Board Level Exercise

Type (I): Very Short Answer Type Questions: **01 Mark Each**

1. Write the principal value of $sec^2(-2)$.

2. If $tan^{-1}\sqrt{3} + cot^{-1}(x) = \frac{\pi}{2}$,find x.

Type (II): Short Answer Type Questions : **02 Mark Each**

1. If $sin^{-1}(x) + cos^{-1}(\frac{1}{2}) = \frac{\pi}{2}$. then find x.

2. Solve for x: $cos(2sin^1 x) = \frac{1}{9}, x > 0$.

Type (III): Long Answer Type Questions: **04 Mark Each**

1. Solve the following for x : $tan^1 \left[\frac{1+x}{1-x}\right] = \frac{\pi}{4} + tan^1 x, 0 < x < 1$.

2. Solve the x : $cos^{-1}x + sin^1(\frac{x}{2}) = \frac{\pi}{6}$.

3. Prove the following : $2tan^{-1}\frac{1}{3} + tan^{-1}\frac{1}{7} = \frac{\pi}{4}$.

4. Prove the following : $cos\left(sin^{-1}\frac{3}{5} + cot^{-1}\frac{3}{2}\right) = \frac{6}{5\sqrt{13}}$.

Type (IV): Very Long Answer Type Questions : **06 Mark Each**

1. if $tan^{-1}x + tan^{-1}y + tan^{-1}z = \pi$, prove that $x + y + z = xyz$.

2. Prove that : $sin^{-1}\left(\frac{4}{5}\right) + sin^{-1}\left(\frac{5}{13}\right) + sin^{-1}\left(\frac{16}{65}\right) = \frac{\pi}{2}$.

3. Solve the following for x : $tan^{-1}x + 2cot^{-1}x = \frac{2\pi}{3}$

4. Solve for x : $tan^{-1}\frac{x}{2} + tan^{-1}\frac{x}{3} = \frac{\pi}{4}; \sqrt{6} > x > 0$.

5. Prove that : $tan^{-1}\frac{1}{4} + tan^{-1}\frac{2}{9} = \frac{1}{2}tan^{-1}\frac{4}{3}$.

6. Prove that : $2tan^{-1}\frac{3}{4} - tan^{-1}\frac{17}{31} = \frac{\pi}{4}$.

7. Solve for x : $tan^{-1}\left(\frac{2x}{1-x^2}\right) + cot^{-1}\left(\frac{1-x^2}{2x}\right) = \frac{\pi}{3}, -1 < x < 1$.

8. Solve for x :$tan^{-1}(x+2)+tan^{-1}(x-2)=tan^{-1}\left(\frac{8}{79}\right);x>0$.

9. Prove that : $tan^{-1}(1)+tan^{-1}(2)+tan^{-1}(3)=\pi$.

10. Prove the following : $tan^{-1}\left(\frac{3}{4}\right)+tan^{-1}\left(\frac{3}{5}\right)-tan^{-1}\left(\frac{8}{19}\right)=\frac{\pi}{4}$.

Exercise # 1

PART - I : SUBJECTIVE QUESTIONS

A Definition, graphs and fundamentals

A-1 Find the simplified value of each of the following inverse trigonometric terms :

(i) $sin^{-1}\left(\frac{1}{2}\right)$

(ii) $cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

(iii) $cosec^{-1}\left(-\frac{1}{2}\right)$

(iv) $cos^{-1}\left(-\frac{1}{2}\right)$

(v) $sec^{-1}(-\sqrt{2})$

A-2 Find the simplified value of the following expressions :

(i) $sin\left[\frac{\pi}{3}-sin^{-1}\left(-\frac{1}{2}\right)\right]$

(ii) $tan\left[cos^{-1}\frac{1}{2}+tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)\right]$

(iii) $sin^{-1}\left[\left\{sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right\}cos\right]$

A-3 Draw the graph of the following functions :

(i) $y=sin^{-1}(x+1)$

(ii) $y=cos^{-1}(3x)$

(iii) $y=tan^{-1}(2x-1)$

A-4 Solve the following inequalities :

(i) $sin^{-1}x>-1$

(ii) $cos^{-1}x<2$

(iii) $cot^{-1}x<-\sqrt{3}$

A-5

(i) If $\sum_{i=1}^{n}cos^{-1}\alpha_i=0$, then find the value of $\sum_{i=1}^{n}i\alpha_i$

(ii) If $\sum_{i=1}^{2n}sin^{-1}x_i=n\pi$, then show that $\sum_{i=1}^{2n}x_i=2n$

B Trig. (trig -1 x), trig -1 (trig x) trig (-x)

B-1 Evaluate the following expressions :

(i) $sin\left(cos^{-1}\frac{3}{5}\right)$

(ii) $tan\left(cos^{-1}\frac{1}{3}\right)$

(iii) $cosec\left(sec^{-1}\frac{\sqrt{41}}{5}\right)$

(iv) $tan\left(cosec^{-1}\frac{65}{63}\right)$

(v) $sec\left(tan\left\{tan^{-1}\left(-\frac{\pi}{3}\right)\right\}\right)$ $costan^{-1}sincot^{-1}\frac{1}{2}$

B-2 Evaluate the following inverse trigonometric expressions :

(i) $sin^{-1}\left(sin\frac{7\pi}{6}\right)$

(ii) $tan^{-1}\left(tan\frac{2\pi}{3}\right)$

(iii) $cos^{-1}\left(cos\frac{5\pi}{4}\right)$

(iv) $sec^{-1}\left(sec\frac{7\pi}{4}\right)$

B-3 Find the value of the following inverse trigonometric expressions :

(i) $sin^{-1}(sin4)$

(ii) $cos^{-1}(cos10)$

(iii) $tan^{-1}(tan(-6))$

(iv) $cot^{-1}(cot(-10))$

(v) $cos{-}1\left(\frac{1}{\sqrt{2}}\left(cos\frac{9\pi}{10}-sin9\pi10\right)\right)$

B-4

Express $sin^1(sin\theta, cos1(cos\theta, tan1(tan\theta$ and $cot1(cot\theta$ in terms of linear expression of θ for $\theta\epsilon\left[\frac{3\pi}{2}, 3\pi\right]$

C Property "$\frac{\pi}{2}$" , Addition and subtraction rule, miscellaneous formula, summation of series

C-1 Find the value of following expressions :

(i) $cot(tan^{-1}a + cot^{-1}a)$

(ii) $sin(sin^{-1}x + cos^{-1}x), |x| \leq 1$

(iii) $tan\left[cos^{-1}\left(\frac{3}{4}\right) + sin^{-1}\left(\frac{3}{4}\right) - sec^{-1}3\right]$

C-2 Prove that

(i) $sin^{-1}\left(\frac{3}{5}\right) + sin^{-1}\left(\frac{8}{17}\right) = sin^{-1}\frac{77}{85}$

(ii) $cos^{-1}\frac{4}{5} + cos^{-1}1213 = cos^{-1}\frac{33}{65}$

(iii) $sin^{-1}\left(\frac{1}{\sqrt{5}}\right) + cot^{-1}3 = \frac{\pi}{4}$

(iv) $tan^{-1}\left(\frac{1}{3}\right) + tan^{-1}\left(\frac{1}{5}\right) + tan^{-1}\left(\frac{1}{7}\right) + tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}$

C-3 Simplify $tan^{-1}\left\{\frac{1}{2}sin^{-1}\left(\frac{2x}{1+x^2}\right)+\frac{1}{2}cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right\}$, **if** $x > y > 1$.

C-4 Find the value of $sin^{-1}(cos(sin^{-1}x)) + cos^{-1}(sin(cos^{-1}x))$

D Inverse trigonometric function Equations :

D-1 Solve for x

(i) $cos(2sin^{-1}x = \frac{1}{3}$

(ii) $cot^{-1}x + tan^{-1}3 = \frac{\pi}{2}$

D-2 Solve the following equations :

(i) $tan^{-1}\left(\frac{x-1}{x-2}\right)+\frac{1}{2}tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$

(ii) $sin^{-1}x + sin^{-1}2x = \frac{2\pi}{3}$

D-3 Solve the following equations :

(i) $tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}tan^{-1}x, (x > 0)$

(ii) $3tan^{-1}\left(\frac{1}{2+\sqrt{3}}\right)-tan^{-1}\left(\frac{1}{x}\right)=tan^{-1}\left(\frac{1}{3}\right)$