18/01/2021

Module - 3

- ⇒ CFG
- ⇒ Derivation
- ⇒ Parse tree
- ⇒ Ambiguous grammar

- ⇒ PDA
- ⇒ Design of PDA
- ⇒ CFG to PDA

Normal forms

- ⇒ CNF
- ⇒ GNF

⇒ CFG

$G = (V, T, P, S)$

⇒ 1) write a CFG for $L = \{a^n \mid n \geq 0\}$

$n = 0$	ϵ
$n = 1$	a
$n = 2$	aa
$n = 3$	aaa

$S \rightarrow aS \mid \epsilon$

2) $L = \{a^n \mid n \geq 2\}$

$S \rightarrow aS \mid aa.$

3) $L = \{a^n b^n \mid n \geq 1\}$

⇒ $S \rightarrow aSb \mid ab$

4) $L = \{\underbrace{a^n}_{A} \underbrace{b^m}_{B} \mid n \geq 0, m \geq 0\}$

⇒

$S \rightarrow AB$

$A \rightarrow aA \mid \epsilon$

$B \rightarrow bB \mid \epsilon$

5) $L = \{a^n b^m \mid m =$ [redacted] $+2,\ n \ge 0\}$

$\Rightarrow$

$$S \to aSb \mid bb$$

6) $L = \{a^i b^j c^k, \mid i = j+k;\ j,k \ge 0\}$

$\Rightarrow$

$$a^{j+k}\, b^j\, c^k$$

$$a^j a^k\, b^j\, c^k$$

$$\underbrace{a^k\, \underbrace{a^j b^j}_{A}\, c^k}$$

$\Rightarrow S \to aSc \mid A \mid \epsilon$

$A \to aAb \mid \epsilon$

7) $L = \{a^i b^j c^k, \mid i = j+k;\ j,k \ge 1\}$

$\Rightarrow$

$$S \to aSc \mid aAC$$

$$A \to aAb \mid ab$$

8) $L = \{a^i b^j c^k \mid j = i+k;\ i,k \ge 1\}$

$\Rightarrow$

$$\underbrace{a^i b^i}_{A}\, \underbrace{b^k c^k}_{B}$$

$S \to AB$

$A \to aAb \mid ab$

$B \to bBc \mid bc$

9) $L = \{a^i b^j c^k \mid k = i+j;\ i,j \ge 1\}$

$\Rightarrow a^i b^j c^i c^j$

$$a^i\, \underbrace{b^j c^j}_{A}\, c^i$$

$S \to aSc \mid aAC$

$A \to bAc \mid bc$

9) $L = \{a^n b^n c^m : n, m \geq 0\}$

$L = \{a^n b^{n+2} : n \geq 0\}$

$L = \{w \in \{a, b\}^*, n_a(w) = n_b(w)\}$

$\cancel{S \to abS\epsilon \mid \epsilon}$

$L = \{\underbrace{a^n b^n}_{A} \underbrace{c^m}_{B}\}$

$S \to AB$
$A \to aAb \mid \epsilon$
$B \to cB \mid \epsilon$

$S \to aSb \mid bSa \mid SS \mid \epsilon$
or $S \to aSbS \mid bSaS \mid \epsilon$

$L = \{a^n b^{n+2} : n \geq 0\}$
$\Rightarrow S \to aSb \mid bb$

10) $L = \{a^n b^{2n} \mid n \geq 1\}$
$\Rightarrow$ $S \to aSbb \mid abb$.

11) $L = \{w \mid w$ is palindrome, $w \in \{a, b\}^*\}$

$\Rightarrow$ even $S \to aSa \mid bSb \mid \epsilon$ (ww^R)
odd $S \to aSa \mid bSb \mid a \mid b$.

$\Rightarrow$ Combiningly :-

$S \to aSa \mid bSb \mid a \mid b \mid \epsilon$

12) $L = \{a^n w w^R b^n \mid n \geq 1, w \in \{a, b\}^*\}$
$\Rightarrow$ $S \to aRb \mid aSb$
$R \to aRa \mid bRb \mid \epsilon$.

13) Balance parenthesis.

⇒. ( ), { }, [ ]

$S \to (S) \mid \{S\} \mid [S] \mid \epsilon \mid SS$

Eg: $w = ()\{[\,]\}$

14) $L = \{w \in \{a,b\}^* \mid$ w containing substring abb$\}$

⇒ $(a+b)^* \underline{abb} (a+b)^*$

A   A.

$S \to A\,abb\,A$

$A \to aA \mid bA \mid \epsilon$.

15) $L = \{$w ends with abb & $w \in \{a,b\}^*\}$

$S \to Aabb$

$A \to aA \mid bA \mid \epsilon$

16) $L = \{a^i b^j c^k \mid i = j$ or $j = k\}$

⇒ Case-1 :- $i = j$

$\underbrace{a^i b^i}_{A} \underbrace{c^k}_{B}$

$S_1 \to AB$

$A \to aAb \mid \epsilon$

$B \to cB \mid \epsilon$

Case-2 :- $j = k$

$\underbrace{a^i}_{C} \underbrace{b^j c^j}_{D}$

$S_2 \to CD$

$C \to aC \mid \epsilon$

$D \to bDc \mid \epsilon$

Finally, $S \to S_1 \mid S_2$

17) $L = \{ \underbrace{0^n 1^n}_{A} \underbrace{0^m 1^m}_{B} \mid n, m \geq 1 \}$

$\Rightarrow$

~~S → ASB~~

$S \to AB$

$A \to 0A1 \mid 01$

$B \to 0B1 \mid 01$

Derivation & derivation tree

→ LMD

→ RMD

1) $E \to E+T \mid T$

$T \to T*F \mid F$

$F \to (E) \mid id$

Derive, $w = (id+id*id)+id$.

LMD, RMD, & draw parse tree

$\Rightarrow$

LMD

$E \Rightarrow \underline{E}+T$

$\Rightarrow T+T \quad (E \to T)$

$\Rightarrow F+T \quad (T \to F)$

$\Rightarrow (E)+T \quad (F \to (E))$

$\Rightarrow (E+T)+T \quad (E \to E+T)$

$\Rightarrow (T+T)+T \quad (E \to T)$

$\Rightarrow (F+T)+T \quad (T \to F)$

$\Rightarrow (id+T)+T \quad (F \to id)$

$\Rightarrow (id+T*F)+T \quad (T \to T*F)$

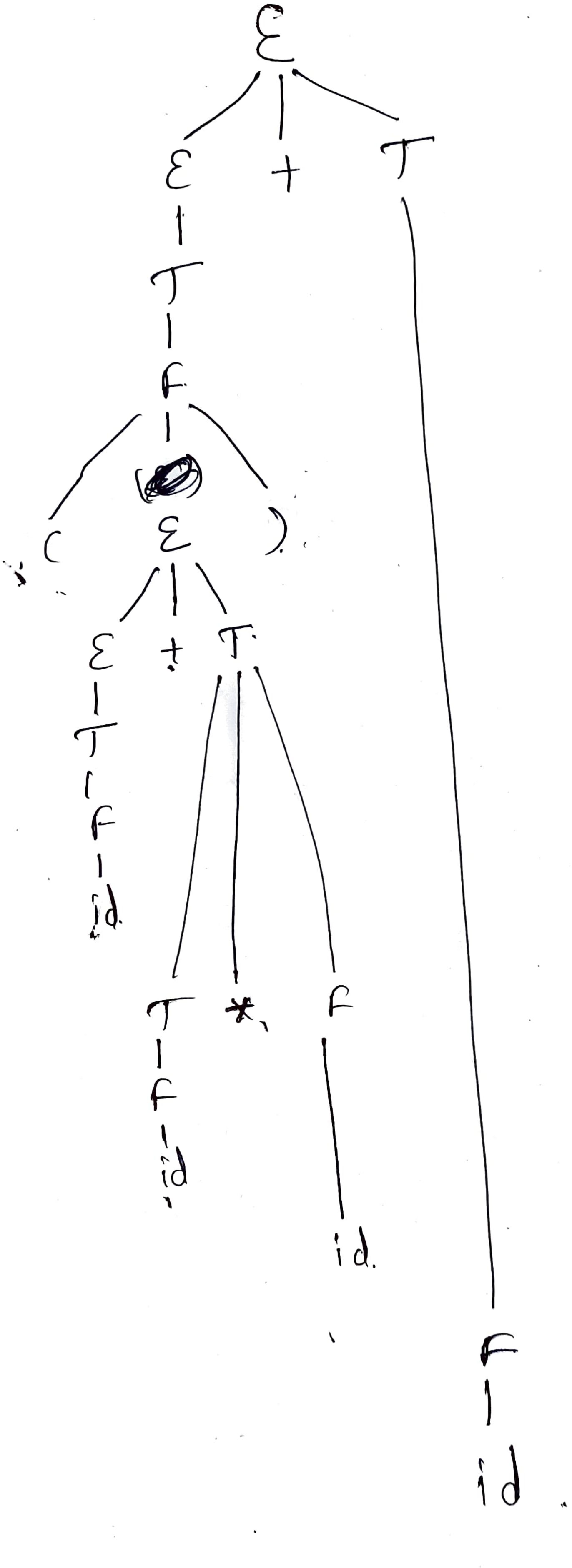
$\Rightarrow (id+F*F)+T \quad (T \to F)$

$\Rightarrow (id+id*F)+T \quad (F \to id)$

$\Rightarrow (id+id*id)+T \quad (F \to id)$

$\Rightarrow (id+id*id)+F \quad (T \to F)$

$\Rightarrow (id+id*id)+id \quad (F \to id)$


E
E + T
T
F
(E)
E + T
T
F
id
T * F
F
id
id
F
id

Ambiguous Grammar :-

→ 2LMD or 2RMD

Ex: $E \to E + E \mid E * E \mid id$.

$w = id + id * id$.

Show that this grammar is ambiguous

⇒ LMD1

$E \Rightarrow E + E$

$\Rightarrow id + E$

$\Rightarrow id + E * E$

$\Rightarrow id + id * E$

$\Rightarrow id + id * id$.

LMD2.

$E \Rightarrow E * E$

$\Rightarrow E + E * E$

$\Rightarrow id + E * E$

$\Rightarrow id + id * E$

$\Rightarrow id + id * id$.

→ Yes, this grammar is ambiguous

⇒ S.T. the following grammar is ambiguous

$S \to iCtS \mid iCtSeS \mid a$

$C \to b$

$S = ibta$

$S = ibtaea$

$S = ibtibtaea$

⇒ $w = ibtibtaea$.

LMD1

$S \Rightarrow iCtS$

$\Rightarrow ibtS$

$\Rightarrow ibtS$

~~$\Rightarrow ibtiCtS$~~

~~$\Rightarrow ibtibtS$~~

$\Rightarrow ibtiCtSeS$

$\Rightarrow ibtibtSeS$

$\Rightarrow ibtibtaeS$

$\Rightarrow ibtibtaea$

LMD2

$S \Rightarrow iCtSeS$

$\Rightarrow ibtSeS$

$\Rightarrow ibtiCtSeS$

$\Rightarrow ibtibtSeS$

$\Rightarrow ibtibtaeS$

$\Rightarrow ibtibtaea$

PDA

Push down Automata.

1) Design a PDA for, $L = \{a^n b^n \mid n \geq 1\}$ & show the ID for $w = aabb$

⇒

Case-1:- $\delta(q_0, a, Z_0) = (q_0, aZ_0)$

Case-2:- $\delta(q_0, a, a) = (q_0, aa)$

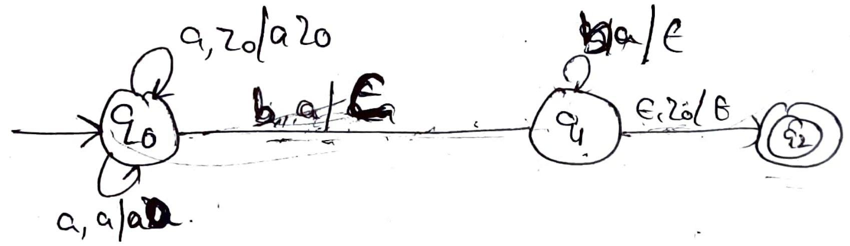
Case-3:- $\delta(q_0, a, a) = (q_0, aaa)$

Case-4:- $\delta(q_0, b, a) = \cancel{B}(q_1, \epsilon)$

Case-5:- $\delta(q_1, b, a) = \cancel{B}(q_1, \epsilon)$

Case-6:- $\delta(q_1, b, a) = \cancel{B}(q_1, \epsilon)$

Case-7:- $\delta(q_1, \epsilon, Z_0) = \cancel{B}(q_2, \epsilon)$

⇒ 

⇒ $q_0, aabb, Z_0 \vdash q_0, abb, aZ_0 \vdash q_0, bb, aaZ_0$

$\vdash q_1, b, aZ_0 \vdash q_1, \epsilon, Z_0 \vdash q_2, \epsilon, \epsilon$

Accepted

2) Design a PDA for $L = \{wcw^R \mid w \in \{a,b\}^*\}$

⇒ Case-1 :-
$$\delta(q_0, a, z_0) = (q_0, az_0)$$
$$\delta(q_0, b, z_0) = (q_0, bz_0)$$

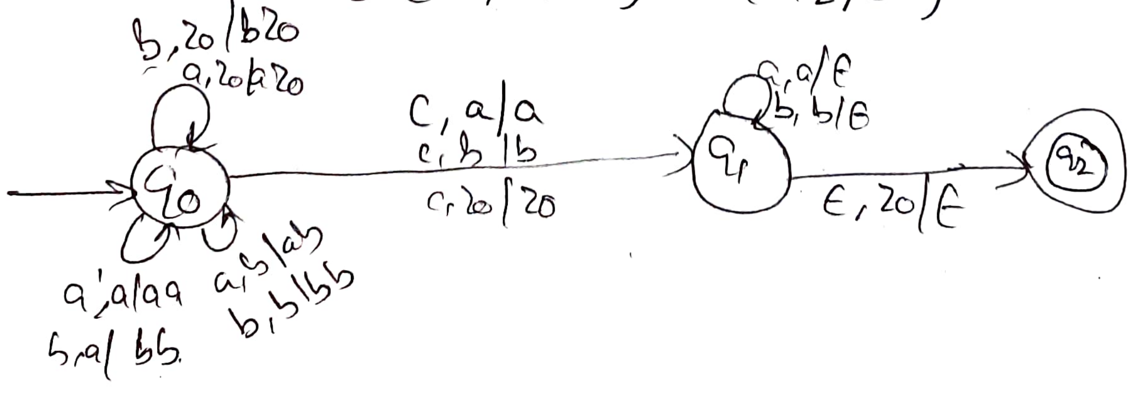
Case-2 :- TOS is a.
$$\delta(q_0, a, a) = (q_0, aa)$$
$$\delta(q_0, b, b) = (q_0, bb)$$

Case-3 :- TOS is b
$$\delta(q_0, b, a) = (q_0, ba)$$
$$\delta(q_0, b, b) = (q_0, bb)$$

Case-4 :- i/p is c.
$$\left.\begin{aligned}\delta(q_0, c, a) &= (q_1, a)\\ \delta(q_0, c, b) &= (q_1, b)\\ \delta(q_0, c, z_0) &= (q_1, z_0)\end{aligned}\right\} \text{skip operation.}$$

Case-5 :-
$$\delta(q_1, a, a) = (q_1, \epsilon)$$
$$\delta(q_1, b, b) = (q_1, \epsilon)$$

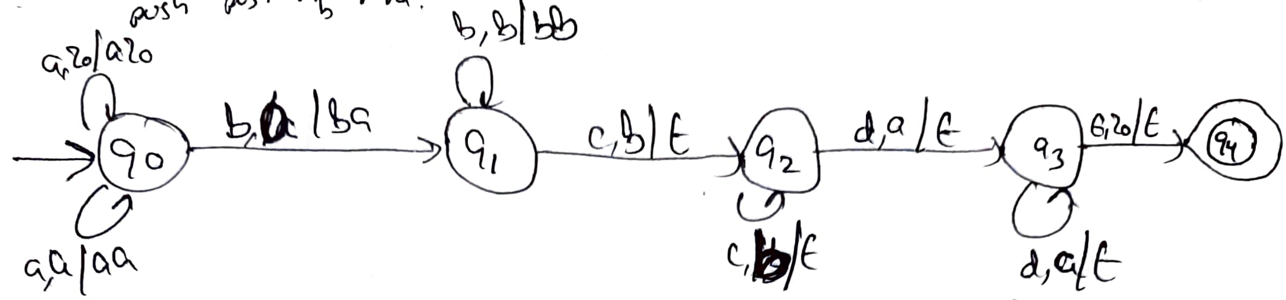
Case-6 :-
$$\delta(q_1, \epsilon, z_0) = (q_2, \epsilon)$$



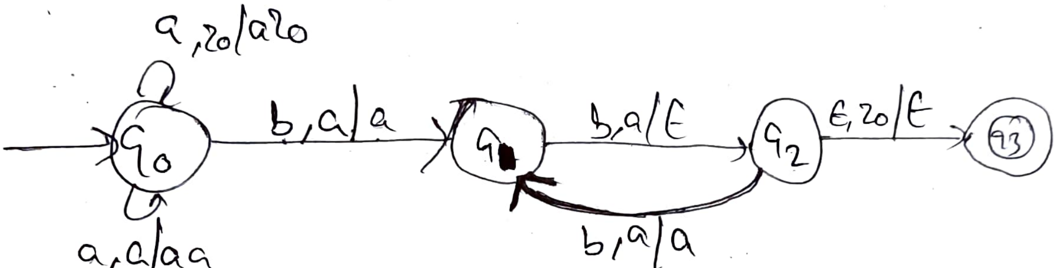
3) $L = \{a^n b^m c^m d^n \mid m, n \geq 1\}$

push push pop b pop a.

⟹



4) $L = \{a^n b^{2n} \mid n \geq 1\}$



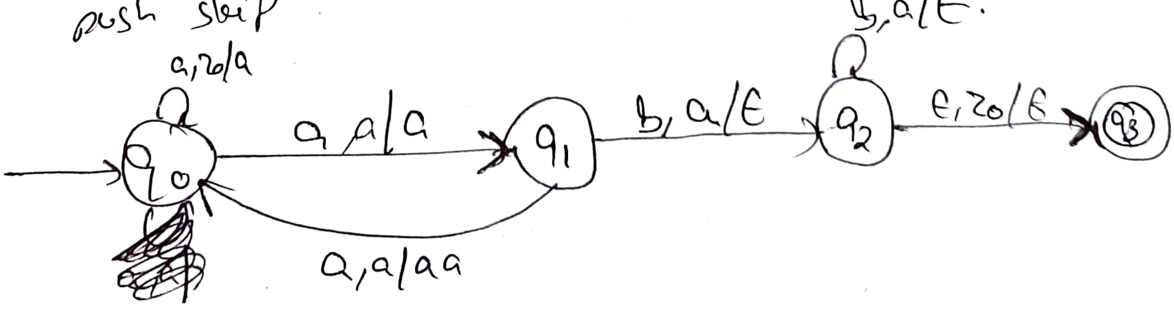
Note :- odd position b skip & even position b pop a.

5) $L = \{a^{2n} b^n \mid n \geq 1\}$

aaaabb

push skip

⟹



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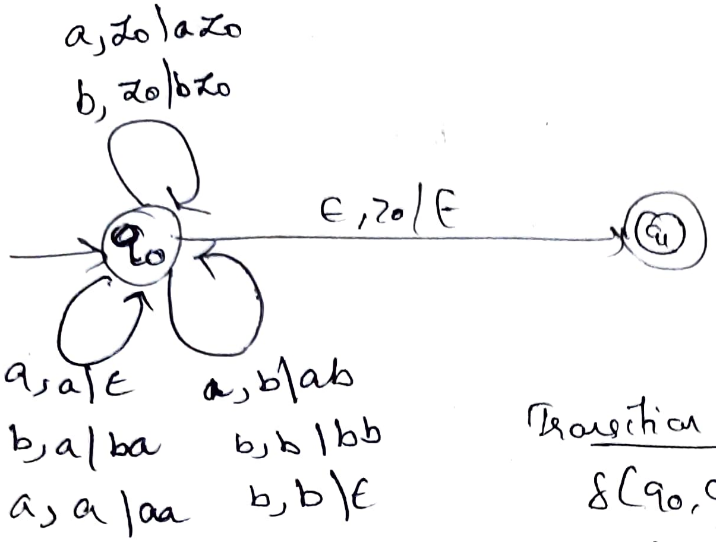
Non-Deterministic PDA (NPDA):

1) Design a PDA for $L = \{ww^R \mid w \in \{a,b\}^*\}$

⇒



1) $\delta(q, a, A) = (q_0, aA)$
(q_0, ϵ)

2) if $\delta(q, \epsilon, A)$ have a transition then $\delta(q, a, A)$ shouldn't have transition

Transition function:-

$\delta(q_0, a, Z_0) = \{(q_0, aZ_0)\}$

$\delta(q_0, b, Z_0) = \{(q_0, bZ_0)\}$

✓ $\delta(q_0, a, a) = \{(q_0, \epsilon), (q_0, aa)\}$

$\delta(q_0, b, a) = \{(q_0, ba)\}$

$\delta(q_0, a, b) = \{(q_0, ab)\}$

✓ $\delta(q_0, b, b) = \{(q_0, \epsilon), (q_0, bb)\}$

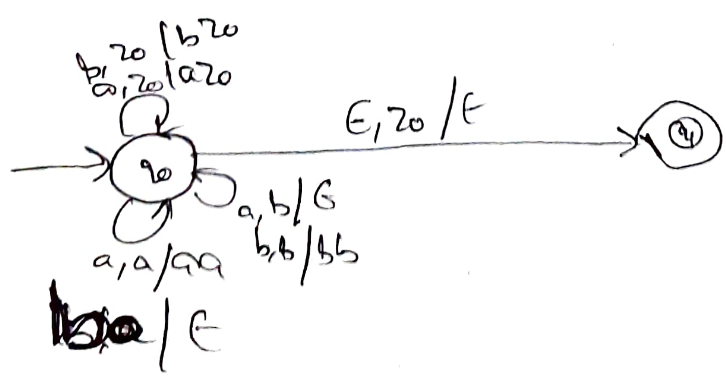
$\delta(q_0, \epsilon, Z_0) = \{(q_1, \epsilon)\}$

∴ This PDA is NPDA

2) Design a PDA for $L = \{w \mid n_a(w) = n_b(w)\}$, ~~Is~~ it DPDA or NPDA?

Ex:- babbaa
push pop push push pop pop

⇒



$\delta(q_0, a, Z_0) = (q_0, aZ_0)$

$\delta(q_0, b, Z_0) = (q_0, bZ_0)$

$\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, b, a) = (q_0, \epsilon)$

$\delta(q_0, b, b) = (q_0, bb)$

$\delta(q_0, a, b) = (q_0, \epsilon)$

$\delta(q_0, \epsilon, Z_0) = (q_1, \epsilon)$

through all transitions ...

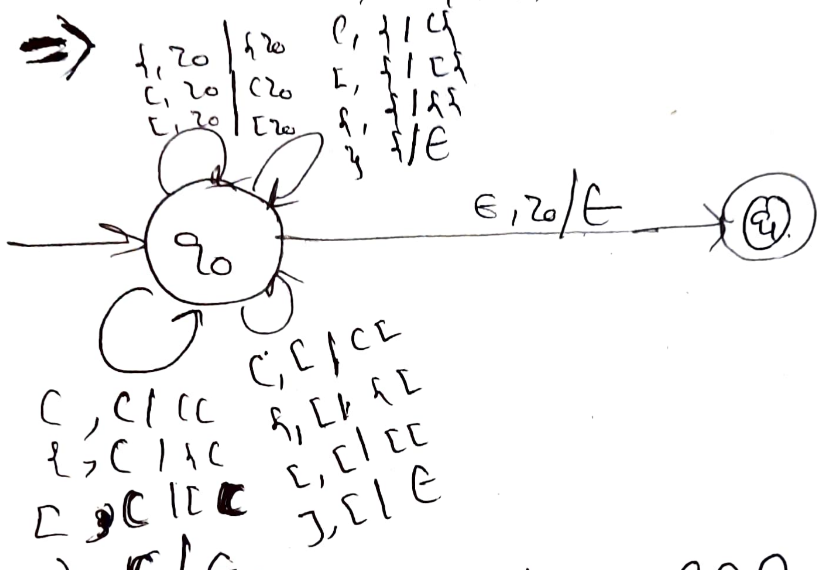
$\delta(q_0, a, z_0) = (q_0, a z_0)$ $\quad [\because \delta(q_1, \epsilon, A) = x$

& $\delta(q_0, \epsilon, z_0) = (q_1, \epsilon)$ $\quad$ & $\delta(q_0, a, A) = y]$

$\therefore$ It is NPDA

3) Design a PDA for L = {Balanced Parenthesis}
( ), { }, [ ].

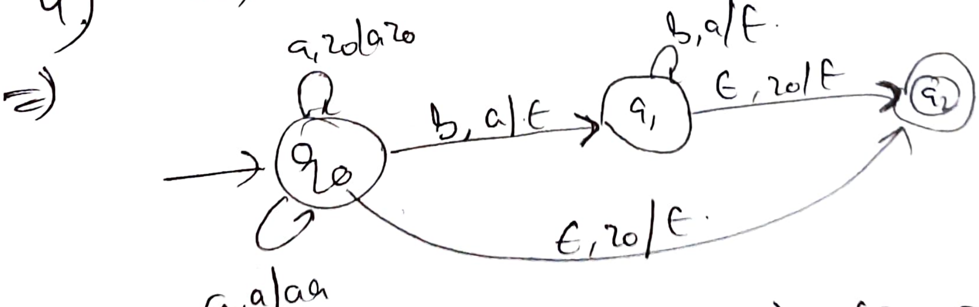
$\Rightarrow$



$\therefore$ It is NPDA

4) Write NPDA for $L = \{a^n b^n \mid n \geq 0\}$.

$\Rightarrow$



$\delta(q_0, a, z_0) = (q_0, a z_0)$

$\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, b, a) = (q_1, \epsilon)$

$\delta(q_1, b, a) = (q_1, \epsilon)$

$\delta(q_0, \epsilon, z_0) = (q_2, \epsilon)$

$\delta(q_1, \epsilon, z_0) = (q_2, \epsilon)$

NPDA

5) Write NPDA for $L = \{wcw^R / w\{a,b\}^*\}$

⇒ It will be always DPDA

CFG to PDA

Ex:- $E \to E+T \mid E-T \mid T$

$T \to T*F \mid F$

$F \to (E) \mid id$

Steps:- 1) for each non-terminal $A \to \beta$

$\delta(q, \epsilon, A) = (q, \beta)$

2) For each terminal

$\delta(q, a, a) = (q, \epsilon)$

⇒ Equivalent PDA :-

$\delta(q, \epsilon, E) = \{(q, E+T), (q, E-T), (q, T)\}$

$\delta(q, \epsilon, T) = \{(q, T*F), (q, F)\}$

$\delta(q, \epsilon, F) = \{(q, (E)), (q, id)\}$

$\delta(q, +, +) = (q, \epsilon)$

$\delta(q, -, -) = (q, \epsilon)$

$\delta(q, *, *) = (q, \epsilon)$

$\delta(q, (, () = (q, \epsilon)$

$\delta(q,),)) = (q, \epsilon)$

$\delta(q, id, id) = (q, \epsilon)$

⇒) How to write unambiguous grammar for a given ambiguous grammar :-

$E \rightarrow E+E \mid E-E \mid E*E \mid E/E \mid id \mid (E)$

Unambiguous grammar :-

level 1	+	−	E
level 2	*	/	A
level 3	id	( )	B

$E \rightarrow E+A \mid E-A \mid A$

$A \rightarrow A*B \mid A/B \mid B$

$B \rightarrow (E) \mid id$

Left recursive grammar

$A \rightarrow A\alpha$

Right recursive grammar

$A \rightarrow \alpha A$.

Steps :-

1) Arrange the operators according to precedence rule in ascending order.

2) Assign a NT to each level

3) If the operators are, left associative, then write left recursive grammar.

$E \rightarrow E+E \mid E-E \mid E*E \mid E/E \mid id \mid (E) \mid E^E$.

$E \rightarrow E+A \mid E-A \mid A$

$A \rightarrow A*B \mid A/B \mid B$

$B \rightarrow C^B \mid C$ → Right recursive grammar

$C \rightarrow (E) \mid id$.

Normal Form :-

CNF

GNF.

CNF :-

$A \to BC$

or

$A \to a$.

Steps to convert to CNF :-

1) Remove ϵ-prodⁿ
2) Remove Unit prodⁿ
3) Remove useless symbol
4) Convert to CNF.

Ex :- $S \to aAC a$
$A \to B | a$
$B \to C$
$C \to cC$.

Step1 :- ~~state~~ No ϵ-prodⁿ

Step2 :-

$A \to B$
$B \to C$.

$\Rightarrow A \to B \to C$.

$\Rightarrow$ $S \to aACa$
$A \to B|a$
$B \to cC$.
$C \to cC$
} Removing C.

$\Rightarrow$ $S \to aACa$
$A \to cC|a$.
$B \to cC$
$C \to cC$.
} Removing B

Step-3 :-

B & C are useless (First check terminal is there and check whether all non-terminal can be reached by S).

S also useless.

∴ Grammar is invalid.

2) $S \to aACa$

$A \to B \mid a$

$B \to C$

$C \to cC \mid \varepsilon$.

⇒ Step-1 :- Remove Epsilon Production

~~S → aACa~~

Null set = {C, B, A}

Remove ε-production

$S \to aACa \mid aCa \mid aAa \mid aa$.

$A \to B \mid a$.

$B \to C$

$C \to cC \mid c$

⇒ Step-2 :- Remove unit production.

$S \to aACa \mid aCa \mid aAa \mid aa$

$A \to cC \mid c \mid a$

$B \to cC \mid c$.

$C \to cC \mid c$.

⇒ Step-3 :- Remove Useless Symbol :-

B is useless (Not Reachable)

$S \to aACa \mid aCa \mid aAa \mid aa$

$A \to cC \mid c \mid a$

$C \to cC \mid c$

Step-4 :- Convert to CNF :- $A \to BC$ or $A \to a$

let $X \to a$ $\quad S \to \cancel{X}ACX \mid \cancel{X}\cancel{C}\cancel{X} \mid XAX \mid \cancel{XX}$

let $P \to XA$ $\quad S \to PQ \mid XQ \mid PX \mid XX$

$Q \to CX$

let $Y \to c$ $\quad A \to YC \mid c \mid a$

$C \to YC \mid c$

Final answer :-

$S \to PQ \mid XQ \mid PX \mid XX$

$\cancel{A \to Y}$

$\cancel{X} \to a$ $\quad Y \to c$

$P \to XA$ $\quad A \to YC \mid c \mid a$

$Q \to CX$ $\quad C \to YC \mid c$

3) Convert the CFG to CNF

$S \to ABC$

$A \to BC \mid a$

$B \to bAC \mid \epsilon$

$C \to cAB \mid \epsilon$

$\Rightarrow$ ~~Step 1 Remove ϵ prod^n~~

~~Null set = $\{A, B, C\}$~~

(PTO)

~~$S \to ABC \mid AC \mid AB \mid BC \mid$~~

~~$A \to a$~~

~~$B \to bAC \mid bA \mid bC$~~

~~$C \to cAB \mid cA \mid cB$~~

~~Step-2:- Remove unit production.~~

Step-1:- Null set = {S, A, B, C}

Remove E - prod:-

$S \to ABC \mid AB \mid BC \mid AC \mid A \mid B \mid C$

$A \to BC \mid a \mid B \mid C$

$B \to bAC \mid bA \mid bC \mid b$

$C \to cAB \mid cA \mid cB \mid c$

Step-2:- Remove unit production

$S \to ABC \mid AB \mid BC \mid AC \mid a \mid bAC \mid bA \mid bC \mid b \mid cAB \mid cA \mid cB \mid c$

$A \to BC \mid a \mid bAC \mid bA \mid bC \mid b \mid cAB \mid cA \mid cB \mid c$

$B \to bAC \mid bA \mid bC \mid b$

$C \to cAB \mid cA \mid cB \mid c$

Step-3:- Remove useless

No useless symbol

step-4 :-

$S \to ZC \mid AB \mid BC \mid AC \mid a \mid XAC \mid XA \mid XC \mid b \mid YZ \mid YA \mid YB \mid C$

$S \to ZC \mid AB \mid BC \mid AC \mid a \mid XP \mid XA \mid XC \mid b \mid YZ \mid YA \mid YB \mid C.$

$X \to b$

$Y \to C.$ $\quad A \to BC \mid a \mid XP \mid XA \mid XC \mid b \mid YZ \mid YA \mid YB \mid C.$

$Z \to AB$

$P \to AC$ $\quad B \to XP \mid XA \mid XC \mid b$

$C \to YZ \mid YA \mid YB \mid c.$