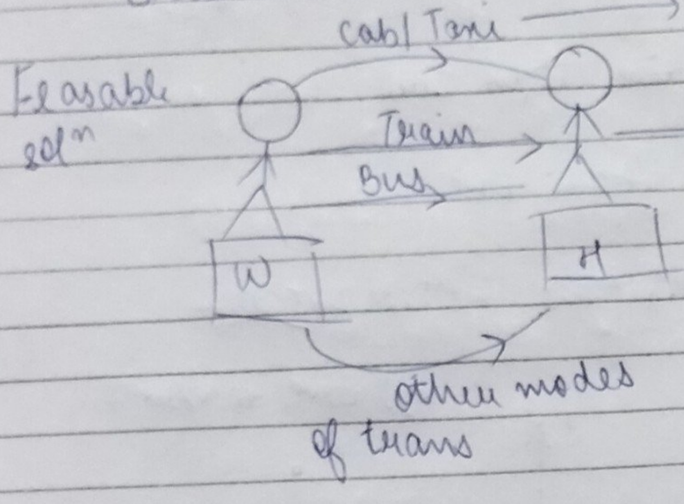
NUMERICAL AND OPTIMIZATION TECHNIQUES-

icon
PAGE NO.
DATE : Kanti Swarup

Q What is optimization?

The process of obtaining best results under given circumstances



cab/Taxi → pay more money but less time

Train → pay less money but consume more time

Best will be optimal sol^n

① Cost } minimise
② Time }

CLASSIFICATION

↓ MAXIMIZATION

↓ MINIMIZATION

Physical problem to Mathematical modelling

Resources ↑ Input

→ Obj functions

→ Constrains

↓

Mathematical formulation $\xrightarrow{\text{Methods}}$ Output

→ decision

Optimal sol^n.

Engineering Applications of optimization

Objective function – Expresses main aim of model which are either to be minimized or maximised

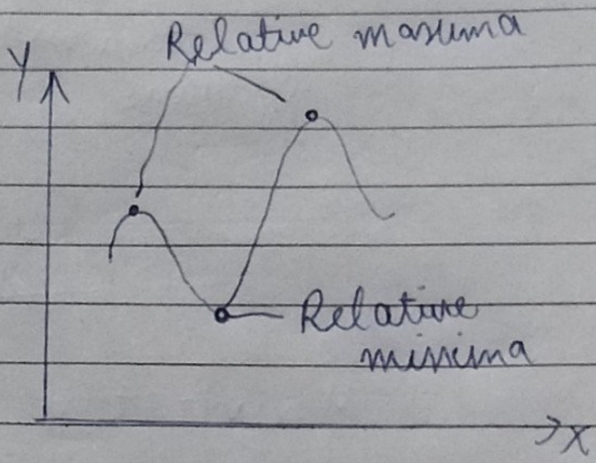
Constrains –

Decision – Always greater than zero.

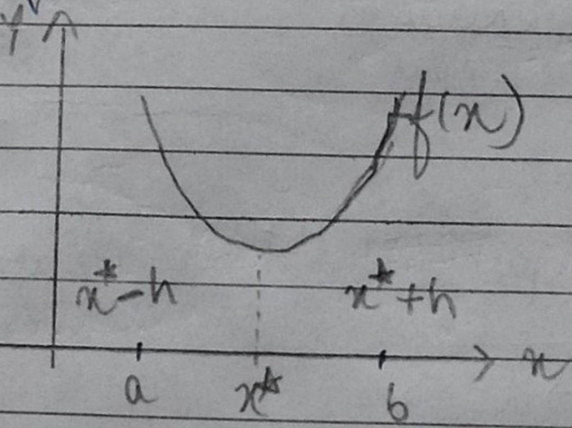
CLASSIFICATION – Study from ppt

SINGLE VARIABLE OPTIMIZATION –

$y = f(x)$



Relative Minima – Let $f(x)$ be function defined on $[a, b]$ then $x = x^*$ is said be relative minimum if $f(x^*) \leq f(x^* + h)$ for sufficiently small +ve and −ve values of h.

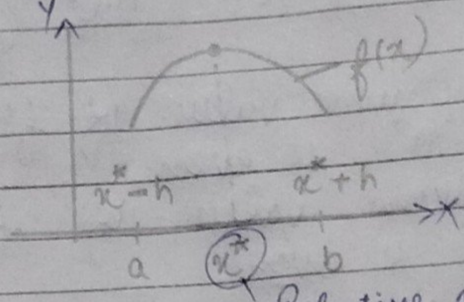


$$f(x^*) \leq f(x^* + h)$$

Relative Maxima – Let $f(x)$ be a function defined on $[a, b]$ then $x = x^*$ is said to be relative maximum if $f(x^*) \geq f(x^* + h)$

for sufficiently small +ve and -ve values of h

$f(x^*) \geq f(x^*+h)$



Relative maxima

(3) Global Minima - $x = x^*$ is said to be global minima if $f(x) \geq f(x^*)$ ~~for~~ for all x in the domain

Relative me apan neighbourhood me dekhte hai aur global me pure ke liye (ie full domain)

(4) Global Maxima - at $x = x^*$ is said to be Global Maxima if $f(x) \leq f(x^*)$ for all x in the domain

Necessary Condition for single variable optimization
If a function $f(x)$ is defined in the interval $a \leq x \leq b$ and has relative minimum at $x = x^*$ where $a < x^* < b$ and if the $f(x)$ exists as a finite number at $x = x^*$ then $f'(x^*) = 0$

either +ve or −ve

derivative

Sufficient Condition - Let $f'(x^*) = f''(x^*) = f'''(x^*) = \dots = f^{(n-1)}(x^*) = 0$ but $f^{n}(x^*) \neq 0$, Then $f(x^*)$ has

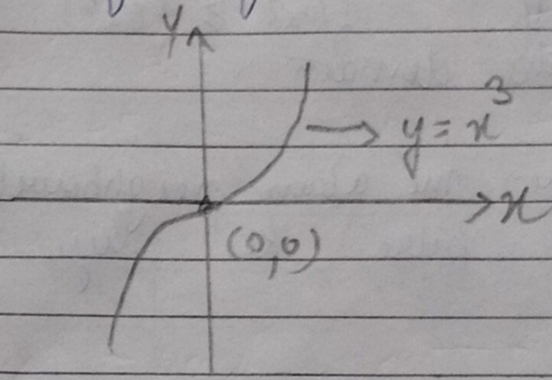
i) a minimum value of $f(x)$ if $f^{(n)}(x^*) > 0$ and 'n' is even.

ii) a maximum value of $f(x)$ if $f^{(n)}(x^*) < 0$ and 'n' is even.

iii) neither maxima nor minima if 'n' is odd

Stationary Points (Point of inflexion)

eg) Let $f(x) = x^3$



Step 1 $f'(x) = 3x^2$

Put = 0 $f'(x) = 0$

$3x^2 = 0$

$\boxed{x = 0}$ CRITICAL POINT

Sufficient condition

$f''(x) = 6x$

$x = 0$

$f^{(iii)}(x) = 6 \neq 0$ — $n = 3 = $ odd So 3rd condition

$6 > 0$

At $f(x) = 0$ it has ~~neigth~~ neither maxima nor minima

$x = 0$ is the stationary pt or pt of inflextio

PAGE NO. icon
DATE : / /

Q) Find maximum and minimum values of function
$f(x) = 12x^5 - 45x^4 + 40x^3 + 5$

Ans)

$f'(x) = 60x^4 - 180x^3 + 120x^2 = 0$

$x^2(60x^2 - 180x + 120) = 0$
$60x^2(x^2 - 3x + 2) = 0$
$x^2 \cancel{x} x - 2x + 2$
$x(x-1) - 2(x-1)$
$x = 1, 2$

$\boxed{x = 0, 0, 1, 2}$

$f''(x) = 240x^3 - 540x^2 + 240x$
$\cancel{240x(x^2}$

At $x=0$ $\quad f''(x) = 0$

~~At x=1~~

$f'''(x) = 720x^2 - 1080x + 240$.
At $x=0$ $\quad f'''(x) = 240$
Since $f'''(x) \neq 0$ so we will apply sufficient condition (function)
n is odd so it has neither maxima nor minima at $x=0$. It is the pt of inflection

At $x=1$ $\quad \cancel{f'''(x) = 60}$

$f''(x) = 240 - 540 + 240$
$480 - 540$
$-60 < 0$

Condition 2 will apply
~~Maximum~~

agar ye 0 atta tab agle wale ko check karte

$45 \times 4 = 180$
$18 \times 3 = 54$
$24 \times 3 = 72$
$54 \times 2 = 108$

PAGE NO.
DATE : / /

The point $x=1$ is point of relative maxima and maximum value $= f(1)$ → Put this value in real function

$12(1) - 45(1) + 40 + 5 = 12$

At $x=2$ $f''(x) = 240(8) - 540(4) + 240(2)$

$1920 - 2160 + 480$

Relative minima $240 > 0$

Put 2 in real function

$12(2)^5 - 45(2)^4 + 40(2)^3 + 5$

$12(32) - 45(16) + 40(8) + 5$

$384 - 720 + 320 + 5$

-11 → Minimum value

$1920 + 480 = 2400$

$2400 - 2160 = 240$

8 4

Q1) $f(x) = 5x^6 - 36x^5 + \frac{165}{2}x^4 - 60x^3 + 36$ — (1)

$f'(x) = 30x^5 - 180x^4 + 330x^3 - 180x^2$ — (2)

$36 \times 5 = 180$

Put $f'(x) = 0$

$165 \times 2 = 330$

$30x^2(x^3 - 6x^2 + 110x - 6) = 0$

$x = 0, 0$

$x^3 - 6x^2 + 110x - 6 = 0$

Step1 By hit and trail $x=1$ is other root

Step2 Write cofficients of eq^n.

	x^3	x^2	x	constant
1	1	−6	11	−6
		1	−5	6
	1	−5	6	0

Mostly last term will come 0

quadratic eq^n ban gayi

BY LONG DIVISION ALSO WE CAN DO IT

$$x^2 - 5x + 6 = 0$$
$$x^2 - 3x - 2x + 6 = 0$$
$$x(x-3) - 2(x-3) = 0$$
$$2, 3$$

So all roots are 0, 0, 1, 2, 3

Home work :-

① $f(x) = 5x^6 - 36x^5 + \frac{165}{2}x^4 - 60x^3 + 36$

② $f(x) = 2x^3 - 6x^2 + 6x + 5$

③ $f(x) = -x^3 + 3x^2 + 9x + 10$ in $[-2, 4]$

Done → ④ $f(x) = 12x^5 - 45x^4 + 40x^3 + 6$

⑤ $f(x) = x^4 - 62x^2 + 120x + 9$

Everyone

Recording messages in this chat.

MANPREET KAUR 10:59 AM
yes

BIRJU BHARTI 10:59 AM
yes mam

SURABHI TIWARI 10:59 AM
yes maam

GURSIMAR SINGH ANAND 10:59 AM
yes mam

ANSHDWIP KUMAR 10:59 AM
yes mam

HW Ques Sol^{n}s —

PAGE NO.
DATE : / /

② $f(x) = 2x^3 - 6x^2 + 6x + 5$

$f'(x) = 6x^2 - 12x + 6$

$f'(x) = 0$

$x^2 - 2x + 1 = 0$

$x^2 - x - x + 1 = 0$

$x(x-1) - 1(x-1) = 0$

$x = 1, 1$ (CRITICAL PTS)

$f''(x) = 12x - 12$

At $x = 1$

$f''(1) = 12 - 12$
$= 0$

$f'''(x) = 12 > 0$

$n = $ odd
So neither minima nor maxima.

③ $f(x) = -x^3 + 3x^2 + 9x + 10$ in $[-2, 4]$

$f'(x) = -3x^2 + 6x + 9$

$f'(x) = 0$

$-3(x^2 - 2x - 3) = 0$

$x^2 + x - 3x - 3 = 0$

$x(x+1) - 3(x+1) = 0$

$x = 3, -1$ (CRITICAL PTS)

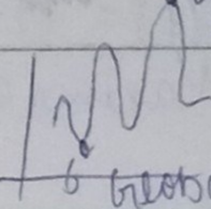
$f''(x) = -6x + 6$

At $x = 3$ global maxima

~~$f'''(x) = -6$~~

$f''(3) = -18 + 6 = -12 < 0$

$n = 2$ is even
Second condition will apply.
local maxima at $x = 3$
So maximum value
$= -(3)^3 + 3(3)^2 + 9(3) + 10$
$= -27 + 27 + 27 + 10$
$= 37$



$f''(x)$ at $x = -1$

$-6(-1) + 6$

$6 + 6 = 12 > 0$

at $x = -1$ minima

So minimum value will be

$-(-1)^3 + 3 - 9 + 10$

$1 + 3 + 1$

$= 5$

global minima

PAGE NO.
DATE: / /

(5) $f(x) = x^4 - 62x^2 + 120x + 9$

$f'(x) = 4x^3 - 124x + 120$

Put $f'(x) = 0$

$x^3 - 31x + 30 = 0$

Hit and trial $x = 1$?

~~8 - 64 + 30~~

So $x = 1$ is one root.

Method 1

	x^3	x^2	x	Cons
1	1	-31	0	30
		1	-30	-30
	1	-30	-30	0

quadratic eq^n

$x^2 - 30x - 30 = 0$

Method 2

$$\begin{array}{r} x^2 + x - 30 \\ x-1 \,\big)\, x^3 - 31x + 30 \\ x^3 - x^2 \\ - \quad + \\ \hline x^2 - 31x + 30 \\ x^2 - x \\ - \quad + \\ \hline -30x + 30 \\ -30x + 30 \\ + \quad - \\ \hline 0 \end{array}$$

$(x-1)(x^2 + x - 30)$

$(x-1)(x^2 + 5x - 6x - 30)$

$(x-1)\big(x(x+5) - 6(x+5)\big) = 0$

$x = 1, -5, 6$ (CRITICAL PTS)

$f''(x) = 12x^2 - 124$

At $x = 1$

$f''(1) = 12 - 124 = -112 < 0$

Maxima at 1

$f(1) = 1 - 62 + 120 + 9$

$130 - 62 = 68$

Max value 68

At $x = -5$

$f''(-5) = 12(25) - 124$

$= 300 - 124$

$= 176 > 0$

Minima

$f(5) = (5)^4 - 62(25) + 120(5) + 9$

$625 - 1550 + 600 + 9$

$-316 < 0$

2) negative defenite when x^* is relative maximum

3) Otherwise x^* is saddle pt.

Q) Find extreme pt and its nature of the function
$$f(x_1, x_2) = x_1^3 + x_2^3 + 2x_1^2 + 4x_2^2 + 6$$

Ans) Necessary condition

$$\frac{\partial f}{\partial x_1} = 0 \qquad 3x_1^2 + 4x_1 = 0$$

$$x_1(3x_1 + 4) = 0$$
$$x_1 = 0 \text{ or } x_1 = \frac{-4}{3}$$

$$\frac{\partial f}{\partial x_2} = 0 \qquad 3x_2^2 + 8x_2 = 0$$

$$x_2(3x_2 + 8) = 0$$
$$x_2 = 0 \text{ or } x_2 = \frac{-8}{3}$$

Extreme pts are $(0,0)$, $\left(0, \frac{-8}{3}\right)$, $\left(\frac{-4}{3}, 0\right)$, $\left(\frac{-4}{3}, \frac{-8}{3}\right)$

$$\cancel{H = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} \\ \partial \end{bmatrix}} \qquad H = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} \end{bmatrix}$$

PAGE NO.
DATE : 4.1

$$H = \begin{bmatrix} 6x_1+4 & 0 \\ 0 & 6x_2+8 \end{bmatrix}$$

~~At (0,0)~~

$$H = \begin{bmatrix} 4 & 0 \\ 0 & 8 \end{bmatrix}$$

$H_1 = 4 > 0$ — First (a_{11}) element

$H_2 = 32 > 0$ — (D) Determinate

(0,0) satisfies the condition of positive definite
(0,0) is relative minima

At $\left(0, -\frac{8}{3}\right)$

$$H = \begin{bmatrix} 4 & 0 \\ 0 & -8 \end{bmatrix}$$

$H_1 = 4 > 0$

$H_2 = -32 < 0$

Indefinite cannot have relative max nor relative minima

At $\left(-\frac{4}{3}, 0\right)$

$$H = \begin{bmatrix} -4 & 0 \\ 0 & 8 \end{bmatrix}$$

Indefinite

$H_1 = -4 < 0$

$H_2 = -32 < 0$

At $\left(-\frac{4}{3}, -\frac{8}{3}\right)$

$$H = \begin{bmatrix} -4 & 0 \\ 0 & -8 \end{bmatrix}$$

$H_1 = -4 < 0$

$H_2 = 32 > 0$

Negative definate

~~Relative maxima~~

Ques) Find the minimum value of $x^2+y^2+z^2$ subject to $x+y+2z=12$

Ans) Min $f(x) = x^2+y^2+z^2$ — (1)

subject to $x+y+2z = 12$ — (2)

We have to use direct substitution method
From (2)

$$2z = 12-x-y$$
$$z = \frac{1}{2}(12-x-y)$$

Put z in eqn (1)

$$f(x) = x^2+y^2+\frac{1}{4}(12-x-y)^2$$

Necessary condition

$$\frac{\partial f}{\partial x} = 2x + (-1)\frac{1}{2}(12-x-y)$$
$$2x - \frac{(12-x-y)}{2} = 0$$
$$2(2x) = 12-x-y$$
$$4x = 12-x-y \quad \text{— (3)}$$
$$\cancel{5x = 12-y}$$

$$\frac{\partial f}{\partial y} = \cancel{\frac{12-x}{5}} \quad \boxed{y = \frac{12-x-y}{4}} \text{ — (4)}$$ Similarly as above

From (3) and (4)

$$\boxed{x=y}$$

Put value of x in eq^n 3

$$4x = 12 - x - y$$
$$4x = 12 - x - x$$
$$4x = 12 - 2x$$
$$6x = 12$$
$$\boxed{x = 2}$$

And $x = y$ So $\boxed{y = 2}$

Value of z

$$z = \frac{1}{2}(12 - 4)$$
$$z = \frac{8}{2} = 4$$

So extreme pt is 2, 2, 4

Apply Hessian Matrix

$$H = \begin{bmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} \end{bmatrix}$$

$$\frac{\partial^2 f}{\partial x^2} = 2 - \frac{1}{2}(-1) = \frac{5}{2}$$

$$H = \begin{bmatrix} 5/2 & 1/2 \\ 1/2 & 5/2 \end{bmatrix}$$

$$H_1 = \frac{5}{2} > 0$$

$$H_2 = \frac{25}{4} - \frac{1}{4} = \frac{24}{4} = 6 > 0$$

Point is relative minima and Hessian Matrix is a +ve definate

PAGE NO.
DATE : / /

Linear Programing

Formulation of LPP

1) Decision variables
2) Find objective function (whether maximise or minimise)
3) Constrains. $\geq$ $\leq$

$x, y \geq 0$ (Non -ve condition)

Ques) A firm can ~~reduce~~ produce 3 types of clothes say A, B and C. Three kinds of wool required. Red, Green and Blue. One unit length of A needs 2m of red, 3m of blue. One unit of B needs 3m of red, 2m of green and 2m of blue. One unit of C need 5m of green, 4m of blue. ~~Th~~ Firm has only stock of 8m of red, 10m of green and 15m of blue wool. Income from A is Rs 3 and from B is Rs 5 and from C is Rs 4. How firm should use material so as to maximise income?

Ans)

		Red	Blue	Green
3	A	2	3	
5	B	3	2	2
4	C		4	5
		8	10	15

$2x + 3y \leq 8$ —— (1)

$3x + 2y + 4z \leq 10$ —— (2)

$2y + 5z \leq 15$ —— (3)

Objective
Maximise
$3x + 5y + 4z$
(Objective function)

$x, y \geq 0$ (Non -ve constrains)

Ques) A diet for a sick person must contain atleast 4000 units of vitamins, 50 units of minerals and 1400 unit of cal. Two foods A and B are available at cost of Rs 4 and Rs 3 per unit resp. one unit of food A contains 200 units of vitamins, 1 unit of minerals and 40 units of calories and one unit of food B contain 100 units of vitamins 2 unit minerals and 40 unit cal. Formulate as LP for minimising cost.

Ans)

		Vita	Min	Cal
4	A	200	1	40
3	B	100	2	40
		4000	50	1400

Objective

$4x + 3y$

$200x + 100y \geq 4000$

$x + 2y \geq 50$

$40x + 40y \geq 1400$

$x, y \geq 0$

Ques) A dealer deals in only 2 items sewing machine and table fans. She has Rs 5760 to invest in store max of 20 items sewing machine cost Rs 360 and table fan Rs 240. He can sell sewing machine at profit of Rs 22 and table fan " " 60. Make LPP

Ans)

	Cost	profit
Sewing machine	360	22
	240	18
	5760	

capital X for x_1, x_2, x_3

GPB lect

Multivariable without constraints

3 variable

$$f(X) = 3x_1^2 + 6x_1x_3 + x_2^2 - 4x_2x_3 + 8x_3^2$$

$$\frac{\partial f}{\partial x_1} = 6x_1 + 6x_3 = 0 \qquad \frac{\partial f}{\partial x_2} = 2x_2 - 4x_3 = 0$$

$$\frac{\partial f}{\partial x_3} = 6x_1 - 4x_2 + 16x_3 = 0 \quad \text{—— (3)}$$

$$x_1 = -x_3 \qquad x_2 = 2x_3$$

Put in eqn (3)

$$-6x_3 - 8x_3 + 16x_3 = 0$$

$$2x_3 = 0 \qquad \boxed{x_3 = 0} \quad \text{so} \quad \boxed{x_1 = 0} \quad \boxed{x_2 = 0}$$

$\therefore$ (0, 0, 0) is the extreme point.

HASSIAN MATRIX

Second ~~other~~ order partial derivatives

$$\frac{\partial^2 f}{\partial x_1^2} = 6 \; ; \; \frac{\partial^2 f}{\partial x_2 \partial x_1} = 0 \; ; \; \frac{\partial^2 f}{\partial x_3 \partial x_1} = 6$$

$$\frac{\partial^2 f}{\partial x_1 \partial x_2} = 0 \; ; \; \frac{\partial^2 f}{\partial x_2^2} = 2 \; ; \; \frac{\partial^2 f}{\partial x_3 \partial x_2} = -4$$

$$\frac{\partial^2 f}{\partial x_1 \partial x_3} = 6 \; ; \; \frac{\partial^2 f}{\partial x_2 \partial x_3} = -4 \; ; \; \frac{\partial^2 f}{\partial x_3^2} = 16$$

$$H = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \frac{\partial^2 f}{\partial x_1 \partial x_3} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & \frac{\partial^2 f}{\partial x_2 \partial x_3} \\ \frac{\partial^2 f}{\partial x_3 \partial x_1} & \frac{\partial^2 f}{\partial x_3 x_2} & \frac{\partial^2 f}{\partial x_3^2} \end{bmatrix} = \begin{bmatrix} 6 & 0 & 6 \\ 0 & 2 & -4 \\ 6 & -4 & 16 \end{bmatrix}$$

$6(32-16)+6(-12)$

PAGE NO.
DATE : / /

$$\begin{array}{r} 16 \\ \underline{6} \\ 96 \end{array}$$

$96-72 = 24$

Principal minors are

$H1 =$ Det of first element $= |6| = 6 > 0$

$H2 =$ Det of first 4 elements $= \begin{vmatrix} 6 & 0 \\ 6 & 2 \end{vmatrix} = 12 > 0$

$H_3 =$ Det of whole matrix $24 > 0$

Since all principle minors are +ve this implies matrix is +ve definate so pt $(0,0,0)$ is pt of relative minima

Since at H we have constant so no need to put value of pt

Min value $= f(0,0,0) = 0$ Ans-

Q2) $f(x,y) = x^2 - y^2$ Ext pt $(0,0)$ Saddle pt

Q3) $f(x,y) = x^4 + y^4 + 2x^3 + 2y^3 + 48$ Ext pt $(0,0)$, $\left(-\frac{3}{2}, -\frac{3}{2}\right)$, $\left(0, -\frac{3}{2}\right)$, $\left(-\frac{3}{2}, 0\right)$

Minima

Saddle

Q4) $f(x) = x_1^2 - x_2^2 - x_1x_2$ $(0,0)$ Extreme pt saddle pt

Q5) $f(x) = 2x_1 - \cancel{x_2} - x_1^2 + x_1x_2 - x_2^2$

HW Ques Sol^{n}s

PAGE NO.
DATE : / /

Ans2) $f(x,y) = x^2 - y^2$

$\frac{\partial f}{\partial x} = 2x = 0$ $\frac{\partial f}{\partial y} = -2y = 0$

$x = 0$ $y = 0$

$\therefore$ Pt $(0,0)$.

Double derivative $\frac{\partial^2 f}{\partial x^2} = 2$, $\frac{\partial^2 f}{\partial y^2} = -2$

$\frac{\partial^2 f}{\partial x \partial y} = 0$ $\frac{\partial^2 f}{\partial y \partial x} = 0$

$H = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$

$H_1 = 2 > 0$ $H_2 = -4 < 0$

Ans3) $f(x,y) = x^4 + y^4 + 2x^3 + 2y^3 + 48$

$\frac{\partial f}{\partial x} = 4x^3 + 6x^2$ $\frac{\partial^2 f}{\partial x^2} = 12x^2 + 12x$ $\frac{\partial f}{\partial y \partial x} = 0$

$\frac{\partial f}{\partial y} = 4y^3 + 6y^2$ $\frac{\partial^2 f}{\partial y^2} = 12y^2 + 12y$ $\frac{\partial f}{\partial x \partial y} = 0$

Put first derivative as 0

$4x^3 + 6x^2 = 0$ $4y^3 + 6y^2 = 0$

$2x^2(2x+3) = 0$ $2y^2(2y+3) = 0$

$\boxed{x = 0}$ $\boxed{x = -\frac{3}{2}}$ $y = 0, y = 0, y = -\frac{3}{2}$

$x = 0$

So Pts = $(0,0)$ $\left(0, -\frac{3}{2}\right)$ $\left(-\frac{3}{2}, 0\right)$ $\left(-\frac{3}{2}, -\frac{3}{2}\right)$

PAGE NO.
DATE : / /

$$\text{Matrix} = \begin{bmatrix} 12x^2+12x & 0 \\ 0 & 12y^2+12y \end{bmatrix}$$

At pt $(0,0)$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$H1 = 0$
$H2 = 0$

At pt $\left(0, -\frac{3}{2}\right)$

$$\begin{bmatrix} 0 & 0 \\ 0 & 12\left(\frac{3}{2}\right)^2 - 18 \end{bmatrix}$$

$H1 = 0$
$H2 = 0$

At pt $\left(-\frac{3}{2}, 0\right)$

$$\begin{bmatrix} 12\left(\frac{3}{2}\right)^2 - 18 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 3(9) - 18 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 27-18 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 9 & 0 \\ 0 & 0 \end{bmatrix}$$

$H1 = 9$ $H_2 = 0$

At pt $\left(-\frac{3}{2}, -\frac{3}{2}\right)$

$$\begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$$

$H_1 = 9 > 0$
$H_2 = 81 > 0$

So $\left(-\frac{3}{2}, -\frac{3}{2}\right)$ is minima.

Q 4) $f(x) = x_1^2 - x_2^2 - x_1 x_2$

$$\frac{\partial f}{\partial x_1} = 2x_1 - x_2 = 0$$

$$\frac{\partial f}{\partial x_2} = -2x_2 - x_1 = 0$$

$2x_1 = x_2$

$2x_2 + x_1 = 0$
$x_1 = -2x_2$

Only possible when $(0,0)$

PAGE NO.
DATE : / /

Conversion of LPP to standard LPP

(1) Write the objective function in the minimization form

$$\text{Max } Z' = \text{Min}(-Z)$$

eg) $\text{Min } Z = -3x_1 + x_2$

$\text{Max } Z = 3x_1 - x_2$

2) Convert all inequalities as eqⁿs (by introducing slack and surplus variable)

$+ \le \qquad - \ge$

eg)

$x_1 + 2x_2 \le 12$

$x_1 + 2x_2 + s_1 = 12$

$x_1 + 2x_2 \ge 12$

$x_1 + 2x_2 - s_1 = 12$

3) The right hand element of each constrain should be made non -ve

eg) $2x_1 + x_2 - s_2 = -15$

$-2x_1 - x_2 + s_2 = 15$

4) All variables must have non -ve values

eg) $x_1 + x_2 \le 3$, $x_1 \ge 0$, x_2 is unrestricted in sign

$x_2 = x_2' - x_2''$

$x_1 + (x_2' - x_2'') \le 3$, $x_1, x_2', x_2'' \ge 0$

$x_1 + (x_2' - x_2'') + s_1 = 3$

PAGE NO.
DATE : / /

① Max $z = 3x_1 + x_2$ sub to $2x_1 + x_2 \leq 2$
$3x_1 + 4x_2 \geq 12$, $x_1, x_2 \geq 0$

Sol

Max $z = 3x_1 + x_2$

$2x_1 + x_2 + s_1 = 2$
$3x_1 + 4x_2 - s_2 = 12$
$x_1, x_2, s_1, s_2 \geq 0$

② Min $z = 4x_1 + 2x_2$ sub to $3x_1 + x_2 \geq 2$, $x_1 + x_2 \geq 21$,
$x_1 + 2x_2 \geq 30$, $x_1, x_2 \geq 0$

Ans) Max $z' = -4x_1 - 2x_2$ sub to $3x_1 + x_2 - s_1 = 2$
$x_1 + x_2 - s_2 = 21$ $x_1 + 2x_2 - s_2 = 30$
and $x_1, x_2, s_1, s_2, s_3 \geq 0$

③ Min $z = x_1 + 2x_2 + 3x_3$ sub to $2x_1 + 3x_2 + 3x_3 \geq 4$
$3x_1 + 5x_2 + 2x_3 \leq 7$ and $x_1, x_2 \geq 0$, x_3 is
unrestricted

Ans)

Max $z' = -x_1 - 2x_2 - 3x_3$ sub to $2x_1 + 3x_2 + 3x_3 - s_1 = 4$ — ①

$3x_1 + 5x_2 + 2x_3 + s_2 = 7$ — ② $x_1, x_2 \geq 0$

$(x_3 = x_3' - x_3'')$
Put in eq^n ①

$2x_1 + 3x_2 + 3(x_3' - x_3'') - s_1 = 4$
Put in eq^n ②
$3x_1 + 5x_2 + 2(x_3' - x_3'') + s_2 = 7$

x_1, x_2

3 Sept 2021

GRAPHICAL METHOD-

Q) Maximise $z = 3x_1 + 4x_2$ sub to constrains $x_1 + x_2 \leq 450$, $2x_1 + x_2 \leq 600$ and $x_1, x_2 \geq 0$

Ans) Convert inequality to equality

$x_1 + x_2 = 450$

Put $x_1 = 0$, $x_2 = 450$

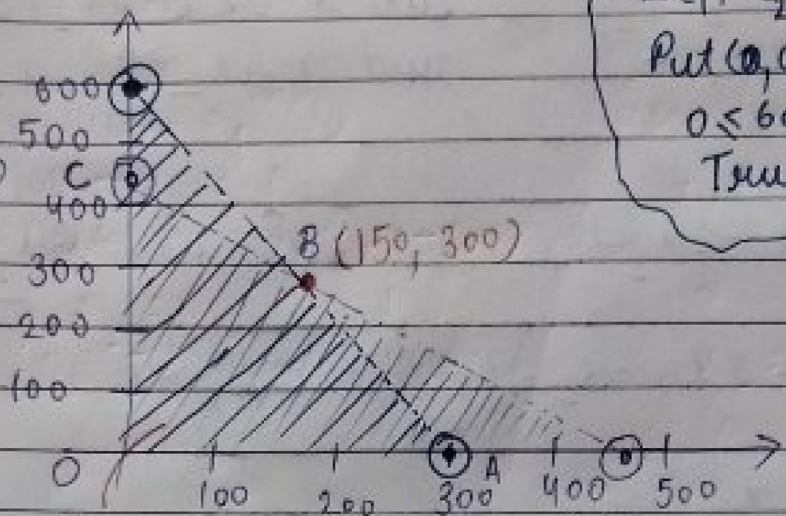
$x_2 = 0$, $x_1 = 450$

$(0, 450)$, $(450, 0)$

$2x_1 + x_2 = 600$

Put $x_1 = 0$, $x_2 = 600$

$(0, 600)$ and $(300, 0)$



Feasible region

$2x_1 + x_2 \leq 600$

Put $(0,0)$

$0 \leq 600$

True

$x_1, x_2 \leq 450$

Put origin in this to check shaded portion

$0 \leq 450$

True so towards

Solving eq^{n}s for intersection,

$x_1 = 450 - x_2$

Put in

$2(450 - x_2) + x_2 = 600$

$900 - x_2 = 600$

$\boxed{300 = x_2}$

$\boxed{x_1 = 450 - 300 = 150}$

Intersection pt = $(150, 300)$

Last date 15

icon PAGE NO. DATE : / /

Put in obj function

Points are

$O(0,0) = 0$

$A(300, 0) = 900$

$B(150, 300) = 1650$

$C(0, 450) = 1800$

Feasible sol^n

best is optimal sol^n

Max. will be 1800 so it is optimal sol^n.

MULTIPLE OPTIMAL SOL

Q) Max $z = 4x_1 + 3x_2$ sub. to $4x_1 + 3x_2 \leq 24$ — (1)

$x_1 \leq 4.5$ — (2)

$x_2 \leq 6$ — (3) and $x_1, x_2 \geq 0$

Sol^n) From (1) $4x_1 + 3x_2 = 24$

Put $x_1 = 0$ $x_2 = 8$

$x_2 = 0$ $x_1 = 6$

$(0,8), (6,0)$

From (3) $x_2 = 6$, $x_1 = 0$

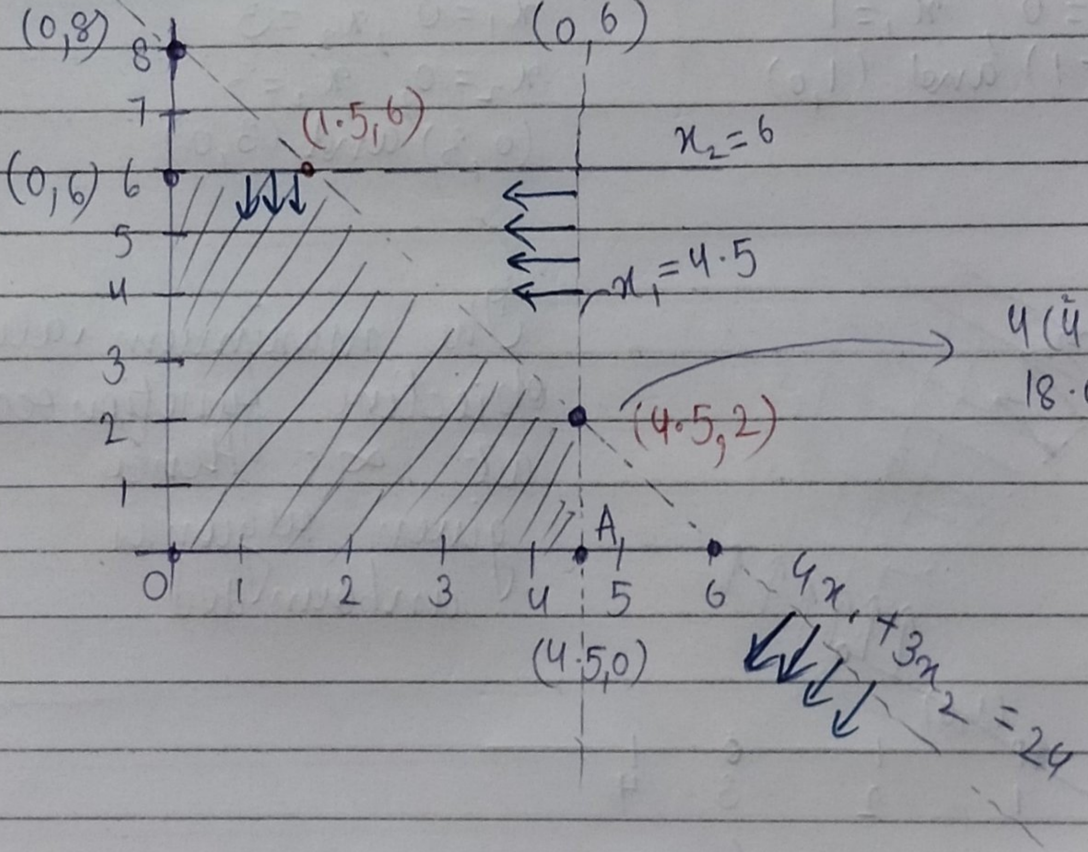
$(0,6)$

From eq^n (2)

~~$x_2 = 6$~~ $x_1 = 4.5$

$x_2 = 0$

$\therefore (4.5, 0)$



$4(4.5) + 3x_2 = 24$

$18.0 + 3x_2 = 24$

$3x_2 = 6$

$\boxed{x_2 = 2}$

Pt $(4.5, 2)$

PAGE NO.
DATE : / /

Pts are

$O(0,0) = 0$

$A(4.5,0) = 18$

$B(4.5,2) = \boxed{24}$ Multiple optimal sol^n.

$C(1.5,6) = \boxed{24}$

$D(0,6) = 18$

Un-bounded solution

Q) Max $z = 3x_1 + 2x_2$

Sub to

$x_1 - x_2 \leq 1$ —— ①

$x_1 + x_2 \geq 3$ —— ②

$x_1, x_2 \geq 0$

Sol^n)

From eq^n ①

$x_1 - x_2 = 1$

Put $x_1 = 0, x_2 = -1$

$x_2 = 0, x_1 = 1$

$(0,-1)$ and $(1,0)$

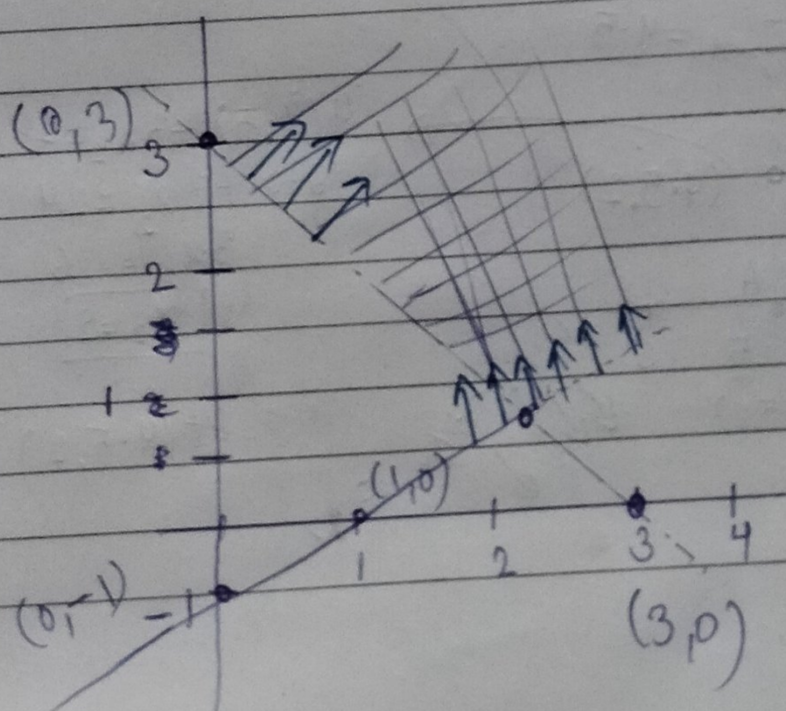
From eq^n ②

$x_1 + x_2 = 3$

$x_1 = 0, x_2 = 3$

$x_2 = 0, x_1 = 3$

$(0,3)$ and $(3,0)$



The maximum value of objective function occurs at ∞. Hence given region is unbounded

NO OPTIMAL SOL^n

Q) Max $z = 3x_1 + 2x_2$ Sub to $x_1 + x_2 \le 1$ — 1

$x_1 + x_2 \ge 3$ — 2

$x_1, x_2 \ge 0$

Ans)

$x_1 + x_2 = 1$

Put $x_1 = 0$ $x_2 = 1$

$x_2 = 0$ $x_1 = 1$

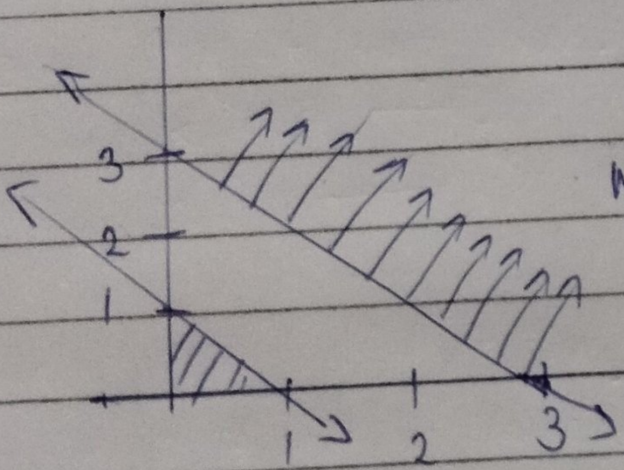
$(1,0), (0,1)$

$x_1 + x_2 = 3$

Put $x_1 = 0$ $x_2 = 3$

$x_2 = 0$ $x_1 = 3$

$(0,3), (3,0)$



No feasible region

1 2 3

4 Sept 2024

Find sol^n of LPP by using SIMPLEX METHOD

Q) Max $Z = 80x_1 + 55x_2$

Sub to
$$4x_1 + 2x_2 \leq 40$$
$$2x_1 + 4x_2 \leq 32$$
$$x_1, x_2 \geq 0$$

LHS + S = RHS

slack

Sol^n STEP 1
$$4x_1 + 2x_2 + S_1 = 40$$
$$2x_1 + 4x_2 + S_2 = 32$$

Max $Z = 80x_1 + 55x_2$

$$x_1, x_2, S_1, S_2 \geq 0$$

STEP 2 Represent system of Linear Eq^n in step 1 in matrix form $AX = 0$

i.e. $\begin{bmatrix} 4 & 2 & 1 & 0 \\ 2 & 4 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ S_1 \\ S_2 \end{bmatrix} = \begin{bmatrix} 40 \\ 32 \end{bmatrix}$

In obj function u have written any slack or surplus variable, cost of these variable is 0

PAGE NO.
DATE

Step 3 Simplex Table 1

Introduce variables created by own

Incoming vector

Outgoing vector

		C_j	80	55	0	0	
Basic variables	Cost of basic variable C_B	Solution Set X_B	Decision Variables X_K x_1	x_2	S_1	S_2	Min
S_1	0	40	4	2	1	0	$\frac{40}{4}$ Key row
S_2	0	32	2	4	0	1	$\frac{32}{2}$
$Z = 0$ $(0\times40 + 0\times32)$ $(C_B \times X_B)$		Z_j	0	0	0	0	
		$Z_j - C_j$ (Δ_j)	-80	-55	0	0	

C_B column with each decision variable

More -ve value

Key column

If all +ve stop
if any -ve entry select that

To Select row

Find min ratio

$\frac{X_B}{X_K}$ — Column which u have calculated

$\frac{40}{4}$

If denomiator was -ve or 0 put hypen not need to find ratio

4 = key element
Pivyot element

Make ~~unity~~ key element unity

$R_1 \rightarrow \frac{1}{4} R_1$

Pivot element ko one banao phir uske neeche walo ko 0 banao

PAGE NO.
DATE : / /

Simplex Table 2

BV	C_B	C_j / X_B	80 / x_1	55 / x_2	0 / S_1	0 / S_2	Min $\left(\frac{X_B}{X_k}\right)$	
x_1	80	10	1	1/2	1/4	0	20	$R_1 \rightarrow \frac{1}{4} R_1$
S_2	0	12	0	3	-1/2	1	4 ⟸	$R_2 \rightarrow R_2 - 2R_1$
Z = 800		z_j	80	40	20	0		
		$z_j - c_j$	0	-15	20	0		

~~32 2 4 0 1~~

Since $z_j - c_j <$ ~~0 will~~ will move to next table

Now make Simplex table 3.

Only 3 operations possible

$R_i \rightleftharpoons R_j$

$R_i \rightarrow \lambda R_j$

$R_i \rightarrow R_i + \lambda R_j$

Old $R_2 \rightarrow$ 32 2 4 0 1

New $2R_1 \rightarrow$ 20 2 1 1/2 0

12 0 3 -1/2 1