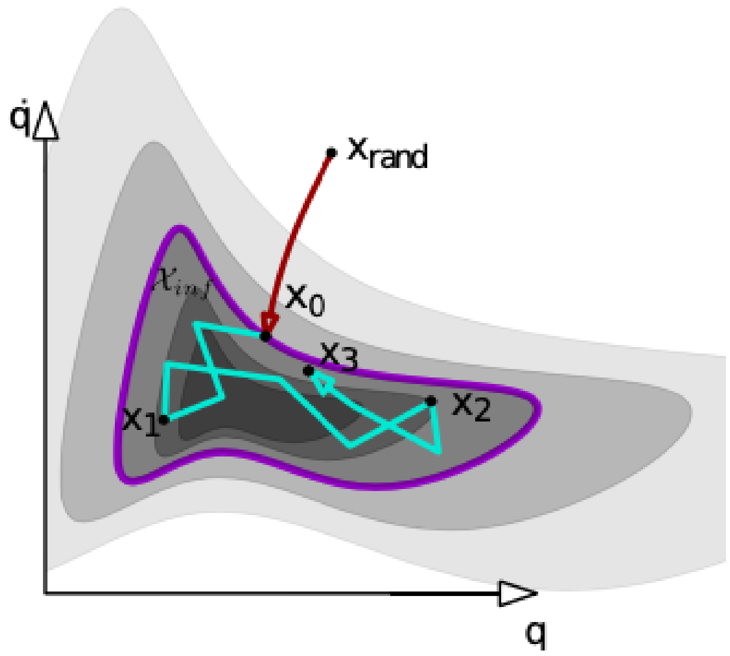


Fig. 1: Algorithmic approach. Cost function is depicted using iso-contours (darker shades reflect lower cost) while the boundary of the informed set is depicted in purple. The root-finding and MCMC algorithms are depicted in red and turquoise, respectively. x_0 lies on the boundary of $\mathcal{X}_{\text{inf}}$

Generalizing Informed Sampling for Non-Euclidean state spaces

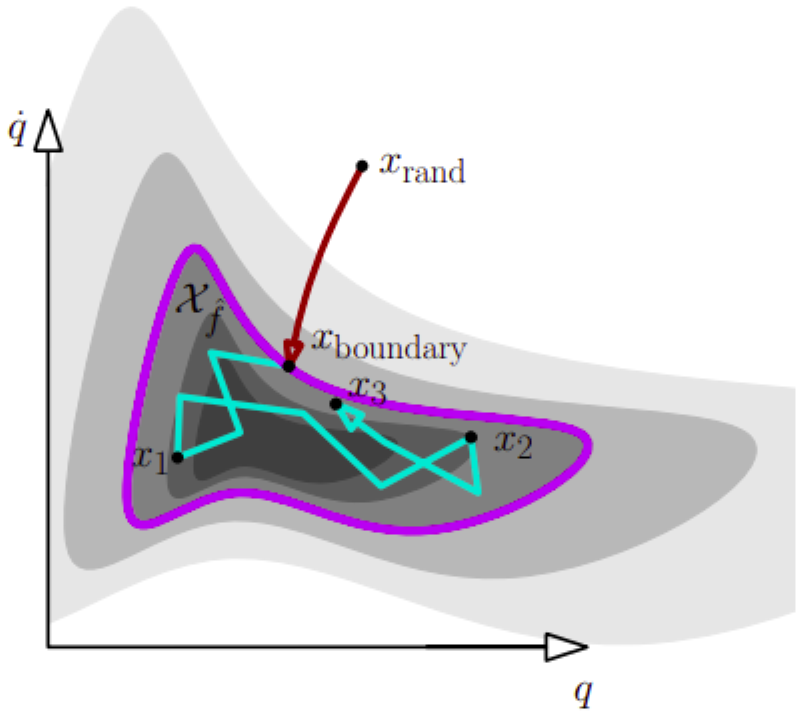
If γ* (x) denotes the optimal path from xs to xg limited to travel via a point x in the state space. Then the informed set X_inf can be defined as

$$X_{inf} = \{x \in X | c(\gamma^*(x)) < c_{best}\}$$

To efficiently producing new samples in an informed set, we use Hit-and-Run, a MCMC sampling technique. Hit-and-Run conducts a sequence of one-dimensional rejection samples that are exceedingly rapid to calculate, even in high-dimensional environments.

Markov chain Monte Carlo (MCMC)

- Markov Chain Monte Carlo is a type of Monte Carlo simulation. The samples are generated by creating a Markov chain with a distribution of points that converges to the desired distribution.
- To efficiently producing new samples in an informed set, we use Hit-and-Run, a MCMC sampling technique.



Hit-and-Run sampling algorithm

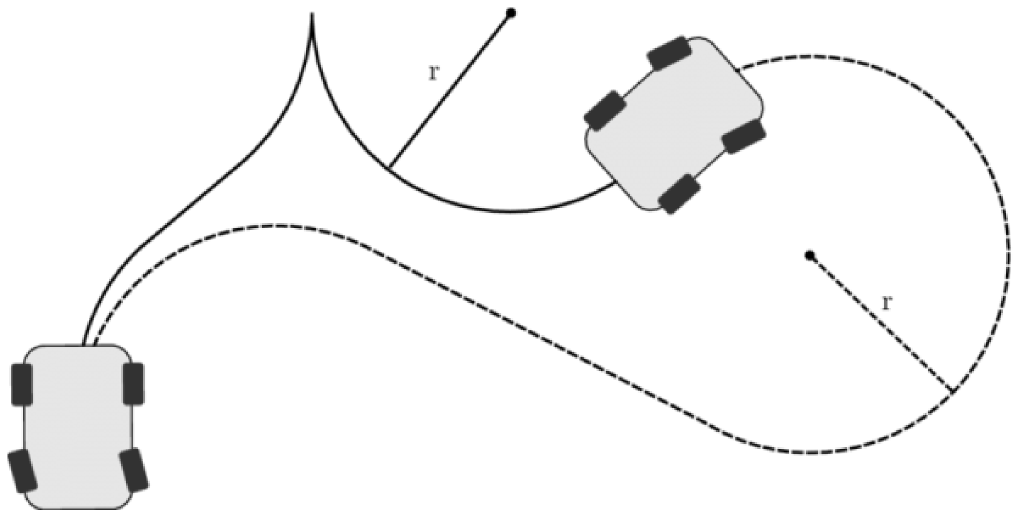
- The sampler first chooses a random direction on a unit sphere centered at the previous sample x_{i−1}. This generates a line L in the direction sampled passing through x_{i−1}.
- The line L is parameterized by a scalar λ, A new point (xi) is sampled on L(λ)by sampling a scalar λ′within the bounds [λ+, λ−].
- We then check if the point lies in the informed set and if it does, we return it or else we update our bounds and repeat the process.
- The algorithm can be viewed as performing rejection sampling along a one-dimensional line passing through the previous sample.

Algorithm 2 Hit-and-Run MCMC (x_{i-1}, c_{best})

$d \leftarrow SampleRandomDirection()$
$L(\lambda) = \{x \mid x = x_{i-1} + \lambda d_i\}$
$\lambda^+ \leftarrow \sup\ L(\lambda)\ \lambda^- \leftarrow \inf\ L(\lambda)$
loop
    $\lambda' \leftarrow$ SampleUniform (λ^-, λ^+)
    $x_i \leftarrow x_{i-1} + \lambda'_i d_i$
    if $c(\gamma^*(x_i)) < c_{best}$ **then**
        return x_i
    end if
    if $\lambda' > 0$ **then**
        $\lambda^+ \leftarrow \lambda^-$
    else
        $\lambda^- \leftarrow \lambda^+$
    end if
end loop

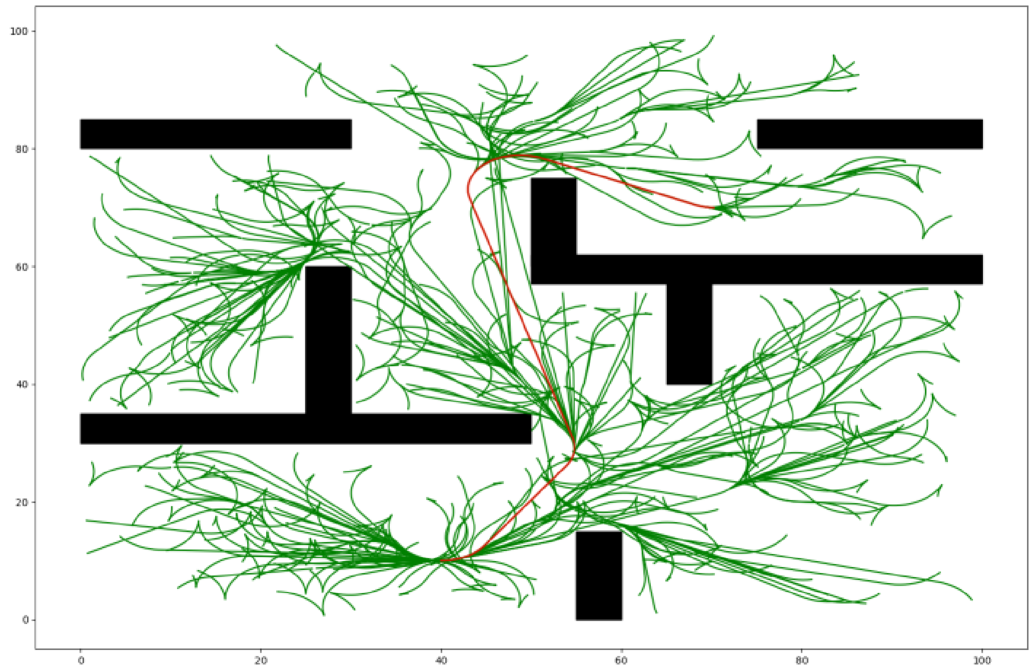
Reeds-Shepp Car

- The state of a reed-shepp's car is 2d position and orientation . It uses motion primitives to optimally steer between start and goal states.
- The shortest curve that connects two points in the two-dimensional Euclidean plane with a constraint on the curvature of the path, and given initial and terminal tangents to the path. If the vehicle can reverse direction, the path will follow the Reeds–Shepp curve. We use Reeds–Shepp curves in steer function to connect states in RRT*.

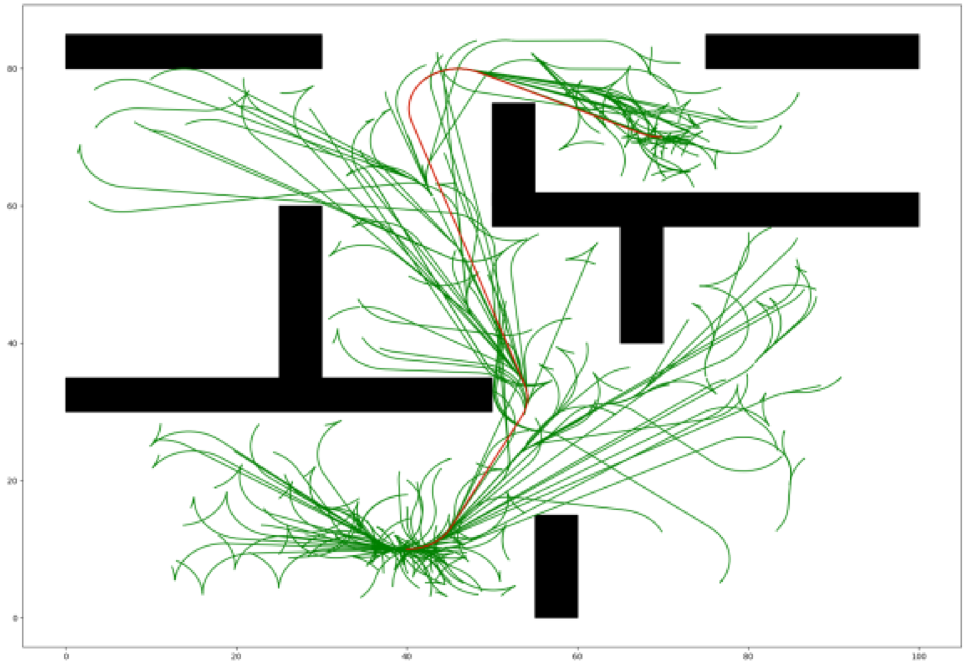


Reeds-Shepp Car's state space: (x, y, theta)

Results:



Reeds-Shepp Car without informed sampler



Reeds-Shepp Car with informed sampler

Quadrotor Trajectory Planning

- A quadrotor model is very nonlinear and has a lot of degrees of freedom. Direct planning on the full dynamics is computationally expensive.
- The quadrotor has been demonstrated to be a deferentially flat system, with the flat output serving as its position x, y, z, and yaw.
- Therefore, it's efficient to employ a simpler integrator-based model during the trajectory planning such as the third order model. Hence the steering function will be giving a minimum jerk solution to generate smooth trajectories for quadrotors. The state space is 9 dimensions.

$$
\begin{aligned}
&\min_{\mathbf{j}(t),T,t\in[0,T]} \left\{ J = \int_{t=0}^{T} m + \mathbf{j}(t)^2 dt \right\} \; s.t. \\
&\mathbf{p}(0) = \mathbf{p}_{ini}, \quad \mathbf{p}(T) = \mathbf{p}_{ref} \\
&\mathbf{v}(0) = \mathbf{v}_{ini}, \quad \mathbf{v}(T) = \mathbf{v}_{ref} \\
&\mathbf{a}(0) = \mathbf{a}_{ini}, \quad \mathbf{a}(T) = \mathbf{a}_{ref} \\
&\dot{\mathbf{p}}(t) = \mathbf{v}(t) \\
&\dot{\mathbf{v}}(t) = \mathbf{a}(t) \\
&\dot{\mathbf{a}}(t) = \mathbf{j}(t) \\
&-\mathbf{v}_{\max} \le \mathbf{v}(t) \le \mathbf{v}_{\max}, \quad \forall t \in [0,T] \\
&-\mathbf{a}_{\max} \le \mathbf{a}(t) \le \mathbf{a}_{\max}, \quad \forall t \in [0,T]
\end{aligned}
$$

Results

https://drive.google.com/drive/folders/1KKWMSkZoGQM39M02epcrLi5Y6nkc20ic?usp=sharing

conclusion

In this project, we demonstrated the effectiveness of using MCMC algorithms, particularly Hit and Run sampling to efficiently produce samples for motion planning algorithms. We have applied it on three kino -dynamic planning problems namely double integrator, Reed-shepps and Quadrotor trajectory generation. Such sampling algorithms are effective in high dimensional state spaces.

References

1) Generalizing Informed Sampling for Asymptotically-Optimal Sampling-based Kinodynamic Planning via Markov Chain Monte Carlo
Daqing Yi, Rohan Thakker, Cole Gulino, Oren Salzman and Siddhartha Srinivasa

2) Kinodynamic RRT*: Optimal Motion Planning for Systems with Linear Differential Constraints Dustin J. Webb, Jur van den Berg

3) Tobias Kunz, Andrea Thomaz and Henrik Christensen. "Hierarchical rejection sampling for informed kinodynamic planning in high-dimensional spaces".

4)Jonathan D, Gammell, Siddhartha S. Srinivasa and Timothy D. Barfoot. "Batch Informed Trees (BIT*): Sampling-based Optimal Planning viathe Heuristically Guided Search of Implicit Random Geometric Graphs".

5) Alejandro Perez, Robert Platt Jr, George Konidaris, Leslie Kaelbling and Tomas Lozano-Perez. "Kinodynamic RRT*: Asymptotically Optimal Motion Planning for Robots with Linear Dynamics".

6)Menglu Lan1, Shupeng Lai2, and Ben M. Chen2 "Towards the Realtime Sampling-based Kinodynamic Planning for Quadcopters".

7)Christopher Xie, Jur van den Berg, Sachin Patil and Pieter Abbeel "Toward Asymptotically Optimal Motion Planning for Kinodynamic
Systems using a Two-Point Boundary Value Problem Solver".