CFT :- Contrext free Grammer

$\alpha \rightarrow \beta$. → (with capital letter can be represented)

α ↓ Non-terminal

β → Terminal.

$\alpha = V_n$ $|V_n| = 1$ →

$\beta = V_n \cup T$

↓ Non-ter ↓ union ↘ Terminal.

for finite automata. we should add one more memory & stack

CFG :- we can create PDA (push down automata).

CFG
↓
PDA → PDA is much more power full than FA.

[Regular Exp] RG | RE.
↓
FA → finite automata.

$0^n 1^n$ $n \geq 0$.

PDA :- Language, $L = \{0^n 1^n \quad n \geq 0\}$.

* WW^R (a.b)

asa | bsb | a | b | ϵ (epsalone)

w (aa) (bb).

asb | bsa | ϵ

Problem [CFL → CFG]

→ CFL to CFG

Imp.

1. $L = \{\underbrace{a^n b^n c^i \mid n \geq 0, i \geq 1}_{L_1}\} \cup \{\underbrace{a^n b^m c^m d^m \mid}$

$\underbrace{n, m \geq 0}_{L_2}\}^*$

$L = aSb \mid c$ [compare $a^n b^n$ as "A" & c^i as B] i.e.,

$a^n b^n \to A$ $n \geq 0,$

$c^i \to B$. $i \geq 0.$

$S \to AB$. So.

$A \to aAb \mid \epsilon$ → Becoz: $n \geq 0$, $a^0 b^0 \Rightarrow \epsilon$.

$B \to cB \mid c$ → $c^i = c^1 = c$.

for L_2

let $a^n b^n = A$ and $c^m d^m = B$ i.e. $n, m \geq 0$.

$S \to AB$.

$A \to aAb \mid \epsilon$.

$B \to cBd \mid \epsilon$

Adding.

→ $S \to AB$ S and A are same.

$A \to aAb \mid \epsilon$

$B \to cB \mid c \mid cBd \mid \epsilon$.

Production Rule:-

Q) $L = \{ w \mid w \in \{0, 1\}^* \}$ with atleast 1 accurance of 101

(zero followed by any no. 1 or 1 followed by any no. of 0)

sep anything separate anything

$\boxed{(0+1)^* 101 (0+1)^*}$ production rule

0|1 0|1 → (Same as A)

$A \to 0A \mid 1A \mid \lambda$

For termination

Ex:-
0101
1101
00011
101011

$T \to 101$

Both T and A are initial production.

$S \to ATA$.

3) $L = \{ a^i b^j c^k \mid i = j+k \quad \Sigma = \{a, b, c\} \}$.

let

$L = \{a^i b^j c^k\}$ but $i = j+k$

$L = \{ a^{j+k} b^j c^k \}$

$L = \{ a^j a^k b^j c^k \}$.

$\underbrace{a^k a^j}_{a} \underbrace{b^j c^k}_{b}$

$A \to a^k a^j \qquad b^j c^k \to B$.

$A \to aAa \qquad B \to bBC$.

$\boxed{a^k a^j b^j c^k}$ S

$0^n 1^n$

$S' \to aS'b \mid ab$ → Termination will be ab

becoz $\Sigma = \{a, b, c\}$.

$S \to aS'c \mid ab S'$

Ans:-

aabc.
aaa bcc
a aaabbcc
aaa bbc.

$L = \{ w : n_a(w) \bmod 2 = 0$ where $w \in \{a, b\}^*$

→ divisable by 2.

Regular Expression :-

RE, $L = \{ b^* a b^* a b^* \}^*$ ex. → comes any no. of length

For given problem

$RE = \{ b^* a \; b^* a \; b^* \}^*$

But Initial will be $S \to BaBaB$

production $B \to bB \mid \epsilon$ $B \to bB \mid \epsilon$

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Initial . $S = BaBaBS \mid \epsilon$

ex:- $\{b^*a\ b^*a\ b^*\}^*$ s Termination (n^{th} tym)

If they ask $n_b(w) \bmod 2$ → Then ans will be

$S = AbAbAS \mid \epsilon$.

$RE = \{a^*b\ a^*b\ a^*\}^*$

5) $L = \{a^n ww^R b^n \quad w \in \{0,1\} \quad n \geq 2\}$

Initial $S \to aaS'bb$. ($n \geq 2$) min 2.

$\{aaSbb\}$

(WWR) → $S' \to 0S0 \mid 1S1 \mid \epsilon \mid 0 \mid 1$ (0.1)

Generally

$WW^R_{(a,b)} = aSa \mid bSb \mid a \mid b \mid \epsilon$

6) $L = \{a^k b^m c^n \mid m+n = k \ \& \ m, n \geq 1\}$

Let $L = a^k b^m c^n$ but $m+n = k$.

$a^{m+n}\ b^m\ c^n$

$a^m\ b^m\ a^n\ c^n$.

$a^m\ a^n\ b^m\ c^n$.

$a^n\ \underbrace{a^m\ b^m}_{[0^n 1^n]}\ c^n$

$(a^m b^m) \Rightarrow a^1 b^1 = ab$

$S' \Rightarrow aS'b \mid ab$ $(n \geq 1)$ $a^n c^n \to S$.

$S \Rightarrow aS'c \mid ac$ $(a^n b^n) = a^1 b^1 = ab$ S

7) $L = \{ w \in \{a\}^* \quad |w| \bmod 3 \neq |w| \bmod 2 \}$.

3mod 3 = 0
3mod2 = 1
so 0 ≠ 1.

3, 9, 15, 21, ...

first rule :-

$S \rightarrow aaaX|$

$X \rightarrow aaaaaaX | \epsilon$.

(OR)

$S \rightarrow aaaX | aaa$.

derivation and parse tree (string)

Imp
Left most derivation & Right most derivation :-

1) $E \rightarrow E*$

$W = \underbrace{(a11 - b0)}_{} \;|\; \underbrace{(b00 - a01)}_{}$

$E = (E - E) \;|\; (E - E)$

$E = (E) | (E)$

$E \rightarrow \underline{(E)}$

a11 → consist string & digit (11, 0, 00, 01)

If length is 2 it will be.

$E = LD$ (string, digit)

$L \Rightarrow a|b$

$D = 0D | 1D | \epsilon$

Ex:- ab11

$E = LD$

$L \rightarrow aL | bL | \epsilon$

$D = 0D | 1D | \epsilon$

2) $E \rightarrow E*T|T$

$T \rightarrow F - T | F$

$F \rightarrow (E) | 0 | 1$

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0 - ((1*0) - 0)

solu'n:- 0 - ((1*0) - 0) (LMD):-

E → E*T | T
T → F - T | F
F → (E) | 0 | 1

T - (T)

0 - ((F - T) - F)

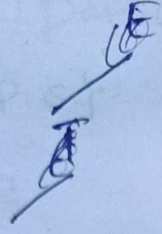
0 - ((1*0) - F)

→ $\underbrace{0}_{1}$ - $\underbrace{((1*0) - 0)}_{2}$

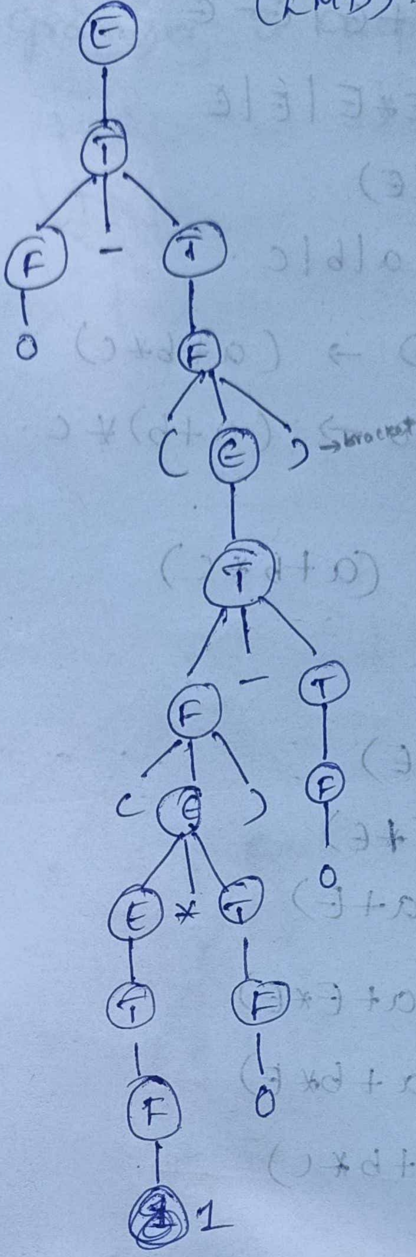
start symbol e
select
1st → E
2nd → T
3rd → F - T
→ 0 - T
→ 0 - F
→ 0 - (E)
→ 0 - (T)
→ 0 - (F - T)
→ 0 - ((E) - T)
→ 0 - ((E*T) - T)
→ 0 - ((T*T) - T)
→ 0 - ((F*T) - T)
→ 0 - ((1*T) - T)
→ 0 - ((1*F) - T)
→ 0 - ((1*0) - T)
→ 0 - ((1*0) - F)
→ 0 - ((1*0) - 0)

(RMD):-
0 - ((1*0) - 0)

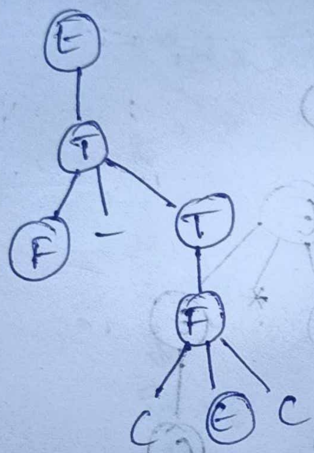
E
→ T
→ F - T
→ F - F
→ F - (E)
→ F - (T)
→ F - (F - T)
→ F - (F - F)
→ F - (F - 0)
→ F - ((E) - 0)
→ F - ((E*T) - 0)
→ F((E*F) - 0)
→ F((E*0) - 0)
→ F - ((T*0) - 0)
→ F - ((F*0) - 0)
→ F - ((1*0) - 0)
→ 0 - ((1*0) - 0)



(LMD).



RMD:-



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2) $E \to E+E \mid E-E$

$E \to E*E \mid E \mid E$

$E \to (E)$

$E \to a \mid b \mid c$.

i) LMD → (a+b*c)

ii) RMD → (a+b)*c.

LMD :- (a+b*c)

→ E

→ (E)

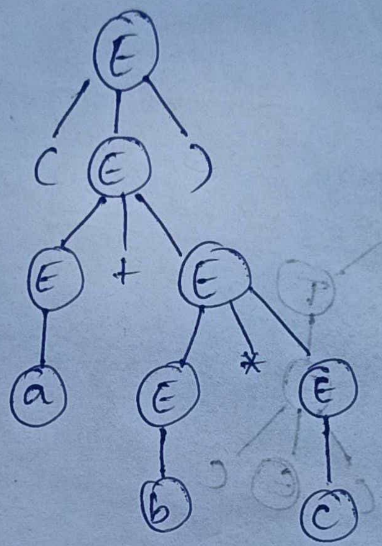
→ (E*E)

→ (a+E)

→ (a+E*E)

→ (a+b*E)

→ (a+b*c)



RMD :- (a+b)*c.

→ E

→ E*E

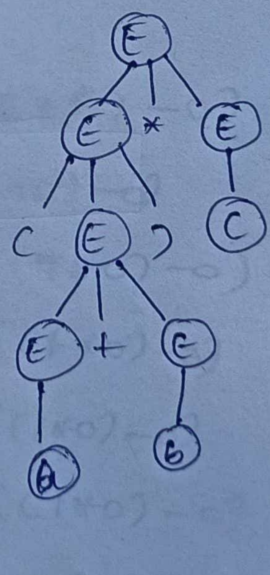
→ E*c

→ (E)*c

→ (E+E)*c

→ (E+b)*c

→ (a+b)*c



IMP

Ambiguity :- A particular grammer is represented by

$G = \{ V, T, P, S \}$

$V = \{ E \}$

$T = \{ id \}$

$P = (E \to E + E$

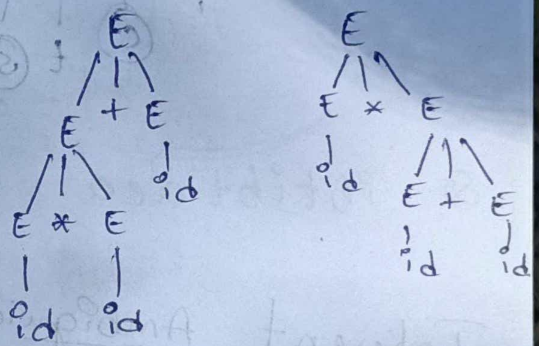
$E \to E * E$

$E \to id)$

Starting symbol, $S = \{ E \}$.

$id * id + id$

E
$\downarrow$
$E * E$
$\downarrow$
$id * E$
$\downarrow$
$id * E + E$
$\downarrow$
$id * id + E$
$\downarrow$
$id * id + id$.



E
$\downarrow$
$E + E$
$\downarrow$
$E * E + E$
$\downarrow$
$id * E + E$
$\downarrow$
$id * id + E$
$\downarrow$
$id * id + id$

1) $S \to ictS$

$S \to ictSeS$ $\qquad C \to b$

$S \to a$

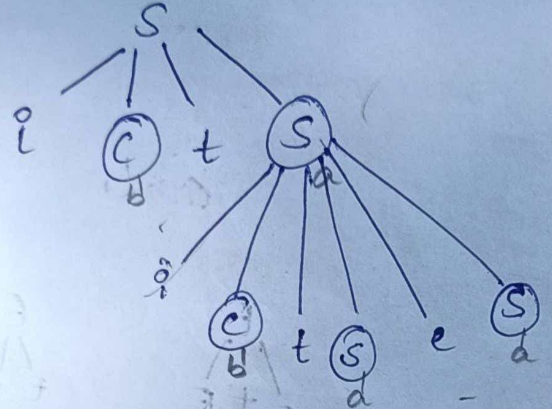
String = icts   S → ictseS

→ ~~ictse~~ · ictictseS.   Replace S as a

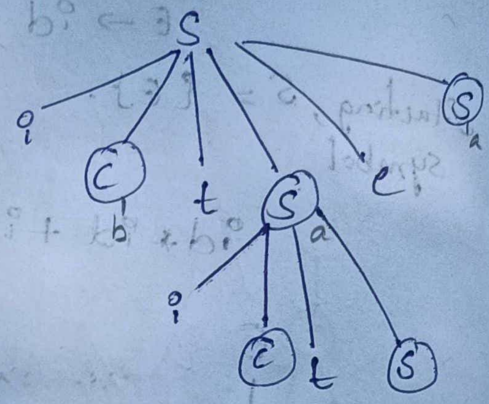
→ ictictaea   Replace C as b

→ ibtibtaea.

S – icts



S – ictSeS.



So ibtibtaea = ibtibtaea

<u>Inherent</u> <u>Ambiguies</u>:-

1) $S \to XYZ \mid aaYbb$

$X \to aaY \mid aa$

$Y \to baZ \mid ba$

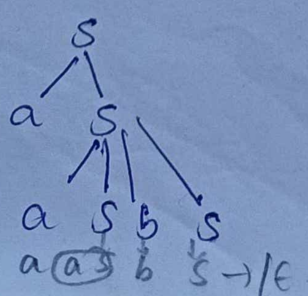
$Z \to bZb \mid bb$

$\boxed{aa \quad ba \quad bb}$

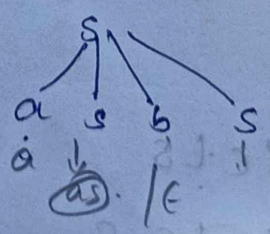
1) $\boxed{aab}$

$\boxed{S \to aS \mid aSbS \mid \epsilon}$

$S \to aS$



$S \to aSbS.$



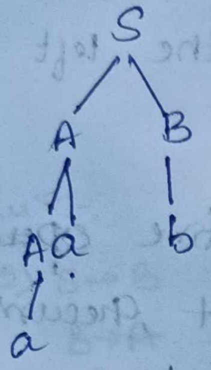
Hence It is a ambeguious.

2) $S \to AB \mid aaB$ string → aab

$A \to a \mid Aa$

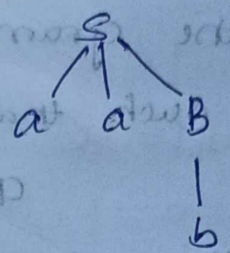
$B \to b$

$S \to AB$



aab

$S \to aaB$



aab

Hence it is an ambeguus.

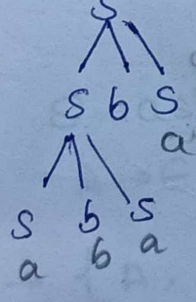
3) $S \to SS \mid (S) \mid \epsilon$ over $w = (()()())$

$S \to SS$

$S \to (S)$

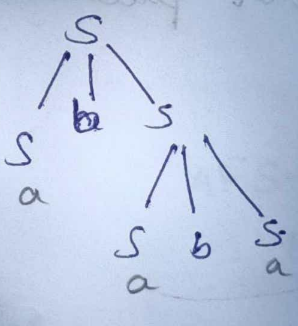
4) $S \to SbS$

$S \to a$



ababa

=



ababa

Ambegious

Removal of ambigious:-

E → E + E

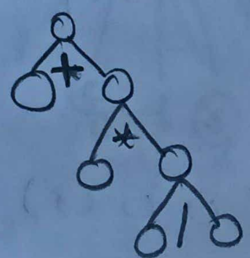
E → E * E

E → id.

* If the grammer has left associative Operator such that (+, *, /) then induce the left recurression.
(~~+~~, *, /)

* If the grammer has right associative operator such that (^) then induce the right recurrssion

LMD:-



E → E + F | F

F → F * T | T

T → id

RMD:-

T → G ^ T | G

G → id.

Elimination of unit production :-

1. S → AB

A → a

B → C | b

C → D

D → E

E → a.

S → A | B

A → ba | bb

B → A | bba.

Soluⁿ:- ① B → C

C → D

$\underbrace{D \to E}$

We should delete this variable and then delete finally we got

S → AB

A → a

B → a | b

② $S \to aA|B$
$A \to ba|bb$
$B \to A|bba.$

$\alpha, \beta \to V$

3) $S \to aS|A$
$A \to \epsilon$

4) $S \to ABC$
$A \to aA|\epsilon$
$B \to bB|\epsilon$
$C \to c.$

Solu'n:-

②) $S \to B$
$B \to A.$
⇓
$S \to aA|B$
$A \to ba|bb$
$B \to A|bba$
↓
$S \to aA|ba|bb|bb$
$A \to ba|bb$
$B \to ba|bbba|bb$ → Neglet 'B' becoz already A have an Item in S

$S \to B \to A.$
Eliminate 'A' → $ba|bb.$

$S \to aA|ba|bba|bb$
$A \to ba|bb$

3) $S \to aS|A$
$A \to \epsilon$
→ $\{A, S\} \to \epsilon$

Solu'n:- $S \to aS|A|a|\epsilon$ → Then aremove
$S \to aS|a|\epsilon$

4) $S \to ABC$, $A \to aA|\epsilon$
$B \to bB|\epsilon$, $C \to c$
→ $\{A, B, \cancel{S}\}$

$S \to ABC|BC|AC|C$
$A \to aA|\epsilon|a$ } Then eliminate the ϵ
ϵ ⇒ Apply ϵ for $B \to bB|\epsilon|b$
A then ABC becomes $C \to c.$
BC
Same as aremaining.

After Elimination ∈ the Production rule will be

S → ABC | BC | AC | c , A → aA | a

B → bB | b , C → c.

5) S → aA | B

A → aB | B

B → aB | b | bC

D → Ea

E → a | aE | bc

Remove useless symbol?

Eliminate D, E and C becoz we doesn't used.

S → aA | B

A → aB | B

B → ab | b | b.

6) S → ABC | BaB

A → aA | BaC | aaa

B → bBb | a | D

C → CA | AC

D → ∈

Eliminate that ∈, unit & useless production?

Solu^n:- a) ~~Useless production~~ [Eliminate D]

S → ABC | BaB

A → aA | BaC | aaa

B → bBb | a | ~~D~~

C → CA | AC

a) ∈ production.

S → ABC | BaB

A → aA | Bac | aaa

B → bBb | a | D

C → CA | AC

D → ∈

⇒ S → ABC | BaB
A → aA | BaC | aaa
B → bBb | a
C → CA | AC.

[eliminate D]

b) Unit production.

[B → D] → But we already eliminated D, then we place the ans D as A.

S → ABC | BaB
A → aA | BaC | aaa.
B → bBb | a | bb
C → CA | AC.

c) Useless production.

→ So remove C.

S → BaB
A → aA | aaa
B → bBb | a | bb

⇓

S → BaB
B → bBb | bb | a

S → ABC ×
S → BaB
A → aA ×
A → BaC ×
A → aaa ×
B → bBb
B → a
B → bb
C → CA ×
C → AC ×

not possible to reach to S.
↓
Infinite

Imp
7) S → aA | aB
A → aaA | B | ∈
B → b | bB
D → B

Eliminate ∈, unit & useless Production?

a) ∈ production :-

⇒ S → aA | aB
A → aaA | B | ∈
B → b | bB
D → B

{A . S . B, D}.

S → aA|aB|ϵ|a

A → aaA|B|ϵ|aa

B → b|bB

D → B

⇓

S → aA|aB|a

A → aaA|B|aa

B → b|bB

D → B

b) unit production:

A → B

D → B.

⇓

S → aA|aB

A → aaA| $\underbrace{b|bB}_{B}$ |ϵ

B → b|Bb

D → B   eliminate D → B

⇓

S → aA|aB

A → aaA|b|bB|ϵ

B → b|Bb   eliminate ϵ

⇓

S → ~~aA~~ aA|aB|a

A → aaA|b|bB

B → b|bB → Not required. B for useless

(c) $S \to aA \mid aB \mid a$ (useless)

$A \to aaA \mid b \mid bB$

8) $S \to AaA \mid CA \mid BaB$

$A \to aaBa \mid CDA \mid aa \mid DC$

$B \to bB \mid bAB \mid bb \mid aS$

$C \to Ca \mid bC \mid D$

$D \to bD \mid \epsilon$

a) unit production:-

$C \to D$

$D \to \epsilon$

⇓

$S \to AaA \mid CA \mid BaB$

$A \to aaBa \mid CDA \mid aa \mid DC$

$B \to bB \mid DAB \mid bb \mid aS.$

$C \to Ca \mid bc \mid D$

$D \to bD \mid \epsilon$

$D \to bD \mid b.$

$C \to Ca \mid bc \mid bD \mid b.$

⇓

$S \to AaA \mid CA \mid BaB \mid$

$A \to aaBa \mid \underline{CDA} \mid aa \mid DC \mid$

$B \to bB \mid DAB \mid bb \mid as$

$C \to Ca \mid bc \mid D$

$D \to bD \mid \epsilon$

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9) S → aAa | bBb | ε (nullable)

A → C | a

B → C | b

C → CDE | ε → C → CDE | DE

D → ~~bDb~~ A | B | ab

⇒ {C, A, B, S}

a) ε

S → aAa | bBb | aa | bb

A → C | a

B → C | b

C → CDE | ~~DE~~ | ε | CE

D → ~~bDb~~ A | B | ab

D → ε
C → ε
B → ε
A → ε

A → C

⇓

S → aAa | bBb | aa | bb

A → CDE | DE | a

B → CDE | DE | b

D → bD | b

Remove D

⇓

S → aAa | bBb | aa | bb

A → CDE | DE | ~~ε~~ | CE | a

B → CDE | DE | ~~ε~~ | CE | b

~~C → CDE | ε | DE | CE~~

D → ab | CDE | DE | CE

C will be eliminated.

S → aAa | bBb | aa | bb

A → a

B → b

D → ab

* Chomsky Normal form:-

→ Non terminal → Non-terminal × Non-terminal

Non terminal → Terminal

1) $S \to 1A \mid 0B$

$A \to 1AA \mid 0S \mid 0$

$B \to 0BB \mid 1$

⇒ $S \to 1A$ (Non Terminal → Terminal)

$S \to 0B$

$A \to 1AA$

$A \to 0S$

$A \to 0$ ✓

$B \to 0BB$

$B \to 1$ ✓

for $S \to 1A$

$A' \to 1$

$S \to A'A$

for $S \to 0B$

$B' \to 0$

$S \to B'B$

for $A \to 1AA$

$A \to SA \mid A \to A'AA$

for $B \to 0BB$

$B \to SB$

for $B \to 1$

No need to change.

for $A \to 0S$

$A \to B'S$

for $A \to 0$

No need to change

2) $S \to bA \mid aB$

$A \to bAA \mid aS \mid a$

$B \to aBB \mid bS \mid b$

Non-terminal → No... & small letter

terminal → Big letter.

⇒ $S \to bA$

$S \to aB$

$A \to bAA$

$A \to aS$

$A \to a$ ✓

$B \to aBB$

$B \to bS$

$B \to b$ ✓

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for $S \to bA$

$S \to B'A$, $B' \to b$

for $S \to aB$

$S \to A'B$, $A' \to a$

for $A \to bAA$

$A \to SA$

for $A \to aS$

$A \to a'S$

for $B \to aBB$

$B \to A'BB$

for $B \to bS$

$B \to B'S$

3) $L = \{a^{4n} \mid n \geq 1\}$

$S \to \underbrace{aaaa}_{a^{4n} \to a^4} S \mid aaaa$ {Production Rule} $a^{4(1)} = a^4$

Ex:- $a^{4n+1}, n \geq 1$

$aaaa\,S \mid aaaaa$

① $a^{2n+1}, n \geq 0$

$aa\,S \mid aaa$

③ $a^{2n}, n \geq 0$

$aa\,S \mid \epsilon$

Replace $A \to a$

Not assumed

$S \to R_1 A$

$R_1 \to R_2 A$

$R_2 \to R_3 A$

(OR)

$S \to A'S$

$A' \to \underset{AAAA}{aaaa}$ so $A' = \underbrace{AA}_{C}\underbrace{AA}_{C}$

$A' \to CC$

$C \to AA$

Puch down Automata :-

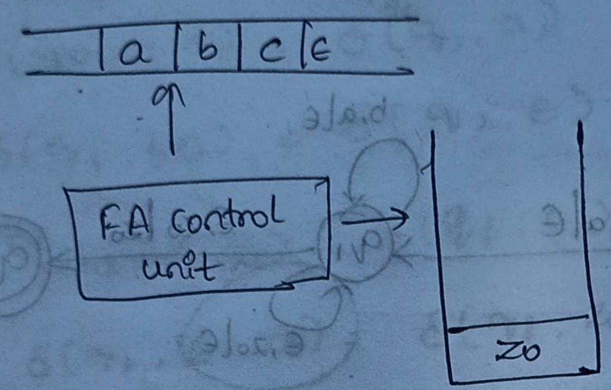
$P(Q, \Sigma, \Gamma, F, S, Z_0)$

Q = finite cet of ctatec

Σ = i/p alphabet

F = final State

S = Start cymbol.



* PDA is a combination of finite automata (FA) and Stuck.

→ Two typec of operation

* puch
* POP and skip (either Puch or POP)

Transition function, $\delta : Q \times \{\Sigma \cup \epsilon\} \times \Gamma \rightarrow Q \times \Gamma^*$

Q → State; $\cup$ → union; ϵ → Epsolone; Γ → Tow; Γ^* → change the top of the stack element

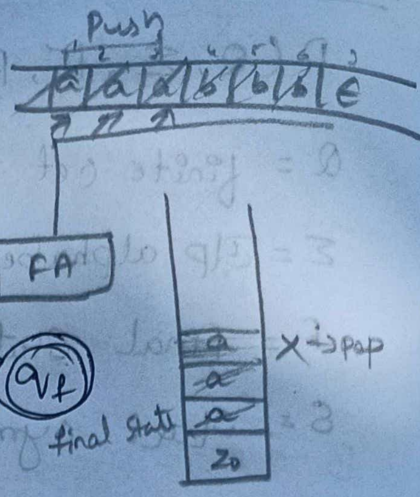
* DPDA → Deterministic Puch down automata
* NPDA → Non -11-

for DPDA :- $\delta : Q \times \{\Sigma \cup \epsilon\} \times \Gamma \rightarrow Q \times \Gamma^*$

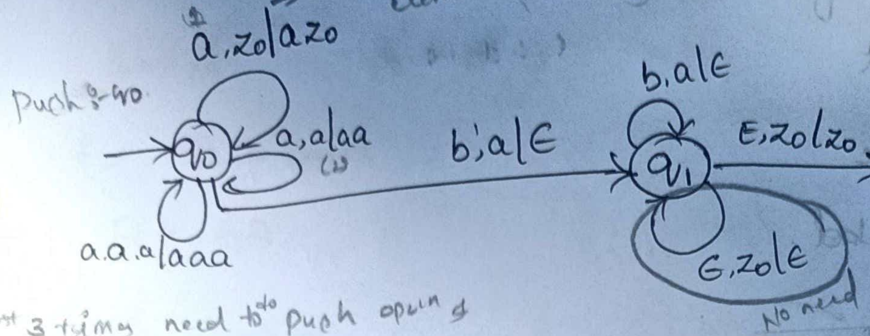
NDPDA :- $\delta : Q \times \{\Sigma \cup \epsilon\} * \Gamma \rightarrow 2^{(Q \times \Gamma^*)}$

Problem:

1) $L = \{a^n b^n \mid n \geq 1\}$ aaa, bbb

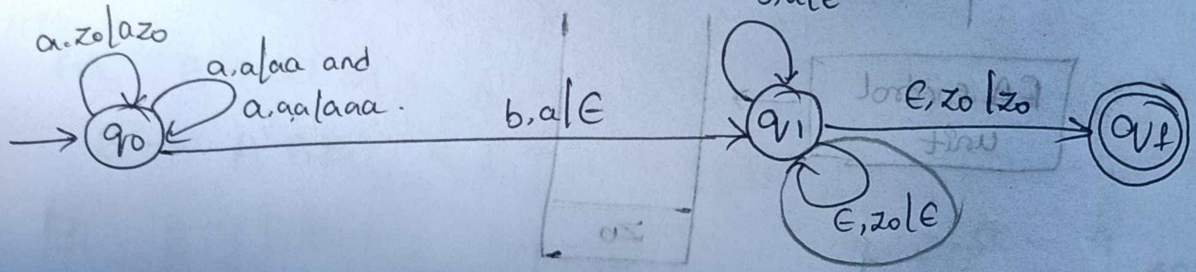


Push:-



1st 3 times need to do push opn & after that do the pop opn

(OR)



$\delta(q_0, a, z_0) = \delta(q_0, az_0)$

$\delta(q_0, a, a) = \delta(q_0, aa)$

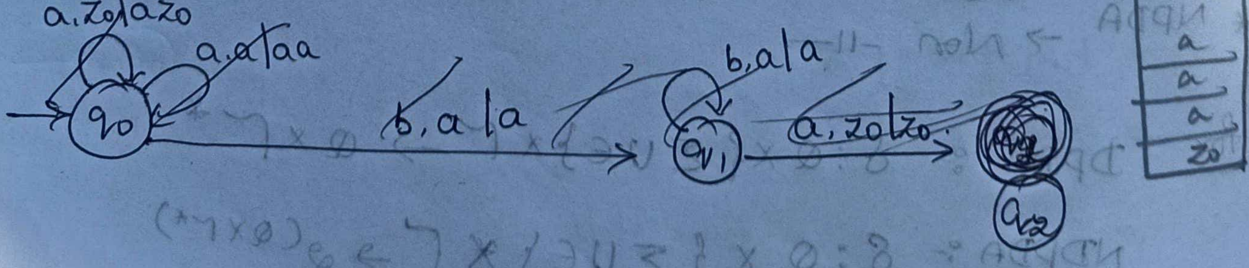
$\delta(q_0, b, a) = \delta(q_1, \epsilon)$

$\delta(q_1, b, a) = \delta(q_1, \epsilon)$

$\delta(q_1, \epsilon, z_0) = \delta(q_f, z_0)$ (OR) $\delta(q_0, \epsilon, z_0) = \delta(q_0, \epsilon)$

2) $L = \{a^n b^{2n} \mid n \geq 1\}$ Ex:- n=2 aa, bbbb.

skip first a means bb



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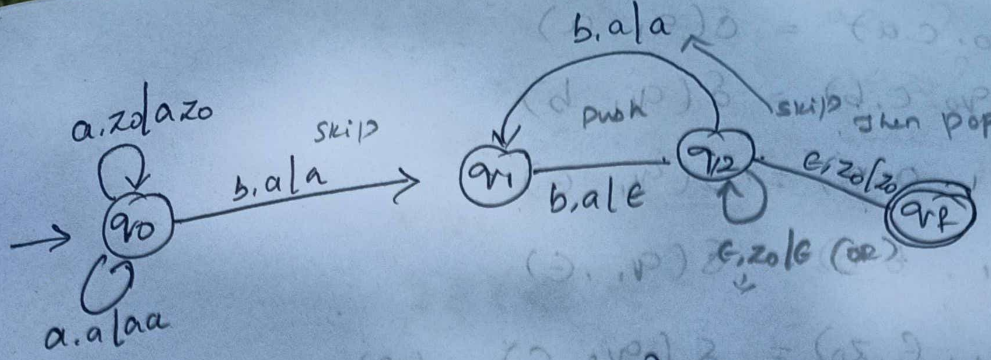


$\delta(q_0, a.z_0) = \delta(q_0. az_0)$

$\delta(q_0, a.a) = \delta(q_0. aa)$

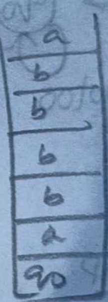
$\delta(q_0, b.a) = \delta(q_1. a)$

$\delta(q_1, b.a) = \delta(q_2, \epsilon)$

$\delta(q_2, \epsilon, z_0) = \delta(q_f, \epsilon)$ (OR)

$\delta(q_2, \epsilon, z_0) = \delta(q_f, z_0)$

3> $L = \{wcw^R \mid w \in (a,b)^*\}$



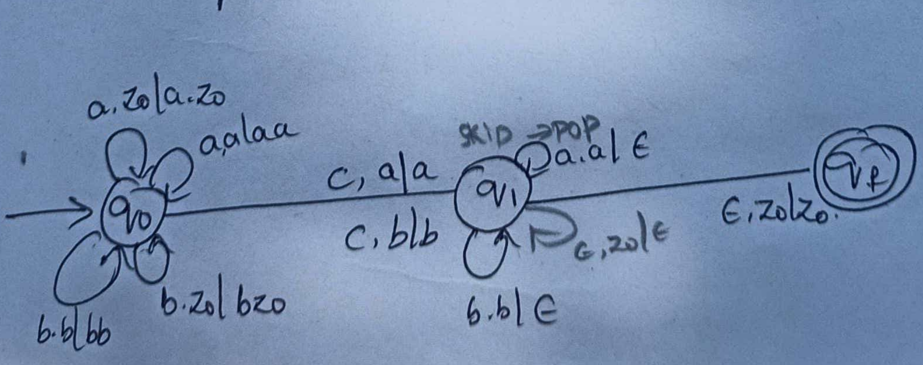
⇒ abb c bba (OR) aabb c bbaa.

Consider aabb c bbaa

{abbaa c aabb

skip

firstly we need to do push op^n.



$\delta(q_0. a, z_0) = \delta(q_0, az_0)$

$\delta(q_0. a.a) = \delta(q_0, aa)$

$\delta(q_0, b.b) = \delta(q_0. bb)$

$\delta(q_0 . b. z_0) = \delta(q_0. bz_0)$

$\delta(q_0, c.a) = \delta(q_1, a)$

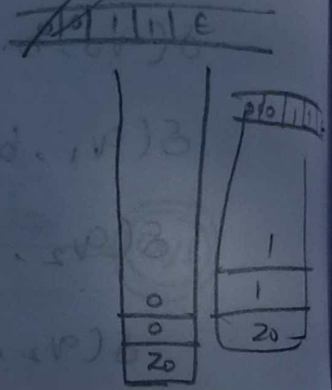
$\delta(q_0, c, b) = \delta(q_1, b)$

$\delta(q_1, a.a) = \delta(q_1, \epsilon)$

$\delta(q_1, b.b) = \delta(q_1, \epsilon)$

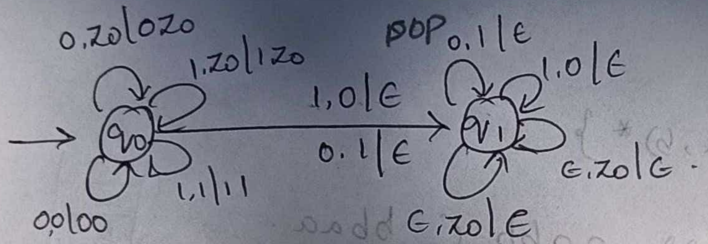
$\delta(q_1, \epsilon.z_0) = \delta(q_f, \epsilon)$ (OR)

$\delta(q_1, \epsilon, z_0) = \delta(q_f, z_0)$



4) $L = \{w \in (0,1)^* \; n_0(w) = n_1(w)\}$

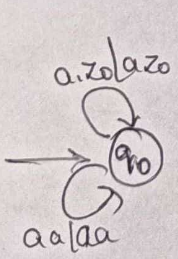
01, 0011, 1100, 10



Imp.

5) $L = \{w : w \in (a+b)^* \; n_a(w) = n_a(w)\}$ for aababb

ab, aabb, bbaa, ba



$\delta(q_0, aababb, z_0) \vdash \delta(q_0, ababb, az_0)$

$\vdash \delta(q_0, babb, aaz_0)$

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$+ \delta(q_0, abb, az_0)$

$+ \delta(q_0, bb, aaz_0)$

$+ \delta(q_0, b, az_0)$

$+ \delta(\quad , \quad (\epsilon, z_0)$

$+ \delta(q_f, z_0)$

6) $L = \{w : w \in (a+b)^* \quad n_a(w) \geq n_b(w)$

7) $0^{2n} 1^n \quad n \geq 1$

8) $L = \{a^i b^j c^k \quad j = i+k \quad i, k \geq 0\}$

Turning Machine :-

* 11 + 111 = 11111 ⇒ 2 + 5 = 7

  ↓

  2 + 3 = 5

$\boxed{11 + 11111}$ = 1111111

11 ~~+~~ +11

1↑

111+11

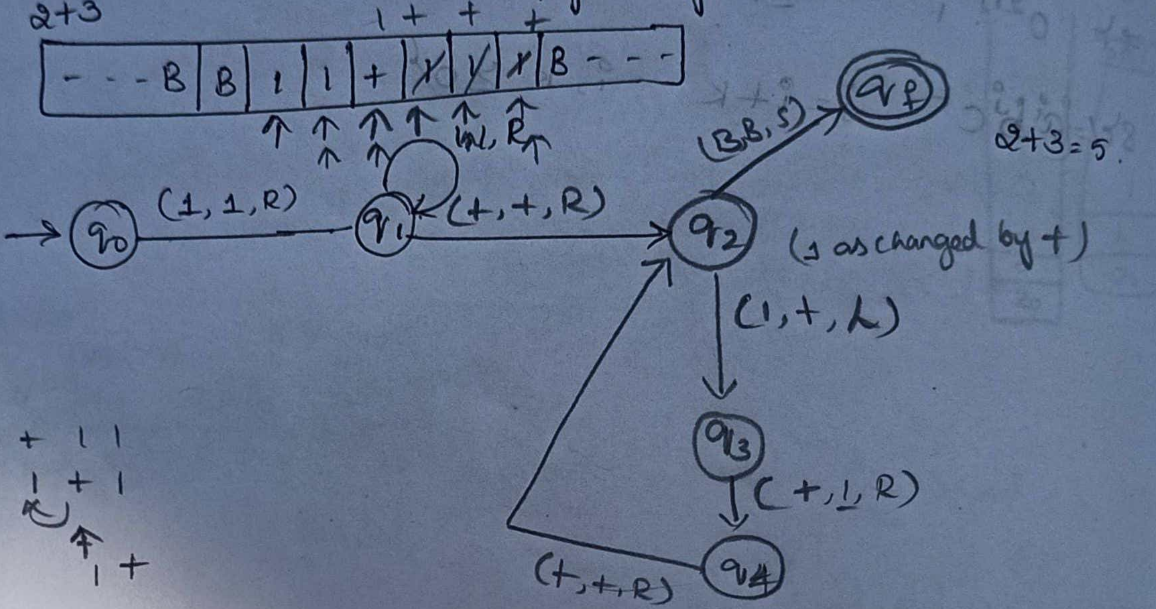
1111+1

11111+B

↑

(empty string)

2+3



111 + 11

1 + 1

↑

1 +