Module - 2

- ⇒ Regular Expression
- ⇒ Pumping lemma
- ⇒ closure Properties
- ⇒ Regular Expression to ENFA
- ⇒ DFSM to RE using state elimination Method.
- ⇒ Theorem(kleen's theorem RE to NDFA
- ⇒ Regular Grammar

$S \to \epsilon \mid a \mid aA$

⇒ Regular expression :-

Basic :-

1) $r = a$
2) $r = \epsilon$
3) $r = \phi$

$a^* = \{\epsilon, a, aa, aaa, \ldots\}$

Operations :-

1) Union $(+, \cdot, |)$ ⇒ $r_1 = a$, $r_2 = b$, $r_1 + r_2 = a+b$
2) Concatenation
3) Closure

1) Write a R.E. over $\Sigma = \{a, b\}$ which will accept any no. of a's & b's.

⇒ R.E ⇒ $(a+b)^*$

2) Write a RE over $\Sigma = \{a, b\}$ where all strings ends with abb.

⇒ RE ⇒ $(a+b)^* \cdot abb$

3) Starts with aa & ends with bb

⇒ RE ⇒ $aa\,(a+b)^*\,bb$

4) Write a R.E over $\Sigma = \{a, b\}$ which contains a substring aa

⇒ $(a+b)^*\, aa\, (a+b)^*$

5) $L = \{a^{2n} \mid n \geq 0\}$

$\Rightarrow$ $n=0 \Rightarrow \epsilon$

$n=1 \Rightarrow aa$

$n=2 \Rightarrow aaaa$

$r = (aa)^*$

6) $L = \{a^{2n+1} \mid n \geq 0\}$

$\Rightarrow r = (aa)^* a$

7) $L = \{a^n b^m \mid n \geq 1, m \geq 0\}$

$\Rightarrow$ $r = a^+ b^*$

8) $\{a^{2n} b^{2m+1} \mid n \geq 1, m \geq 1\}$

$\Rightarrow (aa)^+ (bb)^+ b$

9) $L = \{a^n b^n \mid n \geq 0\}$

Cannot have R.E

10) $L = \{a^n b^m \mid n \geq 2, m \leq 4\}$

$\Rightarrow aa(a)^* (\epsilon + b + bb + bbb + bbbb)$

11) Write a r.e over $\Sigma = \{a, b\}$ where all strings contains 3rd symbol from R.H.S as a

$\Rightarrow$ $\cancel{a(a+b)^*(a+b)}$ $(a+b)^* a (a+b)(a+b)$

12) Write a r.e over $\Sigma = \{0,1\}$ where all strings doesn't end with 11

$\Rightarrow$ $(0+1)^* (00+01+10)$

13) Write a r.e over $\Sigma = \{a,b\}$ for $L = \{w \mid |w| \bmod 3 = 0\}$

$b^2 \Rightarrow bb$

$b^3 = bbb$

$\Rightarrow$ $r.e = ((a+b)^3)^*$

14) Write a r.e over $\Sigma = \{0,1\}$ where all strings does not contain a substring 11.

$\{\cancel{00, 01, 10}$

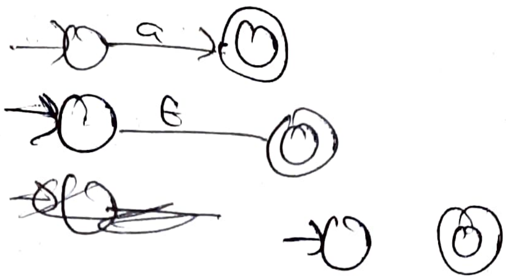
$\Rightarrow$ $r = (10+01)^*$

$\cancel{(01+10)}$

(P.T.O)

RE to NDFSM :-

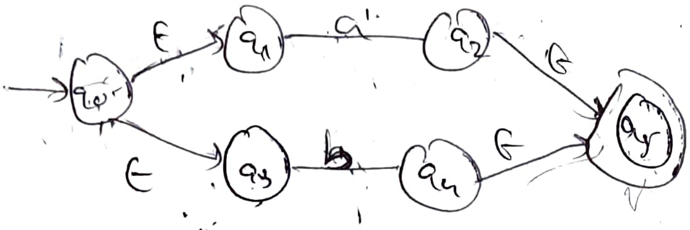
Basic :- 1) $r = a$



Union :-

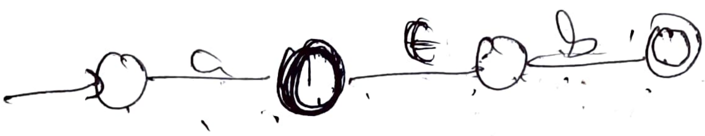
$r_1 = a \quad r_2 = b$

$r = r_1 + r_2 \; (a+b)$



Concatenation :-

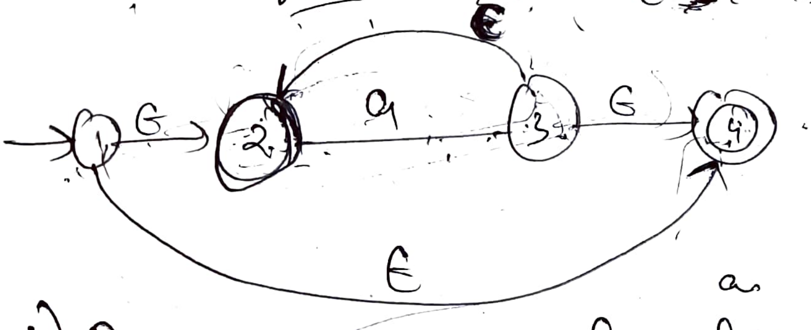
$r_1 r_2 = ab$

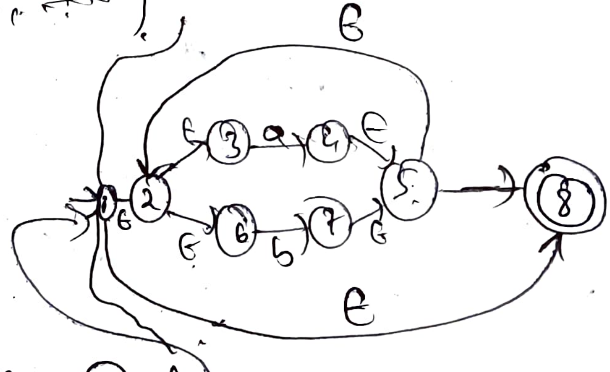


closure :-

$r = a$

$r^* = a^* \{\epsilon, a, aa, \ldots\}$

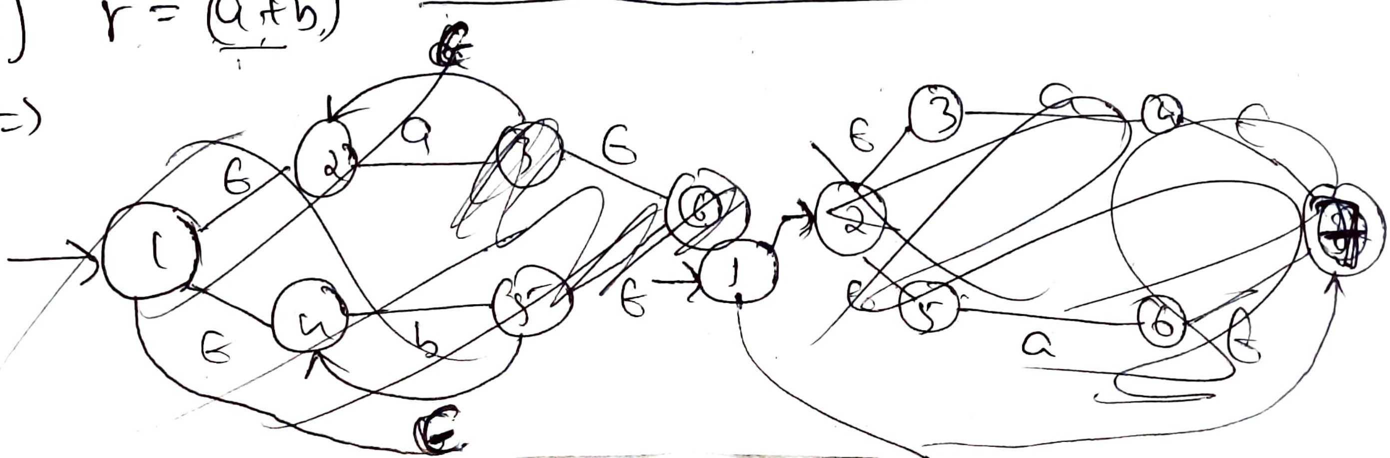




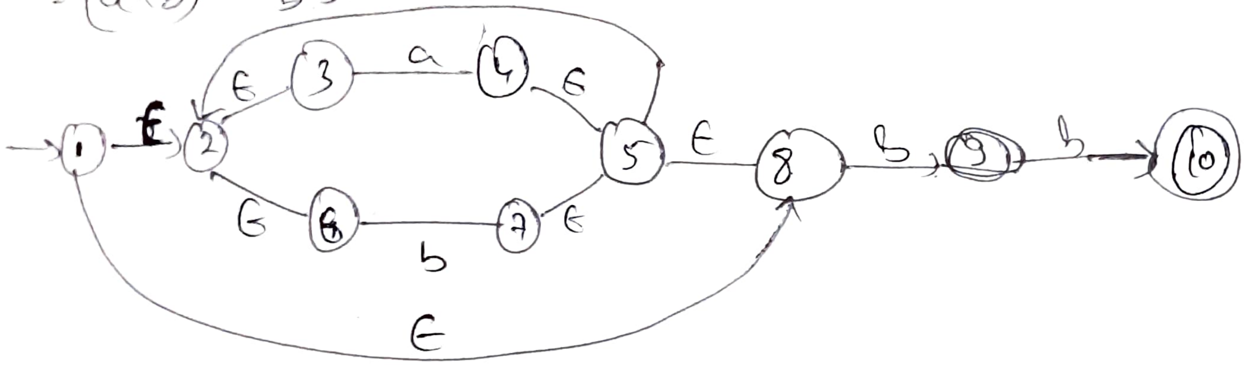
1) Draw NDFSM for following R.E

a) $r = (a+b)^*$

$\Rightarrow$

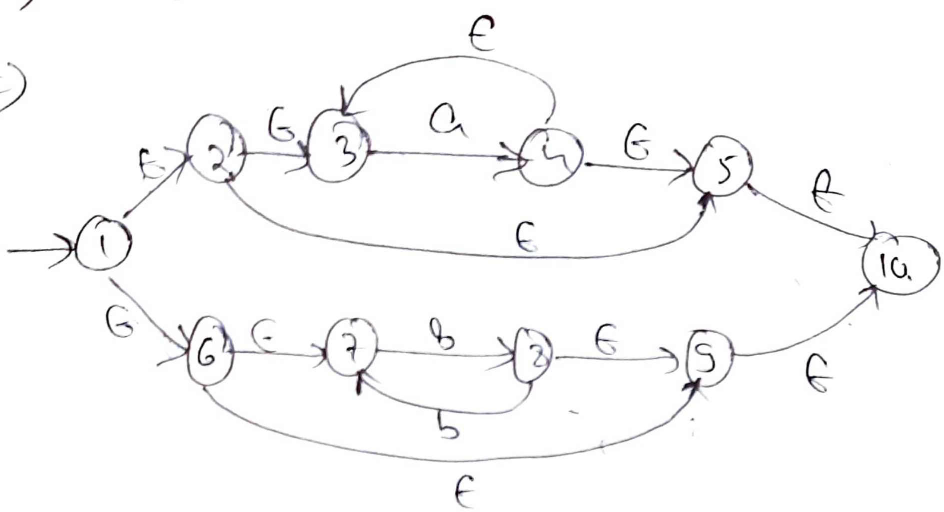


2) $r = (a+b)^* bb$.



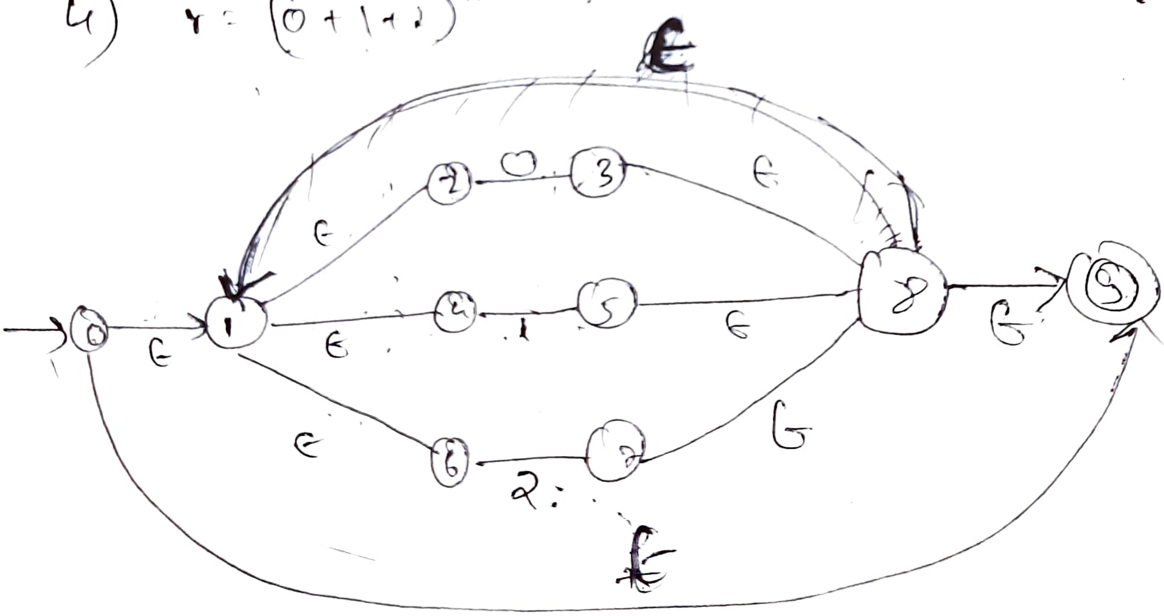
3) $r = a^* + b^*$

$\Rightarrow$



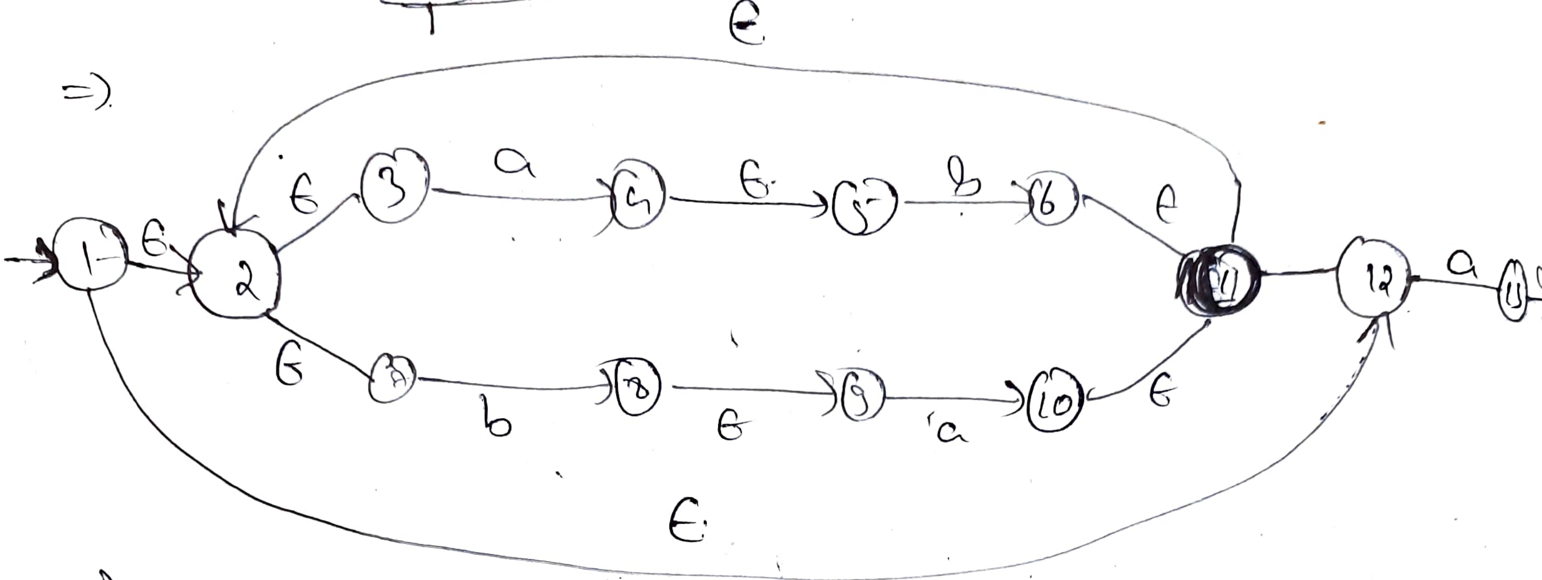
$r = a$

4) $r = (0+1+2)^*$



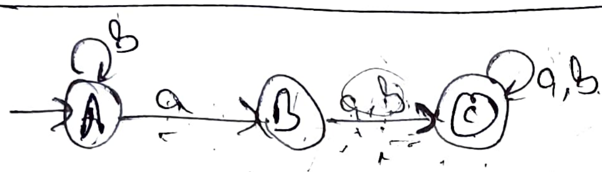
8) $r = (ab+ba)^* aa$.

=>

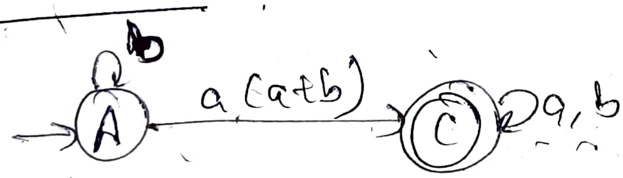


DFSM to RE (By State Elimination Method)

1)

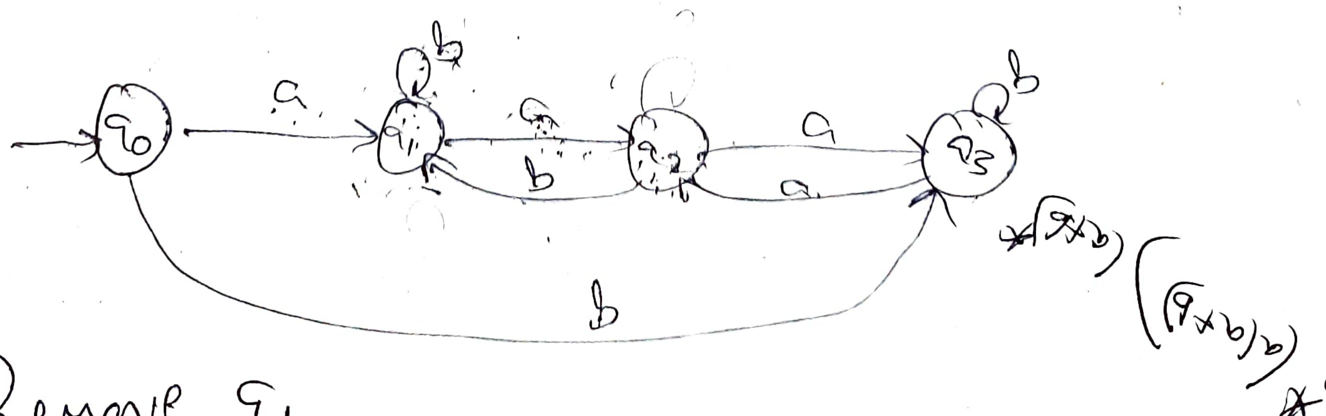


=> Remove B

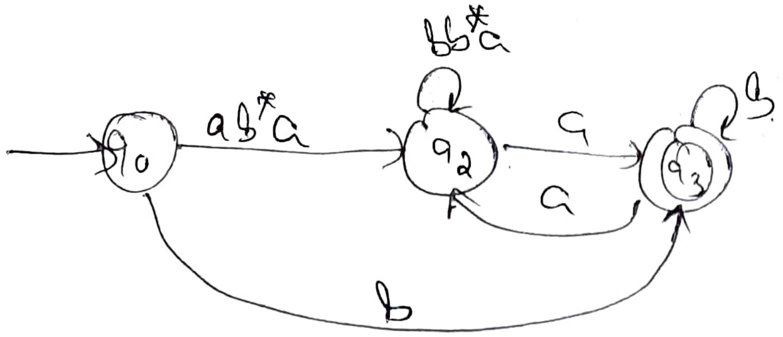


Now R.E => $b^* a(a+b)(a+b)^*$

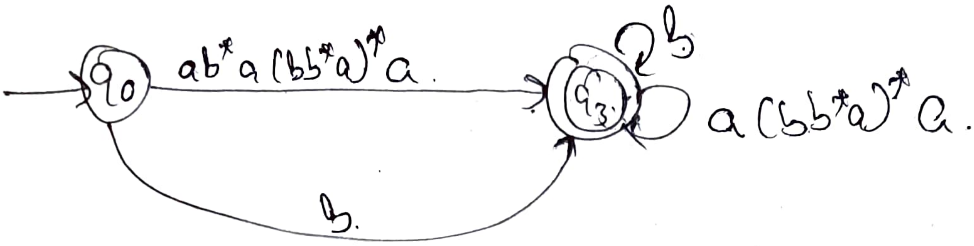
2)



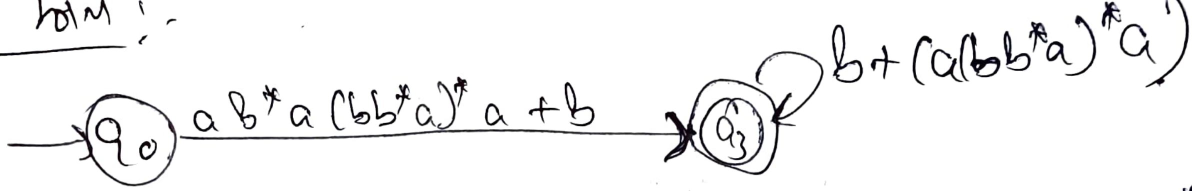
=> Remove q_1



Remove q_2

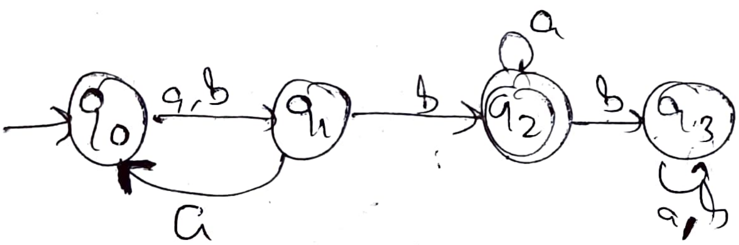


simplified form :-

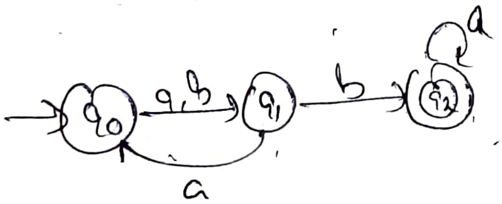


$R.E \Rightarrow (ab^*a(bb^*a)^*a+b)(b+(a(bb^*a)^*a))^*$

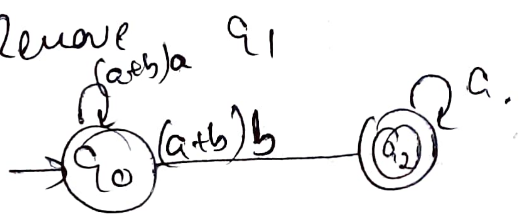
3)



$\Rightarrow$ q_3 is Error state. Remove it

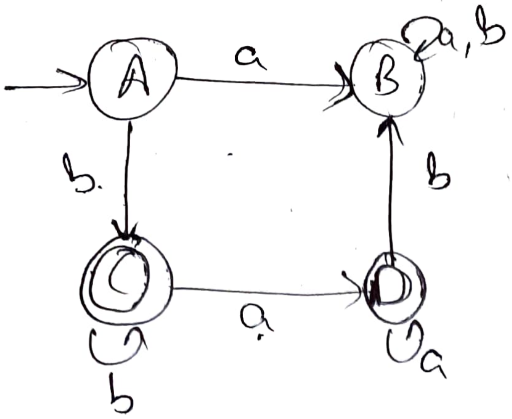


$\Rightarrow$ Remove q_1



$\Rightarrow R.E = ((a+b)a)^*(a+b)b(a)^*$

(4)

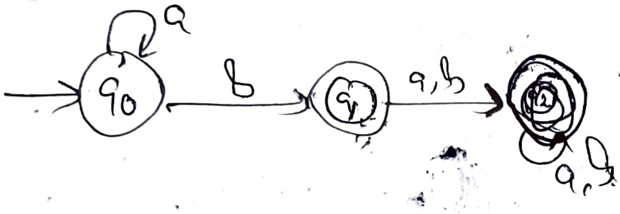


B & D are error states.

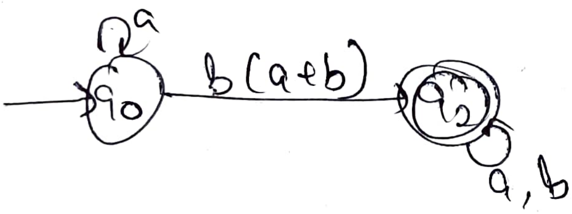
$\Rightarrow$

$r = bb^* = b^+$

5)



$\Rightarrow$ Remove q_1, keep q_2



$r_1 = a^*(b(a+b))(a+b)^*$

$= a^*b(a+b)^+$

Remove q_2 keep q_1

$r_2 = a^*b$

$\therefore r = r_1 + r_2$

$= a^*b(a+b)^+ + a^*b$

$= a^*b((a+b)^+ + \epsilon)$

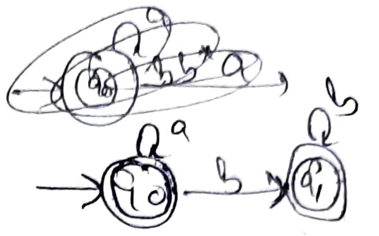
$= a^*b(a+b)^*$

6)

Remove q_1, keep q_2

$r_1 = a^* bb^* a (a+b)^*$

Remove q_2, keep q_1



$r_2 = a^* bb^* = a^* b^+$

Remove both q_1 & q_2.

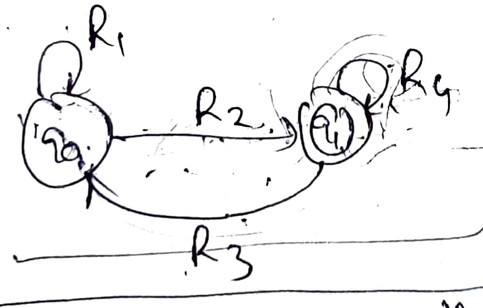
$r_3 = a^*$

$r = r_1 + r_2 + r_3$

$r = a^* bb^* a (a+b)^* + a^* b^+ + a^*$

$r = a^* b^+ a (a+b)^* + a^* b^+ + a^*$

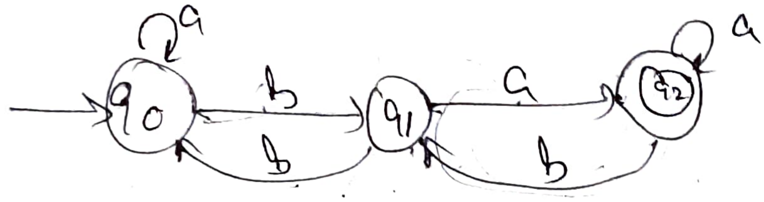
7)



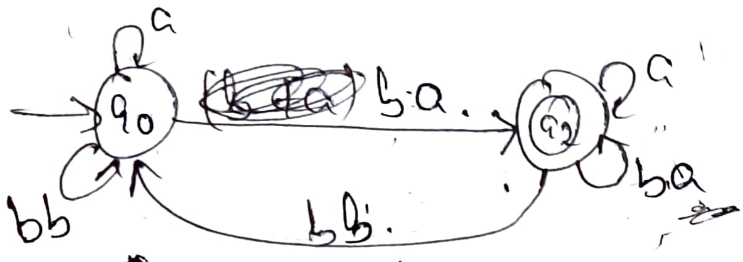
$(R_1 + R_2 R_4^* R_3)^* R_2 R_4^*$

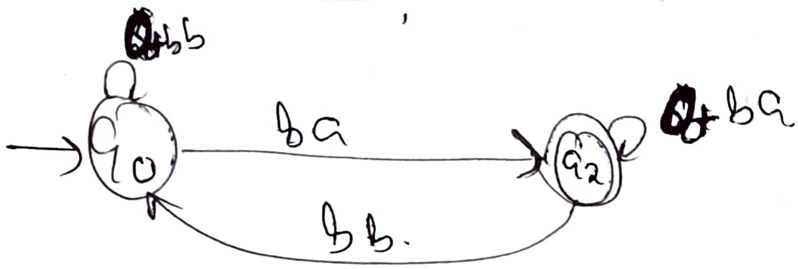
Note $\boxed{R.E \Rightarrow (R_1 + R_2 R_4^* R_3)^* R_2 R_4^*}$

8)



→) Remove q_1 :-

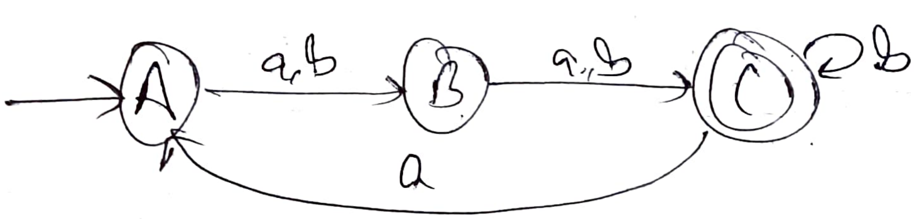




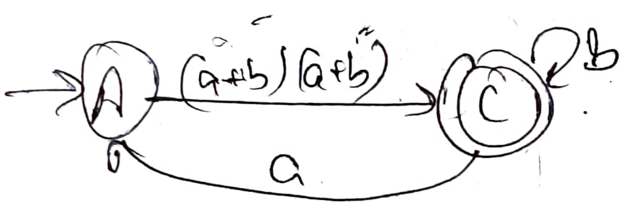
~~R.E, r = ((a+bb)* + ba((a+ba)*)* bb)~~

$$R.E,\ r = \left((a+bb) + ba(a+ba)^* bb\right)^* ba(a+ba)^*$$

9)



→)



$\left\{ \because\ \phi + R = R \quad \phi R = \phi \right\}$

$$RE,\ r = \left(\phi + (a+b)(a+b)b^* a\right)^* (a+b)(a+b)b^*$$

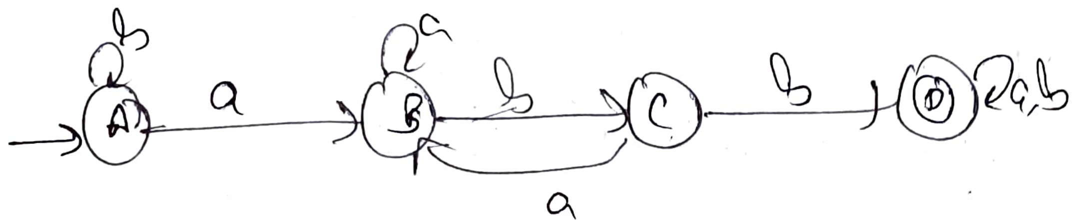
$$r = \left((a+b)^2 b^* a\right)^* (a+b)^2 b^*$$

Regular Grammar :-

1) Write Regular Grammar for the following language.

$L = \{ w \mid w \in \{a,b\}^*, w \text{ contains } abb \text{ as a substring} \}$

$\Rightarrow$



$A \to aB \mid bA$

$B \to aB \mid bC$

$C \to aB \mid bD$

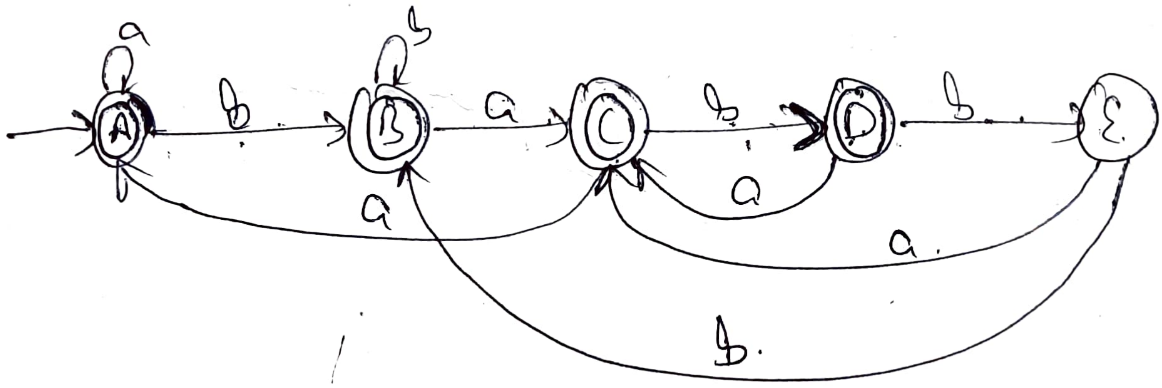
$D \to aD \mid bD \mid \epsilon$

Note $\boxed{A \to a \mid \epsilon \mid aB}$

2) Write Regular Grammar for the following language

$L = \{ w \mid w \in \{a,b\}^*, w \text{ doesn't end with } babb \}$

$\Rightarrow$



Regular Grammar

$A \to aA \mid bB \mid \epsilon$

$B \to aC \mid bB \mid \epsilon$

$C \to aA \mid bD \mid \epsilon$

$D \to aC \mid bE \mid \epsilon$

$E \to aC \mid bB$

Pumping lemma for Regular language.

Ex :- $L = \{a^n b^n \mid n \geq 1\}$

w – string

xyz — divide string in 3 parts.

$|y| > 0$ & xy^iz

let $n = 2$.
aabb

Case-1 :- $w = \underset{x}{a}\ \underset{y}{a}\ \underset{z}{bb}$.

if $i = 3$

a aaa bb $\notin L$.

Case-2 :- $\underset{x}{aa}\ \underset{y}{b}\ \underset{z}{b}$

aabbbb $\notin L$.

Case-3 :- $\underset{x}{a}\ \underset{y}{ab}\ \underset{z}{b}$

a abababb. $\notin L$.

∴ Hence L is not Regular

2) $L=\{ww^R \mid w \in \{a,b\}^*\}$ is not regular.

=) Ex:- abba

Case-1:- $i=4$

$$\underset{x}{a}\ \underset{y}{b}\ \underbrace{b\ a}_{z}.$$

a bbbb ba $\notin L$.

Case-2 $\underset{y}{a}\ \underbrace{b\ b\ a}_{z}$.

aaaa bba. $\notin L$.