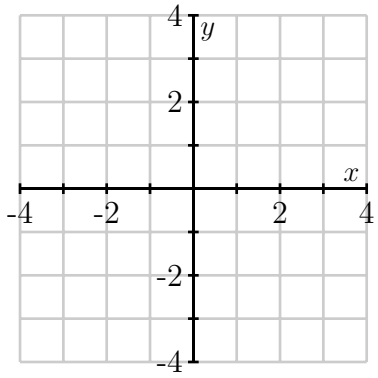
Mathematics 227
Solutions to linear systems

In this first exploration, we aim to develop some intuition for the type of behavior we can expect to see when looking at solutions of systems of linear equations.

1. We will first look at linear equations in two variables.

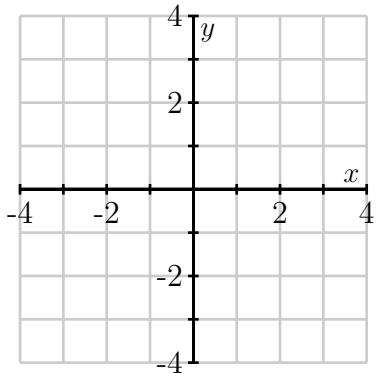
   - On the plot below, graph the line $y = x + 1$. How many points satisfy this equation?



   - On the plot below, graph the lines

$$y = x + 1$$
$$y = 2x - 1$$

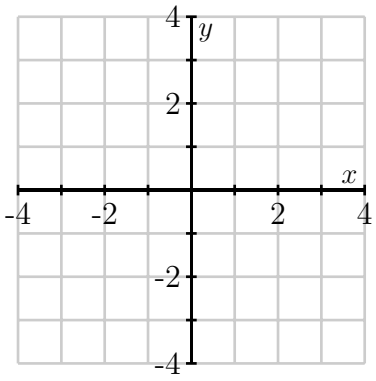
     How many points satisfy both of these equations?



   - On the plot below, graph the lines

$$y = x + 1$$
$$y = 2x - 1$$
$$y = -x$$

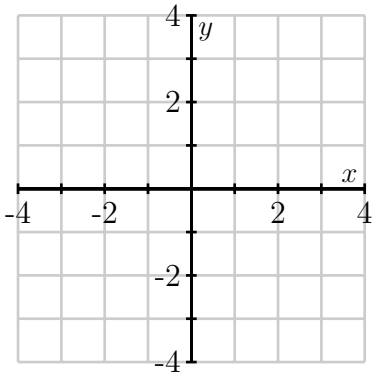
     How many points satisfy all of these equations?



- On the plot below, graph the lines

$$y = x + 1$$
$$y = x - 1$$

How many points satisfy both of these equations?



- Based on what we've seen here, what are the possibilities for the solutions of a system of linear equations in two variables?

- What happens to the size of the solution sets as we add more equations?

2. The solution to a linear equation in three variables x, y, and z is a plane. Use 3×5 cards to study the solutions to systems of linear equations in three variables.

   - Is it possible that there are no solutions to a system of two equations in three variables? If so, explain how or sketch an example.

   - Is it possible that the solutions to a system of two equations in three variables is a single point? If so, explain how or sketch an example.

   - If you are studying a system of two equations in three variables, what would you usually expect for the solution set?

   - If you are studying a system of three equations in three variables, what would you usually expect for the solution set?

   - If you are studying a system of four equations in three variables, what would you usually expect for the solution set?

- Is it possible that four equations in three variables form a line? If how, explain how or sketch an example.

- Is it possible that four equations in three variables form a plane? If how, explain how or sketch an example.

- Based on what we've seen here, what are the possibilities for the solutions of a system of linear equations in three variables?

- What happens to the size of the solution sets as we add more equations?