Home work- 1

ENPM673

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I. PROBLEM 1

A. Part 1

The relation between field of view, focal length and sensor height/width is given by the following relation:

$$\varphi = 2 \times \tan^{-1} (d/2f)$$

Where, φ is the field of view, f is the focal length and d is the sensor dimension.

$$f = 25mm , d = 14mm$$

Since the sensor shape is square, the horizontal and vertical FoV shall be the same. Substituting the values,

$$\varphi = 2 \times \tan^{-1}(25 \times 2) /14)$$
$$\varphi = 31.28^o$$

The horizontal and vertical fields of view are same as square shaped sensor, that is 31.28^o.

B. Part 2

Given the object height (h_o)and the object distance($d_{o)}$ from the camera, the image height ($h_{f)}$ can be calculated using the following relation:

$$h_0 / d_0 = h_f / f$$

$$5 \times 10/ 20 \times 10^3 = hi /25$$
$$h_i = 0.0625 \text{ mm}$$

Since the object and the sensor are square, the height(h_i) and width(w_i) on the image will be equal 0.0625 mm. The area covered by the image will be ($h_i \times w_i$). Let the sensor height and width is denoted by h_s and w_s Let R denote resolution of the camera and P be the number of pixels covered by the image of the square object. Then, by ratio and proportion,

$$R / \text{Area of sensor} = P / \text{Area of image}$$
$$R /h_s \times w_s = P / h_i \times w$$
$$5000000(14*14)/(0.0625)^2$$
$$P = 0.00009964mp$$
$$\sim= 100 \text{ pixels}(1 \text{ mp} = 10^{6)}$$

II. PROBLEM 2

In this section, one curve fitting techniques is done that is Least Square Method,

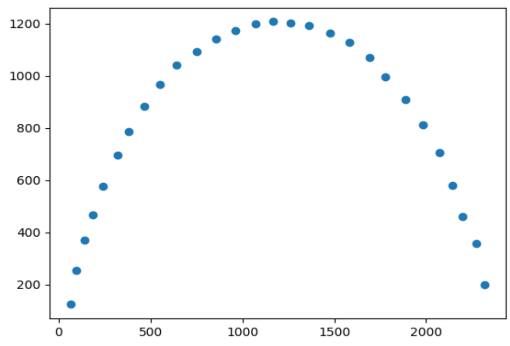
A. Extracting data points

1) Read the video file from the folder.
2) extracting blue channel. as the ball is red it will have low level of blue pixels and background is white
3) Set appropriate thresholds <100 then the we will be able to track the ball
4) Extract the blue pixels, i.e. the pixels occupied by the ball.
5) Find the location of top most and bottom most pixel.
6) Take average of the top and bottom points to get the centre of the ball.
7) These centre points are used to fit a curve, which is assumed as a parabola.

B. Converting the points to a suitable reference frame

Since the image co-ordinate system is different than the one used by NumPy, a transformation is used to converted the image co-ordinates to a suitable frame.

Refer fig. 1 for the extracted data points.



From video 1 (Without noise)

from video 2 (with noise)

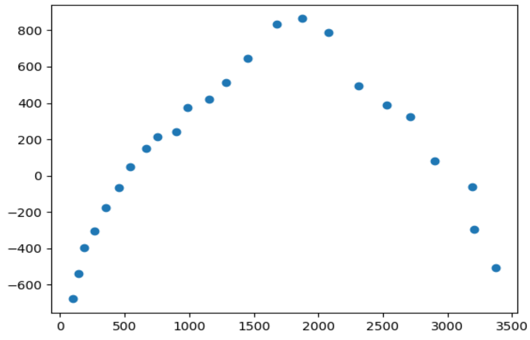


fig.1 .Data extracted from video

C. Curve Fitting

1) Least Square Method(LS): The equation of parabola is given by

$$y = ax^2 + bx + c$$

The error function can be written as,

$$E = \Sigma\, n_n (y - ax^2 - bx - c)^2$$

Let $X = [x^2, x, 1]$, $Y = y$ and $D = [a, b, c]^T$. Then,

$$E = \| (Y - XD) \|^2$$

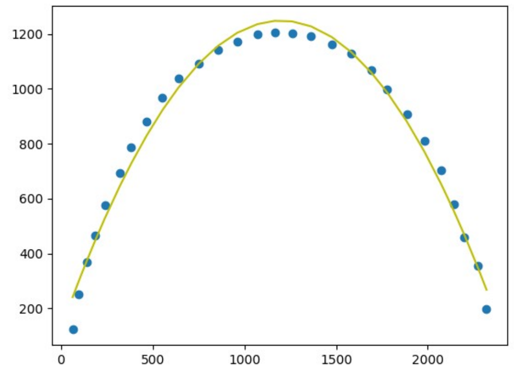
$$E = (Y - XD)^T (Y - XD)$$

$$E = Y^T Y - 2(XD)^T Y + (XD)^T XD$$

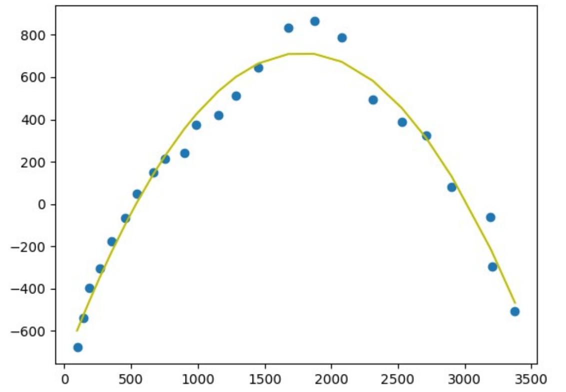
Differentiating wrt D,

$$\delta E / \delta D = -2X^T Y + 2X^T XD$$

Since $\delta E / \delta D = 0$,
$-2X^T Y + 2X^T XD = 0$
$D = (X^T X)^{-1} X^T Y$
Refer fig 2 for the obtained curves.



Video 1(without noise)



video 2 (with noise)

fig 2. least square curve fitting

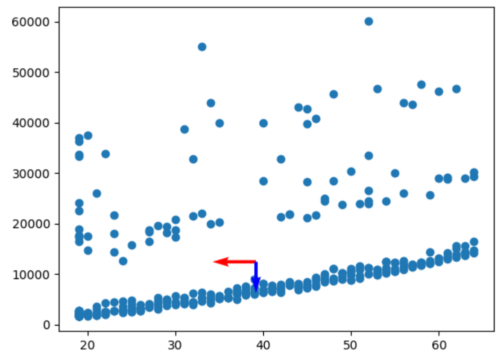
III. PROBLEM 3

part- 1: covariance matrix

In this problem, we are given a bunch of data from which we have to extract data for age and charges, find the variance of each point from the mean and compute a covariance matrix and find its eigen values and vectors and plot the eigen vectors w.r..t to the data point which shows us the direction of eigen vectors in of which has maximum variance and minimum error line and other one which has minimum variance and maximum error line.(Kept age mean and charges mean as origin for ploting)

$$S = \frac{1}{n} \sum_i (x_i - \bar{x})(x_i - \bar{x})^T$$

where $\bar{x}$ is the mean of the set of points $\{x_i\}$



Part-2

B) curve fitting

in this we have use three curve fitting techniques, fit a line to the data using linear least square method, total least square method and RANSAC.

1) linear least square:

It minimizes the mean squared error of the data points with respect to the Residual error

Advantage:

-it is a simple one step learning process and LLS combined with non-linear transforms and regularization can produce incredible results for as simple a process as this

Disadvantage:

-it is susceptible to outliers but not very much.

The equation for line

y = ax + b

The error function can be written as,

E = Σn (ax + b)

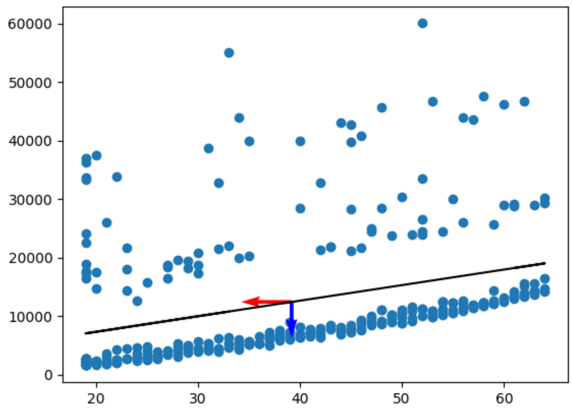
Let X=[x ,1]   ,

Y =y and

D=[a,b]T

Then,

D = (XT X)$^{-1}$ X^{T} Y



Black line represents LLS

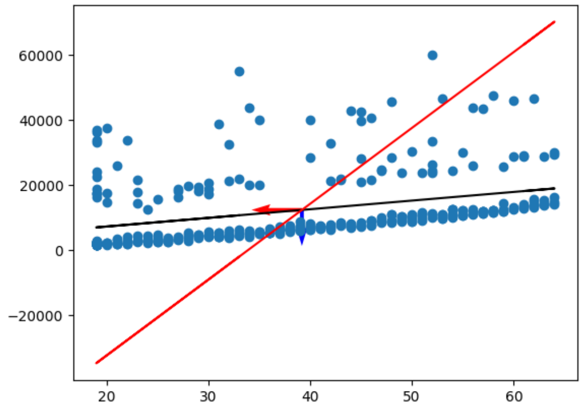
2) Total Least Square Method(TLS):

It minimizes the sum of the squares of the residuals between the observed targets in the dataset, and the targets predicted by the linear approximation. We can find the error line using for y= ax + b

$$E = \sum_{i=1}^{n} (ax_i + by_i - d)^2$$

for matrix

$$U = \begin{bmatrix} x_1 - \bar{x} & y_1 - \bar{y} \\ \vdots & \vdots \\ x_n - \bar{x} & y_n - \bar{y} \end{bmatrix} \quad U^T U = \begin{bmatrix} \sum_{i=1}^{n} (x_i - \bar{x})^2 & \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y}) \\ \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y}) & \sum_{i=1}^{n} (y_i - \bar{y})^2 \end{bmatrix}$$



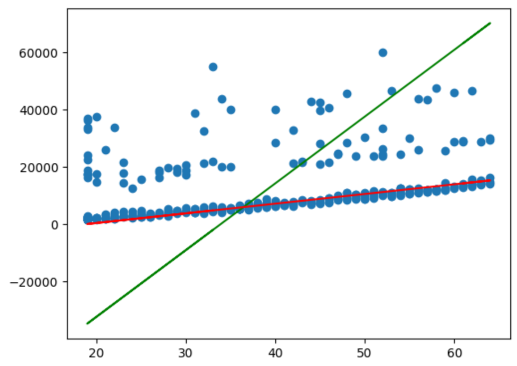
Red line represents TLS

Drawbacks:

-Sensitivity to outliers and Test statistics might be unreliable when the data is not normally distributed

3) RANSAC: In this method, we set the threshold of that line and we take the best line which has most number of points within the threshold. where we see how many points lie within the threshold. A good model depends on user input threshold data and probability. If the threshold is too high it will give a bad model and vice versa.

Number of samples = N $= \log(1 - p)/\log(1 - (1 - e)^S)$



red line represents RANSAC

clearly, for our given data set RANSAC works the best which can infered from our data.

3)
The procedure for LLS has been explained in the previous question. we find the model parameters and we fit a line, in this case it is a straight line so the line is of the form y = mx + c so we only need 2 model parameters. so weight matrix has only two elements which are needed to fit a line. For ransac, The number of iterations are calculated dynamically, i chose to use that with 95 percent probability we get a good sample to calculate the number of iterations with outliners with 0.25 and threshold 100.

Problem 4
 A. Computing SVD of a matrix :
Any matrix can be factored into three pieces: U, Σ, and V .
• U is an orthogonal matrix.
Therefore $U^T.U = I$. (Also called as left singular vectors)

• Σ is a diagonal matrix, where the values are called singular values.
• V is also an orthogonal matrix. Therefore, $V^T.V = I$. (Also called as right singular vectors)
This is physically equivalent to rotation, stretching and rotation.[1] As per the SVD,

$$A = U\Sigma V^T$$

$$A^T A = V\Sigma^T U^T \; U\Sigma V^T$$
$$A^T \; A = V\Sigma^T \Sigma V$$

This equation looks similar to the eigen vector decomposition. Therefore, the columns of V are the orthogonal eigen vectors of A^T A and Σ 2 are the eigen values of A^T A. Lets look at the decomposition again.

$$A = U\Sigma V^T$$

$$AA^T = U\Sigma V^T \, V\Sigma^T U^T$$
$$AA^T = U\Sigma\Sigma^T U^T$$

By the previous logic, columns of U are the orthogonal eigen vectors of AA^T and Σ^2 are the eigen values of AA^T . Also, since V is an orthogonal matrix, $V^{-1} = V^T$ Therefore,

$$AV = U\Sigma$$
All equations are referred from [1], [2], [3].

B. Mathematical computation of SVD for matrix A

The matrix A is

```
[[   -5    -5    -1     0     0     0   500   500   100]
 [    0     0     0    -5    -5    -1   500   500   100]
 [ -150    -5    -1     0     0     0 30000  1000   200]
 [    0     0     0  -150    -5    -1 12000   400    80]
 [ -150  -150    -1     0     0     0 33000 33000   220]
 [    0     0     0  -150  -150    -1 12000 12000    80]
 [   -5  -150    -1     0     0     0   500 15000   100]
 [    0     0     0    -5  -150    -1  1000 30000   200]]
```

1) Calculate AA^T.

```
[[    510051     510000   15520776    6208000   33023501   12008000
     7760776   15520000]
 [    510000     510051   15520000    6208776   33022000   12009501
     7760000   15520776]
 [  15520776   15520000  901062526  360416000 1023067251  372016000
    30021501   60040000]
 [   6208000    6208776  360416000  144188926  409217600  148829651
    12008000   24017501]
 [  33023501   33022000 1023067251  409217600 2178093401  792017600
   511545251 1023044000]
 [  12008000   12009501  372016000  148829651  792017600  288051401
   186008000  372039251]
 [   7760776    7760000   30021501   12008000  511545251  186008000
   225282526  450520000]
 [  15520000   15520776   60040000   24017501 1023044000  372039251
   450520000  901062526]]
```

2) As per eq. 3, U matrix represent the orthogonal eigen vectors of AAT . Therefor, find eigen vectors and eigen values of AAT
3) Arrange the eigen values in descending order and rearrange the eigen vector matrix as per the corresponding eigen value for each eigen vector. The U matrix is,

```
[[ 1.17519867e-02  3.44207228e-04 -5.15532162e-02 -4.66128587e-01
  -2.60345896e-01 -6.78428560e-02  1.08122929e-02 -8.41087769e-01]
 [ 1.17517760e-02  3.43641967e-04 -8.72103737e-02 -4.59351955e-01
  -2.49098952e-01 -8.85591890e-02  7.65455993e-01  3.54169470e-01]
 [ 3.58735699e-01  6.54942912e-01  1.34538659e-02 -4.65084492e-01
   1.70101644e-01  2.93617516e-01 -2.78385484e-01  1.82289869e-01]
 [ 1.43494223e-01  2.61976394e-01 -4.45383120e-01  1.36060221e-01
  -5.00795526e-01 -5.87488150e-01 -2.73099287e-01  1.52897236e-01]
 [ 7.74962678e-01  2.27117371e-02  4.08516159e-01  2.84937362e-01
   3.19642679e-02 -2.35211438e-01  2.62688692e-01 -1.59658351e-01]
 [ 2.81806634e-01  8.24745878e-03 -6.92167142e-01  3.15915567e-01
   1.14149714e-02  5.01908806e-01  2.46628160e-01 -1.69560615e-01]
 [ 1.84643411e-01 -3.16806256e-01  2.48466337e-01 -3.46544961e-02
  -6.98268275e-01  4.67261587e-01 -2.52393736e-01  1.81630339e-01]
 [ 3.69278450e-01 -6.33614920e-01 -2.88917222e-01 -3.93333286e-01
   3.18917542e-01 -1.75016528e-01 -2.61429000e-01  1.52633610e-01]]
```

4) Since Σ 2 is the eigen value of AAT , Σ(singular value) shall be the square roots of the eigen values. Find square roots of the eigen values on AAT and arrange them as a diagonal matrix. The Σ matrix is

```
E: [[6.02148954e+04 0.00000000e+00 0.00000000e+00 0.00000000e+00
  0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
  0.00000000e+00]
 [0.00000000e+00 3.18245207e+04 0.00000000e+00 0.00000000e+00
  0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
  0.00000000e+00]
 [0.00000000e+00 0.00000000e+00 2.60893068e+02 0.00000000e+00
  0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
  0.00000000e+00]
 [0.00000000e+00 0.00000000e+00 0.00000000e+00 1.86219278e+02
  0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
  0.00000000e+00]
 [0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
  1.45606434e+02 0.00000000e+00 0.00000000e+00 0.00000000e+00
  0.00000000e+00]
 [0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
  0.00000000e+00 6.08809411e+01 0.00000000e+00 0.00000000e+00
  0.00000000e+00]
 [0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
  0.00000000e+00 0.00000000e+00 3.89873638e+00 0.00000000e+00
  0.00000000e+00]
 [0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
  0.00000000e+00 0.00000000e+00 0.00000000e+00 8.10241297e-01
  0.00000000e+00]]
```

```
V: [[ 2.84043894e-03  3.14430147e-03 -2.46384735e-01 -1.58554932e-01
  -1.75245114e-01  1.76705635e-01  9.13738625e-01 -1.20261073e-01
   5.31056350e-02]
 [ 2.42121739e-03 -1.28321626e-03 -3.77000733e-01  1.76600215e-01
   6.89508147e-01  5.90273326e-01 -5.29344506e-02 -2.23230964e-03
  -4.91718844e-03]
 [ 2.20891154e-05  1.13495064e-05 -2.37217168e-03 -3.65660431e-03
   5.19584361e-03  7.52000216e-03  6.59901592e-02  7.85970680e-01
   6.14648552e-01]
 [ 1.09109680e-03  1.17416448e-03  6.61240940e-01  3.41172744e-01
   5.01749740e-01 -2.32499365e-01  3.72052359e-01 -4.25903575e-02
   1.77018784e-02]
 [ 1.63479471e-03 -2.90636016e-03  5.74279813e-01 -7.10405325e-02
  -3.14549320e-01  7.49883366e-01 -6.19835401e-02  4.58785873e-03
  -3.93375075e-03]
 [ 1.33908907e-05 -1.14077892e-05  5.80190908e-03 -2.15181510e-03
   2.88147957e-03 -5.73504703e-03 -1.22489909e-01 -6.04930528e-01
   7.86750146e-01]
 [-6.96053715e-01 -7.17961695e-01 -7.57487350e-05 -3.79564118e-03
   2.50334194e-03 -1.50223526e-04  4.37766998e-03 -5.55196291e-04
   2.36025045e-04]
 [-7.17950893e-01  6.96067270e-01  1.62813209e-03 -3.77529395e-03
   2.49754245e-03  3.65599795e-03 -6.00114914e-04  3.48250731e-05
  -4.91718843e-05]
 [-6.16016024e-03  2.29933343e-05 -1.73453679e-01  9.06741660e-01
  -3.78319687e-01  6.21986452e-02  2.52338778e-02 -2.47794676e-03
   7.62164205e-03]]
```

5) Calculate AT A.
6) As per eq. 2, V matrix represent the orthogonal eigen vectors of AT A. Therefor, find eigen vectors and eigen values of AT A.
7) Again. arrange the eigen values in descending order and rearrange the eigen vector matrix as per the corresponding eigen value for each eigen vector. The V matrix is,

1) Computing Homography Matrix: The homogeneous equation is given by

Ax = 0

Where is x is a flattened homograph matrix. Therefore, computing SVD of matrix A can help us calculate the matrix x. Matrix A can be written as,

A = UΣV T

The values for matrix x will be the last value of matrix V. We can also say that the eigen vector of AT A corresponding to the smallest eigen value is the solution for x. Using this relationship, the values for H matrix was computed. Normalizing the matrix, i.e. making the last element as 1(discusses in sec. IV)

```
Homography Matrix: [[-6.48648648e-01  3.81458692e-11  9.72972973e+01]
 [-6.48648648e-01 -4.32432432e-01  9.94594595e+01]
 [-6.48648648e-03 -5.40540541e-03  1.00000000e+00]]
```

a.