Assignment #1

Deliverable # 1 is due by 11:00 PM Thursday, October 2, 2025 on Gradescope. This course permits any usage of large language models (LLM) under the condition that the submission documents the conversations you have had with the LLM and includes the link to the conversations in the submission file. Usage of LLM without documentation is a violation. As long as the documentation is made, the grading will not be based on the level of LLM usage, but only will be based on the submission itself.

Deliverable #1.

Problem 1 In this problem, we will use Monte Carlo simulation to estimate probabilities for a poker variant defined by a slightly non-standard deck and hand-formation rules.

Definition of the Poker Variant

- **Deck Assembly:** The game is dealt from **two standard 52-card decks without Jokers that have been shuffled together into a single stack** of 104 cards. Consequently, there are eight cards of each rank. For the card rank, The Ace is the highest-ranking card, and the 2 is the lowest.
- **Wild Card Rule:** All eight **Jack** (J) cards are designated as **wild**. A wild card can substitute for any rank and suit to form the highest-ranking possible hand.
- **Hand Definition:** A player is dealt a **7-card hand**. From this set of 7 cards, the player must construct **the highest-ranking** (please refer the poker hand ranking section for details) possible 5-card poker hand (a "7-choose-5" format).
- **Poker Hand Ranking**: The hands are ranked from highest to lowest in the following list. The evaluation algorithm must prioritize creating the highest possible hand.
  - a.) **Five of a Kind** (e.g., A-A-A-A-A)
  - b.) **Straight Flush** (e.g., 9-8-7-6-5, all of the same suit)
  - c.) **Four of a Kind** (e.g., K-K-K-K-5)
  - d.) **Full House** (e.g., Q-Q-Q-2-2)
  - e.) **Flush** (Five cards of the same suit, not in sequence)
  - f.) **Straight** (Five cards in sequence, not of the same suit)

g.) **Three of a Kind** (e.g., 7-7-7-J-2)

h.) **Two Pair** (e.g., A-A-8-8-4)

i.) **One Pair** (e.g., 9-9-K-Q-3)

j.) **High Card** (e.g., A-K-5-4-2 of mixed suits)

Example Here is an example for the game. Consider a hand that could form multiple valuable combinations. The highest-ranking one must be chosen.

- **7-Card Hand:** $\{A\heartsuit, A\diamondsuit, J\spadesuit, J\clubsuit, 10\diamondsuit, 7\spadesuit, 4\heartsuit\}$
- **Analysis:** This hand contains two natural Aces and two wild Jacks. One could form a Full House (e.g., A-A-A-10-10 by using one Jack as an Ace and one as a Ten). However, a superior hand is possible: using both Jacks as Aces creates **Four of a Kind** (A-A-A-A-10).

Question Please write code that uses a Monte Carlo simulation with at least $100,000$ trials to solve the following two questions.

a.) Please compute the probability, $P(H)$, where H is the event that the best possible 5-card hand constructible from a randomly dealt 7-card hand is either a **Five of a Kind** or **Four of a Kind**.

b.) Please compute the probability, $P(H)$, where H is the event that the best possible 5-card hand constructible from a randomly dealt 7-card hand is a **High Card**.

Problem 2 A manufacturer of batteries packages them in boxes of 100. It is known that, on the average, the batteries weigh 1 ounce, with a standard deviation of 0.05 ounce. The manufacturer is interested in the total weight of the batteries. Please write code to solve the following problems. You need to decide how many numbers of trials should be done in Monte Carlo Simulation to provide a reasonable result. Denote $(x)_+ = \max\{x, 0\}$ for $x \in \mathbb{R}$.

a.) Assume that the weight distribution of the batteries is i.i.d. Gaussian, use Monte-Carlo Simulation to estimate $\mathbb{P}$(100 batteries weigh more than 101 ounces).

b.) Assume that the weight distribution of the batteries is i.i.d. Gaussian. Let the total weight of a box containing 100 batteries be X. If $X > 100$, the manufacturer need to pay $(X - 100)$ dollars of extra fee. Use Monte Carlo Simulation to estimate the expectation of extra fee conditional on the $X > 100$.

c.) (This Question is Optional.) Assume that the weight distribution of the batteries is i.i.d. Gaussian. Find the analytical value of $\mathbb{E}[(X - 100)_+]$ and compare it to your result in problem c.).

Problem 3 Suppose that you run a food truck on each Wednesday between 11AM and 1PM. There is no customer waiting when the food truck opens at 11AM. Customers arrive at the food truck according to a Poisson process with rate λ persons per minute. There is a single line of queue with a first-come-first-serve rule. Customers that have arrived before 1PM will eventually get served. In other words, customers are not able to join the queue if they arrive later than 1PM. The food truck resumes its operation until all customers who have arrived before 1PM get served. You are the owner of the food truck and there is only one server (yourself). The service time for each customer is independent and identically distributed with some distribution F. The service time includes the time on taking orders and food preparation. The distribution F has a mean of $1/\mu$ and the probability distribution is to be specified. Suppose $\lambda = 2$. Simulate 100 independent days (Wednesdays) for each of the following parts.

a.) Denote E_i as the number of customers that arrive at the food truck between 11:55 AM and 12:05 PM on the i-th day. Do the following:

  i.) Compute $\frac{1}{100}\sum_{i=1}^{100} E_i$ and compute the sample variance for $\mathbb{E}_i$'s.

  ii.) Plot the histogram for $\{E_1, E_2, \ldots, E_{100}\}$.

  iii.) Group all the inter-arrival times over the 100 days, and compute the percentage of inter-arrival times that are longer than 1 minute. How does this value compare to e^{-2}?