CE420 Computational Methods of Structural Analysis

Midterm Project

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Introduction

The structural stiffness method is a systematic analysis of both determinate and indeterminate trusses. The structural stiffness method relates external loads as a function of joint displacements. In this report, a package of MATLAB files will be created to analyze various planar trusses. The report will include the results for these analyses, and the MATLAB files will be appended.

Procedure

In this paper, the procedure for structural analysis of trusses will be:

1. Write a MATLAB code that uses the structural stiffness method to solve trusses for member forces, reaction forces, nodal displacements, and member stresses.
2. Choose a truss, and illustrate an analytical model with numbering systems for joints, members, and degrees of freedom.
3. Create four text files based on the analytical model so MATLAB can read load cases, joint coordinates and their associated degrees of freedom, and member numbers. Each member in the 'members' file is represented by the format [member, beginning joint, end joint, Modulus of Elasticity, Cross-sectional Area]. The 'joints' file has the format [joint, x coordinate, y coordinate, x-DOF, y-DOF]. Supports are represented only by their corresponding DOF (i.e. this truss' supports are 14, 15, and 16), and loads are represented as [joint, x-load, y-load.
4. Load files into MATLAB to generate truss analysis.

Example Problem

To demonstrate the program, an arbitrary 13-member truss and loading case has been conceived. The diagram and loading case of the truss can be seen below in figure 1.

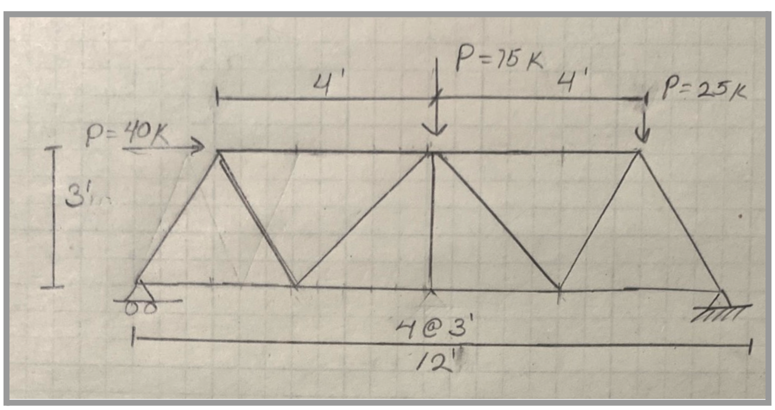

Figure 1:
Truss 1

In order to convert this truss into a format readable by MATLAB, we must first create a numbering system for the members, joints, and degrees of freedom. Together, these numbering systems create our analytical model, seen below.

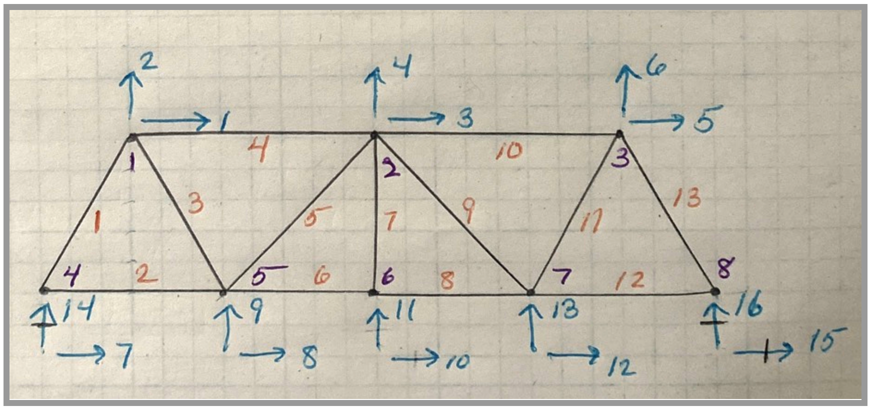

Figure 2:
Truss 1 Analytical Model

Using this model, four files are created to be read by MATLAB. The files follow the format outlined in the ‘procedure’ section of this document. The MATLAB code is then able to output the following values:

1	0.0015
2	-0.0030
3	-7.3668e-04
4	-0.0093
5	-0.0025
6	-0.0033
7	-0.0027
8	-0.0024
9	-0.0058
10	-0.0012
11	-0.0093
12	1.0991e-04
13	-0.0058

Table 1:
Nodal Displacement (ft)

1	-34.2398	-6.8480
2	15.3125	3.0625
3	34.2398	6.8480
4	-70.6250	-14.1250
5	-43.3103	-8.6621
6	61.2500	12.2500
7	0	0
8	61.2500	12.2500
9	-62.7557	-12.5511
10	-56.8750	-11.3750
11	49.6128	9.9226
12	-5.3125	-1.0625
13	-77.5636	-15.5127

Table 2:
Truss Member Force (kip) and Stress (ksi)

14	30.6250
15	-40
16	69.3750

Table 3:
Truss Support Reactions (kip)

These values will be checked by hand-calculations for the purpose of redundancy and surety that the code is working.

Howe Truss

Below is an example of the widely used Howe truss with a hypothetical loading case.

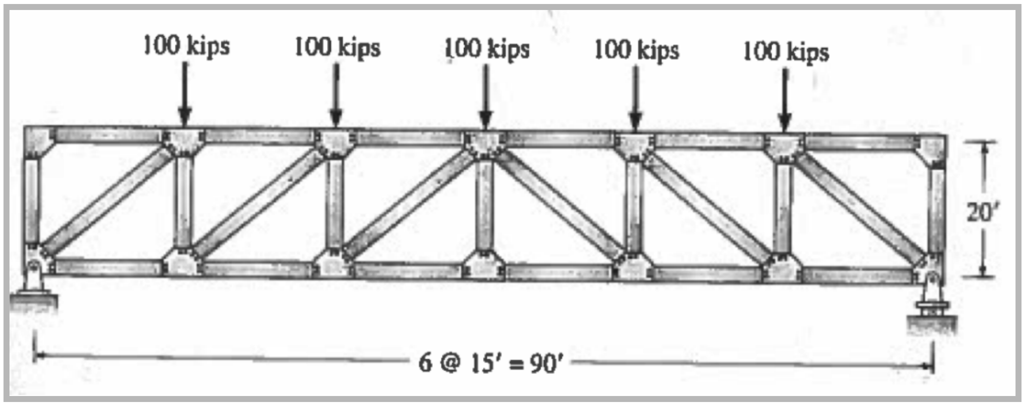

Figure 3: Howe Truss

Just like in figure 2 for the previous truss, an analytical model for the Pratt truss must be created so we can convert it to a MATLAB-readable format. The Howe truss analytical model is shown and labeled below.

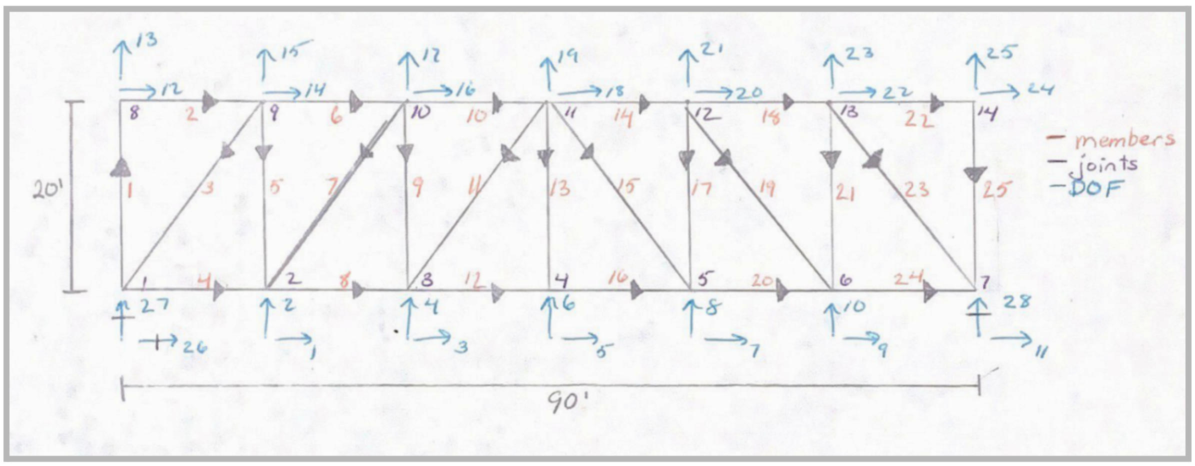

Figure 4: Howe Truss Analytical Model

With all of the heavy lifting done by the MATLAB program, we can quickly calculate all nodal displacements, member axial forces and stresses, and support reaction forces. Below are the tables produced by MATLAB that display these values.

1	0.0194
2	-0.1899
3	0.0504
4	-0.3099
5	0.0853
6	-0.3496
7	0.1203
8	-0.3099
9	0.1513
10	-0.1899
11	0.1707
12	0.1358
13	0
14	0.1358
15	-0.1692
16	0.1164
17	-0.3030
18	0.0853
19	-0.3496
20	0.0543
21	-0.3030
22	0.0349
23	-0.1692
24	0.0349
25	0

Table 4
Howe Truss Nodal Displacement (ft)

1	0	0
2	0	0
3	-312.5000	-62.5000
4	187.5000	37.5000
5	150.0000	30.0000
6	-187.5000	-37.5000
7	-187.5000	-37.5000
8	300.0000	60.0000
9	50.0000	10.0000
10	-300.0000	-60.0000
11	-62.5000	-12.5000
12	337.5000	67.5000
13	0	0
14	-300.0000	-60.0000
15	-62.5000	-12.5000
16	337.5000	67.5000
17	50	10
18	-187.5000	-37.5000
19	-187.5000	-37.5000
20	300	60
21	150.0000	30.0000
22	0	0
23	-312.5000	-62.5000
24	187.5000	37.5000
25	0	0

Table 5
Howe Truss Member Force (kip) and Stress (ksi)

26	0
27	250.0000
28	250.0000

Table 6
Howe Truss Reaction Force (kip)

In table 4, the first column refers to each degree of freedom, labeled in blue in figure 4. The values in the second column represent the displacement, in feet, of nodes at each degree of freedom and in the direction of that degree of freedom.

In Table 5, the first column represents the numbering system for members shown in figure 4. The next two columns represent axial force in kips, and stress in kips/ft^2, respectively.

In Table 6, the first column refers to restrained degrees of freedom found in the analytical model. The second column are reaction forces, in kips, created by the truss' supports.

We can begin to apply checks to the code's results to determine if they seem reasonable. First of all, we can see by inspection that the vertical reactions at joints 1 and 7 should both be 250 kips to resist the 500 kips of symmetrical downward force. This is exactly what the code produced, which is a good verification. It is very convenient that the truss and the hypothetical load case are symmetrical. The truss and load case are symmetrical about member 13. We can see that many members that correspond about the axis of symmetry have the same axial force, i.e. 1 and 25, 2 and 22, 8 and 20, and so on. Again, the results shown in the tables concur with what we know to be true about statics.

Pratt Truss

Below is an example loading case of the very similar Pratt truss.

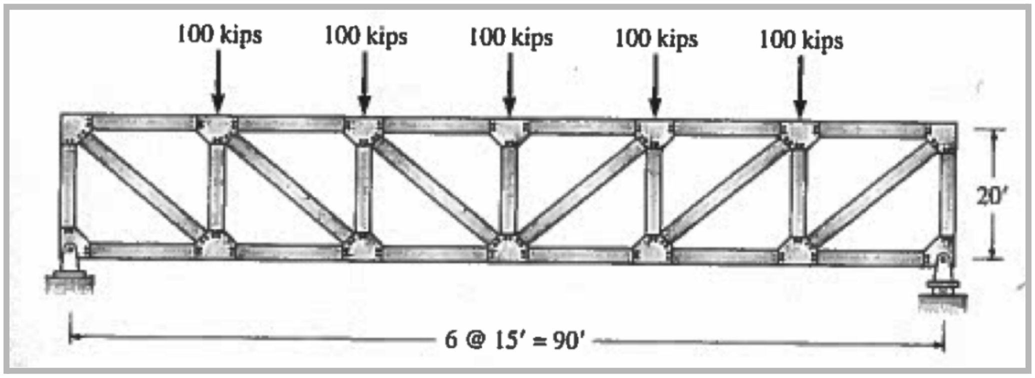

Figure 5: Pratt Truss

Below is the chosen analytical model for the Pratt truss. The numbering system is intentionally similar to the Howe truss model. This is so the comparison between the two results will be easier to read together. It was also convenient that the two trusses share the same files for joints, loads, and supports.

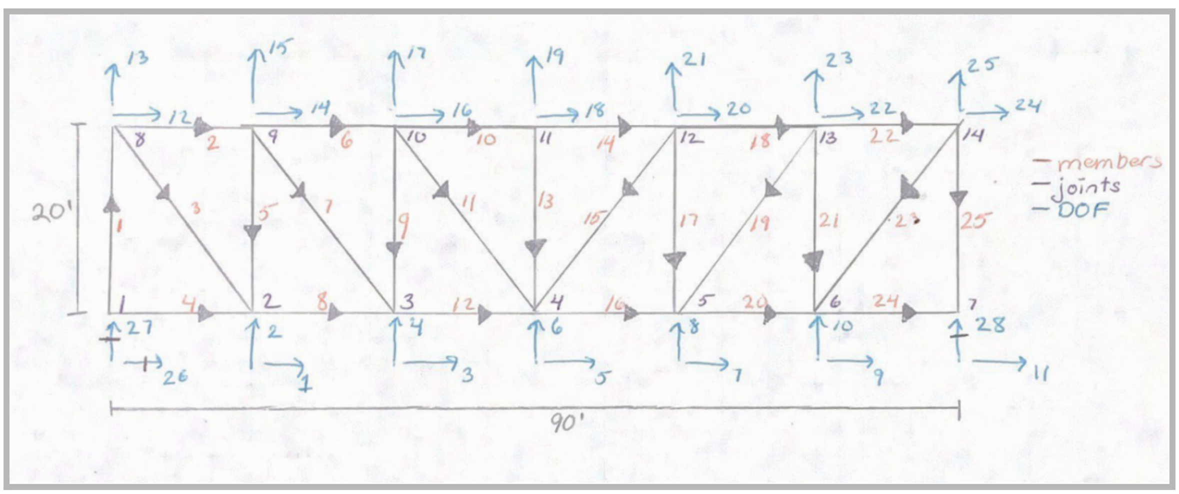

Figure 6: Pratt Truss Analytical Model

1	0
2	-0.2037
3	0.0194
4	-0.3513
5	0.0504
6	-0.4116
7	0.0815
8	-0.3513
9	0.1009
10	-0.2037
11	0.1009
12	0.1358
13	-0.0345
14	0.1164
15	-0.2381
16	0.0853
17	-0.3720
18	0.0504
19	-0.4254
20	0.0155
21	-0.3720
22	-0.0155
23	-0.2381
24	-0.0349
25	-0.0345

Table 7
Pratt Truss Nodal Displacements (ft)

1	-250.0000	-50.0000
2	-187.5000	-37.5000
3	312.5000	62.5000
4	0	0
5	-250.0000	-50.0000
6	-300.0000	-60.0000
7	187.5000	37.5000
8	187.5000	37.5000
9	-150.0000	-30.0000
10	-337.5000	-67.5000
11	62.5000	12.5000
12	300.0000	60.0000
13	-100.0000	-20.0000
14	-337.5000	-67.5000
15	62.5000	12.5000
16	300.0000	60.0000
17	-150.0000	-30.0000
18	-300.0000	-60.0000
19	187.5000	37.5000
20	187.5000	37.5000
21	-250.0000	-50.0000
22	-187.5000	-37.5000
23	312.5000	62.5000
24	0	0
25	-250.0000	-50.0000

Table 8
Pratt Truss Member Force (kip) and Stress (ksi)

26	0
27	250
28	250

Table 9
Pratt Truss Reaction Force (kip)

In these results, the vertical reaction forces are once again 250 kips each, exactly as expected. We also see some symmetry once again in the member forces and stresses. i.e. members 1 and 25, 2 and 22, 11 and 15, etc.

One difference between the Howe and Pratt trusses is that the maximum axial force in the Howe truss is a tension force on members 12 and 16, but for the Pratt truss the maximum axial force is a compressive force on members 10 and 14. The magnitudes of these forces and corresponding stresses are equal. Steel is stronger in tension than in compression, so it is better to have the greatest force be a tensile force rather than a compressive one. Thus, for this loading case, the Howe truss is safer than the Pratt truss. Further analysis may be required to determine if one is objectively better than the other, or if both have their own place in design.

I35W Bridge

Finally, we will move to a real-world example and analyze a slightly simplified 2D model of the I-35W Mississippi River Bridge which famously collapsed in 2007.

Figure 7
I-35W Bridge

Below is the chosen analytical model for the 109-member structure. The symmetrical bridge is shown as the sum of the left side and the right side in order to fit on the same drawing page.

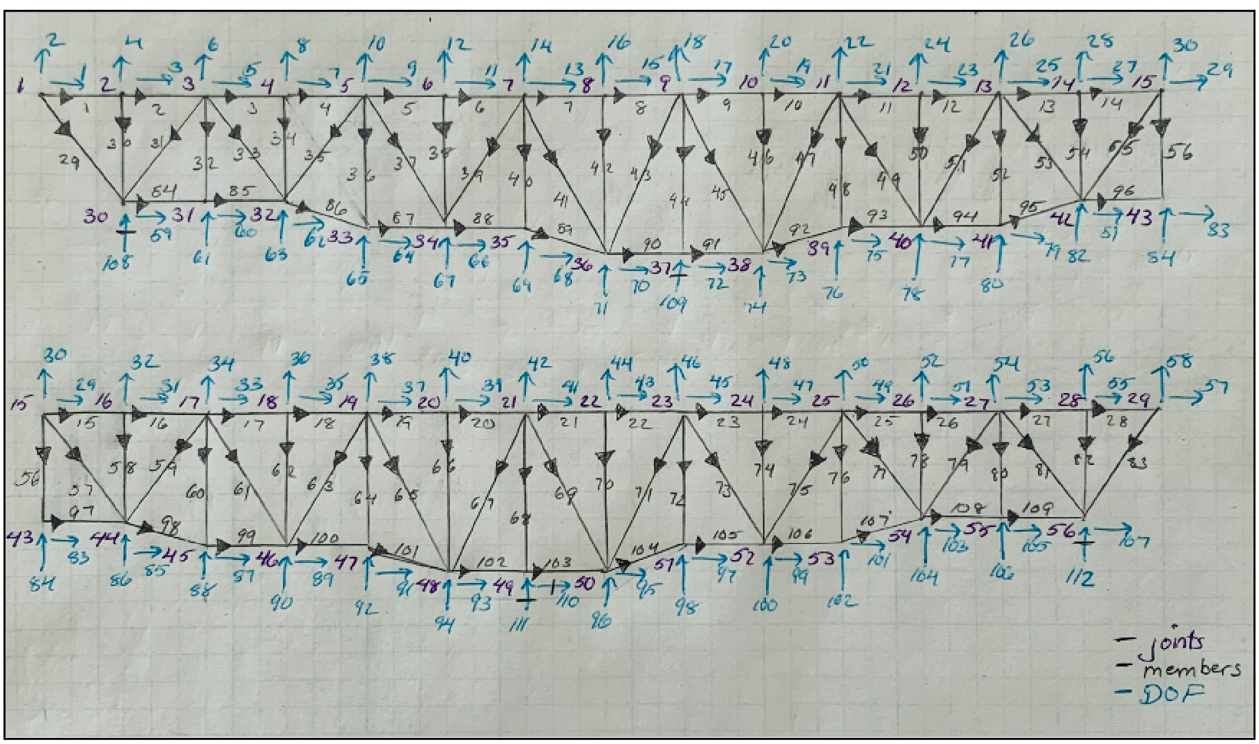

Figure 8

I-35W Bridge Analytical Model

Just as before, this model will be converted into four text files to be loaded into the MATLAB code. Below are the results.

1	461.9211	31	-641.3947	61	-742.1102		
2	461.9211	32	-3.6380e-12	62	-268.9000		
3	-199.6147	33	228.1408	63	1.1137e+03		
4	-199.6147	34	-267.0000	64	265.8414		
5	433.7580	35	275.9434	65	-1.6728e+03		
6	433.7580	36	-60.1485	66	-271.1000		
7	1.6460e+03	37	-612.7746	67	1.4071e+03		
8	1.6460e+03	38	-267.5000	68	372.7833		
9	1.3591e+03	39	982.8431	69	2.2705e+03		
10	1.3591e+03	40	-254.9765	70	272.5000		
11	-752.9998	41	-903.4792	71	403.0770		
12	-752.9998	42	-272.5000	72	-367.5999	91	-1.9944e+03
13	-2.3488e+03	43	1.0500e+03	73	579.7148	92	6.3665e-12
14	-2.3488e+03	44	-2.4003e+03	74	-267.5000	93	-322.0323
15	-2.6810e+03	45	1.5218e+03	75	-209.6464	94	1.5654e+03
16	-2.6810e+03	46	-271.1000	76	-7.7307e-12	95	1.5734e+03
17	-1.6536e+03	47	-1.7414e+03	77	-238.5860	96	2.6850e+03
18	-1.6536e+03	48	-7.2760e-12	78	-267.0000	97	2.6850e+03
19	111.4872	49	1.5573e+03	79	571.0534	98	2.1768e+03
20	111.4872	50	-268.9000	80	26.6795	99	2.1658e+03
21	-390.0706	51	-1.1770e+03	81	-1.0904e+03	100	884.8185
22	-390.0706	52	-157.9044	82	-264.8000	101	923.8915
23	-552.8408	53	1.0791e+03	83	-930.2886	102	-974.4188
24	-552.8408	54	-265.8000	84	348.0179	103	-2.9554e+03
25	-532.8430	55	-463.0201	85	348.0179	104	397.8809
26	-532.8430	56	-9.0949e-12	86	317.4014	105	152.2484
27	691.3253	57	-5.4677	87	311.6501	106	697.7098
28	691.3253	58	-265.8000	88	-790.9475	107	697.7098
29	-624.2492	59	709.6034	89	-831.0301	108	140.7869
30	-264.8000	60	-218.4655	90	-1.9861e+03	109	138.2359

Table 10
I-35W Axial Member Forces (kip)

1	-0.2596	31	-0.2445	61	-0.0558		
2	-0.0179	32	-0.8840	62	-0.2487		
3	-0.2493	33	-0.2693	63	-0.0898		
4	-0.0070	34	-0.7975	64	-0.2388		
5	-0.2383	35	-0.2978	65	-0.0696		
6	-0.0558	36	-0.6822	66	-0.2332		
7	-0.2417	37	-0.3263	67	-0.0371		
8	-0.0968	38	-0.4807	68	-0.2443		
9	-0.2451	39	-0.3249	69	0.0148		
10	-0.0733	40	-0.2511	70	-0.2642		
11	-0.2348	41	-0.3234	71	-0.0072		
12	-0.0453	42	0.0040	72	-0.2778		
13	-0.2245	43	-0.3280	73	-0.2635	90	-0.6735
14	-7.4358e-04	44	-0.0134	74	-0.1283	91	-0.1431
15	-0.2050	45	-0.3326	75	-0.2280	92	-0.4970
16	-0.0169	46	-0.1127	76	-0.2465	93	-0.0482
17	-0.1856	47	-0.3458	77	-0.3336	94	-0.2417
18	-0.0256	48	-0.1793	78	-0.4679	95	-0.1973
19	-0.1679	49	-0.3590	79	-0.3132	96	-0.0038
20	-0.1378	50	-0.1864	80	-0.6385	97	-0.3682
21	-0.1502	51	-0.3681	81	-0.2759	98	-0.0903
22	-0.2465	52	-0.1601	82	-0.8025	99	-0.3661
23	-0.1632	53	-0.3773	83	-0.2506	100	-0.1711
24	-0.4766	54	-0.0665	84	-0.8706	101	-0.3535
25	-0.1761	55	-0.3648	85	-0.2253	102	-0.1864
26	-0.6481	56	-0.0070	86	-0.8763	103	-0.3409
27	-0.1979	57	-0.3530	87	-0.1873	104	-0.1515
28	-0.8102	58	-0.0243	88	-0.7842	105	-0.3541
29	-0.2196	59	-0.2662	89	-0.1591	106	-0.0679
30	-0.8706	60	-0.2577	90	-0.6735	107	-0.3514

Table 11
I-35W Nodal Displacements (ft) by DOF

1	1.0947e+03
2	3.2570e+03
3	-2.6930e-11
4	2.6446e+03
5	1.5950e+03

Table 12
I-35W Reaction Forces (kip)

Conclusion

Computer programs are an invaluable tool for all types of professionals, including structural engineers. The availability of such technology has steered the way that we work. In this report, a computer program was written in MATLAB which functions to analyze both determinate and indeterminate planar trusses of any size using a method that was conceived specifically with computer aid in mind. The program successfully provided all member forces, nodal displacements, and reaction forces for four different trusses, including the infamous I-35W bridge. Several checks were used to ensure that the results were correct.

While it is incredibly convenient to have these programs at our fingertips, there is no replacement for a skilled operator. It is important for engineers to fully understand what is going on behind the scenes and have the knowledge to understand and interpret computer outputs. An engineer's job is to