Quiz 3

Graded

Student

Manshi Sagar

Total Points

6 / 15 pts

Question 1

Q1

3 / 6 pts

+ 6 pts Correct

✔ **+ 3 pts** First part correct

+ 3 pts Second part correct.

+ 2 pts In the second part, checked the derivatives at end points only. That is not sufficient. One needs to prove it for all points in the interval [1,2]

+ 1 pt Only computed derivative for second part.

− 0.5 pts Argued that derivative is less than 1. Ideally one needs to argue that it is some constant less than 1.

+ 0 pts Incorrect.

− 1 pt Did not prove that the derivative i s less than 1, but just mentioned this fact.

− 1.5 pts Used approximate Taylor series.

1 not a proof.

Question 2

Q2

3 / 9 pts

✔ **+ 1.5 pts** Correct Gradient

+ 1 pt Minor errors in gradient calculation.

✔ **+ 1.5 pts** Correct Hessian

− 0.5 pts Minor errors in Hessian

+ 0.5 pts Major errors in Hessian calculation.

+ 2 pts Showed that Hessian is psd

+ 1 pt Showed psd by using diagonal dominance, but no proof of this given.

+ 4 pts correct minimization

+ 2 pts Wrote the condition that gradient is 0 and correct evaluation of gradient. But then the conclusion about minimum is not correct.

+ 3 pts Correct condition on gradient for minimum and partial correct interpretation of minimum (e.g., no non-negativity condition on a_i).

+ 0 pts Incorrect.

Quiz 3 (COL 726)

Name MANSHI SAGAR Entry No. 2020CS50429

increasing function

1. Consider the fixed point iteration $x_{n+1} = (3x_n + 1)^{1/3}$. Show that it has a fixed point in the region $[1, 2]$ (**3 marks**). Show that for any starting value $x_0 \in [1, 2]$, the fixed point iteration converges (**3 marks**).

$f(x) = (3x+1)^{1/3}$

$\cancel{\boxed{x_{n+1} = f(x_n)}}$

$\cancel{y' = \frac{1}{3}(3x+1)^{-2/3} \times 3 - 1}$ $\cancel{= (3x+1)^{-2/3} - 1}$ $\cancel{= \frac{1}{(3x+1)^{2/3}}}$

$\cancel{y'' = -\frac{2}{3}(3x+1)^{-5/3} \times 3}$ $\cancel{= (-2)(3x+1)^{-5/3}}$

let $y = f(x) - x$ has root in $[1,2]$

at $x = 1$

$y = f(x) - x = (3(1)+1)^{1/3} - 1 = 4^{1/3} - 1 \geq 0$

at $x = 2$

$y = f(x) = (3(2)+1)^{1/3} - 2 = 7^{1/3} - 2 \leq 0$

by mean value theorem $\longrightarrow$ $y = f(x) - x$ has a root in $[1,2]$

$\Rightarrow$ $f(x) = x$ has fixed point in $[1,2]$

2. Consider the function $f(x_1, \ldots, x_n) = \ln(e^{x_1} + \ldots + e^{x_n})$. Write down the gradient and the Hessian of this function (No explanations needed) (**3 marks**). Using the Hessian show that the function is convex (**2 marks**). Let $a_1, \ldots, a_n$ be real values. What is the minimum value of $f(x_1, \ldots, x_n) - \sum_i a_i x_i$? (**4 marks**)

gradient = $J_f(x)$ = $\begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \vdots \\ \frac{\partial f}{\partial x_n} \end{bmatrix}$ = $\begin{bmatrix} \left(\frac{1}{e^{x_1} + \ldots e^{x_n}}\right) \times e^{x_1} \\ \vdots \\ \left(\frac{1}{e^{x_1} + \ldots e^{x_n}}\right) \times e^{x_n} \end{bmatrix}$ = $\frac{1}{e^{x_1} + \ldots + e^{x_n}} \begin{bmatrix} e^{x_1} \\ e^{x_2} \\ \vdots \\ e^{x_n} \end{bmatrix}$

$= \begin{bmatrix} \frac{e^{x_1}}{e^{x_1} + c_1} \\ \frac{e^{x_2}}{e^{x_2} + c_2} \\ \vdots \\ \frac{e^{x_n}}{e^{x_n} + c_n} \end{bmatrix}$

~~Hessian = $H_f(x)$ =~~

$c_i = (e^{x_1} + e^{x_2} + \ldots e^{x_n}) - e^{x_i}$

$$\text{Hessian} = H_f(x) = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1 \partial x_1} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \cdots \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \ddots & \\ \vdots & & \frac{\partial f}{\partial x_n \partial x_n} \end{bmatrix}$$

~~da~~ diagonal entries $= \dfrac{c_i \times e^{x_i}}{(e^{x_i}+c_i)^2} = H_{ii}$

non diagonal entries $= H_{ij} = -\dfrac{e^{x_i} \times e^{x_j}}{(e^{x_1}+e^{x_2}+\dots e^{x_n})^2}$

f is convex iff $H_f(x)$ is positive semi definite

$$x^T H x \geq 0$$

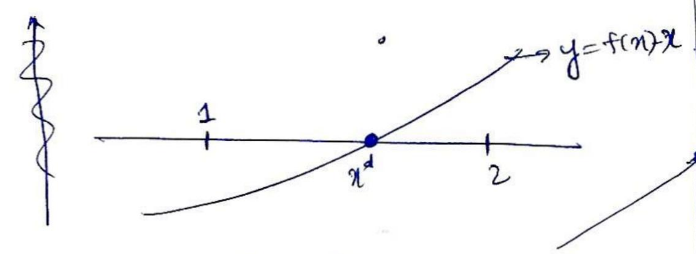
$$[x_1\ x_2\ \dots x_n] \begin{bmatrix} h_1 & h_2 & \cdots & h_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} x^T h_1 & x^T h_2 & \dots & x^T h_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$= \sum_i x^T h_i x_i$$

$$= \left(x_1 \frac{\partial^2 f}{\partial x_1 \partial x_1} + x_2 \frac{\partial^2 f}{\partial x_1 \partial x_2} + x_3 \frac{\partial^2 f}{\partial x_1 \partial x_3} + \cdots \right)$$

$$\left(x_1 \frac{\partial^2 f}{\partial x_1 \partial x_2} + x_2 \frac{\partial^2 f}{\partial x_2 \partial x_2} + x_3 \frac{\partial^2 f}{\partial x_2 \partial x_3} + \cdots \right)$$

Q1



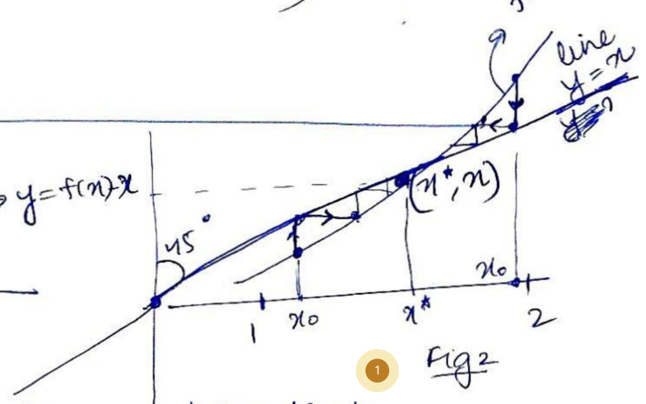


Fig2

Visually, from figure 2, we can see ~~from~~ that if we take $x_0 \in [1,2]$ & use fixed point iteration, we will move towards x^*