Mathematics 227
Gaussian elimination

1. Write the augmented matrix for the following system of equations and perform Gaussian elimination to describe the solution space of the system of equations in as much detail as you can.

$$\begin{aligned} x + 2y - z &= 1 \\ 3x + 2y + 2z &= 7 \\ -x \quad\quad + 4z &= -3 \end{aligned}$$

Suppose that you have a system of linear equations in the unknowns x and y whose augmented matrix is row equivalent to

$$\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right].$$

Write the system of linear equations corresponding to the augmented matrix. Then describe the solution set of the system of equations in as much detail as you can.

Suppose that you have a system of linear equations in the unknowns x and y whose augmented matrix is row equivalent to

$$\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right].$$

Write the system of linear equations corresponding to the augmented matrix. Then describe the solution set of the system of equations in as much detail as you can.

Suppose that the augmented matrix of a system of linear equations has the following shape where $*$ could be any real number.

$$\left[\begin{array}{ccccc|c} * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \end{array}\right].$$

How many equations are there in this system and how many unknowns?

Based on our first class meeting, do you think it's possible that this system has exactly one solution, infinitely many solutions, or no solutions?

2. For each of the following linear systems, form an associated augmented matrix and determine whether there is exactly one solution, infinitely many, or none.

$$x + y + 2z = 1$$
$$2x - y - 2z = 2$$
$$-x + y + z = 0$$

$$-x - 2y + 2z = -1$$
$$2x + 4y - z = 5$$
$$x + 2y = 3$$

$$\begin{aligned} -x - 2y + 2z &= -1 \\ 2x + 4y - z &= 5 \\ x + 2y &= 2 \end{aligned}$$