CHAPTER 1

NUMBER SYSTEM

1 Classification of Numbers

Undoubtedly numbers are the cornerstone of mathematics. There is a huge literature on the development of numbers but that is beyond our scope. We directly jump to the classification of numbers. Remember this classification looks so obvious or trivial today but humans took centuries to reach here. Reader should appreciate this.

- Numbers
  - Real Numbers
    - Rational Numbers
      - Fractions
      - Integers
        - Negative Integers
        - Non-negative Integers (Whole Numbers)
          - $\{0\}$
          - Positive Integers (Natural numbers)
    - Irrational Numbers
  - Imaginary Number

Here are some notations to denote the set of numbers :

$\mathbb{N}$ - Set of Natural Numbers $= \{1, 2, 3, ...\}$
$\mathbb{N}_0$ or $\mathbb{W}$ - Set of whole numbers $= \{0, 1, 2, ...\}$
$\mathbb{Z}$- Set of integers $= \{.... - 5, -4, ... - 1, 0, 1, 2...\}$
$\mathbb{Q}$ - Set of rational numbers $= \{... - 5, \frac{-4}{7}, \frac{-1}{2}, 0, \frac{1}{2}, \frac{1}{3}, \frac{5}{7}, 4, ...\}$

$\mathbb{Q}'$ or $\mathbb{Q}^c$ - Set of irrational Numbers $= \{... - \sqrt{11}, -\sqrt{5}, -\sqrt{7}, -\frac{1}{\sqrt{3}} .... \sqrt{5}, ...\}$

$\mathbb{R}$ - Set of real Number $= \{... - \sqrt{11}, -\sqrt{\frac{5}{7}}, -1.5, -1, 0, \sqrt{2}, \frac{1}{5} ...\}$

A number is said to be rational if it can be written as $\frac{p}{q}$ form where p, q are integers. For example $\frac{1}{2}, \frac{-1}{5}, \frac{1}{-5}, -\frac{2}{7}, -1, 0, 4, -\frac{7}{8}$ are rational numbers. A real number is said to be irrational if it is not rational. For example $\sqrt{2}, \sqrt{3}, \sqrt{5}, -\sqrt{2}, \frac{1}{\sqrt{2}}, -\frac{\sqrt{3}}{\sqrt{2}}$ are irrational number. (It is very interesting to prove that $\sqrt{2}, \sqrt{3}$ etc. cannot be written as $\frac{p}{q}$ form. We shall prove this shortly).

1.1 Exercises

Q.1 Prove that $\frac{a+b}{2}$ lies between a and $b (a < b)$.
Q.2 Find five rational numbers between.

(i) 1 and 10 (ii) 11 and 12 (iii) $\frac{1}{3}$ and $\frac{1}{2}$ (iv) $-\frac{1}{5}$ and $-\frac{1}{7}$ (v) $-\frac{3}{5}$ and $\frac{3}{5}$

Q.3 State true or false

(i) Every rational number is an integer.

(ii) Every natural number is an integer.

(iii) Every integer is an rational number.

(iv) 5 is an non-negative number.

(v) 0 is an non-negative integer.

(vi) Every irrational number is an rational number.

(vii) $-\frac{1}{7}$ and 7 are rational numbers.

Q.4 Express the following in decimal by long division

(i) $\frac{19}{5}$ (iii) $-\frac{19}{2}$ (v) $-\frac{17}{8}$

(ii) $\frac{3}{15}$ (iv) $\frac{2157}{625}$ (vi) $\frac{327}{500}$

Q.5 Show that the decimal expansion of following rational numbers are non-terminating and repeating.

(i) $\frac{8}{3}$ (ii) $\frac{2}{11}$ (iii) $\frac{1}{7}$ (iv) $-\frac{16}{45}$

2 Non-terminating and recurring rational number

There are rational numbers such that when we try to express them in decimal form by division method, we find that no matter how long we divide there is always a remainder. In other words, the division process never comes to an end. This is due to the reason that in the division process the remainder starts repeating after a certain number of steps. In such cases, a digit of a block of digits repeats itself. For example, 0.3333...,0.1666666..., 0.123123123...., 1.2692307692307692307...etc. Such decimals are called non-terminating repeating decimals or non-terminating recurring decimals. These decimal numbers are represented by putting a bar over the first block of the repeating part and omit the other repeating blocks. Thus, we write 0.33333... $= 0.\overline{3}$, 0.16666... $= 0.1\overline{6}$, 0.123123123 ..... $= 0.\overline{123}$ and 1.26923076923076292307... $= 1.2\overline{692307}$.

Fact : Every non terminating and recurring number is a rational number.

Q.1 Express the following rational numbers as decimals

(i) $\frac{2}{3}$ (iii) $-\frac{2}{15}$ (v) $\frac{437}{999}$

(ii) $-\frac{4}{5}$ (iv) $-\frac{22}{13}$ (vi) $\frac{33}{26}$

Q.2 Look at several examples of rational numbers in the form $\frac{p}{q}(q \neq 0)$, where p and q are integers with no common factors other than 1 and having terminating decimal representations. Can you guess what property q must satisfy?

3 Conversion of non-terminating recurring rational numbers in the form $\frac{p}{q}$

Q.1 Express $0.\overline{4}$ in the form $\frac{p}{q}$

Q.2 Express each of the following decimals in the form $\frac{p}{q}$

(i) $0.\overline{35}$ (ii) $0.\overline{585}$

Q.3 Express the following decimals in the form $\frac{p}{q}$

(i) $0.3\overline{2}$ (ii) $0.12\overline{3}$ (iii) $0.003\overline{52}$

Q.4 Express each of the following decimals in the form $\frac{p}{q}$

(i) $0.\overline{4}$ (ii) $0.\overline{37}$ (iii) $0.\overline{54}$ (iv) $0.\overline{621}$ (v) $125.\overline{3}$ (vi) $4.\overline{7}$ (vii) $0.4\overline{7}$

Q.1 Solution :

Let $x = 0.\overline{4} = 0.444$
$10x = 4.444....$
$10x - x = 4$
$9x = 4 = x = \frac{4}{9}$

Q.2 Solution :

(i) Let $x = 0.\overline{35}$
$x = 0.353535......(i)$
Here, we have two repeating digits after the decimal point. So, we multiply sides of (i) by $10^2 = 100$ to get
$100\ x = 35.3535...$ (ii)
Subtracting (i) from (ii), we get
$100x - x = (35.3535...)(0.3535...)$
$99x = 35$
$X = \frac{35}{99}$
Hence, $0.\overline{35} = \frac{35}{99}$

(ii) Let $x = 0.\overline{585}$
$x = 0.585585585...$
Here, we have three repeating digits after the decimal point. So, we multiply both sides of (i) by $10^3 = 1000$ to get
$1000\ x = 585.585585...$
Subtracting (i) from (ii), we get
$1000x - x = (585.585585...) - (0.585585585...)$
$1000x - x = 585$
$999x = 585$
$x = \frac{585}{999}$

Q.3 Solution :

(i) Let $x = 0.3\overline{2}$

Clearly, there is just one digit on the right side of the decimal point which is without bar. So, we multiply both sides of x by 10 so that only the repeating decimal is left on the right side of the decimal point.

i.e $10x = 3.\overline{2}$

$\Rightarrow 10x = 3 + 0.\overline{2}$

$\Rightarrow 10x = 3 + \dfrac{2}{9}$

$\Rightarrow 10x = \dfrac{9 \times 3 + 2}{9} \Rightarrow 10x = \dfrac{29}{9} \Rightarrow x = \dfrac{29}{90}$

(ii) Let $x = 0.12\overline{3}$

Clearly, there are two digits on the righ side of the decimal point which are without bar. So, we multiply both sides of x by $102 = 100$ so that only the repeating decimal is left on the right side of the decimal point.

i.e $100x = 12.\overline{3}$

$\Rightarrow 100x = 12 + 0.\overline{3}$

$\Rightarrow 100x = 12 + \dfrac{3}{9}$

$\Rightarrow 100x = \dfrac{12 \times 9 + 3}{9}$

$\Rightarrow 100x = \dfrac{108 + 3}{9} \Rightarrow 100x = \dfrac{111}{9} \Rightarrow x = \dfrac{111}{900}$

(iii) Let $x = 0.003\overline{52}$

Clearly, there are three digits on the right side of the decimal point which are without bar. So, we multiply both sides of x by $103 = 1000$ so that only the repeating decimal is left on the right side of the decimal point.

i.e $1000x = 3.\overline{52}$

$\Rightarrow 1000x = 3 + 0.52$

$\Rightarrow 1000x = 3 + \dfrac{52}{99}$

$\Rightarrow 1000x = \dfrac{3 \times 99 + 52}{99}$

$\Rightarrow 1000x = \dfrac{297 + 52}{99} \Rightarrow 1000x = \dfrac{349}{99} \Rightarrow x = \dfrac{340}{99000}$

Q.4 Solution :

(i) $\dfrac{4}{9}$

(ii) $\dfrac{37}{99}$

(iii) $\dfrac{6}{11}$

(iv) $\dfrac{23}{37}$

(v) $\dfrac{376}{3}$

(vi) $\dfrac{43}{9}$

(vii) $\dfrac{43}{90}$

Tricky Tip

$$\frac{p}{q}\text{ form} = \frac{\text{Complete Number - Number formed by non - repeating digits}}{\text{No.of 9 as no.of repeating digits after that write no.of 0 as no.of non - repeating digits}}$$

(i) $0.\overline{57} = \dfrac{57 - 0}{99} = \dfrac{57}{99}$

(ii) $0.\overline{347} = \dfrac{347 - 3}{990} = \dfrac{349}{990}$

(iii) $0.53\overline{763} = \dfrac{53763 - 53}{99900} = \dfrac{53710}{99900} = \dfrac{5371}{9990}$

4 Irrational Numbers (finding the square root by division method)

A number which is not rational is known as a irrational number. In other words, a number which cant be written as $\frac{p}{q}$ form.

Fact : Every non-terminating and non-recurring number is irrational number.

$\sqrt{2}, \sqrt{3}, \sqrt{7}, \pi$ are rational number. Lets find the value of $\sqrt{2}, \sqrt{3}$ by division method.

1.

	1.4142135...
1	2.00000000000000
	1
24	100
	96
28	400
	281
2824	11900
	11296
28282	60400
	56564
282841	383600
	282841
2828423	10075900
	8485269
28284265	159063100
	141421325
28284270	17611775

$\Rightarrow \sqrt{2} = 1.4142135...$

2.

	1.732050807...
1	3.00 00 00 00 00 00 00 00
	1
27	200
	189
343	1100
	1029
3462	7100
	6924
346405	1760000
	1732025
34641008	279750000
	277128064
3464101607	26219360000
	24248711249
	1970648751

$\Rightarrow \sqrt{3} = 1.732050807...$

4.1 Exercises

Q.1 Find the square root of following.

(i) 271441 (iii) 522729 (v) 64009 (vii) 454.1161 (ix) 8

(ii) 8281 (iv) 801025 (vi) 4664.89 (viii) 7 (x) 11

5 Proof by Contradiction (Irrationality of $\sqrt{2}$)

We will discuss this topic more thoroughly in the chapter Mathematical Logic. Currently our main motive is to prove the irrationality of $\sqrt{2}$ i.e $\sqrt{2}$ can not be written as $\frac{p}{q}$ form. This method proof by contradiction is used frequently in calculus, one of the most interesting branch of mathematics. In this method we assume the negation of something which is to beproved then use deduce a contradiction which implies that our assumption was wrong. Now lets prove that $\sqrt{2}$ is an irrational number.

Assume that $\sqrt{2}$ is an rational number it means it can be written as $\frac{p}{q}$ form. Let
$\sqrt{2} = \frac{p}{q}$
where p, q are co-prime (p, q have not common factor) Now we have
$2 = \frac{p^2}{q^2}$
$p^2 = 2q^2$

It shows that p^2 is an even integer, since p is an integer it must be an even integer. So take $p = 2m$ for some integer m. now we have

$4m^2 = 2q^2$
$2m^2 = q^2$

It shows q^2 is an even integer hence q is an even integer which contradicts our assumption that p, q are co-prime. Hence our assumption was wrong that $\sqrt{2}$ is an irrational number. $\sqrt{2}$ is an rational number.

5.1 Proposition

For some $a, b, c, d \in \mathbb{Q}$ if $a + b\sqrt{2} = c + d\sqrt{2}$ then $a = c$ and $b = d$.(This result can be generalized if we replace $\sqrt{2}$ by any other irrational numbers).

Proof : We are given

$$a + b\sqrt{2} = c + d\sqrt{2} \tag{1.1}$$

$$\Rightarrow (a - c) = (d - b)\sqrt{2}$$

Since $a, b, c, d \in Q$ then $(a - c) \in \mathbb{Q}$ and $(d - b) \in \mathbb{Q}$ but $(d - b)\sqrt{2} \in \mathbb{Q}'$. So, L.H.S is an rational number and R.H.S is an irrational number. So, equality in (1.1) can be hold only if $a - c = 0$ and $d - b = 0$. So, $a = c$ and $b = d$

5.2 Exercises

Q.1 Prove that $\sqrt{3}$ is an irrational number
Q.2 Prove that $\sqrt{5}$ is an irrational number

Q.3 Prove that $\sqrt{n}$ is an irrational number if n is not a perfect square.
Q.4 Prove that $\sqrt{3}+\sqrt{5}$ is an irrational number.
Q.5 Using the fact that negative of rational number is also a rational number prove that negative of irrational number is an irrational number.
Q.6 Prove that the sum of rational number and irrational number is an irrational number.
Q.7 Prove a disprove that sum of two irrational number is an irrational number.
Q.8 Prove that $\sqrt{ab}$ lies between two positive numbers between a and b $(a < b)$. Hence, find two irrational numbers between

(i) 2 and 3 (ii) $\sqrt{2}$ and $\sqrt{3}$ (iii) 2 and $\sqrt{5}$

Q.9 Find $a, b \in \mathbb{Q}$ if $a + b\sqrt{5} = 3 + 7\sqrt{5}$.
Q.10 Find $x, y \in \mathbb{Q}$ if $(x+y) + (x-y)\sqrt{7} = 7 + 5\sqrt{7}$.

6 Rationalization of numerator/denominator and comparison of surds

6.1 Example

Rationalize the denominator

(i) $\dfrac{1}{3+\sqrt{2}}$ (ii) $\dfrac{7+\sqrt{5}}{7-\sqrt{5}}$ (iii) $\dfrac{3-\sqrt{7}}{2-\sqrt{7}}$

Solution : (i) $\dfrac{1}{3+\sqrt{2}} = \dfrac{1}{3+\sqrt{2}}.\dfrac{(3-\sqrt{2})}{(3-\sqrt{2})} = \dfrac{3-\sqrt{2}}{9-2} = \dfrac{3-\sqrt{2}}{5}$

(ii) $\dfrac{7+\sqrt{5}}{7-\sqrt{5}} = \dfrac{7+\sqrt{5}}{7-\sqrt{5}}.\dfrac{7+\sqrt{5}}{7+\sqrt{5}} = \dfrac{49+5+19\sqrt{5}}{44} = \dfrac{54+14\sqrt{5}}{44} = \dfrac{54}{44} + \dfrac{14\sqrt{5}}{44} = \dfrac{27}{22} + \dfrac{7\sqrt{5}}{22}$

(iii) $\dfrac{3-\sqrt{7}}{2-\sqrt{7}} = \dfrac{3-\sqrt{7}}{3-\sqrt{7}}.\dfrac{2+\sqrt{7}}{2+\sqrt{7}} = \dfrac{6+3\sqrt{7}-2\sqrt{7}+7}{4-7} = \dfrac{13+\sqrt{7}}{-3} = -\dfrac{13}{3} - \dfrac{1\sqrt{7}}{3}$

6.2 Example

Arrange $\sqrt[4]{6}, \sqrt[3]{7}, \sqrt{5}$ in ascending order.

Solution : $\sqrt[4]{6} = (6)^{\frac{1}{4}}, \sqrt[3]{7} = (7)^{\frac{1}{3}}, \sqrt{5} = (5)^{\frac{1}{2}}$
To determine the order of the surds we first make their exponents equal.
L.C.M of denominator of exponents 4,3,2 = 12
So, we have

$6^{\frac{1}{4}} = 6^{\frac{3}{12}} = (6^3)^{\frac{1}{2}} = (216)^{\frac{1}{12}}$

$7^{\frac{1}{3}} = 7^{\frac{4}{12}} = (7^4)^{\frac{1}{12}} = (2401)^{\frac{1}{12}}$

$5^{\frac{1}{2}} = 5^{\frac{6}{12}} = (5^6)^{\frac{1}{12}} = (15625)^{\frac{1}{12}}$

So, $(216)^{\frac{1}{12}} < (2401)^{\frac{1}{12}} < (15625)^{\frac{1}{12}}$
$6^{\frac{1}{4}} < 7^{\frac{1}{3}} < 5^{\frac{1}{2}}$

6.3 Exercises

Q.1 Rationalize the denominator

(i) $\dfrac{2}{\sqrt{7}}$

(ii) $\dfrac{2}{3\sqrt{3}}$

(iii) $\dfrac{5}{3-\sqrt{5}}$

(iv) $\dfrac{5+\sqrt{6}}{5-\sqrt{6}}$

(v) $\dfrac{7+3\sqrt{5}}{7-3\sqrt{5}}$

(vi) $\dfrac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}}$

Q.2 If a, b are rational number, find a and b.

(i) $\dfrac{\sqrt{3}-1}{\sqrt{3}+1} = a + b\sqrt{3}$

(ii) $\dfrac{3+\sqrt{7}}{3-\sqrt{7}} = a + b\sqrt{7}$

(iii) $\dfrac{5+2\sqrt{3}}{7+4\sqrt{3}} = a + b\sqrt{3}$

(iv) $\dfrac{5+\sqrt{3}}{7-4\sqrt{3}} = 47a + \sqrt{3b}$

(v) $\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = a + b\sqrt{15}$

(vi) $\dfrac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = a - b\sqrt{6}$

Q.3 Prove that

(i) $\dfrac{1}{3-\sqrt{8}} - \dfrac{1}{\sqrt{8}-\sqrt{7}} + \dfrac{1}{\sqrt{7}-\sqrt{6}} - \dfrac{1}{\sqrt{6}-\sqrt{5}} + \dfrac{1}{\sqrt{5}-2} = 5$

(ii) $\dfrac{1}{1+\sqrt{2}} + \dfrac{1}{\sqrt{2}+\sqrt{3}} + \dfrac{1}{\sqrt{3}-\sqrt{4}} + ... + \dfrac{1}{\sqrt{8}+\sqrt{9}} = 2$

Q.4 If $x = 2+\sqrt{3}$ find the value of $x^2 + \dfrac{1}{x^2}$

Q.5 If $x = 3+2\sqrt{2}$ find the value of $x^2 + \dfrac{1}{x^2}$

Q.6 If $x = \dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ and $y = x = \dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$, find $x^2 + y^2$

Q.7 If $x = \dfrac{1}{2-\sqrt{3}}$ find the value of $x^3 - 2x^2 - 7x + 5$

Q.8 Arrange in ascending order.

(i) $(10)^{\frac{1}{2}}, (14)^{\frac{1}{3}}, (200)^{\frac{1}{6}}$

(ii) $(17)^{\frac{1}{5}}, (100)^{\frac{1}{10}}, (200)^{\frac{1}{20}}$

7 Algebraic Identities

Here are some identities which is to be used frequently

$(a+b)^2 = a^2 + 2ab + b^2$
$(a-b)^2 = a^2 - 2ab + b^2$
$a^2 - b^2 = (a+b)(a-b)$
$(a+b)^3 = a^3 + b^3 + 3ab(a+b) = a^3 + b^3 + 3a^2b + 3ab^2$
$(a-b)^3 = a^3 - b^3 - 3ab(a-b) = a^3 - b^3 - 3a^2b + 3ab^2$
$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

7.1 Exercises

Q.1 Find the product using identities

(i) $(7x+y)(7x-y)$

(ii) $(x-1)(x+1)(x^2+1)(x^4+1)$

(iii) $\left(x-\frac{1}{x}\right)\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}\right)\left(x^4+\frac{1}{x^4}\right)$

(iv) 103×93

(v) $(97)^2$

(vi) $0.54 \times 0.54 - 0.46 \times 0.46$

(vii) 103×107

(viii) 104×96

Q.2 If $x - \frac{1}{x} = 6$

(i) $x^2 + \frac{1}{x^2}$

(ii) $x^4 + \frac{1}{x^4}$

Q.3 If $\left(x^2 + \frac{1}{x^2}\right) = 27$, find the value of

(i) $x + \frac{1}{x}$

(ii) $x - \frac{1}{x}$

Q.4 If $x + y = 12$ and $xy = 32$, find the value of $x^2 + y^2$.
Q.5 If $4x^2 + y^2 = 40$ and $xy = 6$, find the value of $2x + y$.
Q.6 Simplify

(i) $(2x+3y)^3$

(ii) $(4x+2y)^3 - (4x-2y)^3$

Q.7 If $x + \frac{1}{x} = 7$, find the value of $x^3 + \frac{1}{x^3}$
Q.8 If $x + \frac{1}{x} > 0$ and $x^2 + \frac{1}{x^2} = 7$, find the value of $x^3 + \frac{1}{x^3}$.
Q.9 If $x^2 + \frac{1}{x^2} = 03$, find the value of $x^3 - \frac{1}{x^3} = 7$, find the value of $x^3 - \frac{1}{x^3}$ (Take $x - \frac{1}{x} < 0$)
Q.10 Evaluate using identities.

(i) $23^3 - 17^3$

(ii) $29^3 - 11^3$

Q.11 If $a - b = 4$ and $ab = 45$, find the value of $a^3 - b^3$
Q.12 If $a + b = 10$ and $ab = 21$, find the value of $a^3 + b^3$
Q.13 If $a + b = 7$ and $ab = 12$, find the value of $(a^2 - ab + b^2)$
Q.14 If $a^2 + b^2 + c^2 = 20$ and $a + b + c = 0$, find the value of $ab + bc + ca$
Q.15 If $a + b + c = 9$ and $ab + bc + ca = 40$, find $a^2 + b^2 + c^2$
Q.16 If $a^2 + b^2 + c^2 = 250$ and $ab + bc + ca = 3$, find a+b+c
Q.17 Expand

(i) $(x+2y+4z)^2$

(ii) $(2a-3b-c)^2$

(iii) $(-2x+3y+2z)^2$

Q.18 Simplify

(i) $(a+b+c)^2 + (a-b-c)^2$

(ii) $(a+b+c)^2 - (a-b-c)^2$

Q.19 Prove that

$$a^3 + b^3 + c^3 - 3abc = (a+b+b)(a^2+b^2+c^2-ab-bc-ca)$$

(This is a standard identity, students are advised to remember this).

Hint : $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$

Q.20 By using the last exercise

(i) Show that if $a+b+c=0$ then $a^3+b^3+c^3=3abc$ but converse is false.

(ii) If $a+b+c=6$ then $ab+bc+ca=11$, find the value of $a^3+b^3+c^3-3abc$.

Q.21 By using the first part of last exercise find the value of

(i) $(28)^3+(-15)^3+(-13)^3$

(ii) $(-12)^2+7^3+5^3$

(iii) $30^3+20^3-50^3$

(iv) $\dfrac{(a-b)^3+(b-c)^3+(c-a)^3}{(a-b)(b-c)(c-a)}$

(v) $\dfrac{(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3}{(a-b)(b-c)(c-a)}$

8 Primes and Composites

Definition : A prime number (or simply prime) is a natural number $p>1$ whose only positive divisors 1 and p.

Definition : A natural number greater than 1 which is not prime is called a composite number (or simply composite).

Fact : 1 is the only natural number which is neither prime nor composite.

Fact : If none of the prime number upto $\sqrt{n}$ divides n then n is a prime number. (Try to prove it yourself).

8.1 Exercises

Q.1 Identify which of the following is prime?

(i) 197

(ii) 297

(iii) 499

(iv) 553

(v) 773

Q.2 Find the smallest composite number that has no prime factors less than 10.
Q.3 What is the largest two-digit prime number whose digits are also each prime?
Q.4 How many prime number are perfect cubes?
Q.5 The number 13 is prime. If you reverse the digits you also obtain a prime number, 31. What is the larger of the pair of primes that satisfies this condition and has a sum of 110?
Q.6 What is the smallest prime divisor of 101729 + 729101?
Q.7 A group of 25 coins is arrange into each pile contains a different prime number of coins. What is the greatest number of coins possible in any of the three piles?
Q.8 Is 9409 prime?
Q.9 What are the 5 smallest prime numbers greater than 1000?
Q.10 What is the first year in the twenty-first century that is a prime number?

Exercises

Level 1

1. Which of following fractions lie between $\frac{1}{4}$ and $\frac{1}{5}$

   A $\frac{7}{33}$ B $\frac{4}{11}$ C $\frac{13}{57}$ D $\frac{7}{17}$

   (a) A and B
   (b) A and C
   (c) B,C and D
   (d) A,B and D

2. Express $0.\overline{34}+0.3\overline{4}$ asa single decimal.

   (a) $0.67\overline{88}$
   (b) $0.68\overline{78}$
   (c) $0.6\overline{89}$
   (d) $0.6\overline{87}$

3. If $\sqrt{5^n}=125$, then $5^{\sqrt[n]{64}}=$ _____

   (a) 25
   (b) $\frac{1}{125}$
   (c) 625
   (d) $\frac{1}{25}$

4. If $x^4+1=1297$ and $y^4-1=2400$, then $y^2-x^2=$ _____

   (a) 10
   (b) 25
   (c) 13
   (d) 43

5. What is the value of $4^{(2x-2)}$, if $(16)^{2x+3}=(64)^{x+3}$?

   (a) 64
   (b) 256
   (c) 32
   (d) 512

6. Which of the following have equal values? (where $x\in\mathbb{R}$)_____

   (a) $9^{\frac{x}{2}}, 24^{\frac{x}{3}}$
   (b) $(256)^{\frac{4}{x}}, (4^3)^{\frac{4}{x}}$
   (c) $(343)^{\frac{x}{3}}, (7^4)^{\frac{x}{12}}$
   (d) $(36^2)^{\frac{2}{7}}, (6^3)^{\frac{2}{7}}$

7. The expression $\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{7}-\sqrt{2}\right)$ when simplified becomes a

   (a) simple surd.
   (b) mixed surd.
   (c) compound surd.
   (d) binomial surd

8. If m and n are positive integers, then for a positive number a, $\left\{\sqrt[m]{\left(\sqrt[n]{a}\right)}\right\}^{mn}=$ _____

   (a) a^{mn}
   (b) a
   (c) $a^{m/n}$
   (d) 1

9. If $2^{-m}\times\frac{1}{2^m}=\frac{1}{4}$, then $\frac{1}{14}\left\{(4^m)^{\frac{1}{2}}+\left(\frac{1}{5^m}\right)^{-1}\right\}=$ _____

   (a) $\frac{1}{2}$
   (b) 2
   (c) 4
   (d) $\frac{-1}{4}$

10. The surds $\sqrt{2}, \sqrt[3]{3}$ and $\sqrt[5]{5}$, in their descending order are

    (a) $\sqrt[3]{3}, \sqrt[5]{5}, \sqrt{2}$
    (b) $\sqrt{2}, \sqrt[3]{3}, \sqrt[5]{5}$
    (c) $\sqrt{2}, \sqrt[5]{5}, \sqrt[3]{3}$
    (d) $\sqrt[3]{3}, \sqrt{2}, \sqrt[5]{5}$

11. $2[(16-15)^{-1}+25(13-8)^{-2}]^{-1}+(1024)^0=$ _____

    (a) 2
    (b) 3
    (c) 1
    (d) 5

12. If $x=2$ and $y=4$, then $\left(\frac{x}{y}\right)^{x-y}+\left(\frac{y}{x}\right)^{y-x}=$ _____

    (a) 4
    (b) 8
    (c) 12
    (d) 2

13. In which of the following pairs of surds are the given two surds similar?

    (a) $\sqrt{5}, 7\sqrt{5}$
    (b) $\sqrt[3]{7}, \sqrt[2]{7}$
    (c) $\sqrt{7}, \sqrt{28}$
    (d) Both (a) and (b)

14. Which of the following is the greatest?

    (a) 7^2
    (b) $(49)^{\frac{3}{2}}$
    (c) $\left(\frac{1}{343}\right)^{\frac{-1}{3}}$
    (d) $(2401)^{\frac{-1}{4}}$

15. $(\sqrt[6]{5})(\sqrt[3]{2})(\sqrt{3})(\sqrt[12]{6}) =$ ____

(a) $\sqrt[12]{1749600}$ (c) $\sqrt[12]{177960}$

(b) $\sqrt[3]{2} \times \sqrt[12]{109350}$ (d) Both (a) and (b)

16. If $p = 3$ and $q = 2$,then $(3p-4q)^{q-p} \div (4p-3q)^{2q-p} =$ ____

(a) 1 (c) $\frac{1}{6}$

(b) 6 (d) $\frac{2}{3}$

17. $\left[\frac{(32)^{0.2}+(81)^{0.25}}{(256)^{0.5}-(121)^{0.5}}\right] =$ ____

(a) 2 (c) 1

(b) 5 (d) 11

18. The smallest among the surds $\sqrt{10}-\sqrt{5}$, $\sqrt{19}-\sqrt{14}$, $\sqrt{22}-\sqrt{17}$ and $\sqrt{8}-\sqrt{3}$ is

(a) $\sqrt{10}-\sqrt{5}$ (c) $\sqrt{22}-\sqrt{17}$

(b) $\sqrt{19}-\sqrt{14}$ (d) $\sqrt{8}-\sqrt{3}$

19. If $\sqrt{m} = \sqrt{a}+\sqrt{c}$ and $\sqrt{m}, \sqrt{a}$ and $\sqrt{c}$ are three surds

(a) $\sqrt{m}$ is dissimilar to $\sqrt{a}$ and $\sqrt{c}$

(b) $\sqrt{a}$ and $\sqrt{c}$ are similar to $\sqrt{m}$

(c) only $\sqrt{a}$ is similar to $\sqrt{m}$

(d) None of these

20. The surd obtained after rationalizing the numerator of $\frac{4-\sqrt{25-a}}{a-9}$ is equal to

(a) $\frac{a-9}{4-\sqrt{25-a}}$

(b) $\frac{1}{4-\sqrt{25-a}}$

(c) $\frac{1}{(a-9)(4-\sqrt{25-a})}$

(d) $\frac{1}{4+\sqrt{25-a}}$

21. If $\sqrt{13-x\sqrt{10}} = \sqrt{8}+\sqrt{5}$, then what is the value of x?

(a) -5 (c) -4

(b) -6 (d) -2

22. If the surds $\sqrt[4]{4}, \sqrt[6]{5}, \sqrt[8]{6}$ and $12\sqrt{8}$ are aranged in ascending order from left to right, then the third surd from the left is

(a) $\sqrt[12]{8}$ (c) $\sqrt[8]{6}$

(b) $\sqrt[4]{4}$ (d) $\sqrt[6]{5}$

23. $\sqrt{11\sqrt{11\sqrt{11...4 \text{ terms}}}} =$ ____

(a) $\sqrt[16]{11^5}$ (c) $\sqrt[16]{11^{14}}$

(b) $\sqrt[16]{11}$ (d) $\sqrt[16]{11^{15}}$

24. If $\frac{5+\sqrt{3}}{2+\sqrt{3}} = x+\sqrt{3}$, then (x, y) is

(a) (13,-7) (c) (-13,-7)

(b) (-13,7) (d) (13,7)

25. The simplest form of $\sqrt{125}+\sqrt{125}-\sqrt{845}$ is

(a) $\sqrt{15}$ (c) $-\sqrt{5}$

(b) $2\sqrt{5}$ (d) $-2\sqrt{5}$

26. Which of following statement is true?

I. If x is a conjugate surd of y, then x can be a RF of y

II. If x is a RF of y, then x need not to be the conjugate of y

(a) Only I (c) Both I and II

(b) Only II (d) Neither I nor II

27. If $\frac{3-2\sqrt{5}}{6-\sqrt{5}} = a+b\sqrt{5}$ where a and b are rational numbers, then what are the values of a and b?

(a) $\frac{8}{35}, \frac{-9}{35}$ (c) $\frac{-8}{31}, \frac{9}{31}$

(b) $\frac{8}{31}, \frac{-9}{31}$ (d) $\frac{-8}{35}, \frac{9}{35}$

28. If $\frac{3^{5x} \times (81)^2 \times 6561}{3^{2x}} = 3^7$, then $x =$ ____

(a) 3 (c) $\frac{1}{3}$

(b) -3 (d) $\frac{-1}{3}$

29. If $\sqrt{2^n} = 1024$, then $3^{2\left(\frac{n}{4}-4\right)} =$ ____

(a) 3
(b) -3
(c) 27
(d) 81

30. If $\left[\left\{\left(\frac{1}{7^2}\right)^{-2}\right\}^{\frac{-1}{3}}\right]^{\frac{2}{4}} = 7^m$, then $m =$ ____

(a) $\frac{-1}{3}$
(b) $\frac{1}{4}$
(c) -3
(d) 2

Level 2

31. $\left[\left\{\left(\frac{1}{x^{a^2-b^2}}\right)^{\frac{1}{a-b}}\right\}^{a+b}\right]^{\frac{1}{(a+b)^2}} =$ ____

(a) x^2
(b) $\frac{1}{x}$
(c) 7^3
(d) $\frac{1}{x^2}$

32. If $\frac{2^{m+n}}{2^{m-n}}$ and $a = 2^{\frac{1}{10}}$, then $\frac{(a^{2m+n-p})^2}{(a^{m-2n+2p})^{-1}} =$ ____

(a) 2
(b) $\frac{1}{4}$
(c) 9
(d) $\frac{1}{8}$

33. $\left[(p^{-1}+q^{-1})(p^{-1}-q^{-1}) + \left(\frac{1}{p^{-1}} - \frac{1}{q^{-1}}\right)\left(\frac{1}{p^{-1}} + \frac{1}{q^{-1}}\right)\right](pq)^2$

(a) $(pq)^2$
(b) -1
(c) $-(pq)^2$
(d) 1

34. If $x = \frac{2}{\sqrt{10}-\sqrt{8}}, y = \frac{2}{\sqrt{10}-2\sqrt{2}}$, then $(x-y)^2 =$ ____

(a) $4\sqrt{2}$
(b) 32
(c) $8\sqrt{2}$
(d) 64

35. If $a = \sqrt{6}-\sqrt{3}, b = \sqrt{3}-\sqrt{2}$ and $c = \sqrt{2}-\sqrt{6}$, then find the value of $a^3+b^3+c^3-2abc$.

(a) $3\sqrt{2}-5\sqrt{3}-\sqrt{6}$
(b) $3\sqrt{2}-5\sqrt{3}-\sqrt{6}$
(c) $3\sqrt{2}-4\sqrt{3}+\sqrt{6}$
(d) $3\sqrt{2}+4\sqrt{3}+\sqrt{6}$

36. $\sqrt{\frac{81}{64}\sqrt{\frac{81}{64}\sqrt{\frac{81}{64}\sqrt{\frac{81}{64}....\infty}}}} =$

(a) $\frac{81}{64}$
(b) $\frac{9}{8}$
(c) $\frac{3}{2}$
(d) $\frac{3}{2\sqrt{2}}$

37. If $a^p = b^q = c^r = abc$, then $pqr =$ ____

(a) p^2q+q^2r
(b) $pq+qr+pr$
(c) $(pq+qr+rp)^2$
(d) $pq(qr+rp)$

38. The value of $\left[(23+2^2)^{\frac{2}{3}}+(140-29)^{\frac{1}{2}}\right]^2$ is ____

(a) 196
(b) 289
(c) 324
(d) 400

39. If $x = \sqrt{6}+\sqrt{5}$, then $x^2+\frac{1}{x^2}-2 =$ ____

(a) $2\sqrt{6}$
(b) $2\sqrt{5}$
(c) 24
(d) 20

40. $\sqrt{6+\sqrt{6+\sqrt{6+......\infty}}}$ is equal to ____

(a) -3
(b) 3
(c) 6
(d) 2

41. Simplify $\frac{1}{\sqrt{19-\sqrt{360}}} - \frac{1}{\sqrt{21-\sqrt{440}}} + \frac{2}{\sqrt{20-\sqrt{396}}} =$

(a) 1
(b) 2
(c) 0
(d) Nome of these

42. If $a = \sqrt{17}-\sqrt{16}$ and $b = \sqrt{16}\sqrt{15}$, then

(a) $a < b$
(b) $a > b$
(c) $a = b$
(d) None of these

43. $\left(\sqrt[6]{15-2\sqrt{56}}\right).\left(\sqrt[3]{\sqrt{7}+2\sqrt{56}}\right) = $ ____

(a) 0
(b) 1
(c) -1
(d) 2

44. $\sqrt{\sqrt{63}+\sqrt{56}} = $ ____

(a) $\sqrt[4]{7}(\sqrt{3}+\sqrt{5})$
(b) $\sqrt[4]{7}(\sqrt{3}+1)$
(c) $\sqrt[4]{7}(\sqrt{3}+\sqrt{5})$
(d) $\sqrt[4]{7}(\sqrt{2}+1)$

45. If $\dfrac{\sqrt{7}+2\sqrt{3}}{2\sqrt{7}-\sqrt{5}} = \dfrac{c+\sqrt{p+\sqrt{q}+\sqrt{r}}}{23}(p<q<r)$, where p, q, r are rational numbers, then $q+r-p = $ ____

(a) 361
(b) 302
(c) 418
(d) 426

46. The following are the steps involved in finding the value of $x-y$ from $\dfrac{8-\sqrt{5}}{8+\sqrt{5}} = x-y\sqrt{40}$. Arrange them in sequential order.

(A) $\dfrac{13-2\sqrt{40}}{8-5} = x-y\sqrt{40}$

(B) $\dfrac{(\sqrt{8})^2+(\sqrt{5})^2-2(\sqrt{8})(\sqrt{5})}{(\sqrt{8})^2+(\sqrt{5})^2} = x-y\sqrt{40}$

(C) $x-y = \dfrac{11}{3}$

(D) $x = \dfrac{13}{3}$ and $\dfrac{2}{3}$

(E) $\dfrac{(\sqrt{8}-\sqrt{5})(\sqrt{8}+\sqrt{5})}{(\sqrt{8}+\sqrt{5})(\sqrt{8}-\sqrt{5})} = x-y\sqrt{40}$

(a) EABDC
(b) EBADC
(c) ABDEC
(d) DEBAC

47. The following are the steps involved in finding the least among $\sqrt{3}$, $\sqrt[3]{4}$ and $\sqrt[6]{15}$. Arrange them in sequential order.

(A) $\sqrt[6]{5}$ is the smallest.

(B) $3^{\frac{1}{2}} = 3^{\frac{3}{6}}, 4^{\frac{2}{6}}, 15^{\frac{1}{6}} = 15^{\frac{1}{6}}$

(C) The LCM of the denominators of the exponents is 6.

(D) $\sqrt{3} = 3^{\frac{1}{2}}, 3\sqrt{4} = 4^{\frac{1}{3}}, \sqrt{3} = 15^{\frac{1}{6}}$

(E) $\sqrt{3} = \sqrt[6]{27}$, $\sqrt[3]{4} = \sqrt[6]{16}$, $\sqrt[6]{15} = \sqrt[6]{15}$

(a) DCABE
(b) DABEB
(c) DCBEA
(d) DBCAE

48. $y = 3-\sqrt{8}$, then $\left(y-\dfrac{1}{y}\right)^2 = $

(a) 9
(b) 81
(c) 4
(d) 32

49. The following steps are involved in finding the value of $a+b$ from $\dfrac{2+\sqrt{3}}{2-\sqrt{3}} = a+b\sqrt{3}$. Arrange them in sequential order.

(A) $\dfrac{2^2+(\sqrt{3})^2+2\times 2\times\sqrt{3}}{2^2-(\sqrt{3})^2} = a+b\sqrt{3}$

(B) $a+b = 7+4 = 11$

(C) $\dfrac{\left(2+\sqrt{3}\right)\left(2+\sqrt{3}\right)}{\left(2-\sqrt{3}\right)\left(2+\sqrt{3}\right)} = a+b\sqrt{3}$

(D) $\dfrac{7+4\sqrt{3}}{4-3} = a+b\sqrt{3}$

(E) $a = 7$ and $b = 4$

(a) CDAEB
(b) CAEBD
(c) CADEB
(d) CEDAB

50. The following are the steps involved in finding the greatest among $\sqrt[3]{2}$, $\sqrt[6]{3}$ and $\sqrt{6}$. Arrange them in sequential order.

(A) The LCM of the denominators of the exponents is 6.

(B) $\sqrt[6]{216}$ i.e $\sqrt{6}$ is the greatest.

(C) $3\sqrt{2} = 2^{\frac{1}{3}}, 6\sqrt{3} = 3^{\frac{1}{6}}, \sqrt{6} = 6^{\frac{1}{2}}$

(D) $2^{\frac{1}{3}=2^{\frac{2}{6}}}, 3^{\frac{1}{6}=3^{\frac{1}{6}}}, 6^{\frac{1}{2}=6^{\frac{3}{6}}}$

(E) $\sqrt[3]{2} = \sqrt[6]{4}$, $\sqrt[6]{3} = \sqrt[6]{3}$, $\sqrt{6} = \sqrt[6]{216}$

(a) CADEB
(b) CDABE
(c) DCAEB
(d) DACBE

51. If $x = \dfrac{1}{\sqrt{3}+2}$, then $\left(x+\dfrac{1}{x}\right)^2 =$ _____
    (a) 16 (c) 12
    (b) 3 (d) 6

52. If $\sqrt[x]{3} \times \sqrt[y]{5} = 10125$, then $12xy =$ _____
    (a) 1 (c) 2
    (b) $\dfrac{1}{3}$ (d) $\dfrac{1}{2}$

53. If $x = \dfrac{1}{5+2\sqrt{6}}$, then $x^2 - 10x + 1 =$ _____
    (a) 1 (c) 0
    (b) -1 (d) 10

54. If $x = \dfrac{2}{\sqrt{3}-\sqrt{5}}$ and $y = \dfrac{2}{\sqrt{3}+\sqrt{5}}$, then $x + y =$ _____
    (a) 3 (c) $-2\sqrt{3}$
    (b) $4\sqrt{3}$ (d) 6

55. $\dfrac{3}{7}$ lies between the fractions _____
    (a) $\dfrac{4}{9}, \dfrac{5}{9}$ (c) $\dfrac{42}{99}, \dfrac{4}{9}$
    (b) $\dfrac{43}{99}, \dfrac{4}{9}$ (d) $\dfrac{41}{99}, \dfrac{42}{99}$

Level 3

56. If $\sum_{k=4}^{143} \dfrac{1}{\sqrt{k}+\sqrt{k+1}} = a - \sqrt{b}$, then a and b respectively are
    (a) 10 and 0 (c) 1o and 4
    (b) -10 and 4 (d) -10 and 0

57. The surd $\dfrac{12}{3+\sqrt{5}+2\sqrt{2}}$, after rationalizing the denominators becomes
    (a) $\sqrt{5}-\sqrt{2}+\sqrt{10}+1$ (c) $\sqrt{10}+\sqrt{2}+\sqrt{5}+1$
    (b) $\sqrt{5}+\sqrt{10}+\sqrt{2}+1$ (d) $\sqrt{5}-\sqrt{10}-\sqrt{2}-1$

58. If $A^{\frac{1}{A}} = B^{\frac{1}{B}} = C^{\frac{1}{C}}, A^{BC} + B^{AC} + C^{AB} = 729$. Which of the following equals $A^{\frac{1}{A}}$?
    (a) $\sqrt[ABC]{81}$ (c) $\sqrt[ABC]{27}$
    (b) $\sqrt{2}$ (d) $\sqrt[ABC]{9}$

59. If $x = \dfrac{1}{2-\sqrt{3}}$, the value of $x^3 - 2x^2 - 7x + 10$ is equal to
    (a) $2+\sqrt{3}$ (c) $7+2\sqrt{3}$
    (b) 10 (d) 8

60. If $x = 1 + 5^{\frac{1}{3}} + 5^{\frac{2}{3}}$, then find the value of $x^3 - 3x^2 - 12x + 6$.
    (a) 22 (c) 16
    (b) 20 (d) 14

61. $\dfrac{4}{\sqrt{10-2\sqrt{21}}} =$ _____
    (a) $\dfrac{1}{4}\left(\sqrt{7}+\sqrt{3}\right)$ (c) $\sqrt{7}+\sqrt{3}$
    (b) $\dfrac{1}{4}\left(\sqrt{7}-\sqrt{3}\right)$ (d) $\sqrt{7}-\sqrt{3}$

62. If $y = 3^{\frac{1}{3}} + 3$, then $y^3 - 9y^2 + 27y =$ _____
    (a) 27 (c) -30
    (b) -27 (d) 30

63. $\dfrac{1}{\sqrt{8+2+\sqrt{15}}} =$ _____
    (a) $\dfrac{1}{2}\left(\sqrt{5}+\sqrt{3}\right)$ (c) $\dfrac{1}{2}\left(\sqrt{5}+1\right)$
    (b) $\dfrac{1}{2}\left(\sqrt{5}-\sqrt{3}\right)$ (d) $\dfrac{1}{2}\left(\sqrt{5}-1\right)$

64. If $x = 2^{\frac{1}{3}} - 2$, then $x^3 + 6x^2 + 12x =$ _____
    (a) 6 (c) 8
    (b) -6 (d) -8

65. $\dfrac{3}{\sqrt{19-2\sqrt{88}}} - \dfrac{8}{\sqrt{14+2\sqrt{33}}} =$ _____
    (a) $\sqrt{19+2\sqrt{33}}$
    (b) $\sqrt{14-2\sqrt{88}}$
    (c) $\sqrt{11+2\sqrt{24}}$
    (d) $\sqrt{11-2\sqrt{55}}$

66. $\sqrt{\sqrt[x]{2^x \sqrt[x^2]{3^{x^3} \sqrt[x^3]{6^{x^6} \sqrt[x^4]{9^{x^{10}}}}}}} =$ _____
    (a) 18 (c) 24
    (b) 54 (d) 36

67. $\sqrt{7+2\sqrt{6}}+\sqrt{7-2\sqrt{6}}=$ _____

(a) 14 (c) $2\sqrt{6}$
(b) $\sqrt{6}$ (d) 7

68. $\sqrt{3^2\sqrt{9^2\sqrt{(81)^2\sqrt{(16)^{16}}}}}=$ _____

(a) 6×2^4 (c) $6^3\times 2^3$
(b) $3^3\times 2$ (d) $6^3\times 2$

69. $\sqrt[6]{15-2\sqrt{56}}.\sqrt[3]{\sqrt{7}+2\sqrt{2}}=$ _____

(a) 0 (c) 1
(b) $\sqrt{2}$ (d) $6\sqrt{2}$

70. If $p=7-4\sqrt{3}$, then $\dfrac{p^2+1}{7p}=$ _____

(a) 2 (c) 7
(b) 1 (d) $\sqrt{3}$

Answers

4.1 Exercises

Q.1 (i) 521 (ii) 91 (iii) 723 (iv) 895 (v) 253 (vi) 68.3 (vii) 21.31 (viii) 2.64575... (ix) 2.82842... (x) 3.31662...

6.3 Exercises

Q.1 (i) $\frac{2}{7}\sqrt{7}$ (ii) $\frac{2}{9}\sqrt{3}$ (iii) $\frac{5}{4}(3+\sqrt{5})$ (iv) $\dfrac{31+10\sqrt{6}}{19}$ (v) $\dfrac{47+21\sqrt{5}}{4}$ (vi) $\dfrac{18-2\sqrt{10}-4\sqrt{6}-3\sqrt{15}}{19}$

Q.2 (i) $a=2,b=-1$ (ii) $a=8,b=3$ (iii) $a=11,b=-6$ (iv) $a=1,b=27$ (v) $a=4,b=1$ (vi) $a=2,b=-\dfrac{5}{6}$

Q.3 Q.4 14 Q.5 34 Q.6 $5+2\sqrt{6}$ Q.7 3

Q.8 (i) $(14)^{\frac{1}{3}},(200)^{\frac{1}{6}},(10)^{\frac{1}{2}}$ (ii) $(200)^{\frac{1}{20}},(100)^{\frac{1}{10}},(17)^{\frac{1}{5}}$

7.1 Exercises

Q.1 (i) $49x^2-y^2$ (ii) x^8-1 (iii) $x^8-\dfrac{1}{x^8}$ (iv) 9991 (v) 9409 (vi) 0.08 (vii) 11021 (viii) 9984

Q.2 (i) 34
(ii) 1154

Q.3 (i) $\pm\sqrt{29}$
(ii) ± 5

Q.4 80

Q.5 ± 8

Q.6 (i) $8x^3 + 27y^3 + 36x^2y + 54xy^2$ (ii) $16y^3 + 192x^2y$

Q.7 322

Q.8 18

Q.9 -756

Q.10 (i) 7254
(ii) 23058

Q.11 604

Q.12 370

Q.13 13

Q.14 -10

Q.15 1

Q.16 ± 16

Q.17 (i) $x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8zx$ (ii) $4a^2 + 9b^2 + c^2 - 12ab + 6bc - 4ac$

Q.18 (i) $2(a^2 + b^2 + c^2 + 2abc)$ (ii) 4a(b+c)

Q.19 (ii) 18

Q.20 (i) 1638 (ii) -1260 (iii) -90000

8.1 Exercises

Q.1 197, 499 and 773 are primes.

Q.2 121

Q.3 73

Q.4 No prime number is perfect cube.

Q.5 73

Q.6 2

Q.7 17

Q.8 No

Q.9 1009, 1013, 1019, 1021, 1031

Q.10 2003

Level 1

1. (b)	4. (c)	7. (c)	10. (d)	13. (d)	16. (c)	19. (b)	22. (d)	25. (c)	28. (b)
2. (d)	5. (b)	8. (b)	11. (a)	14. (b)	17. (c)	20. (d)	23. (d)	26. (c)	29. (b)
3. (a)	6. (a)	9. (a)	12. (b)	15. (d)	18. (c)	21. (c)	24. (a)	27. (b)	30. (a)

Level 2

31. (b)	34. (b)	37. (b)	40. (b)	43. (b)	46. (b)	49. (c)	52. (a)	55. (c)
32. (a)	35. (c)	38. (d)	41. (c)	44. (d)	47. (c)	50. (a)	53. (c)	
33. (b)	36. (a)	39. (d)	42. (a)	45. (a)	48. (d)	51. (a)	54. (c)	

Level 3

56. (a)	58. (b)	60. (a)	62. (d)	64. (b)	66. (a)	68. (d)	70. (a)
57. (b)	59. (d)	61. (c)	63. (b)	65. (c)	67. (c)	69. (c)	