ENPM-662 HW2

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1) Homogenous transformation

1.1) R, ϕ about world x axis

PRE multiply world and post multiply current axis

R → by θ about world Z-axis

R → by ψ // current x-axis

$$H = R_{z\theta} R_{x\phi} T_{yy} R_{x\psi}$$

$$R_{z\theta} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$R_{x\phi} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi & 0 \\ 0 & \sin\phi & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{y.y} = \begin{bmatrix} 1 & 0 & 0 & a_y \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_{xy} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\psi & -\sin\psi & 0 \\ 0 & \sin\psi & \cos\psi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\phi & \sin\phi & 0 \\ 0 & \sin\phi & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\psi & -\sin\psi & 0 \\ 0 & \sin\psi & \cos\psi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Translation of y'' along y' axis & rotation of ϕ and x_n w.r.t to current axes.

∴ Therefore post multiplied.

2) ~~H~~ The Rotation occurs as

$$R_z . R_y R_x$$

$$H = T \times R_z \times R_y \times R_x$$

drone translates from $(0,0,0)$ to (d_x, d_y, d_z)

$$H = \begin{bmatrix} C\phi C\phi & (C\phi S\theta S\psi - C\psi S\phi) & (C\phi S\theta C\psi + S\phi C\psi) & d_x \\ S\theta S\phi & (S\phi S\theta S\psi + C\phi C\psi) & (S\phi S\theta C\psi - C\phi S\psi) & d_y \\ -S\theta & C\theta S\psi & C\theta C\psi & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The new equation will become

$$[H x]^T_{1\times4} . S . [H x]_{4\times1} = 0$$

1×1 matrix gives equation of elipse

cone equation $\rightarrow x^2 + y^2 = a z^2$

$$x^2 + y^2 = z^2 \quad \begin{pmatrix} a = \tan\alpha \\ \alpha = 45^\circ \end{pmatrix}$$

$$\rightarrow x^2 + y^2 - z^2 = 0$$

matrix form

$$[x\ y\ z\ 1]\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&0\end{bmatrix}\begin{bmatrix}x\\y\\z\\1\end{bmatrix}=0$$

$P^T \quad K \quad P$

~~There~~ HP^TKP gives us the equation of ~~the~~ ellipsoid

By ~~subst~~ solving & comparing with the equation

$ax^2 + 2bxy + cy^2 + dx + 2ey + f = 0$

and substituting them in below eqn

$$A = \frac{-\pi}{(ac-b^2)^{3/2}}\begin{vmatrix}a&b&d\\b&c&e\\d&e&f\end{vmatrix}$$

refer the code for the calculation

1.3)

~~Sol~~

axes are rotated through ϕ w.r.t z axis

therefore new co-ordinates (x', y', z')

$x' = x\cos\phi - y\sin\phi$

$y' = x\sin\phi + y\cos\phi$

$z' = z$

Translation matrix(T) = $\begin{bmatrix} 1 & 0 & 0 & x'-x \\ 0 & 1 & 0 & y'-y \\ 0 & 0 & 1 & z'-z \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & (xC\phi - yS\phi) - x \\ 0 & 1 & 0 & (xS\phi + yS\phi) - y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation Matrix(R) = $\begin{bmatrix} C\phi & -S\phi & 0 & 0 \\ S\phi & C\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Homogenous transformation matrix (H)

$= R \times T$

$$H = \begin{bmatrix} C\phi & -\sin\phi & 0 & 0 \\ S\phi & C\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & (xC\phi - yS\phi - x) \\ 0 & 1 & 0 & (xS\phi + yC\phi - y) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H = \begin{bmatrix} C\phi & -S\phi & 0 & (xC\phi - yS\phi - x) \\ S\phi & C\phi & 0 & (xS\phi + y\phi - y) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2.1)

As drone in constant motion we get new axes are

$xyz \Rightarrow x'y'z'$

Rota which are rotated by

$\psi_g = 35°$ $\theta_g = 15°$, $\varphi_g = 20°$

we can assume a vector P which is made an angle 'γ' w.r.t to the drone. this meter is same as the rotation metric

After transformation we can get the rotation matrix 'R_T' by

$$R_T = R_{z\theta} \cdot R_{y\theta} \cdot R_{x\varphi}$$

γ we find by

$$\gamma = \cos^{-1}\left(\left[\frac{\text{trance}(R) - 1}{2}\right]\right)$$

$$\gamma \simeq 40.56°$$

$$p = \frac{1}{2\sin\gamma}\left[\text{skew}(R_T)\right]$$

$$\simeq \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix}$$

In euler Representation

$$\text{Rotation} = I + \sin(\theta) \cdot \hat{p} + (1 - \cos\theta)(\hat{p})^2$$

we calculate

$\theta = -\text{asin}(\text{rotation}[2,0])$

$\phi = \text{atan}(\text{rotation}[1,0]$ w.r.t $[0,0])$

$\psi = \text{atan}(\text{rotation}[2,1]$ w.r.t $[2,2])$

The graph would look like at t = 20 secs

& $\theta_0 = 0$, $\phi_0 = 0$ $\psi_0 = 0$

The trajectory angles are plotted θ to ψ to ϕ.