QUiz 1

Graded

Student

Manshi Sagar

Total Points

6 / 15 pts

Question 1

Q1

4 / 4 pts

✔ **– 0 pts** Correct

– 2 pts Example using overflow

– 3 pts Informal proof: No concrete example given

– 4 pts Incorrect: $\frac{a+b}{2}$ lies in range $[a, b]$

– 4 pts Incorrect

Question 2

Q2

Resolved **2** / 4 pts

+ 0 pts Incorrect

+ 4 pts Correct U, Σ, V.

+ 2 pts One (u, σ, v) combination correct.

✔ **+ 1 pt** Correct σ_1

✔ **+ 1 pt** Correct σ_2

+ 0.5 pts Correct u_1

+ 0.5 pts Correct u_2

+ 0.5 pts Correct v_1

+ 0.5 pts Correct v_2

Regrade Request **Submitted on: Mar 03**

I was following the conventional approach for calculating SVD by calculating the eigen vectors of AAT and ATA, there is small calculation mistake. Please go through the solution and give some partial marks.

Regraded.

Reviewed on: Mar 11

Question 3

Q3(i)

0 / 2 pts

– 0 pts Correct

✔ **– 2 pts** Incorrect

Question 4

Q3(ii)

0 / 5 pts

– 0 pts Correct

✔ **– 5 pts** Incorrect

25/01/24

Quiz 1 (COL 726)

Name Mansi Sagar Entry No. 2020CS50429

1. Let $a < b$ be two floating point numbers (i.e., they can be represented in the finite precision number system). Give an example to show that computed value $(a+b)/2$ may not lie in the range $[a, b]$. This computation is done by (first adding a and b) and then (dividing by 2.) You can assume that the extra precision digits are chopped during computation (Use decimal numbers for your example). **(4 marks)**

let $= a = 9.98 = fl(a)$ let precision be 3 digits

$b = 9.99 = fl(a)$

$a+b = 19.97$ $fl(a+b) = 19.9 = 1.99 \times 10^1$

$\frac{fl(a+b)}{2} = \frac{19.9}{2} = 9.95$

$fl\left(\frac{fl(a+b)}{2}\right) = 9.95$ does not lie in the range $[9.98, 9.99]$

2. Write an SVD of the matrix $A = \begin{bmatrix} 1 & 0 \\ 0 & -2 \\ 0 & 3 \end{bmatrix}$ (i.e., specify the matrices U, V, Σ in a singular value decomposition). **(4 marks)**

~~Extending A to make it square matrix~~ $\cancel{A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 3 & 0 \end{bmatrix}}$

$AA^T = \begin{bmatrix} 1 & 0 \\ 0 & -2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & +4 & -6 \\ 0 & -6 & 9 \end{bmatrix}$ 2 colums are Linearly dep.

$\det(AA^T - \lambda I) = 0$ $\det\begin{bmatrix} (1-\lambda) & 0 & 0 \\ & (+4-\lambda) & -6 \\ 0 & -6 & 9-\lambda \end{bmatrix} = 0 = (1-\lambda)\left((4-\lambda)(9-\lambda)-36\right)$ $\cancel{\lambda=1}$

$\lambda - 1 = 0 \Rightarrow \boxed{\lambda = 1}$

$\cancel{\lambda^2 - 2(4)(9)}$ $(\lambda-4)(\lambda-9) - 36 = 0$ $\boxed{\lambda = 0}$ $\boxed{\lambda = 13}$

$\lambda^2 \cancel{+} -13\lambda + \cancel{36}^{-36} = 0$

$\Sigma = \begin{bmatrix} \sqrt{13} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\text{Rank}(A) = \text{Rank}(\Sigma) = 2$

to get V $\rightarrow$ $(AA^T - \lambda I)(v) = 0$

to get U $\rightarrow$ $(A^T A - \lambda I)(u) = 0$

or $\cancel{U^T A V}$

3. Let A be an $m \times n$ matrix with singular values $\sigma_1 \geq \sigma_2 \geq \sigma_3 \geq \cdots$. Let q_1 and q_2 be two orthonormal vectors in $\Re^n$. Show that :

let ~~Σ~~ let $\Sigma = \begin{bmatrix} \sigma_1 & & \\ & \sigma_2 & \\ & & \ddots \sigma_n \end{bmatrix}$

- For any unit vector $x \in \Re^n$, $\langle q_1, x\rangle^2 + \langle q_2, x\rangle^2 \leq 1$ **(2 marks)**

- $|Aq_1|_2^2 + |Aq_2|_2^2 \leq \sigma_1^2 + \sigma_2^2$. **(5 marks)**

q_1 and q_2 are orthonormal

$\Rightarrow q_1 \cdot q_2 = 0$ (dot product)

$q_1 \cdot q_1 = 1$, "

$q_2 \cdot q_2 = 1$ "

$$|Aq_1|_2^2 = (Aq_1)(Aq_1)^T = A(q_1 q_1^T)A^T = AA^T = \|A\|_2^2$$

similarly

$$|Aq_2|^2 = \|A\|_2^2$$

$$|Aq_1|^2 + |Aq_2|^2 = \cancel{Aq} \; 2|A|_2^2 = \cancel{2 \times \sigma_1^2} = 2|\Sigma|_2^2 = 2\sigma_1^2$$

$$\cancel{Aq_1 = \begin{bmatrix} \sigma_1 q_{11} & & \\ & \sigma_2 q_{12} & \\ & \ddots & \sigma_n q_{1n} \end{bmatrix}} \quad \cancel{Aq_2 = \begin{bmatrix} \sigma_1 q_{21} & 0 & 0 & \cdots \\ & \sigma_2 q_{22} & & \\ & & \ddots & \sigma_n q_{2n} \end{bmatrix}}$$

$$\Rightarrow |Aq_1|^2 + |Aq_2|^2 = 2\sigma_1^2 \geq \sigma_1^2 + \sigma_2^2$$

as $\sigma_1 \geq \sigma_2$