

$$\begin{array}{rcl} ax + by + cz & = & 2 \\ dx + ey + fz & = & 3 \\ gx + hy + iz & = & 4 \end{array}$$

$$\begin{bmatrix} a & b & c & 2 \\ d & e & f & 3 \\ g & h & i & 4 \end{bmatrix} \quad \times$$

Augmented

Rank      Unique

$$V = [x_1, x_2, \dots, x_k]$$

$$\begin{array}{l} a_1 x_1 + a_2 x_2 + \dots + a_k x_k = 0 \\ a_1, a_2, a_3, \dots, a_k \neq 0 \end{array}$$

$$a_1 = 0, a_2 = 0, a_3 = \dots$$

$$a_k = 0$$

$$S = \{(2, 2), (1, 2)\} \quad \text{L.I.}$$

$$\begin{cases} 2c_1 + c_2 = 0 \\ 2c_1 + 2c_2 = 0 \end{cases}$$

$$\begin{pmatrix} 2 & 1 & 0 \\ 2 & 2 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 & 0 \\ 0 & -1 & 0 \end{pmatrix}$$

Unique.

$$2c_1 + c_2 = 0$$

$$c_2 = 0$$

$$c_2 = 0, c_1 = 0$$

Product of Eigen values =  $|A|$   
 Sum of Eigen values = trace of  $A$   
 $A - \lambda I = Q$

$$\lambda =$$

$$\begin{vmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & 4 & 3-\lambda \end{vmatrix} = 0$$

$$\lambda = 0, 15, 3$$

$$Ax = \lambda x$$

$$(A - \lambda I)x = 0$$

$$Ax = 0$$

$$\begin{pmatrix} -6 & 2 & 2 \\ -6 & 7 & 2 \\ -4 & -4 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 7 & -4 \\ -4 & 3 \end{pmatrix} = \begin{pmatrix} -6 & 2 \\ 2 & 3 \end{pmatrix}$$

$$\boxed{\frac{x_1}{3} = \frac{x_2}{10} = \frac{x_3}{10}} \quad \begin{pmatrix} -6 & 7 \\ 2 & 2 \end{pmatrix}$$

$$x_1 = x_2 = \frac{x_3}{2}$$

$$x_1 = \frac{x_2}{2} = \frac{x_3}{2}$$

$$\begin{matrix} x_1 \\ x_2 \\ x_2 \end{matrix} = k \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$$

$$LHS$$

$$a_1 t^2 + b_1 t + c_1 \oplus$$

$$a_2 t^2 + b_2 t + c_2$$

$$b = a + 1$$

$$b_1 = a_1 + 1, b_2 = a_2 + 1$$

$$b_1 + b_2 = a_1 + a_2 + 2$$

$$RHS$$

$$(a_1 + a_2)t^2 + (b_1 + b_2)t + (c_1 + c_2)$$

$$a_3 t^2 + b_3 t + c_3$$

$$b_3 = a_3 + 1$$

$$b_1 + b_2 = a_1 + a_2 + 1$$

$$t^2 = a$$

$$t = b$$

$$(a_1 + a_2)t^2 + (a_1 + 1 + a_2 + 1)t + c_1 + c_2$$

$$a_1 t^2 + b_1 t + c_1 \quad b_1 = a_1 + 1$$

$$Coll t = Coll t^2 + 1$$

$$rb = ra - 1$$

$$1 = 0$$

$$0 = 1$$

For every  $r$  this is not true

$$\lambda = 0$$

$$C = 1$$

Vector space  $\mathbb{R}^2[a, b]$

$$1) (a_1, b_1) \oplus (a_2, b_2) \in V$$

$$2) c \odot (a, b) \in V$$

3) zero vector //

$$\begin{array}{rcl} c_1 + c_2 + c_3 & = & 2 \\ 2c_1 + 0 + c_3 & = & 1 \\ c_1 + 2c_2 + 0 & = & 5 \end{array}$$

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \begin{array}{ccc} c_1 & c_2 & c_3 \\ \begin{pmatrix} 1 & 1 & 1 \\ 2 & 0 & 1 \\ 1 & 2 & 0 \end{pmatrix} & \Rightarrow & \end{array}$$

$$I = 2 \times 2$$

$$\begin{array}{l} R_2 = R_2 - 2R_1 \rightarrow 1 \quad -4 \\ R_3 = R_3 - R_1 \rightarrow 5 \quad -2 \end{array}$$

$$R_3 = 2R_3 + R_2$$

~~R2~~

$$f(A, B) = 3$$

$J = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$$f(A) = f(A \oplus B) = n$$

Unique sol

$f(A) = f(A=0) < \infty$   
Infinite set



$$f(A) \neq f(A=B)$$

mc 502

$$\begin{aligned} c_1 + c_2 + c_3 &= 2 \\ 2c_2 - c_3 &= -3 \\ 3c_3 &= 3 \\ c_3 &= 1 \end{aligned}$$

$$c_2 =$$

$$t^2 = 2c_1 + 5c_3 \Rightarrow c_4 = 1$$

$$\begin{bmatrix} 2 & 1 & 5 & -1 & : & 1 \\ 0 & -2 & -5 & -3 & : & 1 \\ 2 & 0 & 2 & -2 & : & 2 \end{bmatrix}$$

$$2R_2 - R_1 \quad R_3 = R_3 - R_1$$

$$\times 5 \begin{bmatrix} 2 & 1 & 5 & -1 & : & 1 \\ 0 & -5 & -15 & -5 & : & 1 \\ 0 & -1 & -3 & -1 & : & 1 \end{bmatrix}$$

$$R_3 = 5R_3 - R_2$$

$$\begin{bmatrix} 2 & 1 & 5 & -1 & : & 1 \\ 0 & -5 & -15 & -5 & : & 1 \\ 0 & 0 & 0 & 0 & : & 4 \end{bmatrix}$$

$$\rho(A) = 2$$

$$\rho(A \ B) = 3 \neq$$

NC & !