(1) $y = ax^2 + bx + c$ $(0, 3)$ $(2, 4)$

$3 = a(0)^2 + b(0) + c$

$c = 3$

$4 = 4a + 2b + 3$

$1 = 4a + 2b$ — (2)

$(1, -1)$

$-1 = a + b + 3$

$-4 = a + b$ — (1)

Solving (1) and (2)

$4a + 2b = 1$

$2a + 2b = -8$

$2a = 9$

$a = 9/2$

$\frac{9}{2} + b = -4$

$b = -\frac{17}{2}$

$y = \frac{9}{2}x^2 - \frac{17}{2}x + 3$

so linear eqn are $a + b = -4$, $4a + 2b = 1$, $c = 3$

The polynomial equation formed is $y = \frac{9}{2}x^2 - \frac{17}{2}x + 3$

(2) If the degree of polynomial is N then we need N+1 points to find equation.
Because if we take example of quadratic equation here, the degree of polynomial is 2 but we need 3 points where 1 of the point will decide whether the parabola is positive curve or negative curve.

(3) $y = ax^2 + bx + c$

- (1, 0)
  $a + b + c = 0$
- (2, 2)
  $4a + 2b + c = 2$
- (3, −6)
  $9a + 3b + c = -6$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 4 & 2 & 1 & 2 \\ 9 & 3 & 1 & -6 \end{array}\right]$$

$R_2 \to R_2 - 4R_1$

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & -2 & -3 & 2 \\ 9 & 3 & 1 & -6 \end{bmatrix}$$

$R_3 \to R_3 - 9R_1$

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & -2 & -3 & 2 \\ 0 & -6 & -8 & -6 \end{bmatrix}$$

$R_2 \to -R_2/2$

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 3/2 & -1 \\ 0 & -6 & -8 & -6 \end{bmatrix}$$

$R_1 \to R_1 - R_2$

$$\begin{bmatrix} 1 & 0 & -1/2 & 1 \\ 0 & 1 & 3/2 & -1 \\ 0 & 0 & 1 & -12 \end{bmatrix}$$

$R_1 \to R_1 + R3/2$

$$\begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 3/2 & -1 \\ 0 & 0 & 1 & -12 \end{bmatrix}$$

$R_2 \to R_2 - 3R3/2$

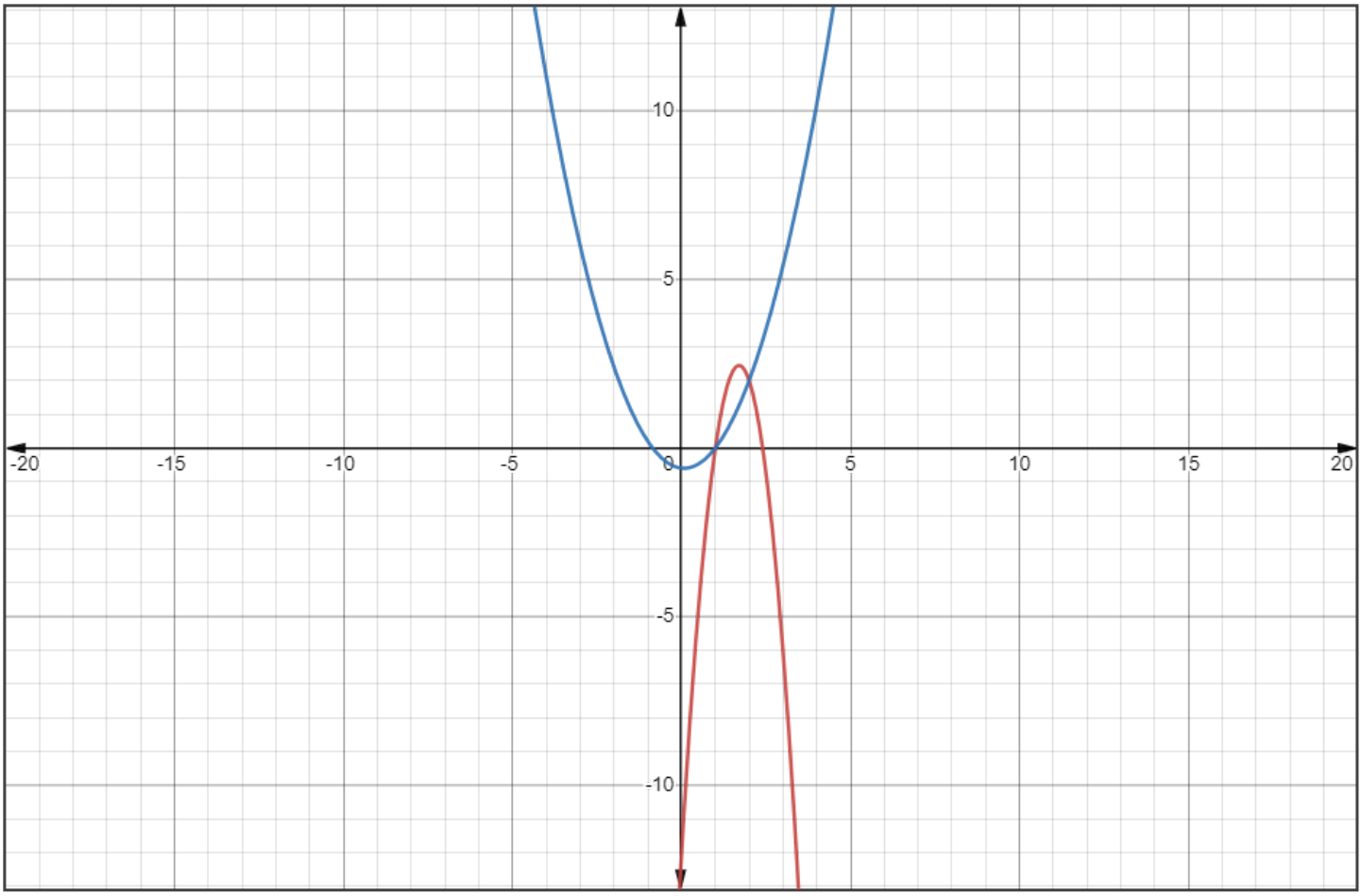
$$\begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & 17 \\ 0 & 0 & 1 & -12 \end{bmatrix}$$

Therefore,

$y = -5x^2 + 17x - 12$

$a = -5$
$b = 17$
$c = -12$

$y = -5x^2 + 17x - 12$



1 $y = -5x^2 + 17x - 12$

2 $y = \frac{7}{10}x^2 - \frac{x}{10} - \frac{3}{5}$

(b) $(-3,6)$ $(1,0)$ $(2,2)$

$y = ax^2 + bx + c$

$6 = 9a - 3b + c$

$0 = a + b + c$

$2 = 4a + 2b + c$

$$\left[\begin{array}{ccc|c} 9 & -3 & 1 & 6 \\ 1 & 1 & 1 & 0 \\ 4 & 2 & 1 & 2 \end{array}\right]$$

$R_2 \to R_2 - \frac{1}{9}R_1$

$$\left[\begin{array}{cccc} 9 & -3 & 1 & 6 \\ 0 & 4/3 & 8/9 & -2/3 \\ 4 & 2 & 1 & 2 \end{array}\right] \quad R_3 \to R_3 - \frac{4}{9}R_1 \quad \left[\begin{array}{cccc} 9 & -3 & 1 & 6 \\ 0 & 4/3 & 8/9 & -2/3 \\ 0 & 10/3 & 5/9 & -2/9 \end{array}\right]$$

$R_2 \leftrightarrow R_3$

$$\left[\begin{array}{cccc} 9 & -3 & 1 & 6 \\ 0 & 10/3 & 5/9 & -2/3 \\ 0 & 4/3 & 8/9 & -2/3 \end{array}\right] \to R_3 \to R_3 - 2/5 R_2 \left[\begin{array}{cccc} 9 & -3 & 1 & 6 \\ 0 & 10/3 & 5/9 & -2/3 \\ 0 & 0 & 2/3 & -2/5 \end{array}\right]$$

$\frac{2}{3}c = -\frac{2}{5}$

$c = -\frac{2}{5} \times \frac{3}{2}$

$\boxed{c = -3/5}$

$\frac{10}{3}b + \frac{5}{9}c = -\frac{2}{3}$

$\frac{10}{3}b + \frac{5}{9}\left(-\frac{3}{5}\right) = -\frac{2}{3}$

$\boxed{b = -1/10}$

$9a - 3b + c = 6$

$9a + \frac{3}{10} - \frac{3}{5} = 6$

$\boxed{a = \frac{7}{10}}$

$y = \frac{7}{10}x^2 - \frac{x}{10} - \frac{3}{5}$

(c) The two points are $(2, 2)$ and $(1, 0)$

(d) The polynomial used will be $y = ax^3 + bx^2 + cx + d$

$(1, 0)$ $a + b + c + d = 0$

$(2, 2)$ $8a + 4b + 2c + d = 2$

$(3, -6)$ $27a + 9b + 3c + d = -6$

$(6, -3)$ $-27a + 9b - 3c + d = 6$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 8 & 4 & 2 & 1 & 2 \\ 27 & 9 & 3 & 1 & -6 \\ -27 & 9 & -3 & 1 & 6 \end{array}\right]$$