modeling assignment - I

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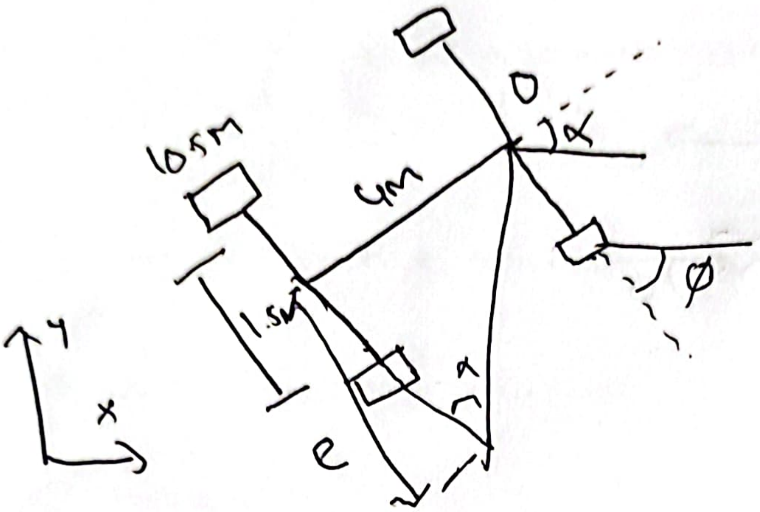


Given Initial Position : (x_i, y_i, ϕ_i)
drive speed : ω
steering angle : α
duration : T

$$\dot{x} = f_1(x, y, \theta, s, \phi)$$
$$\dot{y} = f_2(x, y, \theta, s, \phi)$$
$$\dot{\theta} = f_3(x, y, \theta, s, \phi)$$

At $t = 0$

$$\frac{y_0}{x_0} = \frac{\sin\theta}{\cos\theta}$$

$$x_0 \cos\theta - y_0 \sin\theta = 0$$

$\boxed{x^o = \cos\theta}$ & $\boxed{y^o = \sin\theta}$ — ①

$\therefore x' = u\cos\theta$
$y' = u\sin\theta$ (u is a scalar value of speed)

Let z be distance moved
&
$\rho \rightarrow$ radius of curvature.

$\Rightarrow \tan\alpha = \frac{l}{\rho}$

for

$dz = \rho . d\theta$ (for neglible value θ)

$dz = \frac{l}{\tan\alpha} . d\theta$

$d\theta = \frac{\tan\alpha . dz}{l}$

Dividing dt by both sides

$\frac{d\theta}{dt} = \frac{\tan\alpha}{l} . \frac{dz}{dt}$ ← ③

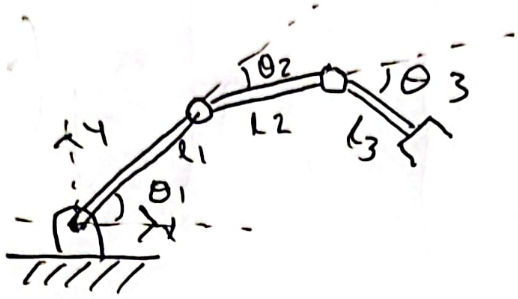
$\theta^o = u . \frac{\tan\alpha}{l}$ $\left(\frac{dz}{dt} = u , \frac{d\theta}{dt} = \theta^o \right)$ — ②

From ① & ②

$$\boxed{\begin{array}{c} x^o = u . \cos\theta \quad , y^o = u\sin\theta \\ \\ \theta^o = \frac{u . \tan\theta}{l} \end{array}}$$

1.2)

a) Forward kinematics equations



The Position's & orientation of end effector & joint co-ordinates can be written as

$$x = l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) + l_3 \cos(\theta_1+\theta_2+\theta_3)$$

$$y = l_1 \sin\theta_1 + l_2 \sin(\theta_1+\theta_2) + l_3 \sin(\theta_1+\theta_2+\theta_3)$$

$$\phi = \theta_1 + \theta_2 + \theta_3$$

To find the joint co-ordinates given the end effector co-ordinates (x, y, ϕ)

Differentiating x, y & ϕ w.r.t time

$$\dot{x} = -(l_1 \theta_1 \sin\theta_1) - l_2(\theta_1+\theta_2)\sin(\theta_1+\theta_2) - l_3(\theta_1+\theta_2+\theta_3)\sin(\theta_1+\theta_2+\theta_3)$$

$$\dot{y} = l_1 \theta_1 \cos\theta_1 + l_2(\theta_1+\theta_2)\cos(\theta_1+\theta_2) + l_3(\theta_1+\theta_2+\theta_3)\cos(\theta_1+\theta_2+\theta_3)$$

$$\phi = (\theta_1+\theta_2+\theta_3)'$$

1b) we use sympy to solve Jacobian matrix.

$$\begin{bmatrix} \frac{\partial x}{\partial \theta_1} & \frac{\partial x}{\partial \theta_2} & \frac{\partial x}{\theta_3} \\ \frac{\partial y}{\partial \theta_1} & \frac{\partial y}{\partial \theta_2} & \frac{\partial y}{\partial \theta_3} \\ \frac{\partial \theta}{\partial \theta_1} & \frac{\partial \theta}{\partial \theta_1} & \frac{\partial \theta}{\partial \theta_3} \end{bmatrix}, \quad q = \begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{pmatrix} \text{(rate at which joint changing)}$$

we get

$P = J\dot{\theta}$

~~we get~~

$$\begin{pmatrix} x^{\circ} \\ y^{\circ} \\ \theta^{\circ} \end{pmatrix} \dot{P} = \begin{bmatrix} -(l_1 \sin\theta_1 + l_2 \sin(\theta_1+\theta_2) + l_3 \sin(\theta_1+\theta_2+\theta_3) & -(l_2 \sin(\theta_1+\theta_2) + l_3 \sin(\theta_1+\theta_2+\theta_3)) & -(l_3 \sin(\theta_1+\theta_2+\theta_3) \\ l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) + l_3 \cos(\theta_1+\theta_2+\theta_3) & l_2 \cos(\theta_1+\theta_2) + l_3 \cos(\theta_1+\theta_2+\theta_3) & l_3 \cos(\theta_1+\theta_2+\theta_3) \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$

J $\quad \dot{\theta}$

$\dot{P} = J.\dot{\theta}$