The prisoner's dilemma game on a spatial lattice

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1 Introduction

In the standard prisoner's dilemma game, agents can choose two kinds of strategy cooperation(C) or defection(D). When both the two interacting players choose to cooperate, they will both get a reward R; if one player cooperates while the other defects, then the cooperate one will gain the temptation to T and the cooperate one will get a sucker's payoff S; when both the players choose to defect, they will have a punishment P. The dilemma matrix is as follow:

$$J = \begin{pmatrix} R & S \\ T & P \end{pmatrix} \tag{1}$$

2 Theory

2.1 Dilemma Matrix

In our experiment, we use a punishment fraction b to describe the defection strength, the dilemma matrix is as follow:

$$J = \begin{pmatrix} 1 & -b \\ b & 0 \end{pmatrix} \tag{2}$$

2.2 Decision Matrix & Wealth Matrix

To study the prisoner's dilemma in a spatial lattice, we suppose two 100*100 matrix to explain the trade process, the former represents the people's strategy (C or D) by value 0

or 1 and the later represents the wealth of these 10000 agents with an initial value of zero. Both of these two matrix will chance after each trading round.

2.3 Transform Possibility

We define a Transform Possibility to describe this kind of phenomenon. The P_x means the transform possibility of x change its strategy to its neighbor y. W_y stands for its neighbor y's wealth. Ω_x means a set of x's neighbors. When it's bigger than 0.5, we suppose x is going to change the strategy. The transform possibility is like:

$$P_{x \to y} = \frac{W_y}{\sum_{z=0}^{\Omega_x} (W_z)} \tag{3}$$

2.4 Defect Possibility

Defect possibility is designed to show the ratio of defect strategy agents in the lattice and is calculated by the mean of last 10% global defect possibilities. Later the result will explain the relation between defection strength b with the final defect possibility.

2.5 Average Wealth

Average wealth is the simply represented the average wealth of the global, cooperators and defectors.

3 Results

3.1 Defect Possibility with *b*

Suppose each agent have 4 neighbors and after 1000 round of prison's dilemma game trading, the plot of defect possibility against defection strength b is as Fig1:

There is a steep rise at b approximately equal from 1.2 to 1.5. After 1.5 the defect possibility is relatively high and approach to the value 1.0

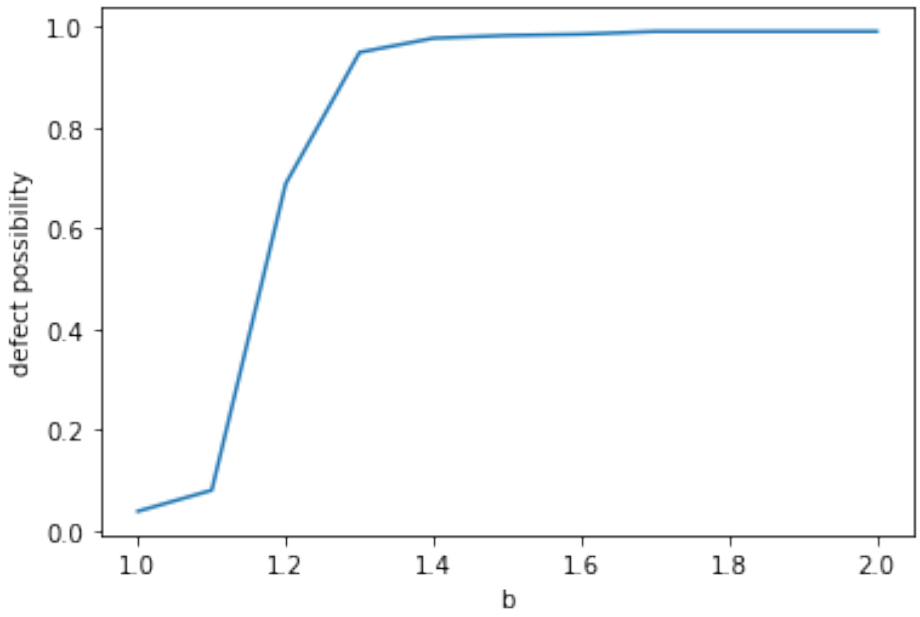

Fig. 1. The fraction of defectors against b

3.2 Average Wealth with *b*

The global average wealth against b is as Fig2. Related to the defect possibility, the defect possibility rises with the average wealth falls.

Fig3 represents the average wealth for defectors and cooperators against game round when b is 1.2, 1.3, 1.4, 1.5, 1.6, 1.7 respectively.

3.3 Strategy Changes through the Game

To figure out the real change of the agents 'strategy changes, in Fig4 5 6 we plot the evolution of Prisoner's Dilemma Game (b=1.5) on a 100*100 spatial lattice to see the evolution process with the snapshot at round 0, 50 and 200.

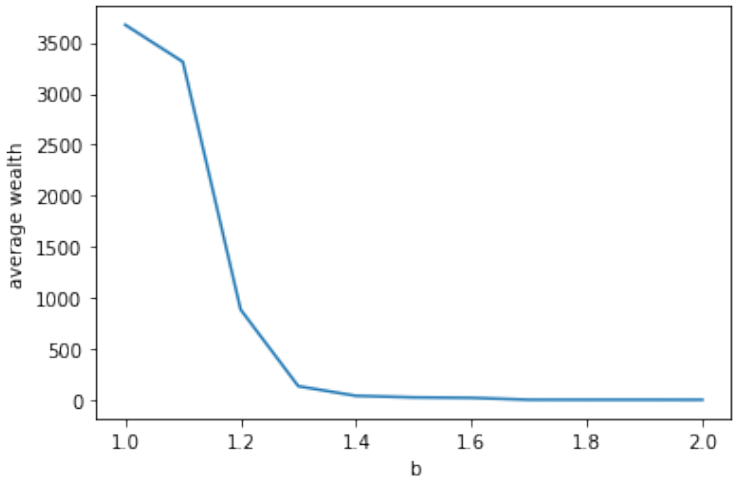

Fig. 2. The average wealth against b

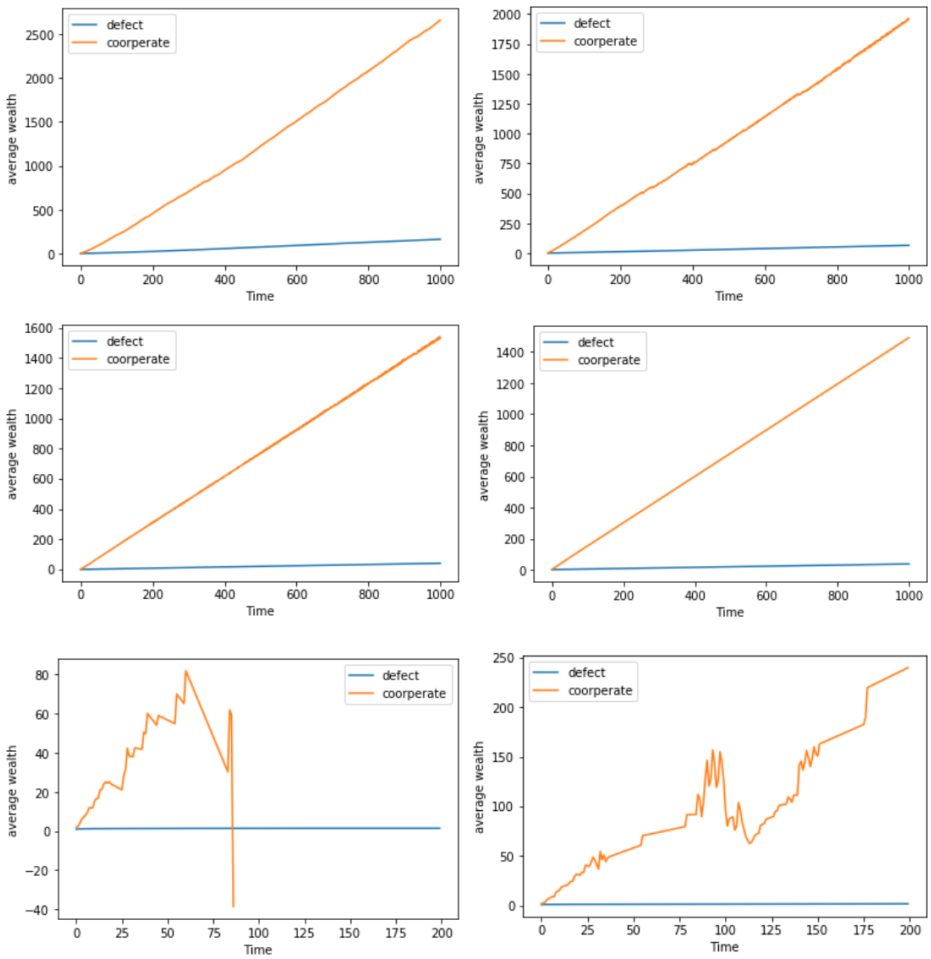

Fig. 3. The average wealth for defectors and cooperators against round at different b

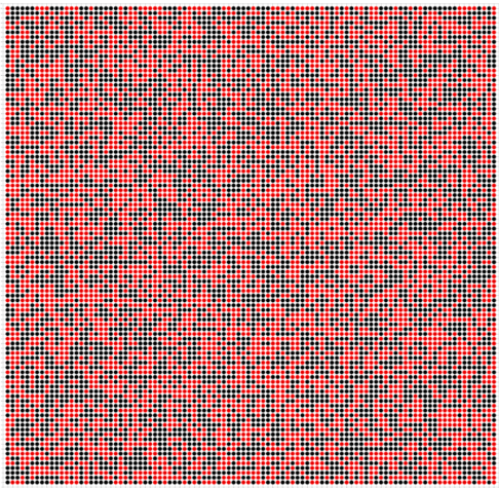

Fig. 4. The evolution of PDG at round 0(b=1.5)

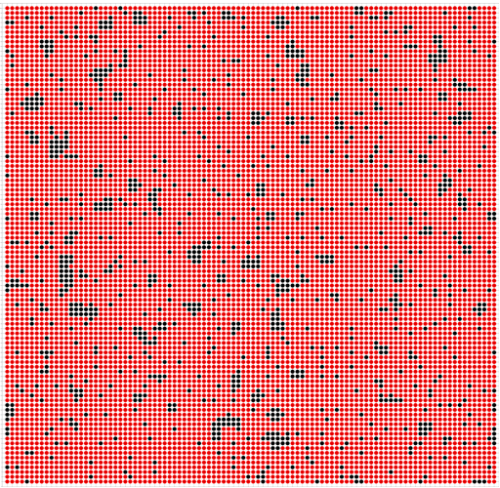

Fig. 5. The evolution of PDG at round 50(b=1.5)

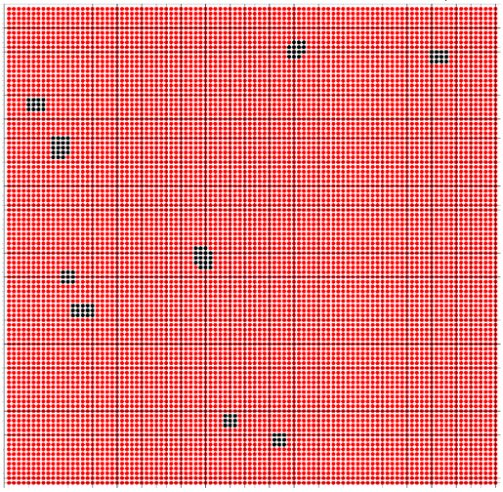

Fig. 6. The evolution of PDG at round 200(b=1.5)

4 Conclusion

4.1 *b*'s influence

Firstly, b's increase will cause the ascent of defect possibility, which can be easily explained that the more temptation will attract people to make a defection choice. Secondly, b's increase will cause the decrease of average wealth and the global average wealth is approaching to zero with a bigger b. This might because of the defector gradually coming to be the dominant and without cooperate, two defectors can't have any payoff from either side. Thirdly, with an increasing b the slope of cooperate wealth reduce at the same time. This might be aroused by the increase defect possibilities. When the environment becoming to the steady state, some of the cooperators aggregate to be the small communities and in these communities, cooperators gain the wealth from each round so that the cooperate wealth increase and shows a linear characteristics.

4.2 Wealth changes

From the Fig3 we can know that when b is in the value of 1.2-1.5 at the beginning there is not much difference between the average wealth for defectors and cooperators. However, as game round increase, the average wealth for cooperators keep increasing, while the value for defectors increases slowly. When b is above 1.5, this kind of situation disappears and it coincident with the increase defector possibility.

References