The Physics of Quantum Mechanics

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This book is a consequence of the vision and munificence of
Walter of Merton, who in 1264 launched something good

Published by Cappella Archive 2008; revised printings 2009, 2010, 2011

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Preface

This book is the fruit of for many years teaching the introduction to quantum mechanics to second-year students of physics at Oxford University. We have tried to convey to students that it is the use of probability amplitudes rather than probabilities that makes quantum mechanics the extraordinary thing that it is, and to grasp that the theory's mathematical structure follows almost inevitably from the concept of a probability amplitude. We have also tried to explain how classical mechanics emerges from quantum mechanics. Classical mechanics is about movement and change, while the strong emphasis on stationary states in traditional quantum courses makes the quantum world seem static and irreconcilably different from the world of every-day experience and intuition. By stressing that stationary states are merely the tool we use to solve the time-dependent Schrödinger equation, and presenting plenty of examples of how interference between stationary states gives rise to familiar dynamics, we have tried to pull the quantum and classical worlds into alignment, and to help students to extend their physical intuition into the quantum domain.

Traditional courses use only the position representation. If you step back from the position representation, it becomes easier to explain that the familiar operators have a dual role: on the one hand they are repositories of information about the physical characteristics of the associated observable, and on the other hand they are the generators of the fundamental symmetries of space and time. These symmetries are crucial for, as we show already in Chapter 4, they dictate the canonical commutation relations, from which much follows.

Another advantage of down-playing the position representation is that it becomes more natural to solve eigenvalue problems by operator methods than by invoking Frobenius' method for solving differential equations in series. A careful presentation of Frobenius' method is both time-consuming and rather dull. The job is routinely bodged to the extent that it is only demonstrated that in certain circumstances a series solution *can* be found, whereas in quantum mechanics we need assurance that *all* solutions can be found by this method, which is a priori implausible. We solve all the eigenvalue problems we encounter by rigorous operator methods and dispense with solution in series.

By introducing the angular momentum operators outside the position representation, we give them an existence independent of the orbital angular-momentum operators, and thus reduce the mystery that often surrounds spin. We have tried hard to be clear and rigorous in our discussions of the connection between a body's spin and its orientation, and the implications of spin for exchange symmetry. We treat hydrogen in fair detail, helium at the level of gross structure only, and restrict our treatment of other atoms to an explanation of how quantum mechanics explains the main trends of atomic properties as one proceeds down the periodic table. Many-electron atoms are extremely complex systems that cannot be treated in a first course with a level of rigour with which we are comfortable.

Scattering theory is of enormous practical importance and raises some tricky conceptual questions. Chapter 5 on motion in one-dimensional step potentials introduces many of the key concepts, such as the connection between phase shifts and the scattering cross section and how and why in resonant scattering sensitive dependence of phases shifts on energy gives rise to sharp peaks in the scattering cross section. In Chapter 12 we discuss fully three-dimensional scattering in terms of the S-matrix and partial waves.

In most branches of physics it is impossible in a first course to bring students to the frontier of human understanding. We are fortunate in being able to do this already in Chapter 6, which introduces entanglement and quantum computing, and closes with a discussion of the still unresolved problem of measurement. Chapter 6 also demonstrates that thermodynamics is a straightforward consequence of quantum mechanics and that we no longer need to derive the laws of thermodynamics through the traditional, rather subtle, arguments about heat engines.

We assume familiarity with complex numbers, including de Moivre's theorem, and familiarity with first-order linear ordinary differential equations. We assume basic familiarity with vector calculus and matrix algebra. We introduce the theory of abstract linear algebra to the level we require from scratch. Appendices contain compact introductions to tensor notation, Fourier series and transforms, and Lorentz covariance.

Every chapter concludes with an extensive list of problems for which solutions are available. The solutions to problems marked with an asterisk, which tend to be the harder problems, are available online[1] and solutions to other problems are available to colleagues who are teaching a course from the book. In nearly every problem a student will either prove a useful result or deepen his/her understanding of quantum mechanics and what it says about the material world. Even after successfully solving a problem we suspect students will find it instructive and thought-provoking to study the solution posted on the web.

We are grateful to several colleagues for comments on the first two editions, particularly Justin Wark for alerting us to the problem with the singlet-triplet splitting. Fabian Essler, Andre Lukas, John March-Russell and Laszlo Solymar made several constructive suggestions. We thank Artur Ekert for stimulating discussions of material covered in Chapter 6 and for reading that chapter in draft form.

June 2012

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[1] `http://www-thphys.physics.ox.ac.uk/people/JamesBinney/QBhome.htm`

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Probability and probability amplitudes

The future is always uncertain. Will it rain tomorrow? Will Pretty Lady win the 4.20 race at Sandown Park on Tuesday? Will the Financial Times All Shares index rise by more than 50 points in the next two months? Nobody knows the answers to such questions, but in each case we may have information that makes a positive answer more or less appropriate: if we are in the Great Australian Desert and it's winter, it is exceedingly unlikely to rain tomorrow, but if we are in Delhi in the middle of the monsoon, it will almost certainly rain. If Pretty Lady is getting on in years and hasn't won a race yet, she's unlikely to win on Tuesday either, while if she recently won a couple of major races and she's looking fit, she may well win at Sandown Park. The performance of the All Shares index is hard to predict, but factors affecting company profitability and the direction interest rates will move, will make the index more or less likely to rise. Probability is a concept which enables us to quantify and manipulate uncertainties. We assign a probability $p = 0$ to an event if we think it is simply impossible, and we assign $p = 1$ if we think the event is certain to happen. Intermediate values for p imply that we think an event may happen and may not, the value of p increasing with our confidence that it will happen.

Physics is about predicting the future. Will this ladder slip when I step on it? How many times will this pendulum swing to and fro in an hour? What temperature will the water in this thermos be at when it has completely melted this ice cube? Physics often enables us to answer such questions with a satisfying degree of certainty: the ladder will not slip provided it is inclined at less than $23.34°$ to the vertical; the pendulum makes 3602 oscillations per hour; the water will reach $6.43°$C. But if we are pressed for sufficient accuracy we must admit to uncertainty and resort to probability because our predictions depend on the data we have, and these are always subject to measuring error, and idealisations: the ladder's critical angle depends on the coefficients of friction at the two ends of the ladder, and these cannot be precisely given because both the wall and the floor are slightly irregular surfaces; the period of the pendulum depends slightly on the amplitude of its swing, which will vary with temperature and the humidity of the air; the final temperature of the water will vary with the amount of heat transferred through the walls of the thermos and the speed of evaporation from the water's surface, which depends on draughts in the room as well as on humidity. If we are asked to make predictions about a ladder that is inclined near its critical angle, or we need to know a quantity like the period of the pendulum to high accuracy, we cannot make definite statements, we can only say something like the probability of the ladder slipping is 0.8, or there is a probability of 0.5 that the period of the pendulum lies between 1.0007 s and 1.0004 s. We can dispense with probability when slightly vague answers are permissible, such as that the period is 1.00 s to three significant figures. The concept of probability enables us to push our science to its limits, and make the most precise and reliable statements possible.

Probability enters physics in two ways: through uncertain data and through the system being subject to random influences. In the first case we could make a more accurate prediction if a property of the system, such as the length or temperature of the pendulum, were more precisely characterised. That is, the value of some number is well defined, it's just that we don't know the value very accurately. The second case is that in which our system is subject to inherently random influences – for example, to the draughts that make us uncertain what will be the final temperature of the water. To attain greater certainty when the system under study is subject to such random influences, we can either take steps to increase the isolation of our system – for example by putting a lid on the thermos – or we can expand the system under study so that the formerly random influences become calculable interactions between one part of the system and another. Such expansion of the system is not a practical proposition in the case of the thermos – the expanded system would have to encompass the air in the room, and then we would worry about fluctuations in the intensity of sunlight through the window, draughts under the door and much else. The strategy does work in other cases, however. For example, climate changes over the last ten million years can be studied as the response of a complex dynamical system – the atmosphere coupled to the oceans – that is subject to random external stimuli, but a more complete account of climate changes can be made when the dynamical system is expanded to include the Sun and Moon because climate is strongly affected by the inclination of the Earth's spin axis to the plane of the Earth's orbit and the Sun's coronal activity.

A low-mass system is less likely to be well isolated from its surroundings than a massive one. For example, the orbit of the Earth is scarcely affected by radiation pressure that sunlight exerts on it, while dust grains less than a few microns in size that are in orbit about the Sun lose angular momentum through radiation pressure at a rate that causes them to spiral in from near the Earth to the Sun within a few millennia. Similarly, a rubber duck left in the bath after the children have got out will stay very still, while tiny pollen grains in the water near it execute Brownian motion that carries them along a jerky path many times their own length each minute. Given the difficulty of isolating low-mass systems, and the tremendous obstacles that have to be surmounted if we are to expand the system to the point at which all influences on the object of interest become causal, it is natural that the physics of small systems is invariably probabilistic in nature. Quantum mechanics describes the dynamics of all systems, great and small. Rather than making firm predictions, it enables us to calculate probabilities. If the system is massive, the probabilities of interest may be so near zero or unity that we have effective certainty. If the system is small, the probabilistic aspect of the theory will be more evident.

The scale of atoms is precisely the scale on which the probabilistic aspect is predominant. Its predominance reflects two facts. First, there is no such thing as an isolated atom because all atoms are inherently coupled to the electromagnetic field, and to the fields associated with electrons, neutrinos, quarks, and various 'gauge bosons'. Since we have incomplete information about the states of these fields, we cannot hope to make precise predictions about the behaviour of an individual atom. Second, we cannot build measuring instruments of arbitrary delicacy. The instruments we use to measure