Number System

1 Section : Classification of Numbers

Undoubtedly numbers are the cornerstone of mathematics. There is a huge literature on the development of numbers but that is beyond our scope. We directly jump to the classification of numbers. Remember this classification looks so obvious or trivial today but humans took centuries to reach here. Reader should appreciate this.

Here are some notations to denote the set of numbers :

N - Set of Natural Numbers $= 1, 2, 3, ...$
N_0 or W - Set of whole numbers $= 0, 1, 2, ...$
Z - Set of integers $= .... -5, -4, ... -1, 0, 1, 2...$
Q - Set of rational numbers $= ... -5, \frac{-4}{7}, \frac{-1}{2}, 0, \frac{1}{2}, \frac{1}{3}, \frac{5}{7}, 4, ...$

* All integers are rational numbers

Q' or Q^c - Set of irrational Numbers $= ... -\sqrt{11}, -\sqrt{5}, -\sqrt{7}, -\frac{1}{\sqrt{3}....\sqrt{5}, ...}$
R - Set of real Number.

A number is said to be rational if it can be written as $\frac{p}{q}$ form where p, q are integers. For example $\frac{1}{2}, \frac{-1}{5}, \frac{1}{-5}, -\frac{2}{7}, -1, 0, 4, -\frac{7}{8}$ are rational numbers. A real number is said to be irrational if it is not rational. For example $\sqrt{2}, \sqrt{3}, \sqrt{5}, -\sqrt{2}, \frac{1}{\sqrt{2}}, -\frac{\sqrt{3}}{\sqrt{2}}$ are irrational number. (It is very interesting to prove that $\sqrt{2}, \sqrt{3}$ etc cannot be written as $\frac{p}{q}$ form. We shall prove this shortly).

1.1 Exercises

Q.1 Find five rational numbers between

a. 1 and 10
b. 11 and 12
c. $\frac{1}{3}$ and $\frac{1}{2}$
d. $-\frac{1}{5}$ and $-\frac{1}{7}$
e. $-\frac{3}{5}$ and $\frac{3}{5}$

Q.2 State true or false

a. Every rational number is an integer.
b. Every natural number is an integer.
c. Every integer is an rational number.
d. 5 is an non-negative number.
e. 0 is an non-negative integer.
f. Every irrational number is an rational number.
g. $-\frac{1}{7}$ and 7 are rational numbers.

Q.3 Express the following in decimal by log division

a. $\frac{19}{5}$
b. $\frac{3}{15}$
c. $-\frac{19}{2}$
d. $\frac{2157}{625}$
e. $-\frac{17}{8}$
f. $\frac{327}{500}$

Q.4 Show that the decimal expansion of following rational numbers are non-terminating and repeating.

a. $\frac{8}{3}$ b. $\frac{2}{11}$ c. $\frac{1}{7}$ d. $-\frac{16}{45}$

2 Section : Non-terminating and recurring rational number

There are rational numbers such that when we try to express them in decimal form by division method, we find that no matter how long we divide there is always a remainder. In other words, the division process never comes to an end. This is due to the reason that in the division process the remainder starts repeating after a certain number of steps. In such cases, a digit of a block of digits repeats itself. For example, 0.3333...,0.1666666..., 0.123123123...., 1.26923076923076923 07...etc. Such decimals are called non-terminating repeating decimals or non-terminating recurring decimals. These decimal numbers are represented by putting a bar over the first block of the repeating part and omit the other repeating blocks. Thus, we write $0.33333... = 0.3$, $0.16666... = \overline{0.16}$, $0.123123123 ..... = \overline{0.123}$ and $1.26923076923076292307... = \overline{1.2692307}$.

Fact : Every non terminating and recurring number is a rational number.

1) Express the following rational numbers as decimals

a. $\frac{2}{3}$ c. $-\frac{2}{15}$ e. $\frac{437}{999}$

b. $-\frac{4}{5}$ d. $-\frac{22}{13}$ f. $\frac{33}{26}$

2) Look at several examples of rational numbers in the form $\frac{p}{q}(\neq 0)$, where p and q are integers with no common factors other than 1 and having terminating decimal representations. Can you guess what property q must satisfy?