DIGITAL SIGNALS

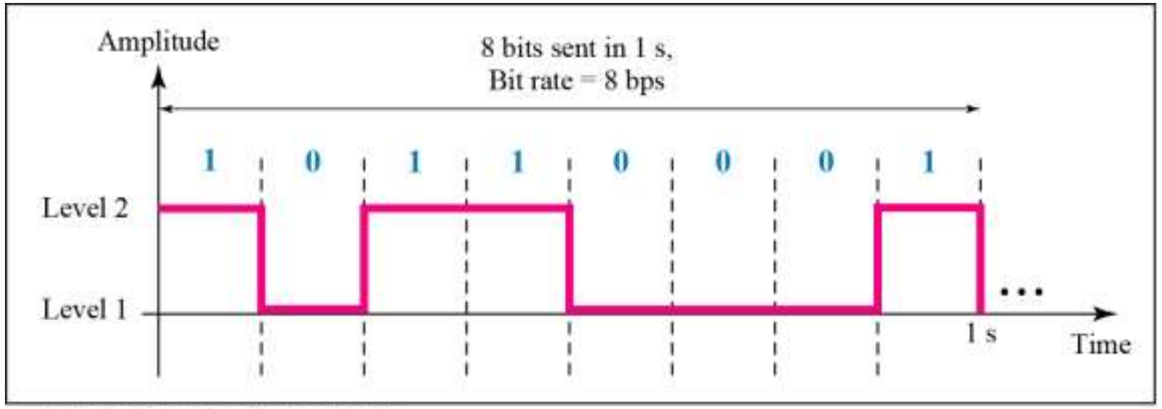
Lecture 4

Chapter 3 Data and Signals

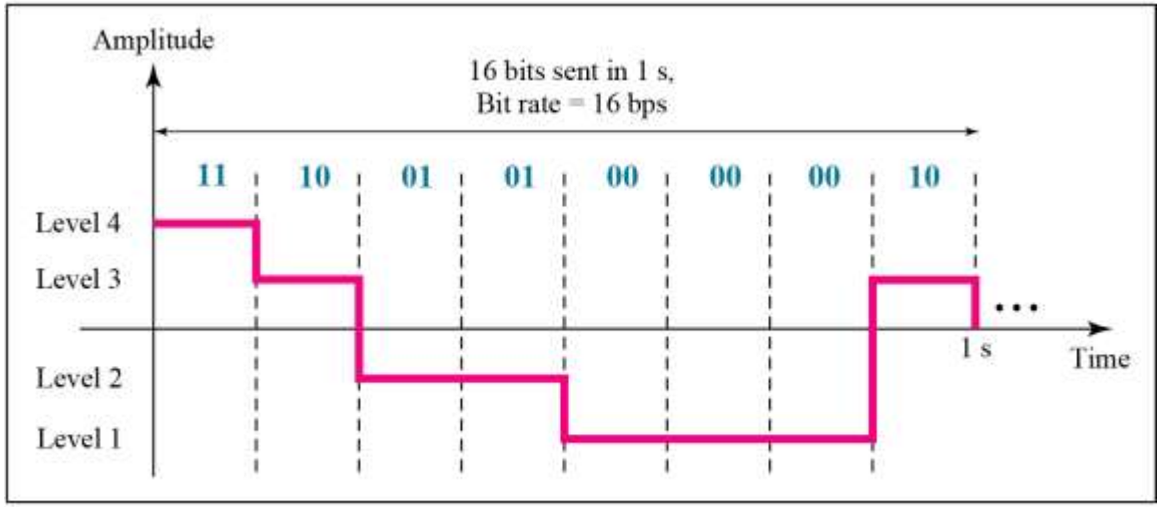
3-3 DIGITAL SIGNALS

- In addition to being represented by an analog signal, information can also be represented by a digital signal. For example, a 1 can be encoded as a positive voltage and a 0 as zero voltage. A digital signal can have more than two levels. In this case, we can send more than 1 bit for each level.
- Topics discussed in this section:
  - Bit Rate
  - Bit Length
  - Digital Signal as a Composite Analog Signal
  - Application Layer

Figure 3.16 Two digital signals: one with two signal levels and the other with four signal levels



a. A digital signal with two levels



b. A digital signal with four levels

Example 3.16

- A digital signal has eight levels. How many bits are needed per level? We calculate the number of bits from the formula

$$\text{Number of bits per level} = \log_2 8 = 3$$

- Each signal level is represented by 3 bits.

Example 3.17

- A digital signal has nine levels. How many bits are needed per level? We calculate the number of bits by using the formula. Each signal level is represented by 3.17 bits. However, this answer is not realistic. The number of bits sent per level needs to be an integer as well as a power of 2. For this example, 4 bits can represent one level.

Example 3.18

- Assume we need to download text documents at the rate of 100 pages per sec. What is the required bit rate of the channel?

Solution

- A page is an average of 24 lines with 80 characters in each line. If we assume that one character requires 8 bits (ascii), the bit rate is

$$100 \times 24 \times 80 \times 8 = 1{,}636{,}000 \text{ bps} = 1.636 \text{ Mbps}$$

Example 3.19

- A digitized voice channel, as we will see in Chapter 4, is made by digitizing a 4-kHz bandwidth analog voice signal. We need to sample the signal at twice the highest frequency (two samples per hertz). We assume that each sample requires 8 bits. What is the required bit rate?

Solution

- The bit rate can be calculated as

$$2 \times 4000 \times 8 = 64{,}000 \text{ bps} = 64 \text{ kbps}$$

Example 3.20

- What is the bit rate for high-definition TV (HDTV)?

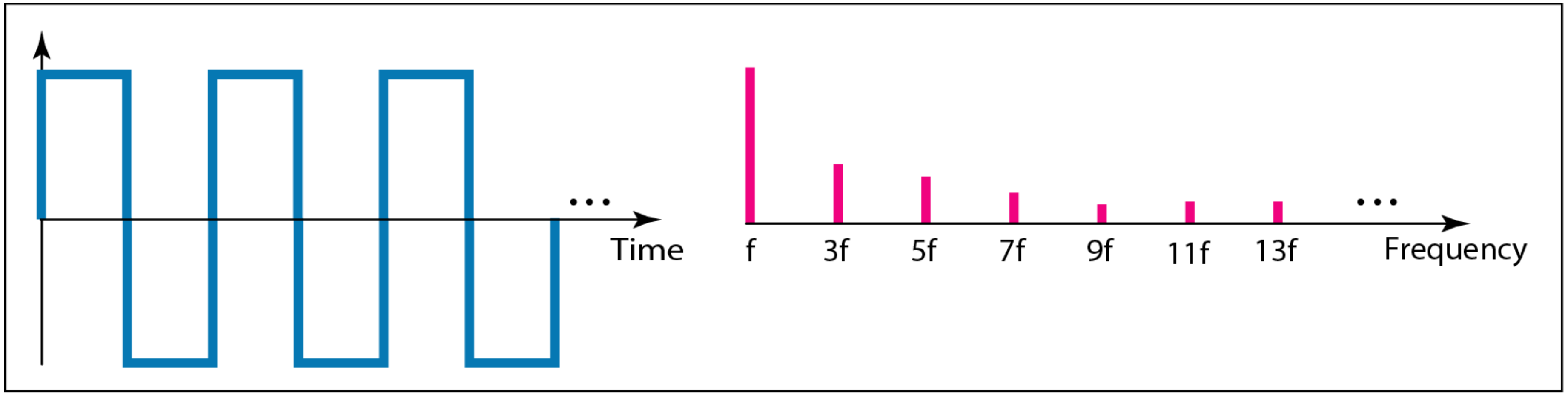
Solution

- HDTV uses digital signals to broadcast high quality video signals. The HDTV screen is normally a ratio of 16 : 9. There are 1920 by 1080 pixels per screen, and the screen is renewed 30 times per second. Twenty-four bits represents one color pixel.

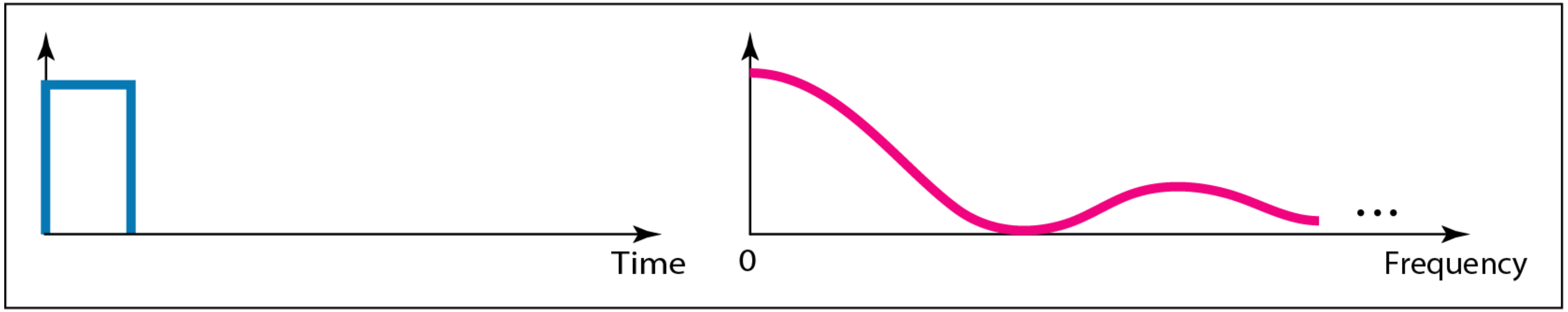
$$1920 \times 1080 \times 30 \times 24 = 1{,}492{,}992{,}000 \text{ or } 1.5 \text{ Gbps}$$

- The TV stations reduce this rate to 20 to 40 Mbps through compression.

Figure 3.17 The time and frequency domains of periodic and nonperiodic digital signals

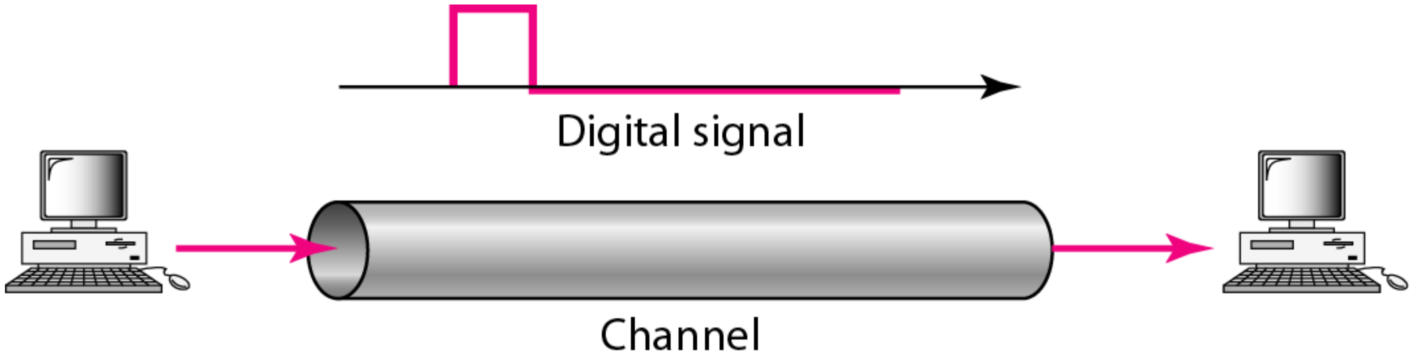


a. Time and frequency domains of periodic digital signal



b. Time and frequency domains of nonperiodic digital signal

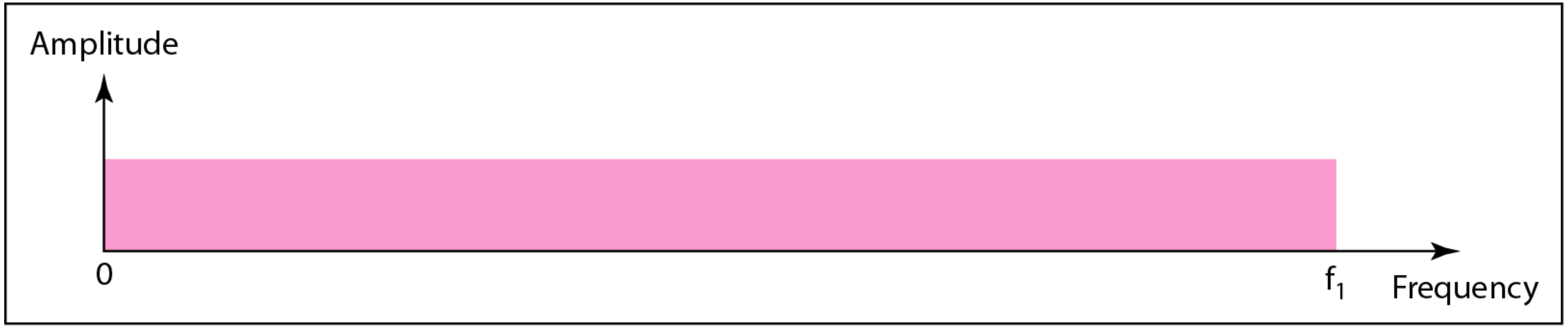
Figure 3.18 Baseband transmission



NOTE

- A digital signal is a composite analog signal with an infinite bandwidth.

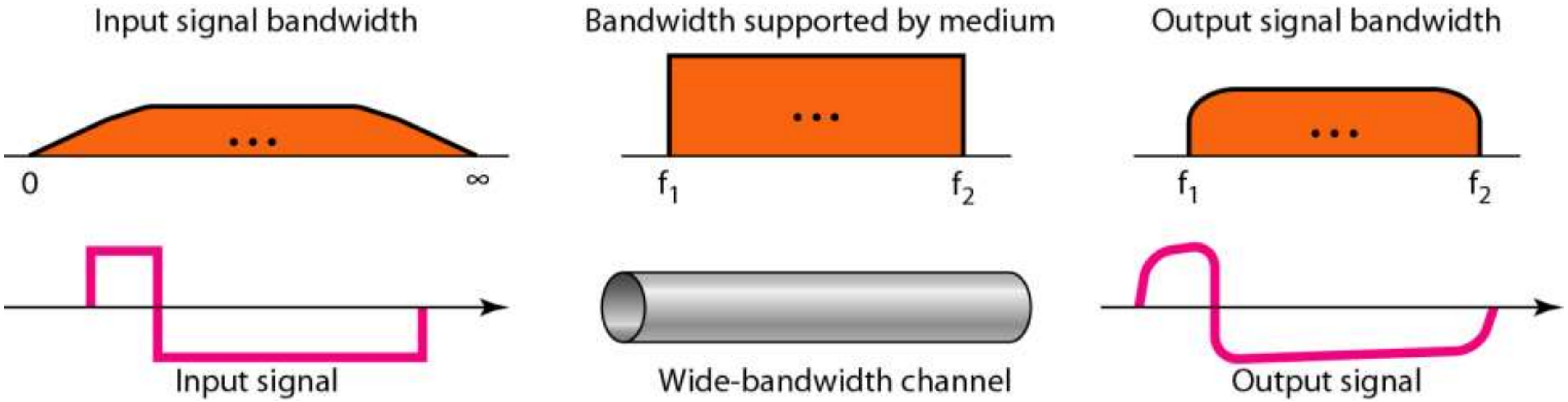
Figure 3.19 Bandwidths of two low-pass channels



a. Low-pass channel, wide bandwidth

b. Low-pass channel, narrow bandwidth

Figure 3.20 Baseband transmission using a dedicated medium



NOTE

- Baseband transmission of a digital signal that preserves the shape of the digital signal is possible only if we have a low-pass channel with an infinite or very wide bandwidth.

Example 3.21

- An example of a dedicated channel where the entire bandwidth of the medium is used as one single channel is a LAN. Almost every wired LAN today uses a dedicated channel for two stations communicating with each other. In a bus topology LAN with multipoint connections, only two stations can communicate with each other at each moment in time (timesharing); the other stations need to refrain from sending data. In a star topology LAN, the entire channel between each station and the hub is used for communication between these two entities.

Figure 3.21 Rough approximation of a digital signal using the first harmonic for worst case

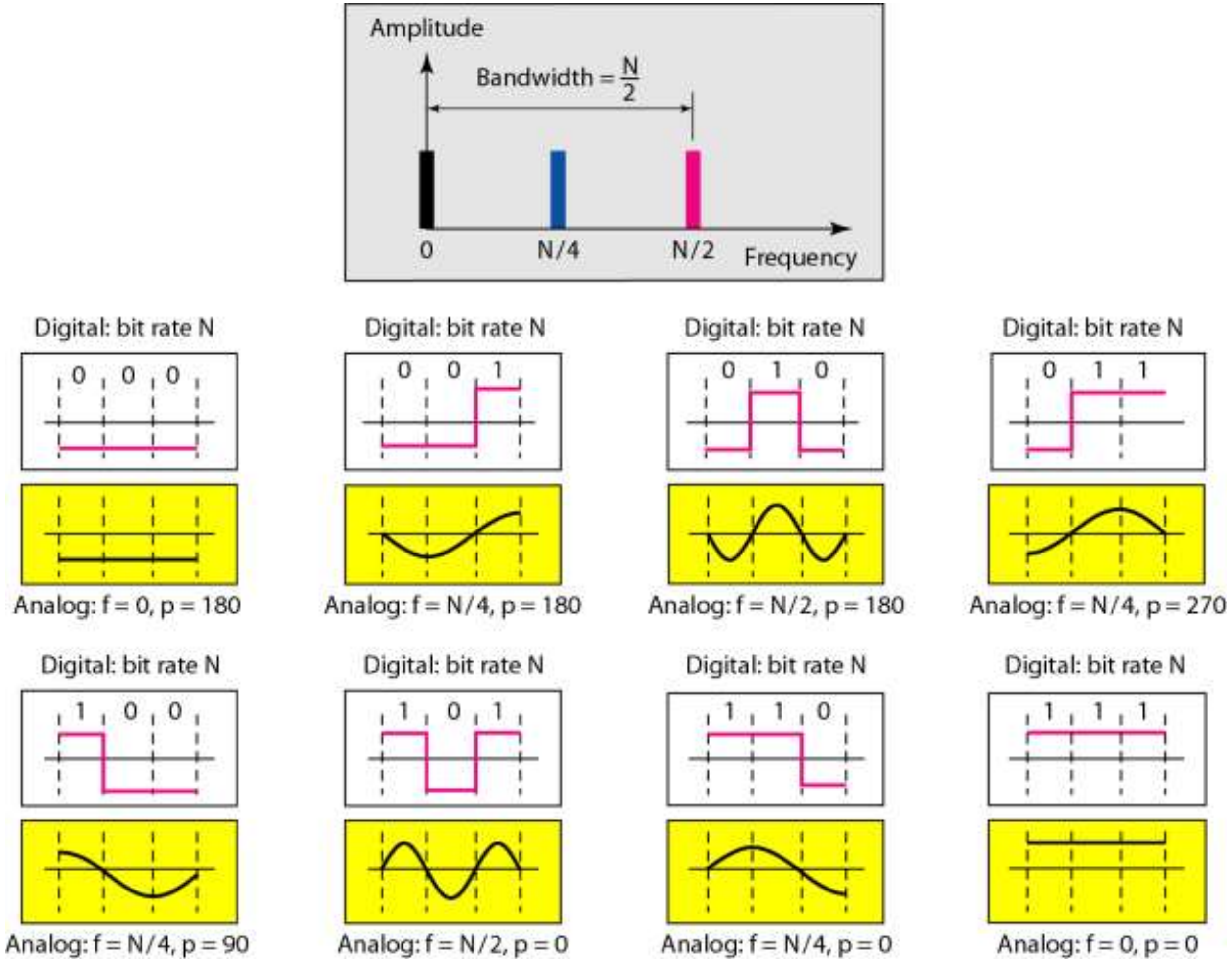
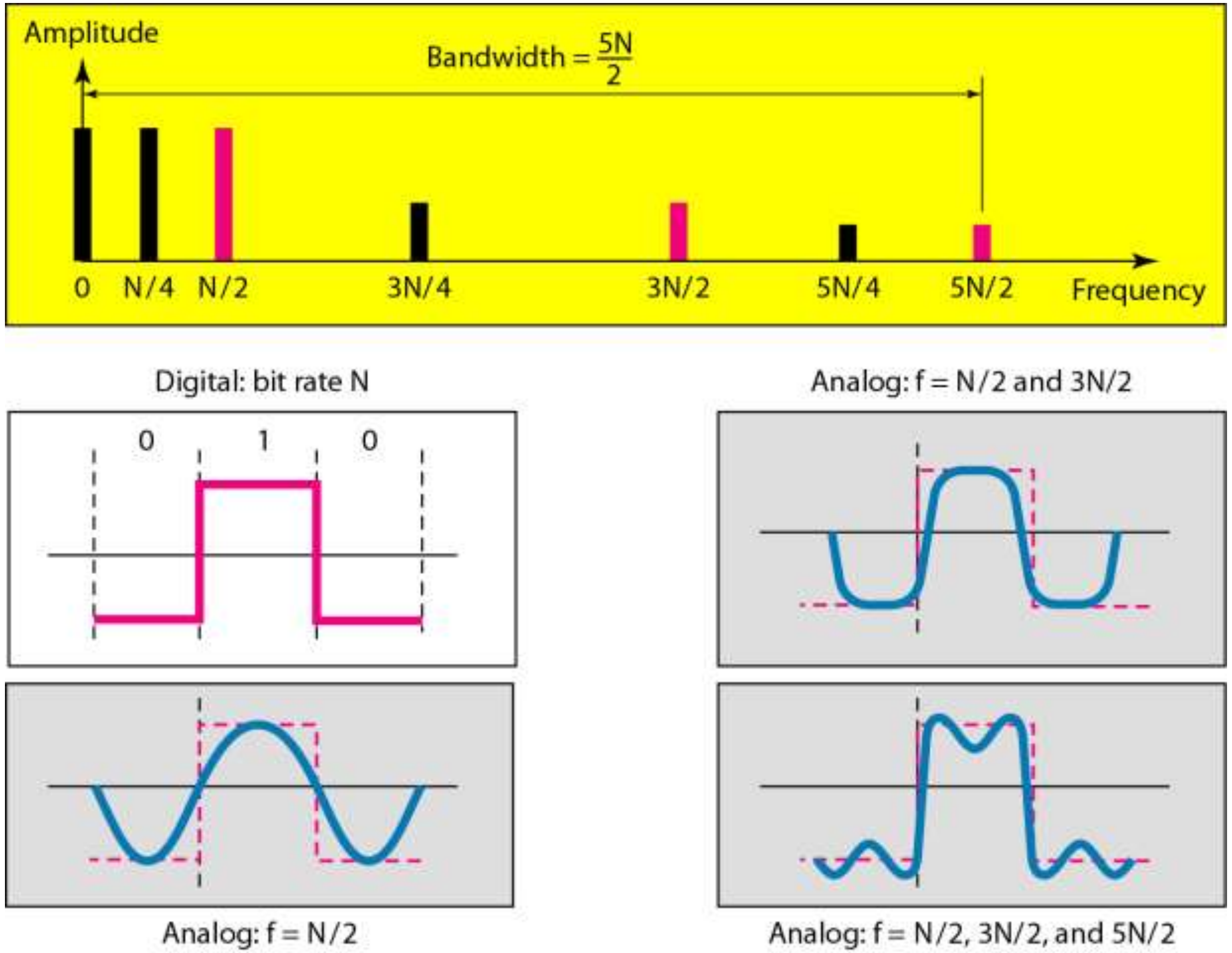


Figure 3.22 Simulating a digital signal with first three harmonics



NOTE

- In baseband transmission, the required bandwidth is proportional to the bit rate;
- if we need to send bits faster, we need more bandwidth.

Table 3.2 Bandwidth requirements

Bit Rate	*Harmonic 1*	*Harmonics 1, 3*	*Harmonics 1, 3, 5*
$n = 1$ kbps	$B = 500$ Hz	$B = 1.5$ kHz	$B = 2.5$ kHz
$n = 10$ kbps	$B = 5$ kHz	$B = 15$ kHz	$B = 25$ kHz
$n = 100$ kbps	$B = 50$ kHz	$B = 150$ kHz	$B = 250$ kHz

Example 3.22

- What is the required bandwidth of a low-pass channel if we need to send 1 Mbps by using baseband transmission?

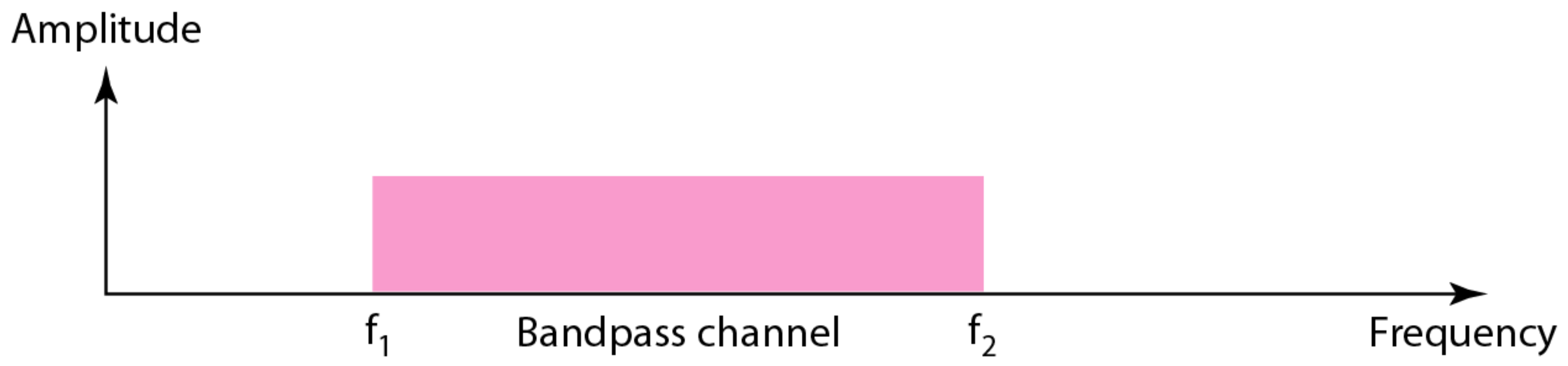
- Solution
- The answer depends on the accuracy desired.
- The minimum bandwidth, is B = bit rate /2, or 500 kHz.
- A better solution is to use the first and the third harmonics with $B = 3 \times 500$ kHz = 1.5 MHz.
- Still a better solution is to use the first, third, and fifth harmonics with $B = 5 \times 500$ kHz = 2.5 MHz.

Example 3.22

- We have a low-pass channel with bandwidth 100 kHz. What is the maximum bit rate of this channel?

- Solution
- The maximum bit rate can be achieved if we use the first harmonic. The bit rate is 2 times the available bandwidth, or 200 kbps.

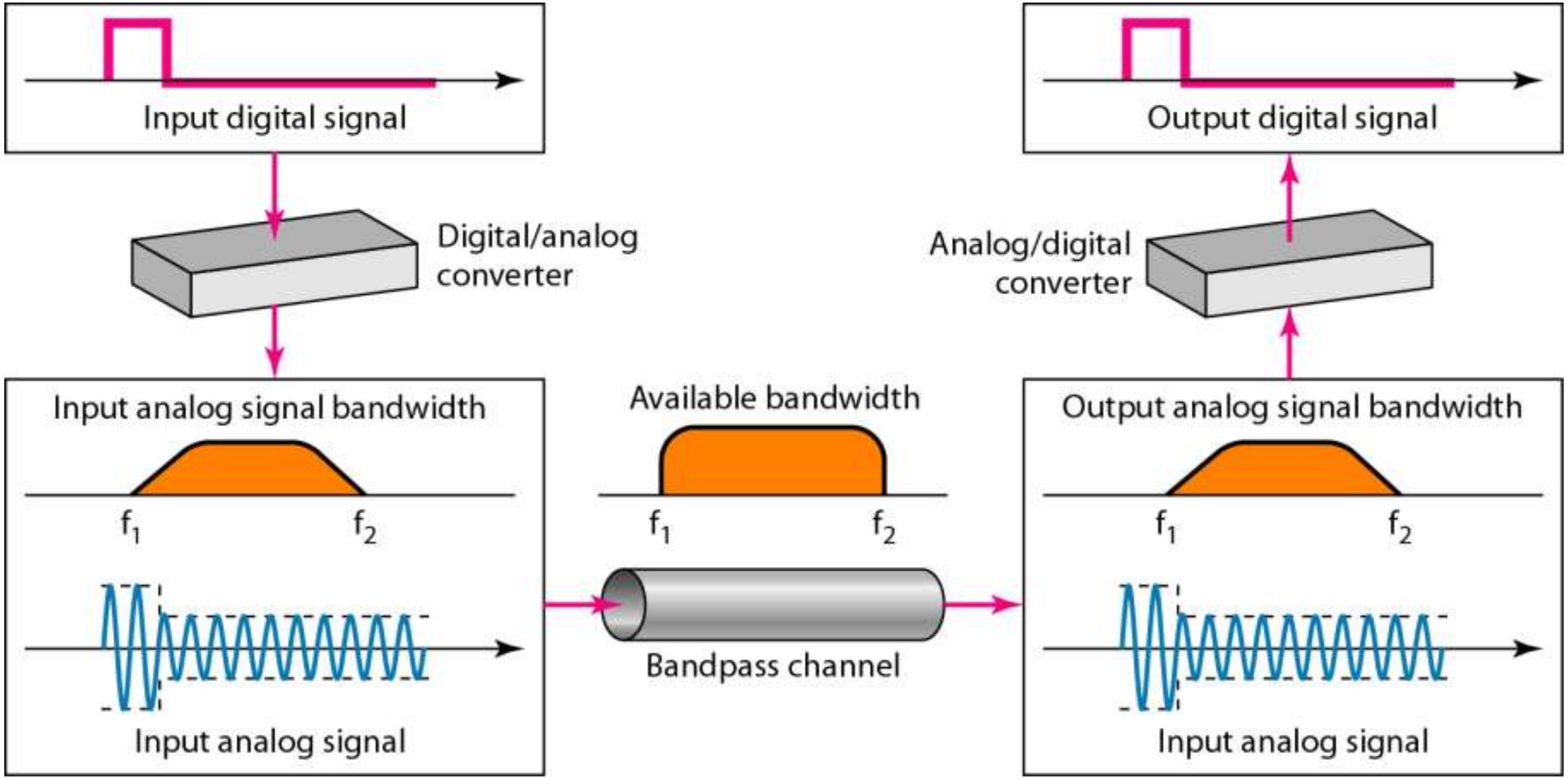
Figure 3.23 Bandwidth of a bandpass channel



NOTE

- If the available channel is a bandpass channel, we cannot send the digital signal directly to the channel; we need to convert the digital signal to an analog signal before transmission.

Figure 3.24 Modulation of a digital signal for transmission on a bandpass channel



Example 3.24

- An example of broadband transmission using modulation is the sending of computer data through a telephone subscriber line, the line connecting a resident to the central telephone office. These lines are designed to carry voice with a limited bandwidth. The channel is considered a bandpass channel. We convert the digital signal from the computer to an analog signal, and send the analog signal. We can install two converters to change the digital signal to analog and vice versa at the receiving end. The converter, in this case, is called a modem which we discuss in detail in Chapter 5.

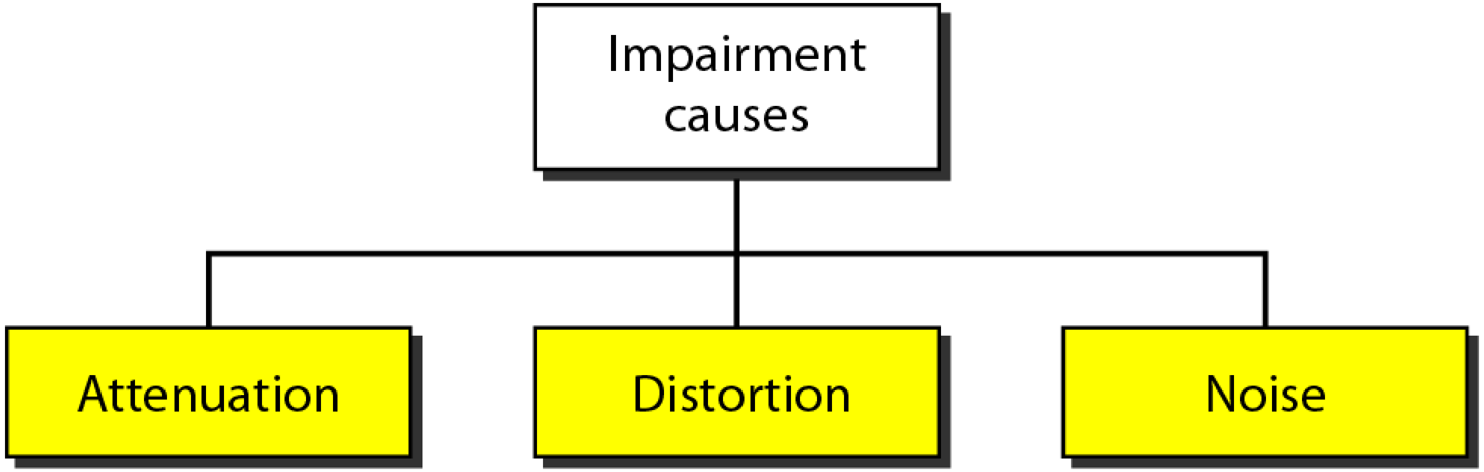
Example 3.25

- A second example is the digital cellular telephone. For better reception, digital cellular phones convert the analog voice signal to a digital signal. Although the bandwidth allocated to a company providing digital cellular phone service is very wide, we still cannot send the digital signal without conversion. The reason is that we only have a bandpass channel available between caller and callee. We need to convert the digitized voice to a composite analog signal before sending.

3-4 TRANSMISSION IMPAIRMENT

- Signals travel through transmission media, which are not perfect. The imperfection causes signal impairment. This means that the signal at the beginning of the medium is not the same as the signal at the end of the medium. What is sent is not what is received. Three causes of impairment are attenuation, distortion, and noise.
- Topics discussed in this section:
  - Attenuation
  - Distortion
  - Noise

Figure 3.25 Causes of impairment



Attenuation

- Means loss of energy -> weaker signal
- When a signal travels through a medium it loses energy overcoming the resistance of the medium
- Amplifiers are used to compensate for this loss of energy by amplifying the signal.

Measurement of Attenuation

- To show the loss or gain of energy the unit “decibel” is used.

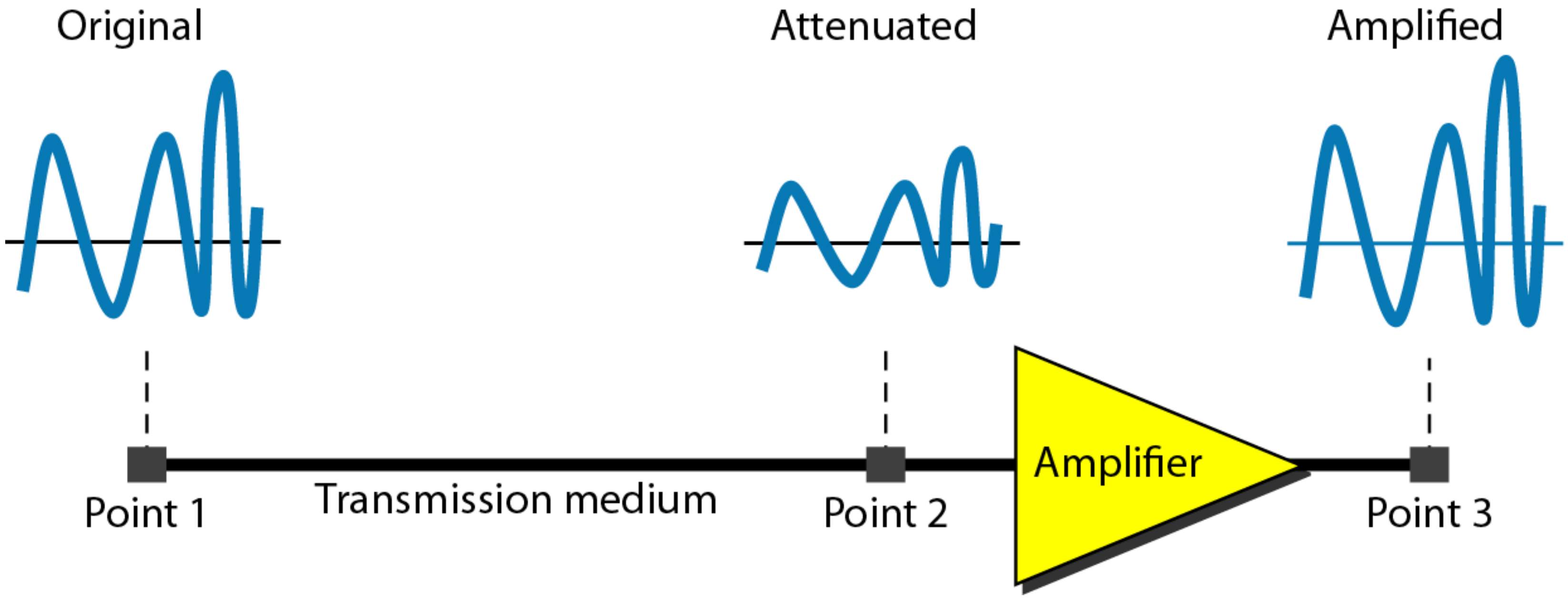
$$dB = 10 \log_{10}\left(\frac{P_2}{P_1}\right)$$

where

P_1 - input signal

P_2 - output signal

Figure 3.26 Attenuation



Example 3.26

- Suppose a signal travels through a transmission medium and its power is reduced to one-half. This means that P2 is (1/2)P1. In this case, the attenuation (loss of power) can be calculated as

$$10 \log_{10} \frac{P_2}{P_1} = 10 \log_{10} \frac{0.5 P_1}{P_1} = 10 \log_{10} 0.5 = 10(-0.3) = -3 \text{ dB}$$

- A loss of 3 dB (–3 dB) is equivalent to losing one-half the power.

Example 3.27

- A signal travels through an amplifier, and its power is increased 10 times. This means that $P_2 = 10P_1$. In this case, the amplification (gain of power) can be calculated as

$$10 \log_{10} \frac{P_2}{P_1} = 10 \log_{10} \frac{10P_1}{P_1}$$

$$= 10 \log_{10} 10 = 10(1) = 10 \text{ dB}$$

Example 3.28

- One reason that engineers use the decibel to measure the changes in the strength of a signal is that decibel numbers can be added (or subtracted) when we are measuring several points (cascading) instead of just two. In Figure 3.27 a signal travels from point 1 to point 4. In this case, the decibel value can be calculated as

$$dB = -3 + 7 - 3 = +1$$

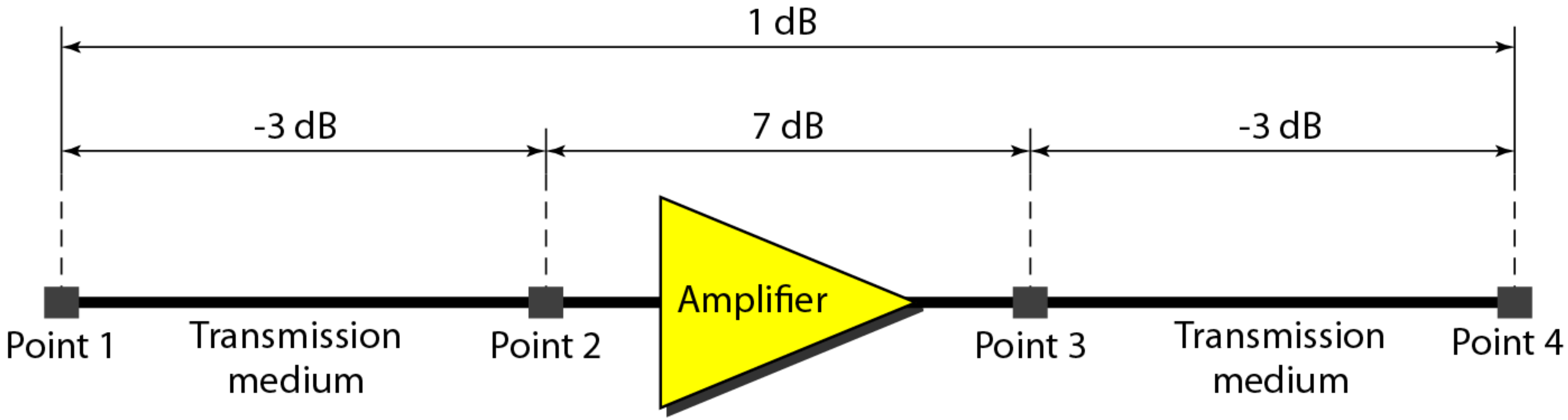


Figure 3.27 Decibels for Example 3.28

Example 3.29

- Sometimes the decibel is used to measure signal power in milliwatts. In this case, it is referred to as dB_m and is calculated as $dB_m = 10 \log10 P_m$, where P_m is the power in milliwatts. Calculate the power of a signal with $dB_m = -30$.

Solution

- We can calculate the power in the signal as

$$dB_m = 10 \log_{10} P_m = -30$$

$$\log_{10} P_m = -3 \qquad P_m = 10^{-3} \text{ mW}$$

Example 3.30

- The loss in a cable is usually defined in decibels per kilometer (dB/km). If the signal at the beginning of a cable with −0.3 dB/km has a power of 2 mW, what is the power of the signal at 5 km?
- Solution
- The loss in the cable in decibels is 5 × (−0.3) = −1.5 dB. We can calculate the power as

$$\text{dB} = 10 \log_{10} \frac{P_2}{P_1} = -1.5$$

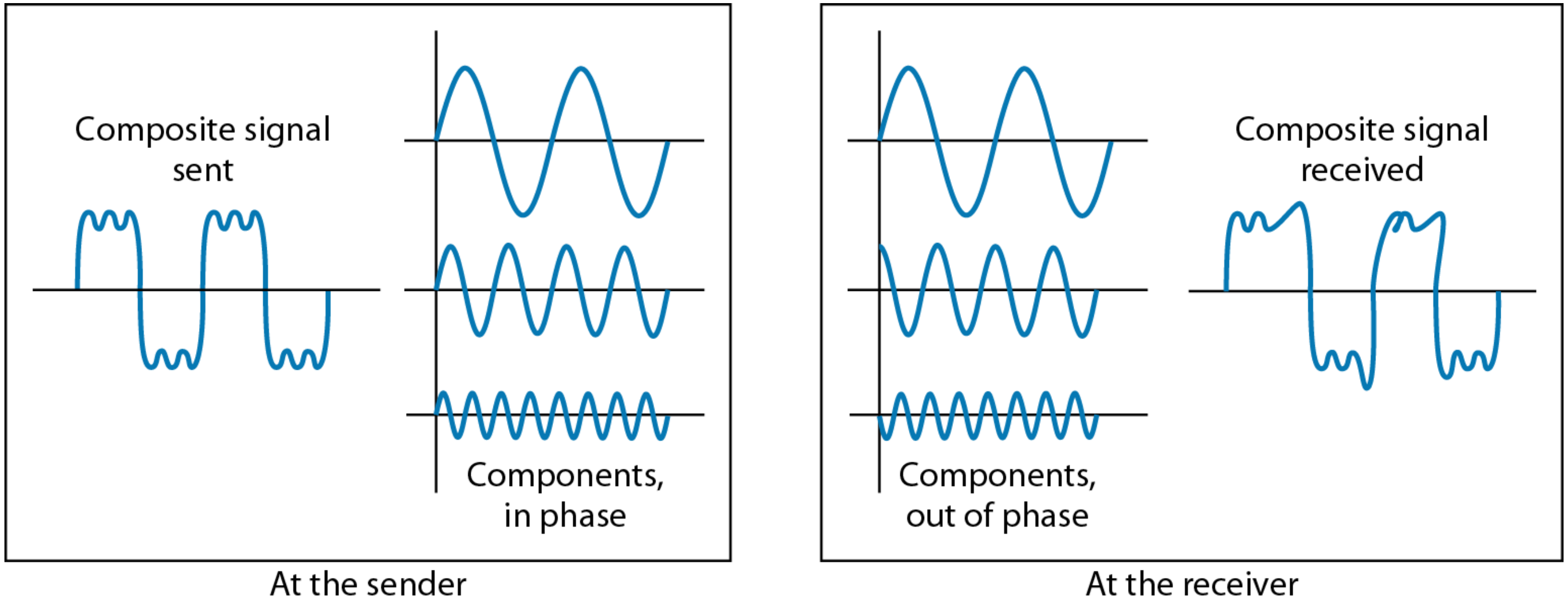
$$\frac{P_2}{P_1} = 10^{-0.15} = 0.71$$

$$P_2 = 0.71 P_1 = 0.7 \times 2 = 1.4 \text{ mW}$$

Distortion

- Means that the signal changes its form or shape
- Distortion occurs in composite signals
- Each frequency component has its own propagation speed traveling through a medium.
- The different components therefore arrive with different delays at the receiver.
- That means that the signals have different phases at the receiver than they did at the source.

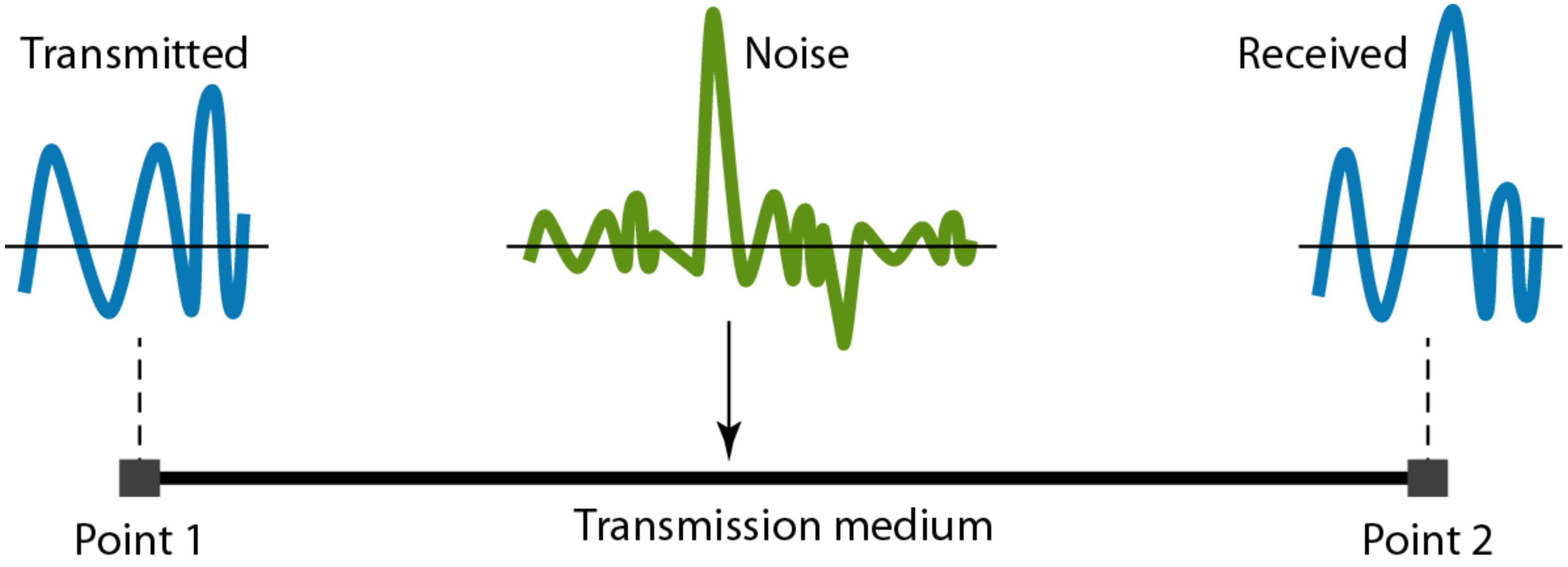
Figure 3.28 Distortion



Noise

- There are different types of noise
  - Thermal - random noise of electrons in the wire creates an extra signal
  - Induced - from motors and appliances, devices act are transmitter antenna and medium as receiving antenna.
  - Crosstalk - same as above but between two wires.
  - Impulse - Spikes that result from power lines, lighning, etc.

Figure 3.29 Noise



Signal to Noise Ratio (SNR)

- To measure the quality of a system the SNR is often used. It indicates the strength of the signal wrt the noise power in the system.
- It is the ratio between two powers.
- It is usually given in dB and referred to as SNR_{dB}.

Example 3.31

- The power of a signal is 10 mW and the power of the noise is 1 μW; what are the values of SNR and SNR_{dB}?

- Solution
- The values of SNR and SNRdB can be calculated as follows:

$$SNR = \frac{10{,}000\ \mu W}{1\ mW} = 10{,}000$$

$$SNR_{dB} = 10 \log_{10} 10{,}000 = 10 \log_{10} 10^4 = 40$$

Example 3.32

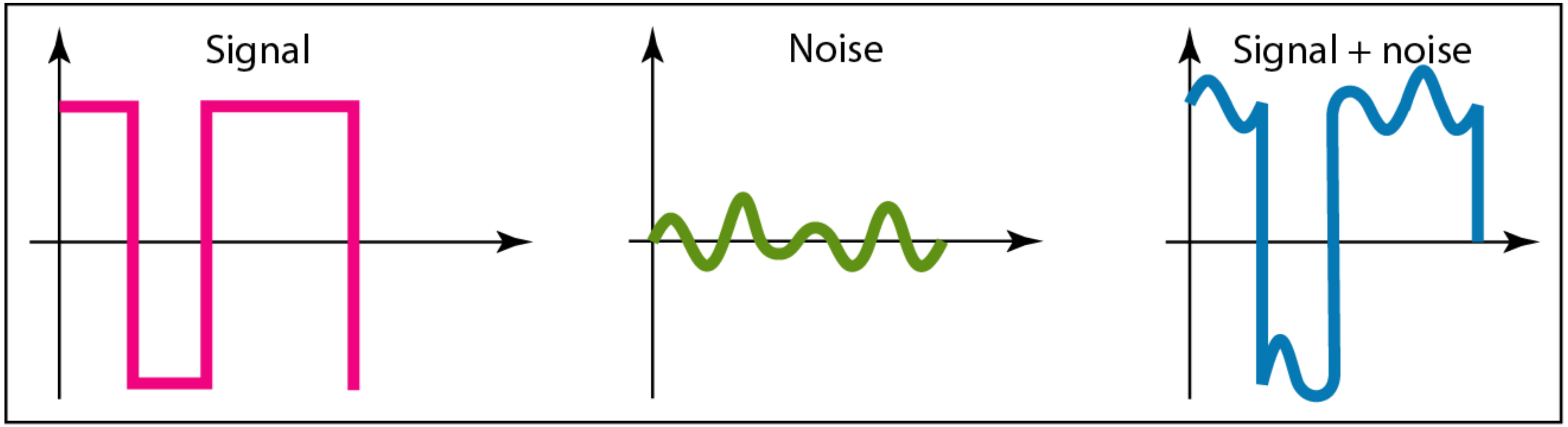
- The values of SNR and SNR_{dB} for a noiseless channel are

$$SNR = \frac{\text{signal power}}{0} = \infty$$

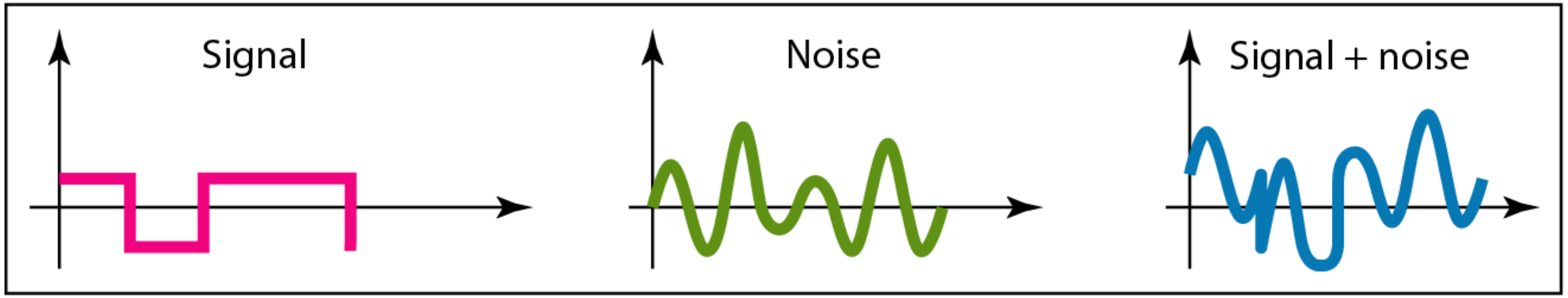
$$SNR_{dB} = 10 \log_{10} \infty = \infty$$

- We can never achieve this ratio in real life; it is an ideal.

Figure 3.30 Two cases of SNR: a high SNR and a low SNR



a. Large SNR



b. Small SNR

3-5 DATA RATE LIMITS

- A very important consideration in data communications is how fast we can send data, in bits per second, over a channel. Data rate depends on three factors:
  1. The bandwidth available
  2. The level of the signals we use
  3. The quality of the channel (the level of noise)

- Topics discussed in this section:
  - Noiseless Channel: Nyquist Bit Rate
  - Noisy Channel: Shannon Capacity
  - Using Both Limits

NOTE

- Increasing the levels of a signal increases the probability of an error occurring, in other words it reduces the reliability of the system. Why??

Capacity of a System

- The bit rate of a system increases with an increase in the number of signal levels we use to denote a symbol.
- A symbol can consist of a single bit or “n” bits.
- The number of signal levels = 2n.
- As the number of levels goes up, the spacing between level decreases -> increasing the probability of an error occurring in the presence of transmission impairments.

Nyquist Theorem

- Nyquist gives the upper bound for the bit rate of a transmission system by calculating the bit rate directly from the number of bits in a symbol (or signal levels) and the bandwidth of the system (assuming 2 symbols/per cycle and first harmonic).
- Nyquist theorem states that for a noiseless channel:
- $C = 2\ B \log_2 2n$
- C= capacity in bps
- B = bandwidth in Hz

Example 3.33

- Does the Nyquist theorem bit rate agree with the intuitive bit rate described in baseband transmission?

Solution

- They match when we have only two levels. We said, in baseband transmission, the bit rate is 2 times the bandwidth if we use only the first harmonic in the worst case. However, the Nyquist formula is more general than what we derived intuitively; it can be applied to baseband transmission and modulation. Also, it can be applied when we have two or more levels of signals.

Example 3.34

- Consider a noiseless channel with a bandwidth of 3000 Hz transmitting a signal with two signal levels. The maximum bit rate can be calculated as

$$\text{BitRate} = 2 \times 3000 \times \log_2 2 = 6000 \text{ bps}$$

Example 3.35

- Consider the same noiseless channel transmitting a signal with four signal levels (for each level, we send 2 bits). The maximum bit rate can be calculated as

$$\text{BitRate} = 2 \times 3000 \times \log_2 4 = 12{,}000 \text{ bps}$$

Example 3.36

- We need to send 265 kbps over a noiseless channel with a bandwidth of 20 kHz. How many signal levels do we need?

Solution

- We can use the Nyquist formula as shown:

$$265{,}000 = 2 \times 20{,}000 \times \log_2 L$$

$$\log_2 L = 6.625 \qquad L = 2^{6.625} = 98.7 \text{ levels}$$

- Since this result is not a power of 2, we need to either increase the number of levels or reduce the bit rate. If we have 128 levels, the bit rate is 280 kbps. If we have 64 levels, the bit rate is 240 kbps.

Shannon's Theorem

- Shannon's theorem gives the capacity of a system in the presence of noise.

$$C = B \log_2(1 + SNR)$$

Example 3.37

- Consider an extremely noisy channel in which the value of the signal-to-noise ratio is almost zero. In other words, the noise is so strong that the signal is faint. For this channel the capacity C is calculated as

$$C = B \log_2 (1 + \mathrm{SNR}) = B \log_2 (1 + 0) = B \log_2 1 = B \times 0 = 0$$

- This means that the capacity of this channel is zero regardless of the bandwidth. In other words, we cannot receive any data through this channel.

Example 3.38

- We can calculate the theoretical highest bit rate of a regular telephone line. A telephone line normally has a bandwidth of 3000. The signal-to-noise ratio is usually 3162. For this channel the capacity is calculated as

$$C = B \log_2 (1 + \text{SNR}) = 3000 \log_2 (1 + 3162) = 3000 \log_2 3163$$
$$= 3000 \times 11.62 = 34{,}860 \text{ bps}$$

- This means that the highest bit rate for a telephone line is 34.860 kbps. If we want to send data faster than this, we can either increase the bandwidth of the line or improve the signal-to-noise ratio.

Example 3.39

- The signal-to-noise ratio is often given in decibels. Assume that $SNR_{dB} = 36$ and the channel bandwidth is 2 MHz. The theoretical channel capacity can be calculated as

$$SNR_{dB} = 10 \log_{10} SNR \quad \rightarrow \quad SNR = 10^{SNR_{dB}/10} \quad \rightarrow \quad SNR = 10^{3.6} = 3981$$

$$C = B \log_2 (1 + SNR) = 2 \times 10^6 \times \log_2 3982 = 24 \text{ Mbps}$$

Example 3.40

- For practical purposes, when the SNR is very high, we can assume that SNR + 1 is almost the same as SNR. In these cases, the theoretical channel capacity can be simplified to

$$C = B \times \frac{\mathrm{SNR_{dB}}}{3}$$

- For example, we can calculate the theoretical capacity of the previous example as

$$C = 2\ \mathrm{MHz} \times \frac{36}{3} = 24\ \mathrm{Mbps}$$

Example 3.41

- We have a channel with a 1-MHz bandwidth. The SNR for this channel is 63. What are the appropriate bit rate and signal level?

Solution

- First, we use the Shannon formula to find the upper limit.

$$C = B \log_2 (1 + \text{SNR}) = 10^6 \log_2 (1 + 63) = 10^6 \log_2 64 = 6 \text{ Mbps}$$

Example 3.41 (continued)

- The Shannon formula gives us 6 Mbps, the upper limit. For better performance we choose something lower, 4 Mbps, for example. Then we use the Nyquist formula to find the number of signal levels.

$$4 \text{ Mbps} = 2 \times 1 \text{ MHz} \times \log_2 L \quad \rightarrow \quad L = 4$$

NOTE

- The Shannon capacity gives us the upper limit; the Nyquist formula tells us how many signal levels we need.

3-6 PERFORMANCE

- One important issue in networking is the performance of the network—how good is it? We discuss quality of service, an overall measurement of network performance, in greater detail in Chapter 24. In this section, we introduce terms that we need for future chapters.
- Topics discussed in this section:
  - Bandwidth - capacity of the system
  - Throughput - no. of bits that can be pushed through
  - Latency (Delay) - delay incurred by a bit from start to finish
  - Bandwidth-Delay Product

NOTE

- In networking, we use the term bandwidth in two contexts.
  - The first, bandwidth in hertz, refers to the range of frequencies in a composite signal or the range of frequencies that a channel can pass.
  - The second, bandwidth in bits per second, refers to the speed of bit transmission in a channel or link. Often referred to as Capacity.

Example 3.42

- The bandwidth of a subscriber line is 4 kHz for voice or data. The bandwidth of this line for data transmission
- can be up to 56,000 bps using a sophisticated modem to change the digital signal to analog.

Example 3.43

- If the telephone company improves the quality of the line and increases the bandwidth to 8 kHz, we can send 112,000 bps by using the same technology as mentioned in Example 3.42.

Example 3.44

- A network with bandwidth of 10 Mbps can pass only an average of 12,000 frames per minute with each frame carrying an average of 10,000 bits. What is the throughput of this network?

Solution

- We can calculate the throughput as

$$\text{Throughput} = \frac{12{,}000 \times 10{,}000}{60} = 2 \text{ Mbps}$$

- The throughput is almost one-fifth of the bandwidth in this case.

Propagation & Transmission delay

- Propagation speed - speed at which a bit travels though the medium from source to destination.
- Transmission speed - the speed at which all the bits in a message arrive at the destination. (difference in arrival time of first and last bit)

Propagation and Transmission Delay

- Propagation Delay = Distance/Propagation speed

- Transmission Delay = Message size/bandwidth bps

- Latency = Propagation delay + Transmission delay + Queueing time + Processing time

Example 3.45

- What is the propagation time if the distance between the two points is 12,000 km? Assume the propagation speed to be 2.4 × 108 m/s in cable.

Solution

- We can calculate the propagation time as

$$\text{Propagation time} = \frac{12{,}000 \times 1000}{2.4 \times 10^8} = 50 \text{ ms}$$

- The example shows that a bit can go over the Atlantic Ocean in only 50 ms if there is a direct cable between the source and the destination.

Example 3.46

- What are the propagation time and the transmission time for a 2.5-kbyte message (an e-mail) if the bandwidth of the network is 1 Gbps? Assume that the distance between the sender and the receiver is 12,000 km and that light travels at 2.4 × 108 m/s.

Solution

- We can calculate the propagation and transmission time as shown on the next slide:

Example 3.46 (continued)

$$\text{Propagation time} = \frac{12{,}000 \times 1000}{2.4 \times 10^8} = 50 \text{ ms}$$

$$\text{Transmission time} = \frac{2500 \times 8}{10^9} = 0.020 \text{ ms}$$

- Note that in this case, because the message is short and the bandwidth is high, the dominant factor is the propagation time, not the transmission time. The transmission time can be ignored.

Example 3.47

- What are the propagation time and the transmission time for a 5-Mbyte message (an image) if the bandwidth of the network is 1 Mbps? Assume that the distance between the sender and the receiver is 12,000 km and that light travels at 2.4×10^8 m/s.

Solution

- We can calculate the propagation and transmission times as shown on the next slide.

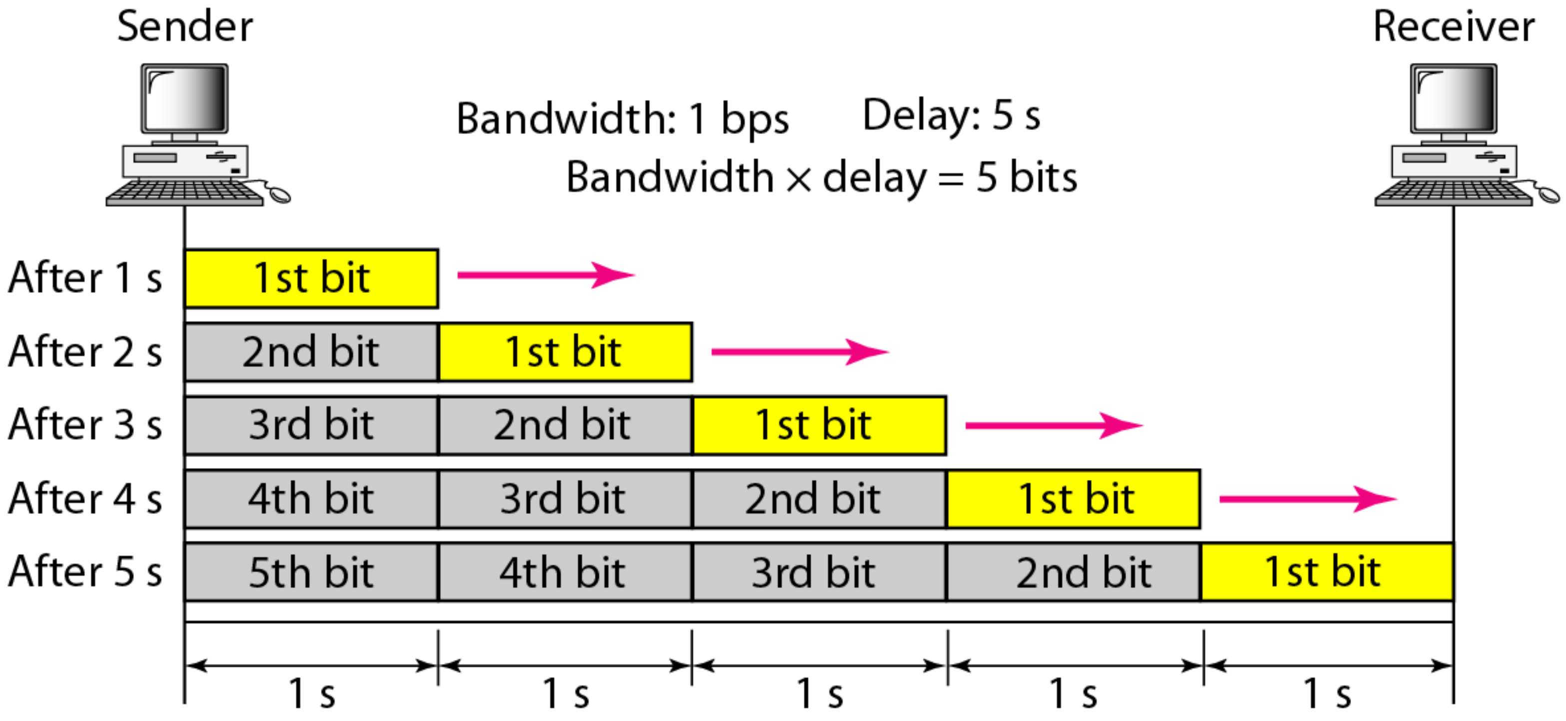
Example 3.47 (continued)

$$\text{Propagation time} = \frac{12{,}000 \times 1000}{2.4 \times 10^8} = 50 \text{ ms}$$

$$\text{Transmission time} = \frac{5{,}000{,}000 \times 8}{10^6} = 40 \text{ s}$$

- Note that in this case, because the message is very long and the bandwidth is not very high, the dominant factor is the transmission time, not the propagation time. The propagation time can be ignored.

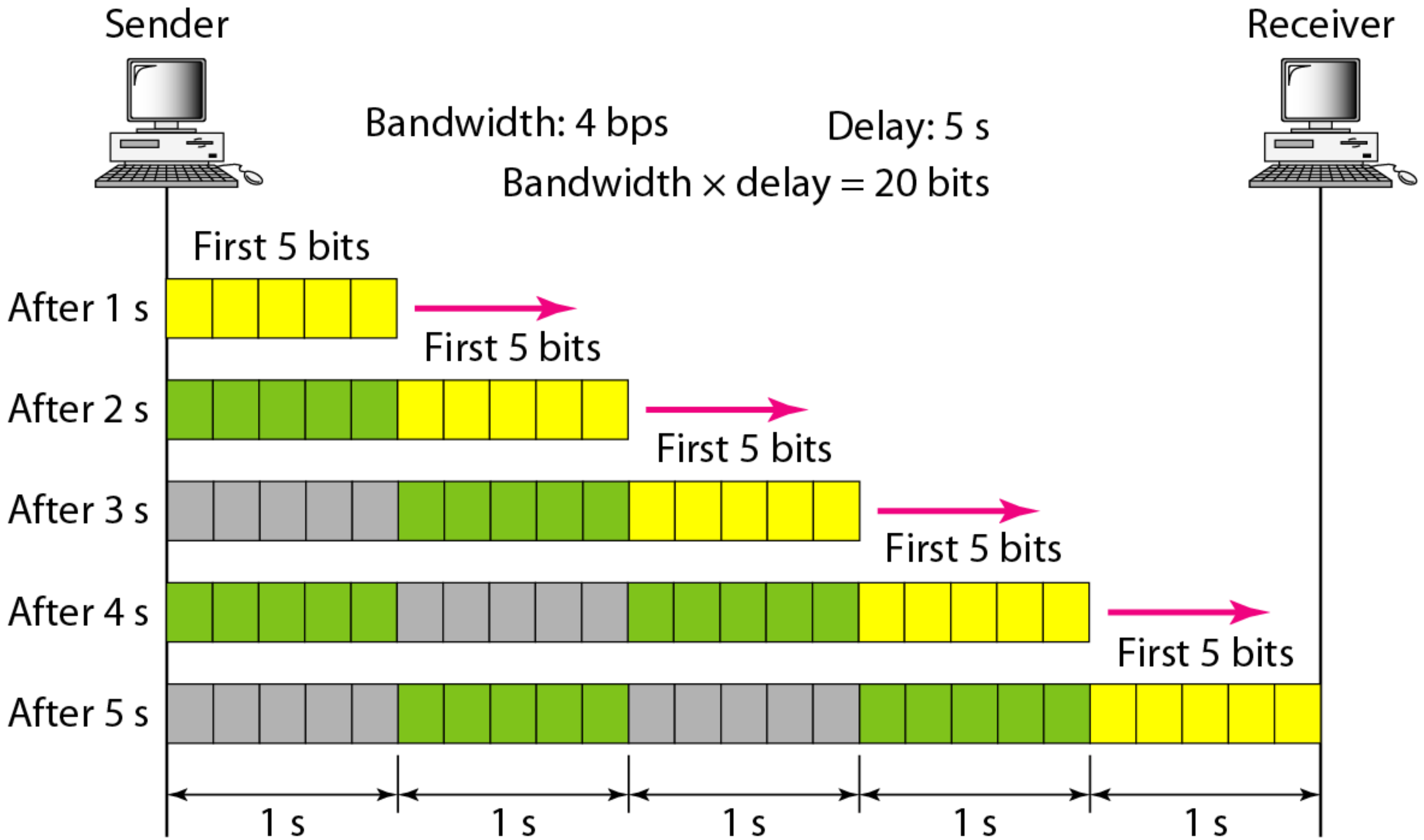
Figure 3.31 Filling the link with bits for case 1



Example 3.48

- We can think about the link between two points as a pipe. The cross section of the pipe represents the bandwidth, and the length of the pipe represents the delay. We can say the volume of the pipe defines the bandwidth-delay product, as shown in Figure 3.33.

Figure 3.32 Filling the link with bits in case 2



NOTE

- The bandwidth-delay product defines the number of bits that can fill the link.

Figure 3.33 Concept of bandwidth-delay product