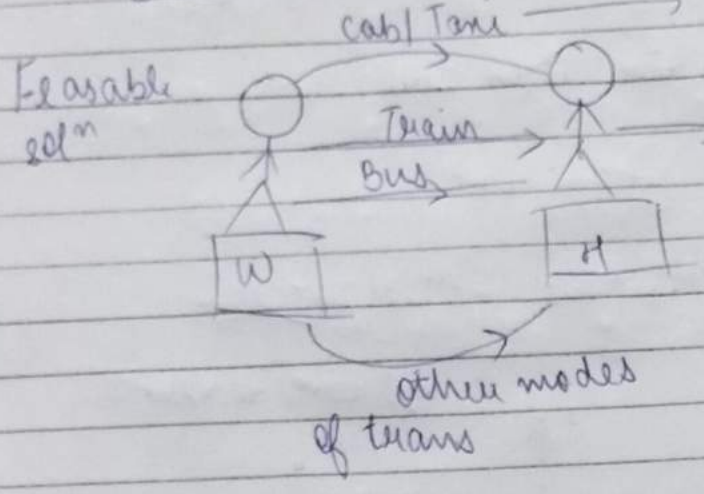
NUMERICAL AND OPTIMIZATION TECHNIQUES

PAGE NO.
DATE: Kartikeswarup

Q What is optimization?
The process of obtaining best results under given circumstances



Best will be optimal sol^n

① Cost } minimise
② Time

CLASSIFICATION

MAXIMIZATION

MINIMIZATION

Physical problem to Mathematical modelling

Resources
Input

→ Obj functions
→ Constrains

Mathematical formulation

Methods

Output

→ decision
Optimal sol^n.

Engineering Applications of optimization

PAGE NO.
DATE: / /

Objective function – Expresses main aim of model which are either to be minimized or maximised

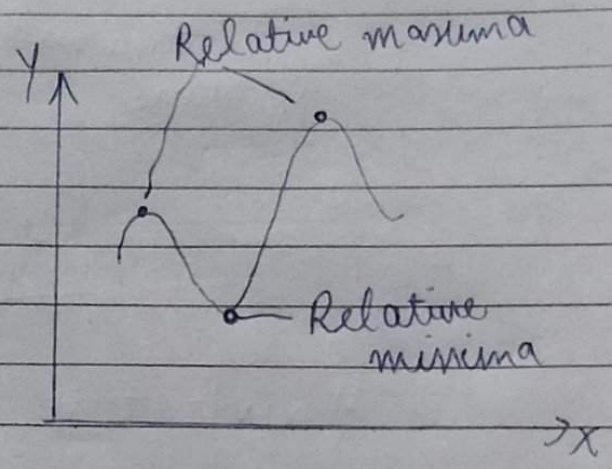
Constrains –

Decision – Always greater than zero.

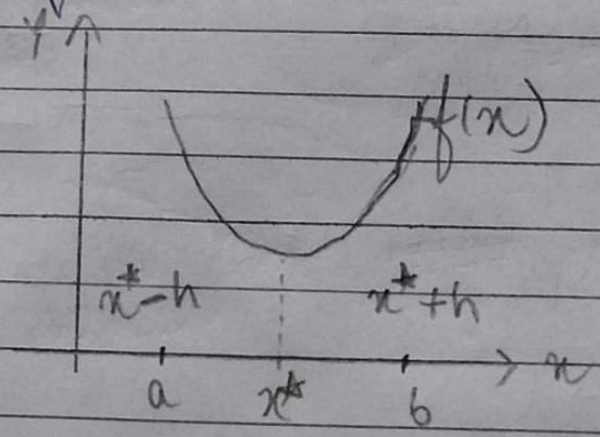
CLASSIFICATION – Study from ppt

SINGLE VARIABLE OPTIMIZATION –

$y = f(x)$



Relative Minima – Let $f(x)$ be function defined on $[a,b]$ then $x = x^*$ is said be relative minimum if $f(x^*) \leq f(x^*+h)$ for sufficiently small +ve and −ve values of h.



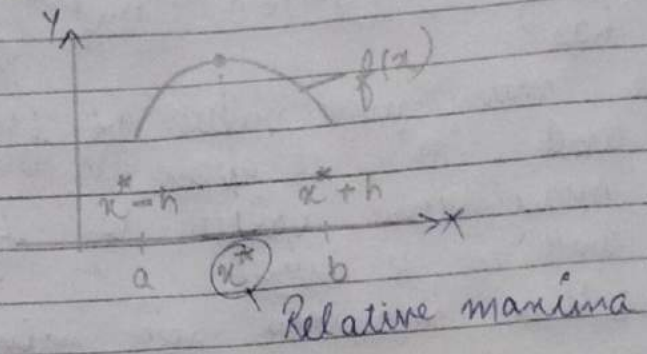
$f(x^*) \leq f(x^*+h)$

Relative Maxima – Let $f(x)$ be a function defined on $[a, b]$ then $x = x^*$ is said to be relative maximum if $f(x^*) \geq f(x^*+h)$

PAGE NO.
DATE : / /

for sufficiently small +ve and -ve values of h

$f(x^*) \geq f(x^*+h)$



(3) Global Minima- $x = x^*$ is said to be global minima if $f(x) \geq f(x^*)$ ~~for~~ for all x in the domain

Relative me apan neighbourhood me dekhte hai aur global me pure ke liye (ie full domain)

(4) Global Maxima- at $x = x^*$ is said to be Global Maxima if $f(x) \leq f(x^*)$ for all x in the domain.

Necessary Condition for single variable optimization.
If a function $f(x)$ is defined in the interval $a \leq x \leq b$ and has relative minimum at $x = x^*$ where $a < x^* < b$ and if the $f(x)$ exists as a finite number at $x = x^*$ then $f'(x^*) = 0$

PAGE NO.
DATE : / /

derivative

either +ve or −ve

Sufficient Condition: Let $f'(x^*) = f''(x^*) = f'''(x^*) = \dots = f^{(n-1)}(x^*) = 0$ but $f^{n}(x^*) \neq 0$, then $f(x^*)$ has

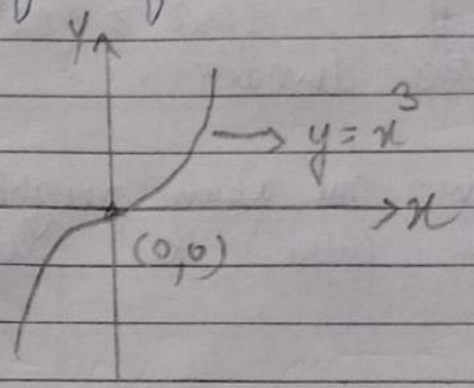
i) a minimum value of $f(x)$ if $f^{(n)}(x^*) > 0$ and 'n' is even.

ii) a maximum value of $f(x)$ if $f^{(n)}(x^*) < 0$ and 'n' is even.

iii) neither maxima nor minima if 'n' is odd

Stationary Points (Point of inflexion)

Eg) Let $f(x) = x^3$



Step 1 $f'(x) = 3x^2$

Put = 0 $f'(x) = 0$

$3x^2 = 0$

$\boxed{x = 0}$ CRITICAL POINT

Sufficient condition

$f''(x) = 6x$

$x = 0$

$n = 3 = \text{odd}$ So 3rd condition

$f^{(III)}(x) = 6 \neq 0$

$6 > 0$

At $f(x) = 0$ it has ~~neigh~~ neither maxima nor minima

$x = 0$ is the stationary pt or pt of inflexio

PAGE NO.
DATE: / /

Q) Find maximum and minimum values of function

$$f(x) = 12x^5 - 45x^4 + 40x^3 + 5$$

Ans)

$$f'(x) = 60x^4 - 180x^3 + 120x^2 = 0$$

$$x^2(60x^2 - 180x + 120) = 0$$

$$60x^2(x^2 - 3x + 2) = 0$$

$$x^2 \cancel{-3}x - 2x + 2$$

$$x(x-1) - 2(x-1)$$

$$x = 1, 2$$

$$\boxed{x = 0, 0, 1, 2}$$

$$\begin{array}{r} 45 \\ \times 4 \\ \hline 180 \end{array} \qquad \begin{array}{r} 18 \\ \times 3 \\ \hline 54 \end{array} \qquad \begin{array}{r} 24 \\ \times 3 \\ \hline 72 \end{array} \qquad \begin{array}{r} 54 \\ \times 2 \\ \hline 108 \end{array}$$

$$f''(x) = 240x^3 - 540x^2 + 240x \cancel{-0}$$

$$\cancel{240x(x^2}$$

At $x=0$ $\quad f''(x) = 0$

~~At x=1~~ $\quad \cancel{2}$

$$f'''(x) = 720x^2 - 1080x + 240$$

At $x=0$ $\quad f'''(x) = 240$

Since $f'''(x) \neq 0$ so we will apply sufficient condition (function)

n is odd so it has neither maxima nor minima at $x=0$. It is the pt of inflection

At $x=1$ $\quad \cancel{f''(x) = 60}$

$$f''(x) = 240 - 540 + 240$$

$$480 - 540$$

$$-60 < 0$$

Condition 2 will apply ~~Maximum~~

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PAGE NO.
DATE : / /

The point $x=1$ is point of relative maxima and maximum value $= f(1)$ → Put this value in real function

$12(1) - 45(1) + 40 + 5 = 12$

At $x=2$ $\quad f''(x) = 240(8) - 540(4) + 240(2)$

$1920 - 2160 + 480$

Relative minima $240 > 0$

Put 2 in real function

$12(2)^5 - 45(2)^4 + 40(2)^3 + 5$

$12(32) - 45(16) + 40(8) + 5$

$384 - 720 + 320 + 5$

-11 → Minimum value

$$\begin{array}{r} 1920 \\ 480 \\ \hline 2400 \\ 2160 \\ \hline 240 \end{array}$$

84

Q1) $f(x) = 5x^6 - 36x^5 + \frac{165}{2}x^4 - 60x^3 + 36$ — (1)

$f'(x) = 30x^5 - 180x^4 + 330x^3 - 180x^2$ — (2)

$$\begin{array}{r} 36 \\ 5 \\ \hline 180 \end{array}$$

$$\begin{array}{r} 165 \\ 2 \\ \hline 330 \end{array}$$

Put $f'(x) = 0$

$30x^2(x^3 - 6x^2 + 110x - 6) = 0$

$x = 0, 0$

$x^3 - 6x^2 + 110x - 6 = 0$

Step1 By hit and trail $x=1$ is other root

Step2 Write cofficients of eq^n.

	x^3	x^2	x	constant
	1	−6	11	−6
1	1	1	−5	6
	1	−5	6	0

quadratic eq^n ban gayi

Mostly last term will come 0

By LONG DIVISION ALSO WE CAN DO IT

$x^2 - 5x + 6 = 0$

$x^2 - 3x - 2x + 6 = 0$

$x(x-3) - 2(x-3) = 0$

$2, 3$

So all roots are $0, 0, 1, 2, 3$

Content | Fri_2_Tutorial(B) - Bb Collabo
eu.bbcollab.com/collab/ui/session/join/666eadcc931545ecb769f82dbd8508b6
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Home work:-

(1) $f(x) = 5x^6 - 36x^5 + \frac{165}{2}x^4 - 60x^3 + 36$

(2) $f(x) = 2x^3 - 6x^2 + 6x + 5$

(3) $f(x) = -x^3 + 3x^2 + x + 10$ in $[-2, 4]$

pre → (4) $f(x) = 12x^5 - 45x^4 + 40x^3 + 8$

(5) $f(x) = x^4 - 62x^2 + 120x + 9$

Everyone
Recording messages in this chat.
MANPREET KAUR 10:59 AM
yes
BIRJU BHARTI 10:59 AM
yes mam
SURABHI TIWARI 10:59 AM
yes maam
GURSIMAR SINGH ANAND 10:59 AM
yes mam
ANSHDWIP KUMAR 10:59 AM
yes mam
Say something
Send
Manpreet K...
35
Type here to search
31°C ENG 11:01 20-08-2021

HW Ques Sol^{n}s —

PAGE NO.
DATE : / /

(2) $f(x) = 2x^3 - 6x^2 + 6x + 5$

$f'(x) = 6x^2 - 12x + 6$

$f'(x) = 0$

$x^2 - 2x + 1 = 0$

$x^2 - x - x + 1 = 0$

$x(x-1) - 1(x-1) = 0$

$x = 1, 1$ (CRITICAL PTS)

$f''(x) = 12x - 12$

At $x = 1$

$f''(1) = 12 - 12$
$= 0$

$f'''(x) = 12 > 0$

$n = \text{odd}$

So neither minima nor maxima.

(3) $f(x) = -x^3 + 3x^2 + 9x + 10$ in $[-2, 4]$

$f'(x) = -3x^2 + 6x + 9$

$f'(x) = 0$

$-3(x^2 - 2x - 3) = 0$

$x^2 + x - 3x - 3 = 0$

$x(x+1) - 3(x+1) = 0$

$x = 3, -1$ (CRITICAL PTS)

$f''(x) = -6x + 6$

At $x = 3$ global maxima

~~$f'''(x) = -6$~~

$f''(3) = -18 + 6 = -12 < 0$

$n = 2$ ie even

Second condition will apply.

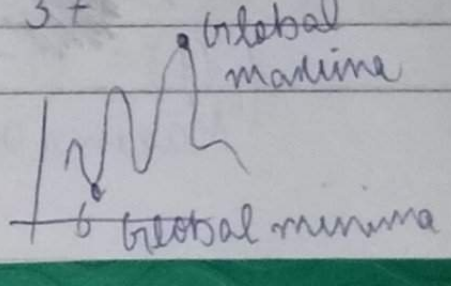
Local maxima ~~minima~~ at $x = 3$

So maximum value

$= -(3)^3 + 3(3)^2 + 9(3) + 10$

$= -27 + 27 + 27 + 10$

$= 37$



$f''(x)$ at $x = -1$

$-6(-1) + 6$

$6 + 6 = 12 > 0$

at $x = -1$ minima

So minimum value will be

$-(-1)^3 + 3 - 9 + 10$

$1 + 3 + 1$

$= 5$

global minima

(5) $f(x) = x^4 - 62x^2 + 120x + 9$

PAGE NO.
DATE : / /

$f'(x) = 4x^3 - 124x + 120$

Put $f'(x) = 0$

$x^3 - 31x + 30 = 0$

Hit and trial $x = 1$

~~8 - 64 + 30~~

So $x = 1$ is one root.

Method 2

$$\begin{array}{r|l} & x^2 + x - 30 \\ x-1 & x^3 - 31x + 30 \\ & x^3 - x^2 \\ & - \quad + \\ & x^2 - 31x + 30 \\ & x^2 - x \\ & - \quad + \\ & -30x + 30 \\ & -30x + 30 \\ & + \quad - \\ & 0 \end{array}$$

$(x-1)(x^2 + x - 30)$

$(x-1)(x^2 + 5x - 6x - 30)$

$(x-1)(x(x+5) - 6(x+5)) = 0$

$x = 1, -5, 6$ (CRITICAL PTS)

$f''(x) = 12x^2 - 124$

At $x = 1$

$f''(1) = 12 - 124 = -112 < 0$

Maxima at 1

Method 1

	x^3	x^2	x	Cons
1	1	-31	0	30
		1	-30	-30
	1	-30	-30	0

quadratic eq^n

$x^2 - 30x - 30 = 0$

$f(1) = 1 - 62 + 120 + 9$

$130 - 62 = 68$

Max value $= 68$

At $x = -5$

$f''(-5) = 12(25) - 124$

$= 300 - 124$

$= 176 > 0$

Minima

$f(5) = (5)^4 - 62(25) + 120(5) + 9$

$625 - 1550 + 600 + 9$

$-316 < 0$

PAGE NO.
DATE: / /

2) negative definite when x^* is relative maximum

3) Otherwise x^* is saddle pt.

Q) Find extreme pt and its nature of the function

$$f(x_1, x_2) = x_1^3 + x_2^3 + 2x_1^2 + 4x_2^2 + 6$$

Ans)

Necessary condition

$$\frac{\partial f}{\partial x_1} = 0 \qquad 3x_1^2 + 4x_1 = 0$$

$$x_1(3x_1 + 4) = 0$$

$$x_1 = 0 \text{ or } x_1 = \frac{-4}{3}$$

$$\frac{\partial f}{\partial x_2} = 0 \qquad 3x_2^2 + 8x_2 = 0$$

$$x_2(3x_2 + 8) = 0$$

$$x_2 = 0 \text{ or } x_2 = \frac{-8}{3}$$

Extreme pts are $(0,0)$, $\left(0, \frac{-8}{3}\right)$, $\left(\frac{-4}{3}, 0\right)$, $\left(\frac{-4}{3}, \frac{-8}{3}\right)$

$$\cancel{H = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} \\ \partial \end{bmatrix}} \qquad H = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} \end{bmatrix}$$

$$H = \begin{bmatrix} 6x_1+4 & 0 \\ 0 & 6x_2+8 \end{bmatrix}$$

PAGE NO.
DATE: 4.1

At (0,0)

$$H = \begin{bmatrix} 4 & 0 \\ 0 & 8 \end{bmatrix}$$

$H_1 = 4 > 0$ (First element)
$H_2 = 32 > 0$ (D Determinate)

(0,0) satisfies the condition of positive definite
(0,0) is relative minima

At $\left(0, \frac{-8}{3}\right)$

$$H = \begin{bmatrix} 4 & 0 \\ 0 & -8 \end{bmatrix}$$

$H_1 = 4 > 0$
$H_2 = -32 < 0$

Indefinite cannot have relative more non relative minima

At $\left(\frac{-4}{3}, 0\right)$

$$H = \begin{bmatrix} -4 & 0 \\ 0 & 8 \end{bmatrix}$$

Indefinite

$H_1 = -4 < 0$
$H_2 = -32 < 0$

At $\left(\frac{-4}{3}, \frac{-8}{3}\right)$

$$H = \begin{bmatrix} -4 & 0 \\ 0 & -8 \end{bmatrix}$$

$H_1 = -4 < 0$
$H_2 = 32 > 0$

Negative definate

~~Relative maxima~~

PAGE NO.
DATE : / /

Ques) Find the minimum value of $x^2+y^2+z^2$ subject to $x+y+2z=12$

Ans) $\text{Min } f(x) = x^2+y^2+z^2$ — (1)

subject to $x+y+2z = 12$ — (2)

we have to use direct substitution method

From (2)

$$2z = 12-x-y$$
$$z = \frac{1}{2}(12-x-y)$$

Put z in eqn (1)

$$f(x) = x^2+y^2+\frac{1}{4}(12-x-y)^2$$

Necessary condition

$$\frac{\partial f}{\partial x} = 2x + (-1)\frac{1}{2}(12-x-y)$$

$$2x - \frac{(12-x-y)}{2} = 0$$

$$2(2x) = 12-x-y$$

$$4x = 12-x-y \quad \text{— (3)}$$

$$\cancel{5x = 12-y}$$

$$\frac{\partial f}{\partial y} = \cancel{\frac{12-x}{5}} \quad \boxed{y = \frac{12-x-y}{4}} \quad \text{— (4)}$$

Similarly as above

From (3) and (4)

$$\boxed{x = y}$$

Put value of x in eq^n (3)

$$4x = 12 - x - y$$
$$4x = 12 - x - x$$
$$4x = 12 - 2x$$
$$6x = 12$$
$$\boxed{x = 2}$$

And $x = y$ So $\boxed{y = 2}$

Value of z

$$z = \frac{1}{2}(12 - 4)$$
$$z = \frac{8}{2} = 4$$

So extreme pt is 2, 2, 4

Apply Hessian Matrix

$$H = \begin{bmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} \end{bmatrix}$$

$$\frac{\partial^2 f}{\partial x^2} = 2 - \frac{1}{2}(-1) = \frac{5}{2}$$

$$H = \begin{bmatrix} 5/2 & 1/2 \\ 1/2 & 5/2 \end{bmatrix}$$

$$H_1 = \frac{5}{2} > 0$$

$$H_2 = \frac{25}{4} - \frac{1}{4} = \frac{24}{4} = 6 > 0$$

Point is relative minima and Hessian Matrix is a +ve definate

PAGE NO.
DATE: / /

Linear Programming

Formulation of LPP

1. Decision variables
2. Find objective function (whether maximise or minimise)
3. Constrains. $\geq$ $\leq$

$x, y \geq 0$ (Non -ve condition)

Ques) A firm can ~~reduce~~ produce 3 types of clothes say A, B and C. Three kinds of wool required. Red, Green and Blue. One unit length of A needs 2m of red, 3m of blue. One unit of B needs 3m of red, 2m of green and 2m of blue. One unit of C need 5m of green, 4m of blue. ~~The~~ Firm has only stock of 8m of red, 10m of green and 15m of blue wool. Income from A is Rs 3 and from B is Rs 5 and from C is Rs 4. How firm should use material so as to maximise income?

Ans)

	Red	Blue	Green
3 A	2	3	
5 B	3	2	2
4 C		4	5
	8	10	15

$2x + 3y \leq 8$ —— (1)

$3x + 2y + 4z \leq 10$ —— (2)

$2y + 5z \leq 15$ —— (3)

Objective
Maximise
$3x + 5y + 4z$
(Objective function)

$x, y \geq 0$ (Non -ve constrains)

PAGE NO.
DATE : / /

Ques) A diet for a sick person must contain atleast 4000 units of vitamins, 50 units of minerals and 1400 unit of cal. Two foods A and B are available at cost of Rs 4 and Rs 3 per unit resp. one unit of food A contains 200 units of vitamins, 1 unit of minerals and 40 units of calories and one unit of food B contain 100 units of vitamins 2 unit minerals and 40 unit cal. Formulate as LP for minimising cost.

Ans)

		Vita	Min	Cal
4	A	200	1	40
3	B	100	2	40
		4000	50	1400

Objective

$4x + 3y$

$200x + 100y \geq 4000$

$x + 2y \geq 50$

$40x + 40y \geq 1400$

$x, y \geq 0$

Ques) A dealer deals in only 2 items sewing machine and table fans. She has Rs 5760 to invest in store max of 20 items sewing machine cost Rs 360 and table fan Rs 240. He can sell sewing machine at profit of Rs 22 and table fan " " 60. Make LP.

Ans)

	Cost	profit
Sewing machine	360	22
	240	18
	5760	

capital X for x_1, x_2, x_3

GPB lect

Multivariable without constraints

PAGE NO.
DATE: / /

3 variable

$$f(X) = 3x_1^2 + 6x_1x_3 + x_2^2 - 4x_2x_3 + 8x_3^2$$

$$\frac{\partial f}{\partial x_1} = 6x_1 + 6x_3 = 0 \qquad \frac{\partial f}{\partial x_2} = 2x_2 - 4x_3 = 0$$

$$\frac{\partial f}{\partial x_3} = 6x_1 - 4x_2 + 16x_3 = 0 \quad \text{—— (3)}$$

$$x_1 = -x_3 \qquad x_2 = 2x_3$$

Put in eq^n (3)

$$-6x_3 - 8x_3 + 16x_3 = 0$$

$$2x_3 = 0 \qquad \boxed{x_3 = 0} \text{ so } \boxed{x_1 = 0} \quad \boxed{x_2 = 0}$$

$\therefore$ (0,0,0) is the extreme point.

HASSIAN MATRIX

Second ~~other~~ order partial derivatives

$$\frac{\partial^2 f}{\partial x_1^2} = 6 \; ; \; \frac{\partial^2 f}{\partial x_2 \partial x_1} = 0 \; ; \; \frac{\partial^2 f}{\partial x_3 \partial x_1} = 6$$

$$\frac{\partial^2 f}{\partial x_1 \partial x_2} = 0 \; ; \; \frac{\partial^2 f}{\partial x_2^2} = 2 \; ; \; \frac{\partial^2 f}{\partial x_3 \partial x_2} = -4$$

$$\frac{\partial^2 f}{\partial x_1 \partial x_3} = 6 \; ; \; \frac{\partial^2 f}{\partial x_2 \partial x_3} = -4 \; ; \; \frac{\partial^2 f}{\partial x_3^2} = 16$$

$$H = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \frac{\partial^2 f}{\partial x_1 \partial x_3} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & \frac{\partial^2 f}{\partial x_2 \partial x_3} \\ \frac{\partial^2 f}{\partial x_3 \partial x_1} & \frac{\partial^2 f}{\partial x_3 x_2} & \frac{\partial^2 f}{\partial x_3^2} \end{bmatrix} = \begin{bmatrix} 6 & 0 & 6 \\ 0 & 2 & -4 \\ 6 & -4 & 16 \end{bmatrix}$$

$6(32-16)+6(-12)$

PAGE NO. DATE: / /

$$\begin{array}{r} 16 \\ \times\ 6 \\ \hline 96 \end{array}$$

$96-72 = 24$

Principal minors are

$H1 =$ Det of first element $= |6| = 6 > 0$

$H2 =$ Det of first 4 elements $= \begin{vmatrix} 6 & 0 \\ 6 & 2 \end{vmatrix} = 12 > 0$

$H_3 =$ Det of whole matrix $24 > 0$

Since all principle minors are +ve this implies matrix is +ve definate so pt $(0,0,0)$ is pt of relative minima

Since at H we have constant so no need to put value of pt

Min value $= f(0,0,0) = 0$ Ans -

Q2) $f(x,y) = x^2 - y^2$ Extt pt $(0,0)$ Saddle pt

Q3) $f(x,y) = x^4 + y^4 + 2x^3 + 2y^3 + 48$ Ext pt $(0,0)$, $\left(0, -\frac{3}{2}\right)$, $\left(-\frac{3}{2}, 0\right)$

$\left(-\frac{3}{2}, -\frac{3}{2}\right)$ → Minima

Saddle

Q4) $f(x) = x_1^2 - x_2^2 - x_1 x_2$ $(0,0)$ Extreme pt saddle pt

Q5) $f(x) = 2x_1 - x_2 - x_1^2 + x_1 x_2 - x_2^2$

HW Ques Sol^{n}s

PAGE NO.
DATE: / /

Ans2) $f(x,y) = x^2 - y^2$

$\frac{\partial f}{\partial x} = 2x = 0$ $\qquad$ $\frac{\partial f}{\partial y} = -2y = 0$

$x = 0$ $\qquad$ $y = 0$

$\therefore$ Pt $(0,0)$

Double derivative $\frac{\partial^2 f}{\partial x^2} = 2$, $\qquad$ $\frac{\partial^2 f}{\partial y^2} = -2$

$\frac{\partial^2 f}{\partial x \partial y} = 0$ $\qquad$ $\frac{\partial^2 f}{\partial y \partial x} = 0$

$$H = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$$

$H_1 = 2 > 0$ $\qquad$ $H_2 = -4 < 0$

Ans3) $f(x,y) = x^4 + y^4 + 2x^3 + 2y^3 + 48$

$\frac{\partial f}{\partial x} = 4x^3 + 6x^2$ $\qquad$ $\frac{\partial^2 f}{\partial x^2} = 12x^2 + 12x$ $\qquad$ $\frac{\partial f}{\partial y \partial x} = 0$

$\frac{\partial f}{\partial y} = 4y^3 + 6y^2$ $\qquad$ $\frac{\partial^2 f}{\partial y^2} = 12y^2 + 12y$ $\qquad$ $\frac{\partial f}{\partial x \partial y} = 0$

Put first derivative as 0

$4x^3 + 6x^2 = 0$ $\qquad$ $4y^3 + 6y^2 = 0$

$2x^2(2x+3) = 0$ $\qquad$ $2y^2(2y+3) = 0$

$\boxed{x = 0}$ $\quad$ $\boxed{x = -\frac{3}{2}}$ $\qquad$ $y = 0, y = 0, y = -\frac{3}{2}$

$x = 0$

So Pts = $(0,0)$ $\quad$ $\left(0, -\frac{3}{2}\right)$ $\quad$ $\left(-\frac{3}{2}, 0\right)$ $\quad$ $\left(-\frac{3}{2}, -\frac{3}{2}\right)$

Matrix $= \begin{bmatrix} 12x^2+12x & 0 \\ 0 & 12y^2+12y \end{bmatrix}$

PAGE NO.
DATE: / /

At pt $(0,0)$

$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$H_1 = 0$

$H_2 = 0$

At pt $\left(0, -\frac{3}{2}\right)$

$\begin{bmatrix} 0 & 0 \\ 0 & 12\left(\frac{3}{2}\right)^2 - 18 \end{bmatrix}$

$H_1 = 0$

$H_2 = 0$

At pt $\left(-\frac{3}{2}, 0\right)$

$\begin{bmatrix} 12\left(\frac{3}{2}\right)^2 - 18 & 0 \\ 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 3(9) - 18 & 0 \\ 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 27 - 18 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 9 & 0 \\ 0 & 0 \end{bmatrix}$

$H_1 = 9 \qquad H_2 = 0$

At pt $\left(-\frac{3}{2}, -\frac{3}{2}\right)$

$\begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$

$H_1 = 9 > 0$

$H_2 = 81 > 0$

So $\left(-\frac{3}{2}, -\frac{3}{2}\right)$ is minima.

Q 4) $f(x) = x_1^2 - x_2^2 - x_1 x_2$

$\frac{\partial f}{\partial x_1} = 2x_1 - x_2 = 0$

$2x_1 = x_2$

$\frac{\partial f}{\partial x_2} = -2x_2 - x_1 = 0$

$2x_2 + x_1 = 0$

$x_1 = -2x_2$

Only possible when $(0,0)$

PAGE NO.
DATE: / /

Conversion of LPP to standard LPP

1) Write the objective function or in the minimization form

$$\text{Max } Z' = \text{Min}(-Z)$$

eg) $\text{Min } Z = -3x_1 + x_2$

$\text{Max } Z = 3x_1 - x_2$

2) Convert all inequalities as eq^{n}s (by introducing slack and surplus variable)

$+ \leq$ | $- \geq$

eg) $x_1 + 2x_2 \leq 12$ | $x_1 + 2x_2 \geq 12$

$x_1 + 2x_2 + s_1 = 12$ | $x_1 + 2x_2 - s_1 = 12$

3) The right hand element of each constrain should be made non -ve

eg) $2x_1 + x_2 - s_2 = -15$

$-2x_1 - x_2 + s_2 = 15$

4) All variables must have non -ve values

eg) $x_1 + x_2 \leq 3$, $x_1 \geq 0$, x_2 is unrestricted in sign

$x_2 = x_2' - x_2''$

$x_1 + (x_2' - x_2'') \leq 3$, $x_1, x_2', x_2'' \geq 0$

$x_1 + (x_2' - x_2'') + s_1 = 3$

(1) Max $z = 3x_1 + x_2$ sub to $2x_1 + x_2 \leq 2$

$3x_1 + 4x_2 \geq 12$, $x_1, x_2 \geq 0$

PAGE NO.
DATE : / /

Sol

Max $z = 3x_1 + x_2$

$2x_1 + x_2 + s_1 = 2$

$3x_1 + 4x_2 - s_2 = 12$

$x_1, x_2, s_1, s_2 \geq 0$

(2) Min $z = 4x_1 + 2x_2$ sub to $3x_1 + x_2 \geq 2$, $x_1 + x_2 \geq 21$,

$x_1 + 2x_2 \geq 30$, $x_1, x_2 \geq 0$

Ans) Max $z' = -4x_1 - 2x_2$ sub to $3x_1 + x_2 - s_1 = 2$

$x_1 + x_2 - s_2 = 21$ $x_1 + 2x_2 - s_2 = 30$

and $x_1, x_2, s_1, s_2, s_3 \geq 0$

(3) Min $z = x_1 + 2x_2 + 3x_3$ sub to $2x_1 + 3x_2 + 3x_3 \geq 4$

$3x_1 + 5x_2 + 2x_3 \leq 7$ and $x_1, x_2 \geq 0$, x_3 is

unrestricted

Ans)

Max $z' = -x_1 - 2x_2 - 3x_3$ sub to $2x_1 + 3x_2 + 3x_3 - s_1 = 4$ — (1)

$3x_1 + 5x_2 + 2x_3 + s_2 = 7$ — (2) $x_1, x_2 \geq 0$

$(x_3 = x_3' - x_3'')$

Put in eq^n (1)

$2x_1 + 3x_2 + 3(x_3' - x_3'') - s_1 = 4$

Put in eq^n (2)

$3x_1 + 5x_2 + 2(x_3' - x_3'') + s_2 = 7$

x_1, x_2

3 Sept 2021

GRAPHICAL METHOD-

Q) Maximize $z = 3x_1 + 4x_2$ sub to constraints $x_1 + x_2 \leq 450$, $2x_1 + x_2 \leq 600$ and $x_1, x_2 \geq 0$

Ans) Convert inequality to equality

$x_1 + x_2 = 450$

Put $x_1 = 0$, $x_2 = 450$

$x_2 = 0$, $x_1 = 450$

$(0, 450)$, $(450, 0)$

$2x_1 + x_2 = 600$

Put $x_1 = 0$, $x_2 = 600$

$(0, 600)$ and $(300, 0)$

$2x_1 + x_2 \leq 600$

Put $(0,0)$

$0 \leq 600$

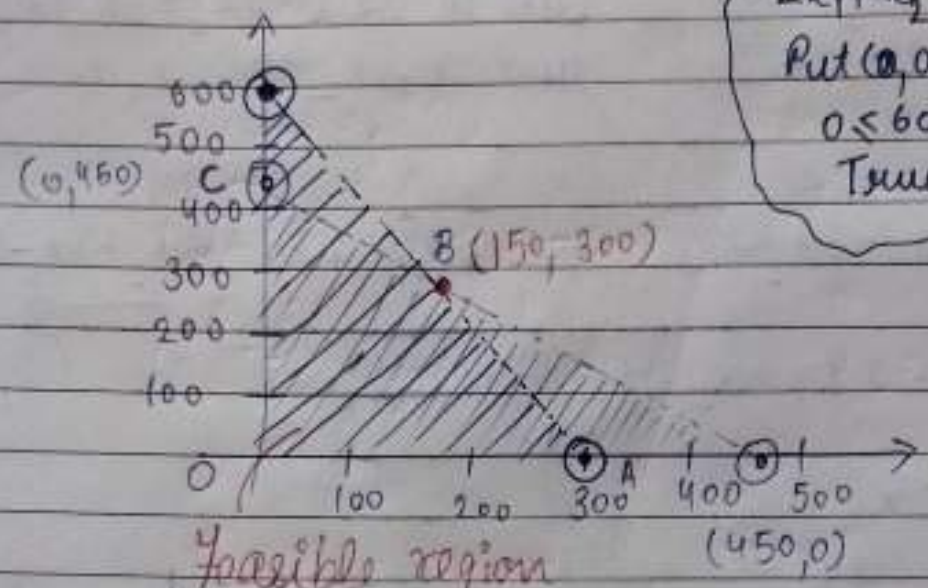
True

$x_1, x_2 \leq 450$

Put origin in this to check shaded portion

$0 \leq 450$

True so taken as



Feasible region

Solving eq^{n}s for intersection,

$x_1 = 450 - x_2$

Put in

$2(450 - x_2) + x_2 = 600$

$900 - x_2 = 600$

$\boxed{300 = x_2}$

$\boxed{x_1 = 450 - 300 = 150}$

Intersection pt = $(150, 300)$

Put in obj function

Points are

$O(0,0) = 0$

$A(300, 0) = 900$

$B(150, 300) = 1650$

$C(0, 450) = 1800$

Feasible sol^n best is optimal sol^n

Max will be 1800 so it is optimal sol^n.

MULTIPLE OPTIMAL SOL

Q) Max $z = 4x_1 + 3x_2$ sub. to $4x_1 + 3x_2 \leq 24$ — (1)

$x_1 \leq 4.5$ — (2)

$x_2 \leq 6$ — (3) and $x_1, x_2 \geq 0$

Sol^n) From (1) $4x_1 + 3x_2 = 24$

Put $x_1 = 0$ $x_2 = 8$

$x_2 = 0$ $x_1 = 6$

$(0, 8), (6, 0)$

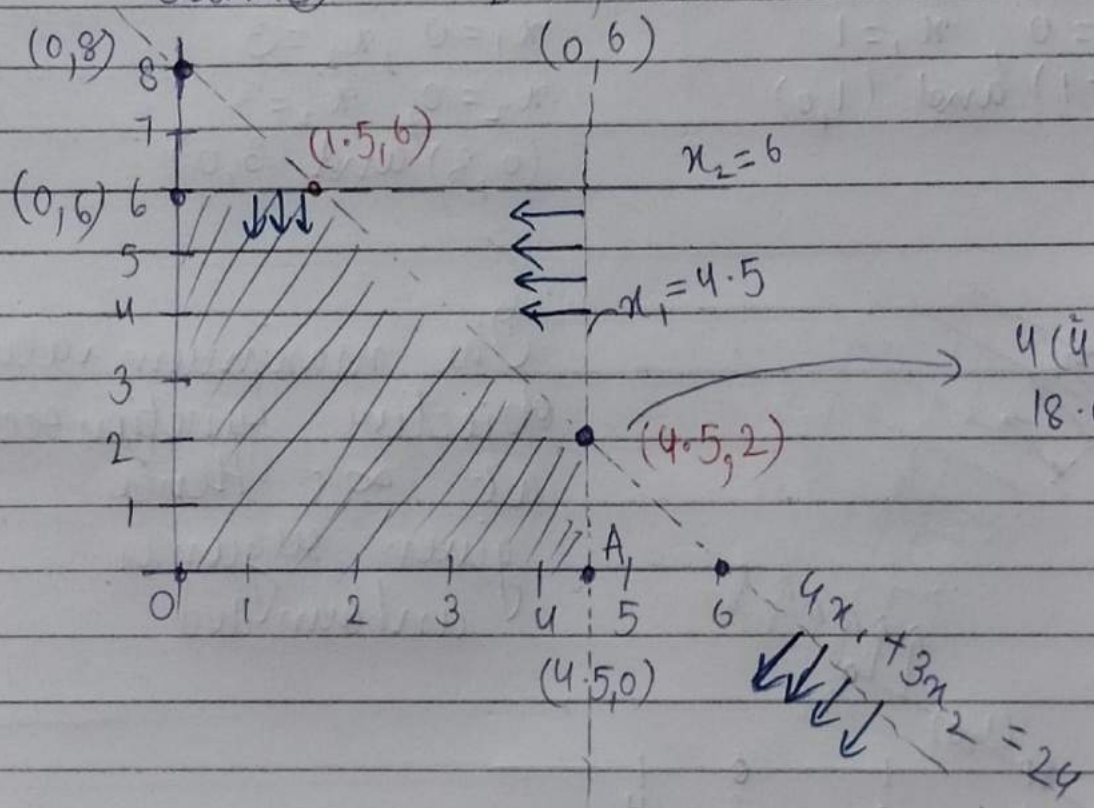
From eq^n (2)

$\cancel{x_2 = 6}$ $x_1 = 4.5$

$x_2 = 0$

$\therefore (4.5, 0)$

From (3) $x_2 = 6$, $x_1 = 0$



$4(4.5) + 3x_2 = 24$

$18.0 + 3x_2 = 24$

$3x_2 = 6$

$\boxed{x_2 = 2}$

Pt $(4.5, 2)$

Pts are

$O(0,0) = 0$

$A(4.5, 0) = 18$

$B(4.5, 2) = \boxed{24}$ Multiple optimal sol^n

$C(1.5, 6) = \boxed{24}$

$D(0, 6) = 18$

Un-bounded solution

Q) Max $z = 3x_1 + 2x_2$

Sub to

$x_1 - x_2 \leq 1$ —— (1)

$x_1 + x_2 \geq 3$ —— (2)

$x_1, x_2 \geq 0$

Sol^n)

From eq^n (1)

$x_1 - x_2 = 1$

Put $x_1 = 0$, $x_2 = -1$

$x_2 = 0$, $x_1 = 1$

$(0, -1)$ and $(1, 0)$

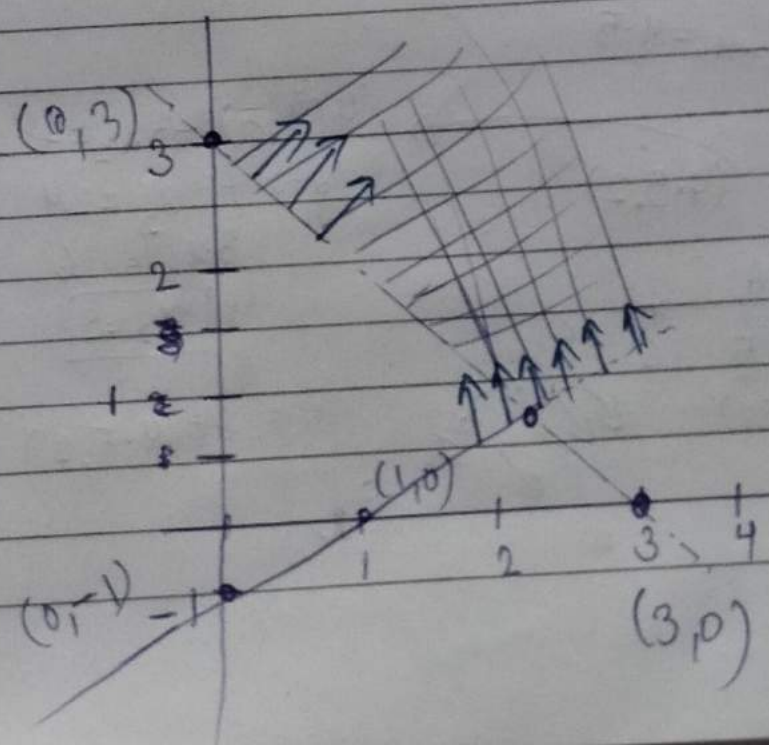
From eq^n (2)

$x_1 + x_2 = 3$

$x_1 = 0$, $x_2 = 3$

$x_2 = 0$, $x_1 = 3$

$(0, 3)$ and $(3, 0)$



The maximum value of objective function occurs at ∞. Hence given region is unbounded

NO OPTIMAL SOL^n

PAGE NO.
DATE: / /

Q) Max $z = 3x_1 + 2x_2$ Sub to $x_1 + x_2 \leq 1$ — (1)

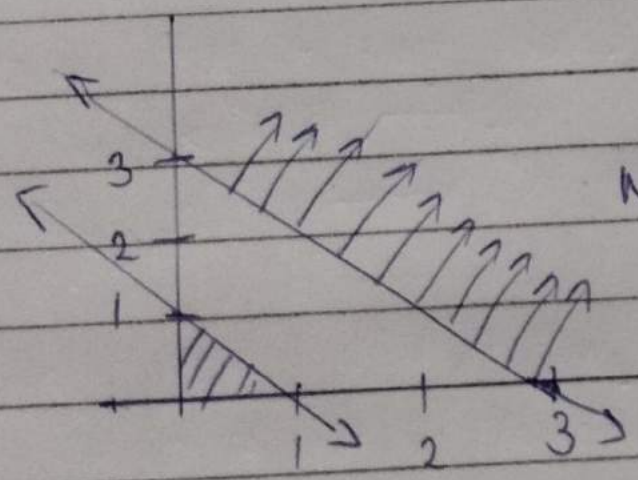
$x_1 + x_2 \geq 3$ — (2)

$x_1, x_2 \geq 0$

Ans)

$x_1 + x_2 = 1$
Put $x_1 = 0$ $x_2 = 1$
$x_2 = 0$ $x_1 = 1$
$(1,0), (0,1)$

$x_1 + x_2 = 3$
Put $x_1 = 0$ $x_2 = 3$
$x_2 = 0$ $x_1 = 3$
$(0,3), (3,0)$



No feasible region

4 Sept 2021

Find sol^n of LPP by using SIMPLEX METHOD

Q) $\text{Max } Z = 80x_1 + 55x_2$

Sub to
$$4x_1 + 2x_2 \leq 40$$
$$2x_1 + 4x_2 \leq 32$$
$$x_1, x_2 \geq 0$$

LHS + (S) = RHS
slack

Sol^n STEP 1

$$4x_1 + 2x_2 + S_1 = 40$$
$$2x_1 + 4x_2 + S_2 = 32$$

$\text{Max } Z = 80x_1 + 55x_2$

$$x_1, x_2, S_1, S_2 \geq 0$$

STEP 2 Represent system of Linear eq^n in step 1 in matrix form $AX = 0$

ie- $\begin{bmatrix} 4 & 2 & 1 & 0 \\ 2 & 4 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ S_1 \\ S_2 \end{bmatrix} = \begin{bmatrix} 40 \\ 32 \end{bmatrix}$

In obj function u have written any slack or surplus variable, cost of these variable is 0

PAGE NO.
DATE

Incoming vector

Step 3 Simplex Table 1

Introduce basic variables curated by own

Outgoing vector

Basic variables	Cost of basic variable C_B	C_j / Solution Set X_B	80	55	0	0	Min
			Decision Variables X_K				
			x_1	x_2	S_1	S_2	
S_1	0	40	4	2	1	0	$\frac{40}{4}$ Key row
S_2	0	32	2	4	0	1	$\frac{32}{2}$
$Z = 0$ $(0 \times 40 + 0 \times 32)$ $(C_B \times X_B)$		Z_j	0	0	0	0	
		$Z_j - C_j$ (Δ_j)	-80	-55	0	0	

C_B column with each decision variable

Key column

More -ve value

If all +ve stop
if any -ve entry select that

To Select row

Find min ratio

$\frac{X_B}{X_K}$ — Column which u have calculated

$\frac{40}{4}$

4 = Key element
Pivot element

If denomiator was -ve or 0 put hypen not need to find ratio

Make ~~unity~~ key element unity

$R_1 \rightarrow \frac{1}{4} R_1$

Pivot element ko one banao phir uske neche walo ko 0 banao

PAGE NO.
DATE : / /

Simplex Table 2

		C_j	80	55	0	0	
BV	C_B	X_B	x_1	x_2	S_1	S_2	$Min\left(\frac{X_B}{X_k}\right)$
x_1	80	10	1	1/2	1/4	0	20
S_2	0	12	0	3	-1/2	1	4
Z=800		z_j	80	40	20	0	
		$z_j - c_j$	0	-15	20	0	

$R_1 \rightarrow \frac{1}{4} R_1$

$R_2 \rightarrow R_2 - 2R_1$

~~32 2 4 0 1~~

Since $z_j - c_j < 0$ ~~will~~ will move to next table

Now make Simplex table 3.

Only 3 operations possible

$R_i \rightleftharpoons \lambda_j$

$R_i \rightarrow \lambda R_j$

$R_i \rightarrow R_i + \lambda R_j$

Old

$R_2 \rightarrow$ 32 2 4 0 1

New

$2R_1 \rightarrow$ 20 2 1 1/2 0

12 0 3 -1/2 1

TABLE 3

BV	CB	C_j	80	55	0	0	
		X_B	x_1	x_2	S_1	S_2	
x_1	80	8	1	0	$1/3$	$-1/6$	$R_2 \to \frac{R}{3}$
x_2	55	4	0	1	$-1/6$	$1/3$	$R_2 \to R$
$Z = 860$		Δ_j	0	0	$35/2$	5	

$$z = 80(8) + 55(4)$$
$$= 860$$

Since all $\Delta_j \geq 0$ basic feasible sol^n is obtained $x_1 = 8$, $x_2 = 4$, $Z = 860$

Ans

PAGE NO.
DATE : / /

$$\boxed{\text{Max } Z = 3x_1 - x_2}$$

Sub to

-ve ko hatane ke liye ie identity banane ke liye

$$a_1 + 2x_1 + x_2 - s_1 = 2$$
$$x_1 + 3x_2 + s_2 = 2$$
$$x_2 + 0 + s_3 = 4$$

$x_1, x_2, s_1, s_2, s_3, a_1 \geq 0$

I) Eliminate artificial variable
II) Optimal sol^n

Phase I

Step 1 Assign a cost -1 to each artificial variable and a cost 0 to all other variable

Step 2 Construct the auxillary LPP in which the new objective function z^* is to be maximised sub to the constrains

Sol^n Auxillary LPP is

$$\text{Max } z^* = 0x_1 + 0x_2 + 0s_1 + 0s_2 + 0s_3 - a_1$$

Sub to constrains

$$2x_1 + x_2 - s_1 + a_1 = 2$$
$$x_1 + 3x_2 + s_2 = 2$$
$$x_2 + 0 + s_3 = 4$$

and $x_1, x_2, s_1, s_2, s_3, a_1 \geq 0$

PAGE NO.
DATE: / /

		C_j	0	0		0	0	0	1	Min $x_k>0$
BV	C_B	X_B	x_1	x_2		s_1	s_2	s_3	a_1	
a_1	-1	2	$\boxed{2}$	1		-1	0	0	1	1
s_2	0	2	1	3		0	1	0	0	2
s_3	0	4	0	1		0	0	1	0	-
$z=-2$		Δ_j	-2	-1		1	0	0	0	

			0	0	0	0	0	-1
BV	C_B	X_B	x_1	x_2	s_1	s_2	s_3	a_1
x_1	0	1	1	$1/2$	$-1/2$	0	0	$1/2$
R_2-R_1 s_2	0	1	0	$3-\frac{1}{2}=\frac{5}{2}$	$1/2$	1	0	$-1/2$
s_3	0	4	0	1	0	0	1	0
$z=0$		Δ_j	0	0	0	0	0	1

Since all Δ_j are +ve so we need to stop

$\Delta_j \geq 0$ Max $z^* = 0$, no artificial vect appear in the basis so we need to proceed to phase 2.

Step3 ① If max $z^* < 0$ and atleast one artificial vector appear in optimum basis at a +ve level. Then given problem does not posses any feasible sol^n.

(2) If Max $z^* = 0$ and atleast one artificial vector appears in optimal basis at zero, then proceed to phase 2.

(3) If Max$(z^*) = 0$ and no Artificial vector appears in basis, then move to phase 2.

z^* is to be maximized subject to constraints.

PAGE NO.
DATE:

Phase 2

$x_* \geq 0$

		C_j	3	-1	0	0	0	
BV	C_B	X_B	x_1	x_2	s_1	s_2	s_3	
x_1	3	1	1	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	—
s_2	0	1	0	$\frac{5}{2}$	$\frac{1}{2}$	1	0	2 ←
s_3	0	4	0	1	0	0	1	—
$Z=3$	Δz		3	$\frac{3}{2}$	$+\frac{3}{2}$	0	0	
	Δj	=	0	$+1+\frac{3}{2}$	$-\frac{3}{2}$	0	0	
	Δj	=	0	$+\frac{5}{2}$	$-\frac{3}{2}$	0	0	

$R_1 \to R_1 + \frac{1}{2} R_2$

$\frac{1}{2} + \frac{2}{2}$

			3	-1	0	0	0
BV	C_B	X_B	x_1	x_2	s_1	s_2	s_3
x_1	3	2	1	3	0	1	0
s_1	0	2	0	5	1	2	0
s_3	0	4	0	1	0	0	1
$Z = 6$		Z	3	2	0	0	0
		$\Delta_j =$	0	10	0	3	1

All $\Delta_j \geq 0$ Optimal sol^n is obtained

$Z = 6, \quad x_1 = 2, \quad x_2 = 0$ Ans

CASE 2 INFEASIBLE SOL^n

Q) $\text{Min } Z = x_1 - 2x_2 - 3x_3$ sub to $-2x_1 + x_2 + 3x_3 = 2$

$2x_1 + 3x_2 + 4x_3 = 1$

and $x_1, x_2, x_3 \geq 0$.

artificial

Ans) $\text{Max } Z = -x_1 + 2x_2 + 3x_3$

Cons $-2x_1 + x_2 + 3x_3 + A_1 = 2$

$2x_1 + 3x_2 + 4x_3 + A_2 = 1$

$x_1, x_2, x_3, A_1, A_2 \geq 0$

$\geq$

~~Add~~ Sub surplus variable and add artificial variable

Phase I

Auxillary LPP

$\text{Max } Z^* = 0x_1 + 0x_2 + 0x_3 - A_1 - A_2$

Sub to

$$-2x_1 + x_2 + 3x_3 + A_1 = 2$$
$$2x_1 + 3x_2 + 4x_3 + A_2 = 1$$

$$x_1, x_2, x_3, a_1, a_2$$

Phase i

BV	C_B	X_B	x_1	x_2	x_3	A_1	A_2	Min
		C_j	0	0	0	-1	-1	
a_1	-1	2	-2	1	3	1	0	$2/3$
a_2	-1	1	2	3	$\boxed{4}$	0	1	$1/4$
		z	0	-4	-7	-1	-1	
		Δj	0	-4	-7	0	0	

$z^* = -2-1$
$= -3$

$R_1 \to R_1 - 3R_2$

BV	C_B	X_B	x_1	x_2	x_3	A_1	A_2	Min
			0	0	0	-1	-1	
a_1	-1	$\boxed{5/4}$ +ve	$-7/2$	$-5/4$	0	1	$-3/4$	
x_3	0	$1/4$	$1/2$	$3/4$	1	0	$1/4$	
		z	$7/2$	$5/4$	0	-1	$3/4$	
		Δj	$7/2$	$5/4$	0	0	$\frac{3}{4}+1$ $\frac{7}{4}$	

$\boxed{z^* = -\frac{5}{4}}$

no feasible soln, no need to go to phase 2

Since all $\Delta j > 0$ so we need to stop here

$1 - \frac{9}{4}$ $\quad -2 - \frac{3}{2}$ $\quad 2 - \frac{3}{4}$

Solve

Q) Max $Z = 5x_1 + 3x_2$ sub to $2x_1 + x_2 \leq 1$

$x_1 + 4x_2 \geq 6$

$x_1, x_2 \geq 0$

PAGE NO.
DATE: / /

Soln

$$\text{Max } z = 5x_1 + 3x_2$$

cons $\quad 2x_1 + x_2 + s_1 = 1$

$$x_1 + 4x_2 - s_2 + A_1 = 6$$

$$x_1, x_2, s_1, s_2, A_1 \geq 0$$

Phase I

Auxillary LPP

$$z^* = 0x_1 + 0x_2 + 0s_1 + 0s_2 - A_1$$

Simplex Table

		c_j	0	0	0	0	-1	
BV	C_B	X_B	x_1	x_2	s_1	s_2	A_1	Min
A_1	-1	6	1	4	0	-1	1	$6/4 = 1.5$
s_1	0	1	2	1	1	0	0	1
$z =$	-6	z	-1	-4	0	1	-1	
		Δ_j	-1	-4	0	1	0	

$R_1 \to R_1 - 4R_2$

			0	0	0	0	-1
BV	C_B	X_B	x_1	x_2	s_1	s_2	A_1
A_1	-1	2	-7	0	-4	-1	1
x_2	0	1	2	1	1	0	0
$Z = -2$		z	7	0	4	1	-1
		Δ_j	7	0	4	1	0

~~We need to stop~~

Phase 2

PAGE NO
DATE:

			5	3	0	0
BV	C_B	X_B	x_1	x_2	s_1	s_2
A_1	0	2	-7	0	-4	-1
x_2	3	1	2	1	1	0
		Z	6	3	3	0
$Z=3$		Δ_j	1	0	3	0

$Z=3, \quad x_2=1 \quad x_1=0$

PAGE NO.
DATE : / /

① Solve the following LPP by using

Big M Penalty Method

Max $z = 3x_1 - x_2$ Sub to $2x_1 + x_2 \leq 2$

$x_1 + 3x_2 \geq 3$

$x_1 \leq 4$

$x_1, x_2 \geq 0$

Ans) $2x_1 + x_2 + s_1 = 2$

$x_1 + 3x_2 - s_2 + A_1 = 3$

$x_1 + s_3 = 4$

$x_1, x_2, s_1, s_2, s_3, A_1 \geq 0$

Very high +ve value

Max $z = 3x_1 - x_2 + 0s_1 + 0s_2 + 0s_3 - MA_1$

			3	-1	0	0	0	-M	
BV	C_B	X_B	x_1	x_2	s_1	s_2	s_3	A_1	Ratio
s_1	0	2	2	1	1	0	0	0	$\frac{2}{1}$
a_1	-M	3	1	$\boxed{3}$	0	-1	0	1	1
s_3	0	4	1	0	0	0	1	0	-
		z_j	-M	-3M	0	M	0	-M	
		Δ_j	-M-3	-3M+1	0	M	0	0	

Take M = 1000 as rough (method)

-1000-3 -3000+1

$R_i \to R_i + \lambda R_j$ $\quad R_i \to \lambda R_i$

$R_i \to R_j$

If artifi variable removes the n

PAGE NO.

DATE :

			3	-1	0	0	0		
BV	C_B	X_B	x_1	x_2	S_1	S_2	S_3	A_1	Min Ratio
S_1	0	1	$5/3$	0	1	$1/3$	0	$-1/3$	$3/5$
x_2	-1	1	$1/3$	1	0	$-1/3$	0	$1/3$	3
S_3	0	4	[illegible]	0	0	0	1	0	4
$Z = -1$		Z	$-1/3$	-1	0	$1/3$	0		
		Δj	$-\frac{1}{3} - 3$	0	0	$1/3$	0		
		$\Delta j =$	$-10/3$	0	0	$1/3$	0		

Skip

BV	C_B	X_B	x_1	x_2	S_1	S_2	S_3
x_1	3	$3/5$	1	0	$3/5$	5	0
x_2	-1	$4/5$	0	1	$-1/5$	$-2/5$	0
S_3	0	0	4	0	0	0	1
	~~Z~~	Z	2	-1	$\frac{9}{5} + \frac{1}{5} = 2$	$15 + \frac{2}{5}$	
$Z = 1$		Δj	0	0	2	1	0

$\frac{15}{5}$

$9 - 4$ $\quad R \to R - \; R_1$

Since all $z_j - c_j >= 0$ so optimal sol^n is obtained

$\max z = 1$ $\quad x_1 = \frac{3}{5}$ $\quad x_2 = \frac{4}{5}$

PAGE NO.
DATE: / /

Sol^n ke liye x_B dekhna hai

Can be done by 2 Phase also

>= or = By 2 Phase or Big M method

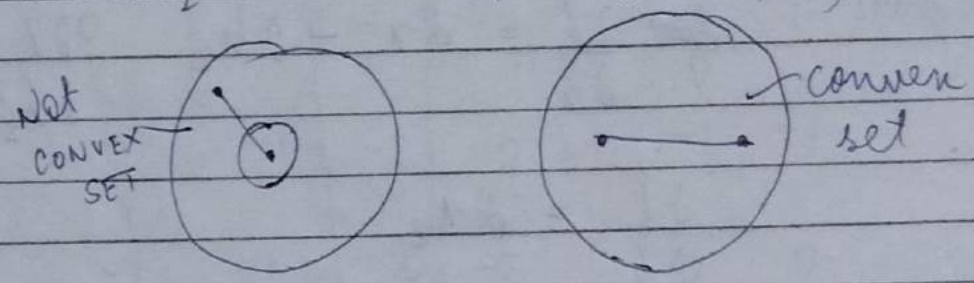
PAGE NO.
DATE : / /

CONVEXITY–

A real valued function $f(x)$ is defined on $[a,b]$ is cld convex set if the line segment b/w any 2 pts on the graph of the function lies above the graph

CONDITION

$$\frac{\partial^2 f_2}{\partial x_1^2} \geq 0 \qquad \frac{\partial^2 f_2}{\partial x_2^2} \geq 0 \qquad \frac{\partial^2 f_2}{\partial x_1^2}\frac{\partial^2 f_2}{\partial x_2^2} - \left(\frac{\partial^2 f}{\partial x_1 \partial x_2}\right)^2 \geq 0$$



Q) $f(x_1, x_2) = (x_1 - x_2)^2$

$$\frac{\partial f}{\partial x_1} = 2(x_1 - x_2) \qquad \frac{\partial f}{\partial x_2} = -2(x_1 - x_2)$$

$$\frac{\partial^2 f_2}{\partial x_1^2} = 2(1) \qquad \frac{\partial^2 f_2}{\partial x_2^2} = +2$$

$$\frac{\partial^2 f}{\partial x_1 \partial x_2} = -2$$

Put in condi $\quad 2(2) - (-2)^2$

$$4 - 4 \geq 0$$

$$\boxed{\frac{\partial^2 f}{\partial x^2} > 0}$$ condition for single variable convex set

$f(x) = x^4 + x^2$

$f'(x) = 4x^3 + 2x$

$f''(x) = 12x^2 + 2 > 0$

Convex set

$f(x) = 10 - x^2$

$f'(x) = -2x$

$f''(x) = -2$

Not convex

PAGE NO.
DATE : / /

Note - A function $f(x)$ is said to be convex function if Hessian matrix is positive semi definate (koi ek minor 0 ho jae)

Q) ST $f(x) = x_1^2 + x_2^2 + x_3^2 - 2x_1x_2$ is convex function.

Ans)

$$\frac{\partial f}{\partial x_1} = 2x_1 - 2x_2 \qquad \frac{\partial f}{\partial x_2} = 2x_2 - 2x_1$$

$$\frac{\partial f}{\partial x_3} = 2x_3$$

$$\begin{bmatrix} 2 & -2 & 0 \\ -2 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$H_1 = 2$

$H_2 = 4 - 4 = 0$

$H_3 = 0$

$2(4) + 2(-4) = 0$

Big M

Q) Max $Z = x_1 - x_2 + 3x_3$ subject to

$x_1 + x_2 \leq 20, \quad x_1 + x_3 = 5, \quad x_2 + x_3 \geq 10$

$x_1, x_2, x_3 \geq 0$

$x_1 + x_2 + s_1 = 20$

$x_1 + x_3 + a_1 = 5$

$x_2 + x_3 - s_2 + a_2 = 10$

PAGE NO.
DATE: / /

BV	Cost	x_1	x_2	x_3	s_1	s_2	a_1	a_2	X_B	Ratio
					0	0	-1	-1		
s_1	0	1	1	0	1	0	0	0	20	-
a_1	-1	1	0	1	0	0	1	0	5	5
a_2	-1	0	1	1	0	-1	0	1	10	10
	z	-1	-1	-2	0	1	-1	-1		
	Δ_j	-1	-1	-2	0	1	0	0		

BV	Cost	x_1	x_2	x_3	s_1	s_2	a_2	X_B	Ratio
s_1	0	1	1	0	1	0	0	20	20
x_3	0	1	0	1	0	0	0	5	-
a_2	-1	-1	+1	0	0	-1	+1	5	5
z	z =	1	-1	0	0	1	-1		
	Δ_j =	1	-1	0	0	0	0		

27/9/2021

UNIT 2 | 23

(a) Duality
(b) Dual simplex method
(c) Transportation method
(d) Argument Prob
(e) Travelling salesman Prob

PAGE NO.
DATE:

DUALITY
(Primal to Dual conversion)

Primal	Standard Primal	Dual
Max	Max $\leq$	Min $\geq$
Min	Min $\geq$	Max $\leq$

Same $\leq$ hone chaiye ni to -1 se multiply karo $\geq$

Ques) Find dual of given primal is $\text{Max } z = 3x_1 + 10x_2 + 2x_3$

Sub to $2x_1 + 3x_2 + 2x_3 \leq 7$ and $3x_1 - 2x_2 + 4x_3 = 3$

$x_1, x_2, x_3 \geq 0$

$3x_1 - 2x_2 + 4x_3 \leq 3$ $\quad$ $3x_1 - 2x_2 + 4x_3 \geq 3$

Sol^n Standard Primal is

$$\begin{bmatrix} 2 & 3 & 2 \\ 3 & -2 & 4 \\ -3 & 2 & -4 \end{bmatrix}^T$$

$$\begin{bmatrix} 2 & 3 & -3 \\ 3 & -2 & 2 \\ 2 & 4 & -4 \end{bmatrix}$$

$\text{Max } z = 3x_1 + 10x_2 + 2x_3$

Sub to

$2x_1 + 3x_2 + 2x_3 \leq 7$

$3x_1 - 2x_2 + 4x_3 \leq 3$

$-3x_1 + 2x_3 - 4x_3 \geq -3$

~~Dual~~ Dual is $\text{Min } Z = 7w_1 + 3w_2 - 3w_3$

Sub to

$2w_1 + 3w_2 - 3w_3 \geq 3$

$3w_1 - 2w_2 + 2w_3 \geq 10$

$2w_1 + 4w_2 - 4w_3 \geq 2$

$w_1, w_2, w_3 \geq 0$

Q) Primal is $\text{Min } z = 3x_1 + 5x_2 + 6x_3$

PAGE NO.
DATE: / /

sub to

$x_1 + 4x_2 + 6x_3 \leq 5$

$2x_1 + 3x_2 + 5x_3 \geq 4$

$3x_1 + x_2 + 7x_3 = 3$

$x_1, x_2 \geq 0$ and x_3 is unrestricted $x_3' - x_3''$

Sol^n $\text{Min } z = 3x_1 + 5x_2 + 6(x_3' - x_3'')$

Sub to

$-x_1 - 4x_2 - 6(x_3' - x_3'') \geq -5$

$2x_1 + 3x_2 + 5(x_3' - x_3'') \geq 4$

$3x_1 + x_2 + 7(x_3' - x_3'') \geq 3$

$-3x_1 - x_2 - 7(x_3' - x_3'') \geq -3$

$x_1, x_2, x_3', x_3'' \geq 0$

$$\begin{bmatrix} -1 & -4 & -6 & 6 \\ 2 & 3 & 5 & -5 \\ 3 & 1 & 7 & -7 \\ -3 & -1 & -7 & 7 \end{bmatrix}^T$$

$$\begin{bmatrix} -1 & 2 & 3 & -3 \\ -4 & 3 & 1 & -1 \\ -6 & 5 & 7 & -7 \\ 6 & -5 & -7 & 7 \end{bmatrix}$$

Dual ~~Min~~ Max $z = -5w_1 + 4w_2 + 3w_3 - 3w_4$

$-w_1 + 2w_2 + 3w_3 - 3w_4 \leq 3$

$-4w_1 + 3w_2 + w_3 - w_4 \leq 5$

$-6w_1 + 5w_2 + 7w_3 - 7w_4 \leq 6$

$6w_1 - 5w_2 - 7w_3 + 7w_3 \leq -6$

$w_1, w_2, w_3, w_4 \geq 0$

Q) Find dual of this prob and hence ~~give~~ find sol^n

A) $\text{Max } z = 5x_1 - 2x_2 + 3x_3$

Sub to $2x_1 + 2x_2 - x_3 \geq 2$

$3x_1 - 4x_2 \leq 3$

$x_2 + 3x_3 \leq 5$

$x_1, x_2, x_3 \geq 0$

Ans)

$$-2x_1 - 2x_2 + x_3 \leq -2$$

$$3x_1 - 4x_2 \leq 3$$

$$x_2 + 3x_3 \leq 5$$

$$\begin{bmatrix} -2 & -2 & 1 \\ 3 & -4 & 0 \\ 0 & 1 & 3 \end{bmatrix}^T$$

$$\begin{bmatrix} -2 & 3 & 0 \\ -2 & -4 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

$$\text{Min } Z = -2w_1 + 3w_2 + 5w_3$$

$$-2w_1 + 3w_2 \geq -2$$

$$-2w_1 - 4w_2 + w_3 \geq 3$$

$$w_1 + 3w_3 \geq 5$$

1 Oct 2021

Dual Simplex Method

PAGE NO.
DATE: / /

① Convert the problem into maximization

② Convert the '≥' type inequalities to '≤' type inequalities by multiplying it with -1

③ Introduce slack variables in the constraints of the given and obtain our initial basic feasible solution

④ Test nature of Δ_j in the starting table

a) If all Δ_j and X_B are non-negative then an optimal basic feasible sol^n has obtained

b) If all Δ_j are non negative and atleast one basic variable x_B is -ve, then go to step 5

c) If atleast one Δ_j is -ve, then method is inappropriate

⑤ Select the most -ve x_B ie outgoing vector (X_r)

⑥ Test nature of X_{ir}

a) If all x_{ir} are non-ve, then there does not exist any feasible sol^n

b) If atleast one x_{ir} is -ve then compute maximum of (Δ_j / x_{ir} Provided $x_{ir} \leq 0$) to determine incoming vector

PAGE NO.
DATE: / /

Ques) Min $z = 3x_1 + x_2$ sub to $x_1 + x_2 \geq 1$

$2x_1 + 3x_2 \geq 2$ and $x_1, x_2 \geq 0$

Solⁿ Max $z' = -3x_1 - x_2$ sub to $-x_1 - x_2 \leq -1$

$-2x_1 - 3x_2 \leq -2$

$x_1, x_2 \geq 0$

Standard form.

Max $z' = -3x_1 - x_2$ sub to $-x_1 - x_2 + S_1 = -1$

$-2x_1 - 3x_2 + S_2 = -2$

$S_1, S_2, x_1, x_2 \geq 0$

BV	Cost	x_1	x_2	S_1	S_2	X_B	Max $\left[\frac{\Delta_j}{x_r} : x_r < 0\right]$
		-3	-1	0	0		
S_1	0	-1	-1	1	0	-1	
← S_2 x_r	0	-2	-3	0	1	-2	Max $\left[\frac{-3}{2}, \frac{-1}{3}\right]$ $= -1/3$
$z'=0$	Δ_j	3	1	0	0		

Isme kabhi Δ_j -ve aaiga hi nahi

$R_2 \to R_2\left(-\frac{1}{3}\right)$

BV	Cost	x_1	x_2	S_1	S_2	X_B
S_1	0	$\frac{2}{3}-1$	0	1	$-1/3$	$\frac{2}{3}-1$
x_2	-1	$2/3$	1	0	$-1/3$	$2/3$

← S_1	0	$-1/3$	0	1	$-1/3$	$-1/3$	Max $\left[-7, \frac{1}{-\frac{1}{3}}\right]$
x_2	-1	$2/3$	1	0	$-1/3$	$2/3$	
	Z	$-2/3$	-1	0	$1/3$	$2/3$	Max $[-7, -1]$
$z' = -\frac{2}{3}$	$\Delta_j =$	$-\frac{2}{3}+3$ $7/3$	0	0	$\frac{1}{3}$		

PAGE NO.
DATE: / /

		-1	-2	0	0		
BV	Cost	x_1	x_2	S_1	S_2	X_B	Max $\left[\frac{-5}{13}, -2 \right]$
S_1	0	$-2+\frac{1}{7}=-\frac{13}{7}$	0	1	$-\frac{1}{7}$	-3	
x_2	-2	$\frac{1}{7}$	1	0	$-\frac{1}{7}$	1	
	z	$-\frac{2}{7}$	-2	0	$\frac{2}{7}$	-2	
	Δ_j	$\frac{5}{7}$	0	0	$\frac{2}{7}$		

		-1	-2	0	0	
BV	Cost	x_1	x_2	S_1	S_2	X_B
x_1	-1	1	0	$-\frac{7}{13}$	$\frac{1}{13}$	$\frac{21}{13}$
x_2	-2	0	1	$\frac{1}{13}$	$-\frac{2}{13}$	$\frac{10}{13}$
$z' = -\frac{41}{13}$	z	-1	-2	$\frac{5}{13}$	$\frac{3}{13}$	
	Δ_j	0	0	$\frac{5}{13}$	$\frac{3}{13}$	

$$\boxed{\text{Max } z = \frac{41}{13}}$$

$\frac{1}{7}$	1	0	$-\frac{1}{7}$	1	$-\frac{1}{7} - \frac{1}{7(13)}$
$-\frac{21}{13} - \frac{20}{13} \frac{1}{7}$	0	$-\frac{1}{13}$	$\frac{1}{7(13)}$	$\frac{3}{13}$	$\frac{-14}{7(13)}$
$-\frac{41}{13}$					$-\frac{1}{13} + \frac{4}{13}$
0	1	$\frac{1}{13}$	$-\frac{2}{13}$	$\frac{10}{13}$	$\frac{7}{13} - \frac{2}{13}$

Ques) $\text{Min } z = 6x_1 + 7x_2 + 3x_3 + 5x_4$ sub to

$$5x_1 + 6x_2 - 3x_3 + 4x_4 \geq 12$$
$$x_2 + 5x_3 - 6x_4 \geq 10$$
$$2x_1 + 5x_2 + x_3 + x_4 \geq 8$$

Ans) $\text{Max } z = -6x_1 - 7x_2 - 3x_3 - 5x_4$

$$-5x_1 - 6x_2 + 3x_3 - 4x_4 + s_1 = -12$$
$$-x_2 - 5x_3 + 6x_4 + s_2 = -10$$
$$-2x_1 - 5x_2 - x_3 - x_4 + s_3 = -8$$

BV	Cost	x_1	x_2	x_3	x_4	s_1	s_2	s_3	X_B	Ratio
		-6	-7	-3	-5	0	0	0		
s_1	0	-5	-6	3	-4	1	0	0	-12	$\frac{-6}{-5}, \frac{-7}{-6}$
s_2	0	0	-1	-5	6	0	1	0	-10	
s_3	0	-2	-5	-1	-1	0	0	1	-8	
		6	7	3	5	0	0	0		

BV	Cost	x_1	x_2	x_3	x_4	s_1	s_2	s_3	X_B	Ratio
		-6	-7	-3	-5	0	0	0		
x_2	-7	$5/6$	1	$-1/2$	$2/3$	$-1/6$	0	0	2	
s_2	0	$5/6$	0	$-11/2$	$20/3$	$-1/6$	1	0	-8	$\frac{1}{5}, \frac{-13}{11}, \frac{1}{20}$
s_3	0	$13/6$	0	$-7/2$	$7/3$	$-5/6$	0	1	2	-7
	z	$-\frac{35}{6}$	-7	$7/2$	$-14/3$	$7/6$	0	0		
	Δ_j	$1/6$	0	$13/2$	$1/3$	$7/6$	0	0		

	-2	-5	-1	-1	0	0	1	-8
	$25/6$	5	$-5/2$	$10/3$	$-5/6$	0	0	10
$\frac{7}{2}+3$	$\frac{-12+25}{6}$	0	$\frac{-2-5}{2}$	$\frac{-3+10}{3}$	$-5/6$	0	1	2
$-\frac{35}{6}+6$	$\frac{13}{6}$	$-\frac{14}{3}+5$	$-7/2$	$7/3$				

PAGE NO.
DATE: / /

$6\overline{)7}^{1.1}$
$\frac{6}{10}$

Ans
$x_1 = 0$
$x_2 = \frac{30}{11}$
$x_3 = \frac{16}{11}$
$x_4 = 0$

$\text{Min } z = \frac{258}{11}$

$\frac{-}{5} \times \frac{-}{3}$ $2(5)$

PAGE NO.
DATE : / /

		-6	-7	-3	-5	0	0		
BV	Cost	x_1	x_2	x_3	x_4	S_1	S_2	S_3	X_B
x_2	-7	0	1	5	$-18/3$	0	-1	0	10
x_1	-6	1	0	$-33/5$	8	$-1/5$	$6/5$	0	$-\frac{48}{5}$
S_3	0	0	0	$\frac{108}{10}$	15	$-2/5$	$-13/5$	1	$\frac{106}{5}$

4/10/21

TRANSPORTATION

PAGE NO.
DATE: / /

PROB

Sources \ Demand	D_1	D_2		D_n	Supply
S_1	C_{11}	C_{12}		C_{1n}	a_1
S_2	C_{21}	C_{22}		C_{2n}	a_2
⋮	⋮	⋮		⋮	⋮
S_m	C_{m1}	C_{m2}		C_{mn}	a_m
Demand	b_1	b_2		b_n	

Balanced Transportation	Unbalanced Transportation
Demand = Supply	(1) supply < demand (2) supply > demand

Dummy row and dummy column

Sol^n of Transportation Problem

1) Find initial basic feasible sol^n.
a) Vogel's Approximation method
b) North west corner rule.
c) Lowest cost entry method
d) Row minima "
e) Column "

2) Perform optimality test
a) Modi method (Modified Distribution method) U−V
b) Stepping stone method)

Vogel's Approximation Method

PAGE NO.
DATE : Penality

	W_1	W_2	W_3	W_4	Capacity				
F_1	19 [5]	30	50	10 [2]	~~7~~ 2 0	9	9	40	40
F_2	70	30	40	60 [2]	~~9~~ 2 0	10	20	20	20
F_3	40	8 [8]	70	20 [10]	~~18~~ 10 0	12	20	50	–
Requirement	5	~~18~~ 0	7	~~14~~ ~~4~~ 2					

21 22 10 10

Highest

21 – 10 10

– – 10 10

– – 10 50

NORTH WEST

Total cost

$19(5) + 10(2) + 40(7) + 60(2) + 8(8) + 20(10)$

$= 779$

–	19 [5]	30 [2]	50	10	~~7~~ 2 0
	70	30 [6]	40 [3]	60	~~9~~ 3 0
	40	8	70 [4]	20 [14]	~~18~~ ~~14~~ 0
	~~15~~ 0	~~8~~ ~~6~~ 0	~~7~~ ~~4~~ 0	14	

Total cost

$19(5) + 30(2) + 30(6) + 70(4) + 20(14) + 40(3)$

$= 1015$

Modi's Method

Q Find an optimal sol^n of

		W_1	W_2	W_3	W_4	Availability
(kg)	F1	19 [5]	30	50	10 [2]	7
	F2	70	30	40 [7]	60 [2]	9
	F3	40	8 [6]	70	20 [10]	18
Req		5	8	7	4	(VAM's)

Ans) Step 1) Find initial basic feasible sol^n Total cost = 779

Step 2) Check for non degeneracy

PAGE NO.
DATE : / /

BFS has $m+n-1$ = allocations in independent positio

rows col

$3+4-1=6$ allocations in independent

Step 3 Calculation of U_i and V_j on occupied cell by using $U_i + V_j = C_{ij}$

19			10	$U_1 = 0$
		40	60	$U_2 = 50$
	8		20	$U_3 = 10$
V_1 19	V_2 −2	V_3 −10	V_4 10	

Marc
K00

$U_1 + V_4 = C_{14}$
$0 + V_4 = 10$
$V_4 = 10$

$U_3 + V_4 = C_{34}$
$U_3 + 10 = 20$
$U_3 = 10$

$U_3 + V_2 = C_{32}$
$10 + V_2 = 8$
$V_2 = -2$

$U_2 + V_4 = C_{24}$
$U_2 + 10 = 60$
$U_2 = 50$

$U_2 + V_3 = C_{23}$
$50 + V_3 = 40$
$V_3 = -10$

$U_1 + V_1 = 19$
$0 + V_1 = 19$
$V_1 = 19$

Step 4 Calculate cost differences for unoccupied cells $d_{ij} = C_{ij} - (U_i + V_j)$

C_{ij}

•	30	50	•
70	30	•	•
40	•	70	•

$U_i + V_j$

•	−2	−10	•	0
69	48	•	•	50
29	•	0	•	10
19	−2	−10	10	

+10 = 0

Inde

Depen

PAGE NO.
DATE : / /

$d_{ij} = C_{ij} - (U_i + V_j)$

•	32	50-(-10)=60	•
1	-18	•	•
11	•	70+0=70	•

Step 5) Optimality test : $d_{ij} < 0$ x_{22} is entering the basis

Step 6) Construction of loop and allocation of unknown quantity θ

5			2
	θ^+	7	$2-\theta$
	$-\theta$ 8		$10+\theta$

Min $\{(2-\theta, 8-\theta)\} = 0$

$2-\theta = 0$

$\boxed{\theta = 2}$

19 [5]			10 [2]
	30 [2]	40 [7]	
	8 [6]		20 [12]

Total Cost =

$19\times5 + 10\times2 + 30\times2 + 40\times7 + 8\times6 + 20\times12$

743

Improved sol^n

19			10	$U_1 = -10$
	30	40		$U_2 = 22$
	8		20	$U_3 = 0$
$V_1 = 29$	$V_2 = 8$	$V_3 = 18$	$V_4 = 20$	

$V_1 - 10 = 19$

$V_1 = 29$

$V_4 + U_3 = 20$ $\boxed{V_4 = 20}$

$20 + U_1 = 10$ $\boxed{U_1 = -10}$

$V_2 + 0 = 8$ $\boxed{V_2 = 8}$

PAGE NO.
DATE: / /

$d_{ij} = C_{ij} - (U_i + V_j)$

C_{ij}

•	30	50	•
70	•	•	60
40	•	70	•

$(U_i + V_j)$

•	-2	8	•	-10
51	•	•	42	22
29	•	18	•	0
29	8	18	20	

d_{ij}

•	32	42	•
19	•	•	18
11	•	52	•

All $d_{ij} > 0$
optimal sol^n is obtained
~~at~~ Total 743

Q) Find the feasible sol^n of transportation prob.

				Supp
	28	17	26	500
	19	12	16	300
Demand	250	250	500	

sol^n

Demand > Supply then add prob is unbalanced dummy row in table T

28	17	26	500
19	12	16	300
0	0	0	200
250	250	50	1000

② If TD < TS add Dummy column

PAGE NO.
DATE : / /

				Pen		
28	17	26 [50]	500	9	9	17 ←
19	12	16	300	4	4	12
0 [200]	0 [50]	0	200	0	–	
250	250	50				
50						
19	12	16				
9	5	10				

Cost = 28 (250) + 17 (200) + 26 (50) + 16 (300)
= 7000 + 3400 + 1300 + 4800
= 16500

Ques)

				Penality		
8	7	3 [60]	600	4	4	–
3 [50]	8	9 [20]	700	5 ←	1	–
11	3 [80]	5	800	2	2	–
50	80	80	200			
50	4	2				
–	4	2				
–	–	6				

If one value we cannot calculate penality

Cost 150 + 240 + 180 + 180
150 + 240 + 360
= 750

Remark – If $m+n-1 \neq$ No. of occupied cells then there is degeneracy in Transportation prob where

$m =$ no. of rows
$n =$ no of col.

In ques

$m+n-1 = 3+3-1 = 5$ i.e. $m+n-1 \neq$ No of occupied

No of occupied cells $= 4$

Degeneracy at initial bvl. cells

8	7	3 [60]
3 [50]	8	9 [20]
11	3 [80]	5 [Δ]

$\Delta \to 0$ from unoccupied, find minimum

Min Cost $60(3) + 50(3) + 20(9) + 80(3) + 5\Delta$

Rs 750

Modi

		3	$u_1 = -6$
3		9	$u_2 = 0$
	3	5	$u_3 = -4$

$v_1 = 3 \quad v_2 = 7 \quad v_3 = 9$

Degeneracy at intermediate

$v_3 + 0 = 9$

$9 + u_3 = 5$
$u_3 = -4$

$v_2 - 4 = 3$
$v_2 = 7$

$9 + u_1 = 3$
$u_1 = -6$

$v_1 + 0 = 3$
$v_1 = 3$

D_{ij}

8	7	0
0	8	0
11	0	5

-3	1	0
0	7	0
-1	0	0

PAGE NO.
DATE: / /

11	6	.
.	1	.
12	.	.

Sol^n is optimal.

Resolution of Degeneracy at intermediate stage

						Supply
	4 [30]	3 [10]	1	2	6	~~40~~ ~~10~~ 0
	5	2 [20]	3 [10] [5]	4	5	~~30~~ ~~10~~ 0
	3	5	6	3 [15]	2	20
	2	4	4	5 [5]	3 [5]	10
Demand	~~30~~ 0	~~30~~ 0	15	20	5	

Step 1 Apply NWCR to find IBFS

Total Cost $= 4\times30 + 3\times10 + 2\times20 + 3\times10 + 6\times5 + 3\times15 + 5\times5 + 3\times5 =$ ~~302~~ 335

Step 2 $m+n-1$ = allocations on independent positions $4+5-1 = 8$

DATE : / /

4	3				$u_1 = 0$
	2	3			$u_2 = -1$
		6	3		$u_3 = 2$
			5	3	$u_4 = 4$
$v_1 = 4$	$v_2 = 3$	$v_3 = 4$	$v_4 = 1$	$v_5 = -1$	

Step 4 $\quad d_{ij} = C_{ij} - (u_i + v_j)$

C_{ij}

•	•	1	2	6
5	•	•	4	5
3	5	•	•	2
2	4	4	•	•

$u_i + v_j$

•	•	4	1	-1	0
3	3			-2	-1
6	5	•	•	1	2
8	7	8	•	•	4
		4	1	-1	

d_{ij}

•	•	-3	1	7
2	•	•	4	7
-3	0	•	•	1
-6	-3	-4	•	•

Optimality test $d_{ij} < 0$

$30^{-\theta}$		$10^{+\theta}$		
		$20-\theta$	$10^{+\theta}$	
			$5^{-\theta}$	$15^{+\theta}$
θ^{+}				$5^{-\theta}$ 5

Min $\{(30-\theta, 20-\theta, 5-\theta\} = 0$ $\quad 5-\theta = 0$

$\boxed{\theta = 5}$

PAGE NO.
DATE: / /

4 25	3 15		2 $\boxed{\Delta}$	
	2 15	3 15		
			3 20	
2 5				3 5

Total cost = $4(25) + 3(15) + 2(15) + 3(15) + 3(20) + 2(5) + 3(5)$

$= 305$

Find u_i and v_j

4	3		2		$u_1 = 0$
	2	3			$u_2 = -1$
			3		$u_3 = 1$
2				3	$u_4 = -2$
$v_1 = 4$	$v_2 = 3$	$v_3 = 4$	$v_4 = 2$	$v_5 = 5$	

$v_4 + 0 = 2$ $\boxed{v_4 = 2}$

$2 + u_3 = 3$ $\boxed{u_3 = 1}$

$\boxed{v_1 = 4}$

$v_5 - 2 = 3$ $\boxed{v_5 = 5}$

$4 + u_4 = 2$ $\boxed{u_4 = -2}$

$3 + u_2 = 2$ $u_2 = -1$

$v_3 - 1 = 3$ $\boxed{v_3 = 4}$

a_{ij}

c_{ij}

•	•	1	•	6
5	•	•	4	5
3	5	6	•	2
2	4	4	5	•

$u_i + v_j$

•	•	4	•	5	0
3	•	•	1	4	-1
5	4	5	•	6	1
2	1	2	0	•	-2
4	3	4	2	5	

Ans 210

PAGE NO.
DATE: / /

d_{ij}

0	0	-3	0	1
2	1	0	3	1
-2	1	1	0	-4 θ
0	3	2	5	0

Not optimal

+θ 25	-θ 15		+θ Δ	
	15	15	-θ 20	θ+ 5 +θ
5 -θ				

$\{(5-\theta,\ 15-\theta,\ 20-\theta)\}$ $\boxed{\theta = 5}$

430	3 10		Δ	
	2 15	3 15	3 15	5
0				3 10

$120 + 30 + 30 + 45 + 45 + 30$

$180 + 75 + 45 + 30$

$180 + 75 + 75$

$150 + 180$

330

Ans 240

11/10/21

Assignment Prob

PAGE NO.
DATE: / /

(Hungarian Method) Subordinates

	I	II	III	IV
A	8	26	17	11
B	13	28	4	56
C	38	19	18	15
D	19	26	24	10

How task is allocated to each person so at to minimise the man hours

Step 1 Sub each element from smallest element of that row.

0	18	9	3
9	24	0	22
23	4	3	0
9	16	14	0

Row Reduction

Step 2 Sub smallest element in each coloumn from ~~of~~ every element of that ~~row~~ Col

	I	II	III	IV
A	0	14	9	3
B	9	20	0	22
C	23	0	3	0
D	9	12	14	0

Row wise then column bhi check

Coloumn Reduction

No. of assignments = ~~No of~~ Order of matrix

So optimal

A – I , B – III , C – II , D – IV

8 + 4 + 19 + 10 = 41 hours

2.) A

18	26	17	11
13	28	14	26
38	19	18	15
19	26	24	10

PAGE NO.
DATE : / /

7	15	6	0
0	15	1	13
23	4	3	0
9	16	14	0

Row

kahi assigned 0 pada hai uss column me

7	11	5	0	③ ✓
0	11	0	13	
23	0	2	0	
9	12	13	0	✓ ① koi bhi assign ni hai

② koi covered 0

Column

Row 4 and col 3 do not have any assignment so it is not optimal ∴ we improve matrix

Unticked rows
Ticked rows

Element

Covered

line (Same)

intersection + x

Uncovered − x

Least element
x = 5

Multiple Optimal Sol^n

PAGE NO.
DATE: / /

	I	II	III	IV
A	2	6	$\boxed{0}$	~~0~~
B	$\boxed{0}$	11	~~0~~	18
C	23	$\boxed{0}$	2	5
D	4	7	8	$\boxed{0}$

A—III B—I C—II D—IV

17 + 13 + 19 + 10

30 + 29

59

Multiple Optimal Sol^n

2	5	4	0
4	3	5	0
5	0	6	0
2	2	4	0

Row reduction

2	5	4	0
4	3	5	0
5	0	6	0
2	2	4	0

Column Reduction

0	5	0	~~0~~
2	3	1	$\boxed{0}$
3	$\boxed{0}$	2	0 ~~0~~
0	2	0	0 ~~0~~

Multiple optimal sol^n

$\boxed{0}$	5	~~0~~	0 ~~0~~
2	3	1	$\boxed{0}$
3	$\boxed{0}$	2	0 ~~0~~
~~0~~	2	$\boxed{0}$	0 ~~0~~

~~0~~	5	$\boxed{0}$	~~0~~
2	3	1	$\boxed{0}$
3	$\boxed{0}$	2	0 ~~0~~
$\boxed{0}$	2	0 ~~0~~	0 ~~0~~

TRAVELLING SALESMAN PROB-

PAGE NO.
DATE : / /

	A	B	C	D	E
A	∞	2	5	7	1
B	6	∞	3	8	2
C	8	7	∞	4	7
D	12	4	6	∞	5
E	1	3	2	8	∞

Row Reduction

A	∞	1	4	6	0
B	4	∞	1	6	0
C	4	3	∞	0	3
D	8	0	2	∞	1
E	0	2	1	7	∞

Column Reduction

A	∞	1	3	6	$\boxed{0}$
B	4	∞	$\boxed{0}$	6	0 X
C	4	3	∞	$\boxed{0}$	3
D	8	$\boxed{0}$	1	∞	1
E	$\boxed{0}$	2	0 X	7	∞

Assignm Optimal Order = Assignm

A → E B → C C → D

D — B E — A

Travelling X

	A	B	C	D	E
A	∞	$\boxed{1}$	3	6	0 X
B	4	∞	$\boxed{0}$	6	$\boxed{0}$
C	4	3	∞	$\boxed{0}$	3
D	8	0 X	1	∞	$\boxed{1}$
E	$\boxed{0}$	2	X0	7	∞

A — B, B — C, C — D, D — E

Cost = 2 + 3 + 4 + 5 + 1

15

NUMERICAL METHOD

Algebraic Eq^{n}s - If an eq^n $f(x)=0$ involves purely polynomials, then eq^n is algebraic eq^n

PAGE NO.
DATE: / /

Eg) $x^2+x+1=0$ $x^4+3=0$

Transcedental eq^n - which involves trigo, log, expo

$x+\cos x=0$ $x-e^{\frac{x}{2}}=0$ etc

METHODS

1) Bisection / Bolzano method Converges 0.5
2) Newton Raphson method Convergence Rate 2

Bisection Method

Based on intermediate value theorem which states continuous function on $[a,b]$ $f(a)<0$ and $f(b)>0$ then $\exists$ atleast one real root of $f(x)=0$ in interval (a,b)

Q) Find real root of $x^4-x-10=0$ using Bisection or direct method

Ans) $f(0) = -10 < 0$
$f(1) = 1-1-10 < 0$
$f(1.5) = -6.4375 < 0$
$f(2) = 4 > 0$

So root lies in interval $[1.5, 2)$

Itteration 1 Take $a=1.5$ $b=2$

Ist approx to root $x_1 = \frac{a+b}{2} = 1.75$

Now $f(1.75) = \cancel{(1.75)^4}$ <0
$f(2)$ >0

So real root lies b/w $(1.75, 2)$

Iteration 2 $a = 1.75$ $b = 2$ $x = 1.856$

PAGE NO.
DATE: / /

2nd approx $x_2 = \frac{1.75 + 2}{2} = 1.875$

$f(1.875) = 0.4846 > 0$
$f(1.75) = -2.3711 < 0$

Roots lie b/w $(1.75, 1.875)$

Iteration - 3 Taking $a = 1.75$, $b = 1.875$

3rd approx given by $x_3 = \frac{1.75 + 1.875}{2} = 1.8125$

$f(1.8125) < 0$
$f(1.875) > 0$ Root lie b/w (1.

$x^3 - 4x - 9 = 0$ 2.6875

$x \log_{10} x = 1.2$ 2.7407

$3x - \cos x - 1 = 0$ By Newton Rap

The process of finding the value of y corresponding to any value of x between x_0 and x_n is called interpolation. Thus the technique of computing intermediate values of function from a given set of tabular values of the function is called interpolation.

27

Assignment

PAGE NO.
DATE:

Langrange's Multiple Method
1) Dynamic Prog
2) knapsack Prog
3) Quadratic Prog
Kulin Tuker condition
Wogle's Method
4) Integer Prog
5) Stockest Prog

UNIT 3

Lagrange's Multiplier Method

Ques) Solve the NLPP
Non Linear Prog Prob

$$\text{Max } z = 4x_1 - x_1^2 + 8x_2 - x_2^2$$

Sub to

$$x_1 + x_2 = 2$$

$$x_1, x_2 \geq 0$$

Sol^n Langrangian function can be written as

$$\mathcal{L}(x_1, x_2, \lambda) = \underbrace{(4x_1 - x_1^2 + 8x_2 - x_2^2)}_{\text{Obj function}} - \lambda \underbrace{(x_1 + x_2 - 2)}_{\text{Constraint}} = 0$$

The necessary condition for Maxima and Minima are

$$\frac{\partial \mathcal{L}}{\partial x_1} = 4 - 2x_1 - \lambda \quad \text{--- (1)}$$

$$\frac{\partial \mathcal{L}}{\partial x_2} = 8 - 2x_2 - \lambda \quad \text{--- (2)}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = -(x_1 + x_2 - 2) \quad \text{--- (3)}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = -h(x)$$

$$h(x) = x_1 + x_2 - 2$$

(No of minors = Degree)

PAGE NO.
DATE : / /

Solving (1) and (2)

$$\begin{array}{r} 4-2x_1-\cancel{\lambda} \\ 8-2x_2-\cancel{\lambda} \\ - \quad + \quad + \\ \hline -4-2x_1+2x_2 \end{array}$$

$4-2x_1=\lambda$, $8-2x_2=\lambda$

$4-2x_1=8-2x_2$

$2(x_2-x_1)=4$

$x_2-x_1=2$ —— (4)

Solve (3) and (4)

$$\begin{array}{r} -x_1-x_2+2 \\ -x_1+x_2-2 \\ \hline -2x_1 \qquad =0 \end{array}$$

$x_1=0$, $x_2=2$ Extreme pts

$\lambda=4$

Sufficient Condition

—
Indefinate

All negative
Minima

+ — + —
Maxima

How to find determinant

PAGE NO.
DATE: / /

The necessary and sufficient condition provide optimal sol^n of prob, the sufficient condition for determinant whether the sol^n results in maxima or minima of the obj function involves sol^n of $(n-1)$ principal minors of the following determinant

$$D_{n+1} = \begin{bmatrix} 0 & \frac{\partial h}{\partial x_1} & \frac{\partial h}{\partial x_2} & \cdots & \frac{\partial h}{\partial x_n} \\ \frac{\partial h}{\partial x_1} & \frac{\partial^2 f}{\partial x_1^2} - \lambda \frac{\partial^2 h}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} - \lambda \frac{\partial^2 h}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_1 x_n} - \lambda \\ \frac{\partial h}{\partial x_2} & \frac{\partial^2 f}{\partial x_2 \partial x_1} - \lambda \frac{\partial^2 h}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} - \lambda \frac{\partial^2 h}{\partial x_2^2} & \cdots & \\ \frac{\partial h}{\partial x_3} & \frac{\partial^2 f}{\partial x_3 \partial x_2} - \lambda \frac{\partial^2 h}{\partial x_3 x_2} & \frac{\partial^2 f}{\partial x_3^2} - \lambda \frac{\partial^2 h}{\partial x_3^2} & & \frac{\partial^2 f}{\partial x_3^2} - \lambda \frac{\partial^2 h}{\partial x_3^2} \\ \frac{\partial h}{\partial x_n} & \frac{\partial^2 f}{\partial x_n \partial x_1} - \lambda \frac{\partial^2 h}{\partial x_n \partial x_1} & \frac{\partial^2 f}{\partial x_n \partial x_2} - \lambda \frac{\partial^2 h}{\partial x_n \partial x_2} & \cdots & \end{bmatrix}$$

$$D_3 = \begin{bmatrix} 0 & 1 & 1 \\ 1 & -2 & 0 \\ 1 & 0 & -2 \end{bmatrix}$$

$4 - 2x_1$
$-(4)$

$8 - 2x_2$
0

$$0 - 1(-2) + 1(2)$$
$$4$$

// Positive hai

Since D_3 is positive the sol^n $x_1 = 0, x_2 = 2$ maximises the obj function and

$$Z_{max} = 4(0) - 0 + 16 - 4$$
$$= 12$$

PAGE NO.
DATE : / /

Ques Solve the following NLPP

$$\text{Max } Z = 5x_1 + x_2 - (x_1 - x_2)^2 \quad \text{St } x_1 + x_2 = 4$$
$$x_1, x_2 \geq 0$$

Ans) $(5x_1 + x_2 - (x_1 - x_2)^2) - \lambda(x_1 + x_2 - 4) = 0$

$$\frac{\partial L}{\partial x_1} = 5 - 2(x_1 - x_2) - \lambda \quad \text{——— (1)}$$

$$\frac{\partial L}{\partial x_2} = 1 \cancel{+2x_2} + 2(x_1 - x_2) - \lambda \quad \text{——— (2)}$$

$$\frac{\partial L}{\partial \lambda} = -(x_1 + x_2 - 4) \quad \text{——— (3)}$$

$$h(x) = x_1 + x_2 - 4 \quad \text{——— (4)}$$

Solving $\lambda = 5 - 2(x_1 - x_2)$

$$0 = 1 + 2(x_1 - x_2) - 5 + 2(x_1 - x_2)$$

$$1 + 4x_1 - 4x_2 - 5 = 0$$

$$4x_1 - 4x_2 - 4 = 0$$

$$\boxed{x_1 - x_2 = 1}$$

$$\boxed{x_1 = 1 + 1.5 = 2.5}$$

$$-\cancel{x_1} - x_2 + 4 = 0$$
$$\cancel{x_1} - x_2 - 1 = 0$$
$$-2x_2 + 3 = 0$$

$$\boxed{x_2 = 1.5}$$

$5-2(1)-\lambda=0$

$3-\lambda=0$

$\boxed{\lambda=3}$

PAGE NO.
DATE: / /

$$D=\begin{bmatrix}0 & 1 & 1\\ 1 & -2 & 2\\ 1 & 2 & -2\end{bmatrix}$$

$-1(-2-2)+1(2+2)$

$+4+4$

8

$$\frac{\partial^2 f}{\partial x_1^2}-\lambda\frac{\partial^2 h}{\partial x_1^2}$$

$5-2(x_1-x_2)-(3)(0)$

-2

$$\frac{\partial^2 f}{\partial x_1 \partial x_2}-\lambda\frac{\partial^2 h}{\partial x_1 \partial x_2}$$

$1-2(x_1-x_2)$

$2-(3)(0)$

$$\frac{\partial^2 f}{\partial x_2 \partial x_1}-\lambda\frac{\partial^2 h}{\partial x_2 \partial x_1}$$

$2-2(0)$

Memorization

DYNAMIC PROGRAMMING

Gives ans in stages

Q) Use DP to solve

$\text{Max } Z = 3x_1+5x_2$

sub to $x_1 \leq 4$ b_1; $x_2 \leq 6$ b_2; $3x_1+2x_2 \leq 18$ b_3 $x_1, x_2 \geq 0$

Sol^n There are 2 variables and hence the prob can be treated as 2 stage dyanamic programming prob both x_1 and x_2 being continuous represent^t ∞ no. of alternatives within feasible space the 3 constraints can be regarded as 3 resources say b_1, b_2 and b_3 which are to be allocated to x_1 and x_2 at diff. stages the state of the system at a stage would be given by the amount of resources allocated at that stage and left for remaining stages. The states of equivalent ~~dyanam~~ dyanamic prog prob (DPP)

No of variables = No. of stages

& Constrains = Resources

PAGE NO.
DATE: / /

Stage I

$b_1 = 4, \quad b_2 = 6, \quad b_3 = 18$

The optimal value of $f_1(b_1, b_2, b_3) = \max\limits_{0 \le x_1 \le b} [3x_1]$

The feasible value of x_1 is non-ve and satisfy all 3 constraints but the maximum value of b that x_1 can assume is

$$\min\left(\frac{\overset{b_1}{4}}{1}, \frac{\overset{b_2}{6}}{0}, \frac{\overset{b_3}{18}}{3}\right) = 4$$

coefficient of x_1 in first constrain

coefficient of x_1 in second const

$$\therefore f_1(4, 6, 18) = \max\limits_{0 \le x_1 \le 4} [3x_1]$$

Second me x_1 hai nahi to 0 ni likhenge

$$= 3 \min\left[4, \frac{18 - 2x_2}{3}\right]$$

First constrain me x_1 ki value

Stage II

The recursive eq^n for optimization of this 2 stage prob is

$$f_2(4, 6, 18) = \max\limits_{0 \le x_2 \le b} [5x_2 + 3x_1]$$

but maximum value of b that x_2 can assume is

$$\min\left\{\frac{6}{1}, \frac{18}{2}\right\} = 6$$

$4 = \frac{18 - 2x_2}{3}$

$x_2 = 3$

Breakpt

$$f_2(4, 6, 18) = \max\limits_{0 \le x_2 \le 6} \left[5x_2 + 3\min\left(4, \frac{18 - 2x_2}{3}\right)\right]$$

PAGE NO.
DATE: / /

$$\min\left(4, \frac{18-2x_2}{3}\right) = \begin{cases} 4 & 0 \le x_2 \le 3 \\ \frac{18-2x_2}{3} & 3 \le x_2 \le 6 \end{cases}$$

$$5x_2 + 3\min\left(4, \frac{18-2x_2}{3}\right) = \begin{cases} 12+5x_2 & 0 \le x_2 \le 3 \\ 18+3x_2 & 3 \le x_2 \le 6 \end{cases}$$

$$\max_{0 \le x_2 \le 6} \begin{cases} 12+5(3) = 27 & 0 \le x_2 \le 3 \\ 36 & 3 \le x_2 \le 6 \end{cases}$$

$\boxed{f_2 = 36}$ $x_2 = 6$

$$\min\left[4, \frac{\overset{2}{\cancel{6}}}{\cancel{3}}\right] \quad \boxed{x_1 = 2}$$

Ques) Solve the following LPP by DP.

$\text{Max } z = 2x_1 + 5x_2$ Sub to $2x_1 + x_2 \le 430$

$2x_2 \le 460$

$x_1, x_2 \ge 0$

Ans) Stage I $b_1 = 430$, $b_2 = 460$

Optimal value of $f_1(b_1, b_2) = \max[2x_1]$

$$x_1 = \min\left(\frac{430}{2}, \frac{460}{0}\right) = 215$$

$$f_1(430, 460) = \max_{0 \le x_1 \le 215}[2x_1] = 2\min\left[\frac{430-x_2}{2}\right]$$

PAGE NO.
DATE : / /

$\frac{430-x_2}{2}$

$2 = 430 - x_2$

$x_2 = 430 - 2$

$x_2 = 428$

Stage 2

$$f_2(430, 460) = \max\left[5x_2 + 2x_1\right]$$

$$\min\left[430 - 2x_1, \cancel{\frac{460}{2}}\ 230\right]$$

$$f_2(430, 460) = \max_{0 \le x_2 \le 230}\left[5x_2 + 2\min\left[\frac{430 - x_2}{2}\right]\right]$$

$$\cancel{\max\left[\frac{430 - x_2}{2}\right] =}$$

$$\min\left[\frac{430 - x_2}{2}\right]$$

Put $x_2 = 230$

$$\frac{430 - 230}{2} = \frac{200}{2} = 100$$

$$f_2(430, 460) = \max_{0 \le x_2 \le 230}\left[5x_2 + 2(100)\right]$$

$$\max_{0 \le x_2 \le 230}(5x_2 + 200)$$

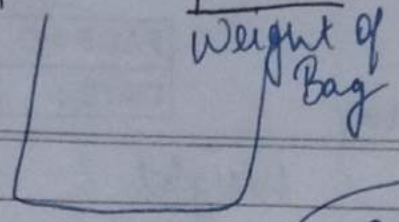
$$5(230) + 200$$

$$1150 + 200$$

$\boxed{x_2 = 230}$ $\boxed{f_2 = 1350}$

$$x_1 = \left(\frac{430 - 230}{2}\right) = 100$$

Knapsack prob -



PAGE NO.
DATE: / /

Objects → Weight
Objects → Profit → Maximise profit
Objects → Divisible → fractional → Profit max, Weight min, Greedy (Ratio)
Objects → Non Divisible → 0/1
(Books can be divided)

Q) Solve the Fractional Knapsack Prob

Obj	1st	2nd	3rd	4	5	6	7
profit	5	10	15	7	8	9	4
Weight	1	3	5	4	1	3	2

Capacity = 15 — Matlab bag me max itna aa sakta hai
N = 7 — Inn saato me se koi bhi daal do

Ans) Profit Maximization

Obj	Profit	Weight	Remaining Weight
3	15	5	15 − 5 = 10
2	10	3	10 − 3 = 7
6	9	3	7 − 3 = 4
5	8	1	4 − 1 = 3
4	7/4 × 3	~~1/4~~ 3	3 − 3 = 0
Igore this 1	5	1	
Igore this 7	4	2	

47.45

Drawback as we are leaving other objects. unka profit zayada tha tab bhi

19 27
27 16
46

Equal weight then profit max

PAGE NO.
DATE: / /

Weight mini

Obj	Profit	Weight	Remaining Weight
5	8	1	15−1=14
1	5	1	14−1=13
7	4	2	13−2=11
2	10	3	11−3=8
6	9	3	8−3=5
4	7	4	5−4=1
3	15	5	1−1=0
	5		
	46		

Ratio $\frac{\text{Profit}}{\text{Weight}}$

GREEDY APPROACH

P/w 5 $10/3 = 3.33$ 3 $7/4 = 1.\ldots$ 8 3 2

Obj	Profit	Weight	Profit/Weight	Remaining
5	8	1	8	15−1=14
1	5	1	5	13
2	10	3	3.33	10
3	15	5	3	5
6	9	3	3	2
7	4	2	2	0
4	7	4	7/4	
	51			

13
25
1

24
25
51

Q)

Capa = 15

Obj	1	2	3	4	5	6	7
Profit	10	5	15	7	6	18	3
weight	2	3	5	7	1	4	1
P/w	5	$5/3 = 1.\ldots$	3	1	6	4.5	3

Any

PAGE NO.
DATE: / /

Obj	Profit	Wei	P/w		
5	6	1	6		14
1	10	2	5		12
6	18	4	4.5		8
3	15	5	3		3
7	3	1	3		2
2	$\frac{5}{3} \times 2$	3	$5/3$		0

3.33

28
15
6
6.33
55.33

55.33

QUADRATIC PROGRAMING

Wolfe's Method (To solve quad prog)
OR
Wolfe's Modified Simplex Method

QPP
- → Kuhn Tucker condition
- → eq^n
- → Add Artificial variables
- → LPP (New objective function
- → Apply 2 phase simplex method to solve LPP
- → Sol^n

Power² multiplication constant² variables

PAGE NO.
DATE: / /

Q) Apply Wolfe's Method to solve QPP

$$\text{Max } z = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$$

Sub to

$$x_1 + 2x_2 \leq 2$$
$$x_1, x_2 \geq 0$$

Step 1 – Firstly convert objective funct. to Max form if Mini multiply by -1

$$\text{Max } z = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$$

Step 2 – Write all constraints & inequalities with $\leq$ sign

$$x_1 + 2x_2 \leq 2$$
$$-x_1 \leq 0$$
$$-x_2 \leq 0$$

Step 3 – Convert inequalities into eq^n's by introducing slack variables i.e q_1^2, r_1^2, r_2^2

$$x_1 + 2x_2 - 2 + q_1^2 = 0$$
$$-x_1 + r_1^2 = 0$$
$$-x_2 + r_2^2 = 0$$

Step 4 – To obtain Kuhn Tucker conditions, we construct Langrangian function

$$L(x_1, x_2, q_1, r_1, r_2, \lambda_1, \mu_1, \mu_2)$$
$$(4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2) - \lambda_1(x_1 + 2x_2 + q_1^2 - 2)$$
$$- \mu_1(-x_1 + r_1^2) - \mu_2(-x_2 + r_2^2) = 0$$

PAGE NO.
DATE:

$$\frac{\partial L}{\partial x_1} = 4 - 2x_2 - \lambda_1 + \mu_1 - 4x_1$$

$$\frac{\partial L}{\partial x_2} = 6 - 2x_1 - 2\lambda_1 + \mu_2 - 4x_2$$

$$\frac{\partial L}{\partial \lambda_1} = -(x_1 + 2x_2 + q_1^2 - 2)$$

Eqn's

Original prob ~~constraints~~ me jitte variables ho. Aur constraint me kamsa multiplier hai

$$4x_1 + 2x_2 + \lambda_1 - \mu_1 = 4, \quad 2x_1 + 4x_2 + 2\lambda_1 - \mu_2 = 6$$

$$x_1 + 2x_2 + q_1^2 = 2$$

Assume $q_1^2 = s_1$

$$x_1 + 2x_2 + s_1 = 2$$

ye entering ne hoga, ye leaving hai

and $\mu_1 x_1 = 0, \quad \mu_2 x_2 = 0, \quad \lambda_1 s_1 = 0$

koi eko hona chahiye

Complementary Slackness Condition

Step 5 To construct the modified linear programming prob introducing artifical variables v_1 and v_2. Modified LPP becomes

$$\text{Max } z^* = -v_1 - v_2$$

Sub to

$$4x_1 + 2x_2 + \lambda_1 - \mu_1 - v_1 = 4$$
$$2x_1 + 4x_2 + 2\lambda_1 - \mu_2 - v_2 = 6$$
$$x_1 + 2x_2 + s_1 = 2$$

where variables are non-ve

μ_1 basic BV me nii hona chahiye

PAGE NO. -1
DATE: / /

BV	Cost	x_1	x_2	λ_1	μ_1	μ_2	s_1	v_1	v_2	X_B	Ratio
		0	0	0	0	0	0				
v_1	-1	$\boxed{4}$	2	1	-1	0	0	1	0	4	1
v_2	-1	2	4	2	0	-1	0	0	1	6	3
s_1	0	1	2	0	0	0	1	0	0	2	2
	z	-6	-6	-3	1	1	0	-1	-1		
	Δj	-6	-6	-3	1	1	0	0	0		

BV	Cost	x_1	x_2	λ_1	μ_1	μ_2	s_1	v_1	v_2	X_B	Ratio
		0	0	0	0	0	0	-1	-1		
x_1	0	1	$1/2$	$1/4$	$-1/4$	0	0	$1/4$	0	1	2
v_2	-1	0	3	$3/2$	$1/2$	-1	0	$-1/2$	1	4	$4/3 = 1.3$
s_1	0	0	$\boxed{3/2}$	$-1/4$	$1/4$	0	1	$-1/4$	0	1	$2/3 = 0.6$
$z = -4$	z	0	-3	$-3/2$	$-1/2$	1	0	$1/2$	-1		
	Δj	0	-3	$-3/2$	$-1/2$	1	0	$3/2$	0		

								-1	-1		
BV	Cost	x_1	x_2	λ_1	μ_1	μ_2	S_1	V_1	V_2	X_B	Ratio
x_1	0	1	0	$\frac{1}{3}$	$-\frac{1}{3}$	0	$-\frac{1}{3}$	$\frac{1}{3}$	0	$\frac{2}{3}$	2
V_2	-1	0	0	2	0	-1	-2	0	1	2	1
x_2	0	0	1	$-\frac{1}{6}$	$\frac{1}{6}$	0	$\frac{2}{3}$	$-\frac{1}{6}$	0	$\frac{2}{3}$	-4
	z	0	0	-2	0	1	2	0	-1		
	Δj	0	0	-2	0	1	2	1	0		

PAGE NO.
DATE: / /

BV	Cost	x_1	x_2	λ_1	μ_1	μ_2	S_1	V_1	V_2	X_B	Ratio
x_1	0	1	0	0	$-\frac{1}{3}$	$\frac{1}{6}$	0	$\frac{1}{3}$	$-\frac{1}{6}$	$\frac{1}{3}$	
λ_1	~~5~~ 0	0	0	1	0	$-\frac{1}{2}$	-1	0	$\frac{1}{2}$	1	
x_2	0	0	1	0	$\frac{1}{6}$	$-\frac{1}{12}$	$\frac{1}{2}$	$-\frac{1}{6}$	$\frac{1}{12}$	$\frac{5}{6}$	

$Z =$ 1 1

$$\boxed{x_1 = \frac{1}{3}} \qquad \boxed{x_2 = \frac{5}{6}}$$

$$Z = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$$

$$4\left(\frac{1}{3}\right) + 5 - 2\left(\frac{1}{9}\right) - 2\left(\frac{5}{18}\right) - 2\left(\frac{25}{36}\right)$$

$$\frac{4}{3} + 5 - \frac{2}{9} - \frac{5}{9} - \frac{25}{18}$$

$$\frac{24 + 90 - 4 - 10 - 25}{18}$$

$$\frac{-1 - 14 + 90}{18}$$

$$\frac{\cancel{90}\ 15}{75}$$

$$\frac{25}{\cancel{15}\ 6}$$

$$\frac{\cancel{18}}{6}$$

$$\frac{2}{3} + \frac{1}{6}$$

$$\frac{4+1}{6}$$

$$\frac{2}{3} - \frac{1}{6}$$

$$\frac{4-1}{2}$$

$$\boxed{Z = \frac{25}{6}}$$

0	1	0	$\frac{1}{3}$	$-\frac{1}{3}$	0	$-\frac{1}{3}$	$\frac{1}{3}$	0	$\frac{2}{3}$		
0	0	0	$\frac{1}{3}$	0	$-\frac{1}{6}$	$-\frac{1}{3}$	0	$\frac{1}{6}$	$\frac{1}{3}$		
0	1	0	0	$-\frac{1}{3}$	$\frac{1}{6}$	0	$\frac{1}{3}$	$-\frac{1}{6}$			
0	0	1	$-\frac{1}{6}$	$\frac{1}{6}$	0	$\frac{2}{3}$	$-\frac{1}{6}$	0	$\frac{2}{3}$		
0	0	0	$-\frac{1}{6}$	0	$\frac{1}{12}$	$\frac{1}{6}$	0	$-\frac{1}{12}$	$-\frac{1}{6}$		
0	0	1	0	$\frac{1}{6}$	$-\frac{1}{12}$	$\frac{1}{2}$	$-\frac{1}{6}$	$\frac{1}{12}$	$\frac{5}{6}$		

PAGE NO.
DATE: / /

Que) Apply wolfe's method for following QPP

~~Max z = $4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$ Sub to $x_1 + 2x_2 \leq 2$~~
~~$x_1, x_2 \geq 0$~~

Max z = $2x_1 + 3x_2 - 2x_1^2$ sub to $x_1 + 4x_2 \leq 4$ $x_1 + x_2 \leq 2$
$x_1, x_2 \geq 0$

Ans)
$$x_1 + 4x_2 + q_1^2 - 4 = 0$$
$$x_1 + x_2 + q_2^2 - 2 = 0$$
$$-x_1 + r_1^2 = 0$$
$$-x_2 + r_2^2 = 0$$

$$L(x_1, x_2, q_1, q_2, r_1, r_2, \lambda_1, \lambda_2, \mu_1, \mu_2)$$

$$(2x_1 + 3x_2 - 2x_1^2) - \lambda_1(x_1 + 4x_2 - 4 + q_1^2) - \lambda_2(x_1 + x_2 - 2 + q_2^2)$$
$$- \mu_1(-x_1 + r_1^2) - \mu_2(-x_2 + r_2^2) = 0$$

$$\frac{\partial L}{\partial x_1} = 2 - 4x_1 - \lambda_1 - \lambda_2 + \mu_1$$

$$\frac{\partial L}{\partial x_2} = 3 - 4\lambda_1 - \lambda_2 + \mu_2$$

$$\frac{\partial L}{\partial \lambda_1} = -x_1 - 4x_2 + 4 - q_1^2$$

$$\frac{\partial L}{\partial \lambda_2} = -x_1 - x_2 + 2 - q_2^2$$

$2 = 4x_1 + \lambda_1 + \lambda_2 - \mu_1$ ✓ $3 = 4\lambda_1 + \lambda_2 - \mu_2$ ✓

$4 = x_1 + 4x_2 + q_1^2$ $2 = x_1 + x_2 + q_2^2$

$4 = x_1 + 4x_2 + s_1$ ✓ $2 = x_1 + x_2 + s_2$ ✓

$\frac{7}{2} \times \frac{1}{4}$

Max $Z = -V_1 - V_2$

$\mu_1 x_1 = 0$ $\mu_2 x_2 = 0$ $\lambda_1 S_1 = 0$ $\lambda_2 S_2 = 0$

PAGE NO.
DATE: / /

$$4x_1 + \lambda_1 + \lambda_2 - \mu_1 - V_1 = 2$$
$$4\lambda_1 + \lambda_2 - \mu_2 - V_2 = 3$$
$$x_1 + 4x_2 + S_1 = 4$$
$$x_1 + x_2 + S_2 = 2$$

since S_1 is in BV so cannot enter

BV	Cost	x_1	x_2	λ_1	λ_2	μ_1	μ_2	V_1	V_2	S_1	S_2	X_B	Ra
								-1	-1				
V_1	-1	$\boxed{4}$	0	1	1	-1	0	-1	0	0	0	2	$\frac{2}{4}$
V_2	-1	0	0	4	1	0	-1	0	-1	0	0	3	$\frac{3}{4}$
S_1	0	1	4	0	0	0	0	0	0	1	0	4	$\frac{4}{1}$
S_2	0	1	1	0	0	0	0	0	0	0	1	2	$\frac{2}{1}$
	$z =$	-4	0	-5	-2	1	1	1	1	0	0		
	$\Delta j =$	-4	0	-5	-2	1	1	2	2	0	0		

BV	Cost	x_1	x_2	λ_1	λ_2	μ_1	μ_2	V_1	V_2	S_1	S_2	X_B	Rat
								-1	-1				
x_1	0	1	0	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	0	$-\frac{1}{4}$	0	0	0	$\frac{1}{2}$	-
V_2	-1	0	0	4	1	0	-1	0	-1	0	0	3	-
S_1	0	0	4	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	0	$\frac{1}{4}$	0	1	0	$\frac{4-\frac{1}{2}}{\frac{7}{2}}$	$\frac{7}{8}$
S_2	0	0	1	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	0	$-\frac{1}{4}$	0	0	1	$\frac{1}{2}$	2
	z	0	0	-4	-1	0	1	0	1	0	0		
	Δj	0	0	-4	-1	0	1	1	2	0	0		

PAGE NO.
DATE : / /

BV	Cost	x_1	x_2	λ_1	λ_2	μ_1	μ_2	v_1	v_2	s_1	s_2	X_B	Ratio
								-1	-1				
x_1	0	1	0	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	0	$-\frac{1}{4}$	0	0	0	$\frac{1}{2}$	2
v_2	-1	0	0	4	1	0	-1	0	-1	0	0	3	$\frac{3}{4}$
x_2	0	0	1	$-\frac{1}{16}$	$-\frac{1}{16}$	$\frac{1}{16}$	0	$\frac{1}{16}$	0	$\frac{1}{4}$	0	$\frac{7}{8}$	=14
s_2	0	0	0	$-\frac{3}{16}$	$-\frac{3}{16}$	$\frac{3}{16}$	0	$-\frac{5}{16}$	0	$-\frac{1}{4}$	1	$-\frac{3}{8}$	2
	Z	0	0	-4	-1	0	1	0	1	0	0		
		0	0	-4	-1	0	1	-1	2	0	0		

BV	Cost	x_1	x_2	λ_1	λ_2	μ_1	μ_2	v_1	v_2	s_1	s_2	X_B	Ratio
								-1	-1				
x_1	0	1	0	0	$\frac{3}{16}$	$-\frac{1}{4}$	$\frac{1}{16}$	$-\frac{1}{4}$	$\frac{1}{16}$	0	0	$\frac{5}{16}$	
λ_1	0	0	0	1	$\frac{1}{4}$	0	$-\frac{1}{4}$	0	$-\frac{1}{4}$	0	0	$\frac{3}{4}$	
x_2	0	0	1	0	$-\frac{3}{64}$	$\frac{1}{16}$	$-\frac{1}{64}$	$\frac{1}{16}$	$-\frac{1}{64}$	$\frac{1}{4}$	0	$\frac{59}{64}$	
s_2	0	0	0	0	$-\frac{9}{64}$	$\frac{3}{16}$	$-\frac{3}{64}$	$-\frac{5}{16}$	$-\frac{3}{64}$	$-\frac{1}{4}$	1	$-\frac{19}{64}$	

$x_1 = \frac{5}{16}$ $x_2 = \frac{59}{64}$

$\boxed{Z = 3.19}$

0	1	0	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	0	$-\frac{1}{4}$	0	0	0	$\frac{1}{2}$
-	0	0	$\frac{1}{4}$	1	0	-1	0	-1	0	0	3
	1	0									

$\frac{1}{4} - \frac{1}{16}$ $\frac{4-1}{16}$ $-\frac{1}{4} + \frac{1}{16}$ $\frac{-4+1}{16}$ $-\frac{1}{4} + \frac{1}{16}$ $\frac{-4+1}{16}$ $\frac{1-7}{8}$ $\frac{4-27}{8}$

$$\text{Max } z = 2x_1 + 3x_2 - 2x_1^2$$

$$2\left(\frac{5}{16}\right) + 3\left(\frac{59}{64}\right) - 2\left(\frac{5}{16}\right)^2$$

$$\frac{5}{8} - \frac{2(25)}{16(16)} + \frac{177}{64}$$

$$\frac{160 - 50 + 708}{256}$$

$$\frac{818}{256}$$

$$128$$

Method for solving Integer Linear Programming

1) Gomory's Cutting plane method.

Find the optimum integer sol^n to the following integer programming problem

Ans should be integer

$$\text{Max } z = x_1 + 2x_2$$

sub to

$$2x_2 \leq 7$$

$$x_1 + x_2 \leq 7$$

$$2x_1 \leq 11$$

$$x_1 \geq 0 \qquad x_2 \geq 0$$

Ans)

$2x_2 + x_3 = 7$

$x_1 + x_2 + x_4 = 7$

$2x_1 + x_5 = 11$

$x_1 + 2x_2$

$x_3, x_4, x_5 \geq 0$

PAGE NO.
DATE : / /

BV	Cost	x_1 (1)	x_2 (2)	x_3 (0)	x_4 (0)	x_5 (0)	X_B	Min
x_3	0	0	$\boxed{2}$	1	0	0	7	$7/2$
x_4	0	1	1	0	1	0	7	7
x_5	0	2	0	0	0	1	11	-
	Δ_j	-1	-2	0	0	0		

BV	Cost	x_1 (1)	x_2 (2)	x_3	x_4	x_5	X_B	Ratio
x_2	2	0	1	$1/2$	0	0	$7/2$	-
x_4	0	1	0	$-1/2$	1	0	$7-\frac{7}{2}=\frac{7}{2}$	$7/2 = 3.5$
x_5	0	2	0	0	0	1	11	$11/2 = 5.\ldots$
	2	0	2	1	0	0		
$z=7$	Δ_j	-1	0	1	0	0		

BV	Cost	x_1 (1)	x_2 (2)	x_3	x_4	x_5	X_B	Ratio
x_2	2	0	1	$1/2$	0	0	$7/2$	
x_1	1	1	0	$-1/2$	1	0	$7/2$	
x_5	0	0	0	1	-2	1	4	
	2	1	2	$\frac{1}{2}$	1	0		
$z=\frac{21}{2}$	Δ_j	0	0	$1/2$	1	0		

$\boxed{x_1 = \frac{7}{2}}$ $\boxed{x_2 = \frac{7}{2}}$ $\boxed{z = \frac{21}{2}}$

PAGE NO.
DATE : / /

Since the obtained optimum sol^n is not integer value so go to next step, $x_1 = \frac{7}{2}$ $x_2 = \frac{7}{2}$

$= 3\frac{1}{2}$ $x_2 = 3\frac{1}{2}$

$= 3 + \frac{1}{2} f_{B_1}$ $3 + \frac{1}{2} f_{B_2}$

If not same take max term

$$\text{Max}\left(f_{B_1}, f_{B_2}\right) = \left(\frac{1}{2}, \frac{1}{2}\right) = \frac{1}{2} = f_{B_1} \text{ or } f_{B_2}$$

Now Gomorey Constaints is to be added

$$-f_{B_1} = -\sum_{j=m+1}^{n} f_{ij}\, x_j + g_i$$

non-ve Gomorian slack variable

$i = 1$, $j = 3, 4, 5$ slack variables adding the

$$-f_{B_1} = -f_{13} x_3 - f_{14} x_4 - f_{15} x_5 + g_1$$

$$-\frac{1}{2} = -\frac{1}{2} x_3 - 0(x_4) - (0)(x_5) + g_1$$

$$-\frac{1}{2} = -\frac{x_3}{2} + g_1$$

equating adjacent values

$g_1 = \frac{-1}{2}$ $x_3 = 0$, $x_4 = 0$, $x_5 = 0$

PAGE NO.
DATE: / /

		1	2	0					
BV	Cost	x_1	x_2	x_3	x_4	x_5	x_6	g_1	Max $\left(\frac{\Delta}{x}\right)$
x_2	2	0	1	$\frac{1}{2}$	0	0	$\frac{7}{2}$	0	
x_1	1	1	0	$-\frac{1}{2}$	1	0	$\frac{7}{2}$	0	
x_5	0	0	0	1	-2	1	4	0	
	Δ_j	0	0	$\frac{1}{2}$	1	0			
g_1	0	0	0	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	1	
		0	0	$\frac{1}{2}$	1	0		0	

PAGE NO.
DATE : / /

BV	cost	x_1	x_2	x_3	x_4	x_5	X_B
		1	2	0			
x_2	2	0	1	0	0	0	3
x_1	1	1	0	0	1	0	4
x_5	0	0	0	0	-2	1	3
x_3	0	0	0	1	0	0	1
$Z = 10$		1	2	0	1	0	
	Δj	0	0	0	1	0	

$x_1 = 4$ $x_2 = 3$