Assignment 1

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Problem 1.

a) Domain R, Codomain Z, Range Z
b) Domain Z, Codomain N, Range N+3
c) Domain R, Codomain R, Range [-1,1]

Problem 2.

d)

Problem 2

→ $f: Z \times Z \to Z$ $f(m,n) = m+n$

Suppos $b \in Z$

we want to show that there exists an element $(m,n) \in Z \times Z$ where $f(m,n) = b$

$m+n = b$

Let $m = 2b$, $n = -b$

$2b - b = b$

$b \in Z$

therefor $F(m,n) = m+n$ is surjective.

e)

2 → We need to show any $b \in Z$ there is there is an element $a \in Z$ where $f(a) = b$ i.e. $f(m,n) = b$

$m^2 + n^2 = b$

Let $m = 0$ $n = \sqrt{b} \lessdot^{+b}_{-b}$

if $n = +b$

$m^2 + n^2 = b$

but if $n = -b$ we get $0^2 + (\sqrt{-b})^2$ so this shows that for any negative value like $f(-a)$ we donot get result

$\therefore F(m,n) = m^2 + n^2$ is not surjective

3 → We need to show that for any $b \in Z$ there is an element $a \in Z$ where $f(a) = b$ i.e. $f(m,n) = b$

let $n = 0$ & $m = b$ then

$f(m,n) = m$ i.e. $f(b,0) = b$

we always get the result for any value so $F(m,n) = m$ is surjective

4 → Suppose $b \in Z$ we want to show there exists an element $(m,n) \in Z \times Z$ where $f(m,n) = b$

$f(m-n) = b$

Let $m = 2b$ $n = b$

then $f(m-n) = 2b - b = b$

therefore $F(m,n) = m-n$ is surjective

Problem 3

1 → Suppose $(m',n') \in Z \times Z$ then $f(m,n) = f(m',n')$ then $(m,n) = (m',n')$ $\therefore$ $m = m'$ & $n = n'$

$2m'+1 = 2m+1$

$2m' = 2m$

$m' = m$

but we cannot say that $n' = n$ always

$f(m,n) = 2m + 1$ is not injective

[illegible] $(1,2)$ & $f(1,3)$ gives same result but we cannot say that $n' = n$ always

$f(m,n) = 2m + 1$ is not injective

Eg: $f(1,2)$ & $f(1,3)$ gives same result

2 → Suppose $m = -1$ & $n = -1$ $f(m,n)$

$3(-1) - 4(-1) = -3 + 4 = 1$

Suppose $m = 3$ & $n = 2$ $f(m,n) = 3(3) - 4(2) = 1$

so $f(-1,-1)$ and $f(3,2)$ both are mapped to 1 and therefore $f(m,n) = 3n - 4m$ is not injective.

f)

3 → We need to show that $f(m,n) = f(k,l)$ then $(m,n) = (k,l)$

Suppose $(m,n), (k,l) \in Z \times Z$ and $f(m,n) = f(k,l)$

$(2n, n+3) = (2l, l+3)$

$2n = 2l$ $n + 3 = l + 3$

subtracting $n+3 = l+3$ from $2n = 2l$

$$\begin{aligned} 2n &= 2l \\ n+3 &= l+3 \\ \hline n-3 &= l-3 \end{aligned}$$

$\boxed{n = l}$

substituting $n = l$ in $f(m,n)$ do not show that $k = m$

infact $(0,1)$ or $(2,1)$ both gets mapped to $(2,4)$

So $f(m,n) = (2n, n+3)$ is not injective.

g)

4 → We need to show that $g(m,n) = g(k,l)$ then $(m,n) = (k,l)$

Suppose $(m,n), (k,l) \in Z \times Z$ & $g(m,n) = g(k,l)$

$(m+n, 2m+n) = (k+l, 2k+l)$

$m + n = k + l$

$2m + n = 2k + l$

Subtracting $m+n = k+l$ from $2m+n = 2k+l$ gives

$m = k$

then substituting $m = k$ in $m + n = k + l$

$k + n = k + l$

$\boxed{n = l}$ $(m,n) = (k,l)$

$\therefore F(m,n) = (m+n, 2m+n)$ is injective

h)

Problem 4

→ We know from problem 3(d) that $g(m,n) = (m+n, 2m+n)$ is an injective function.

for any $(b,c) \in Z \times Z$ there is (m,n) where $g(m,n) = (b,c)$

i.e. $(m+n, 2m+n) = (b,c)$

$m + n = b$ and $2m + n = c$

Solving gives $m = c - b$, $n = 2b - c$

So $(m,n) = (c-b, 2b-c)$

so $g(c-b, 2b-c) = (b,c)$

Therefore, $g(m,n)$ is surjective

$\therefore g(m,n) = (m+n, 2m+n)$ is bijective as it is both injective and surjective.

i)

Problem 5

→ $\{x \in \mathbb{Z} : x \bmod 2 = 0 \ \& \ |x| < 8\}$

Ans – $\{0, 1, 3, 4, 5, 6, 7\}$

→ $\{x \mid x \in N$, x is prime & x is composite

Ans – $\{1\}$

→ $\{x \in \mathbb{Z} : x = 3^y$ for some $y \in \{k \in N : k < 6\}\}$

$y \in \{1, 2, 3, 4, 5\}$

Ans – $\{3, 9, 27, 81, 243\}$

→ $\{x \in \mathbb{Z} \mid x$ is prime and $2 \mid x\}$

Ans – $\{2\}$

→ $\{x \in \mathbb{Q} : 2x \in \mathbb{Z}$ and $x \in [-2, 1)\}$

Ans – $\{-4, -2, 0\}$

Problem 7:

1. $A \cap (B \cap C) = (A \cap B) \cap C$ is False.

Let $A = \{1, 2, 3\}$ $B = \{4, 5, 6\}$ $C = \{7, 8, 9\}$

$A \cap B = \{\}$ $B \cap C = \{\}$

$A \cap (B \cap C) = \{1, 2, 3\}$ $(A \cap B) \cap C = \{7, 8, 9\}$

we know $A \cap (B \cap C) \neq (A \cap B) \cap C$

so $\boxed{\text{False}}$

2) $A \setminus B \subseteq A$

Let $A = \{1, 2, 3, 4, 5\}$ $B = \{2, 3, 4, 5, 6, 7\}$

$A \setminus B = \{1\}$

$\{1\}$ is a subset of A

so $A \setminus B \subseteq A$ is true.

→ If $A \subseteq B$ & $B \subseteq C$ then $A \subseteq C$ is $\boxed{\text{true}}$

Ans → Let $A = \{1\}$ $B = \{1, 2\}$ $C = \{1, 2, 3\}$

$A \subseteq B$ $B \subseteq C$ we see that $A \subseteq C$

→ TRUE

→ $A \times (B \times C) = (A \times B) \times C$

Let $A = \{1, 2\}$ $B = \{3, 4\}$ $C = \{5\}$

$B \times C = \{(3,5), (4,5)\}$ $A \times B = \{(1,3), (1,4), (2,3), (2,4)\}$

$A \times (B \times C)$

$\{(1,(3,5)), (1,(4,5)), (2,(3,5)), (2,(4,5))\}$

$(A \times B) \times C$

$\{((1,3),5), ((1,4),5), ((2,3),5), ((2,4),5)\}$

$\therefore$ $A \times (B \times C) = (A \times B) \times C$ is not true

Problem 8

→ $\{6, 7, 8\} = \{\phi, \{6\}, \{7\}, \{8\}, \{6,7\}, \{7,8\}, \{8,6\}, \{6,7,8\}\}$.

$P(A) = 2^3 = \boxed{8}$

j) → $\{\phi\}$ $P(A) = \{\phi, \{\phi\}\} = 2^1 = \boxed{2}$

→ $\{\phi, \{\phi\}, \{\phi, \{\phi\}\}\}$ $P(A) = 2^3 = \boxed{8}$

Ans → $\{\phi, \{\phi\}, \{\{\phi\}\}, \{\{\phi, \{\phi\}\}\}, \{\phi, \{\phi\}\}, \{\{\phi\}, \{\phi, \{\phi\}\}\}, \{\phi, \{\phi, \{\phi\}\}\}, \{\phi, \{\phi\}, \{\phi, \{\phi\}\}\}\}$

Problem 9

→ $\{1, 2, 3\} \times \{7, 8, 9\}$

Ans – $\{(1,7), (1,8), (1,9), (2,7), (2,8), (2,9), (3,7), (3,8), (3,9)\}$

→ $\{a, b\} \times \{x, y, z\}$

Ans – $\{(a,x), (a,y), (a,z), (b,x), (b,y), (b,z)\}$

→ $\{1\} \times \{0, 1\} \times \{0, 1\}$

$\{(1,0), (1,1)\} \times \{0, 1\}$ →

Ans :- $\{((1,0),0), ((1,0),1), ((1,1),0), ((1,1),1)\}$

$\rightarrow \{a,b\}^3 \Rightarrow \{a,b\} \times \{a,b\} \rightarrow \{(a,a),(a,b),(b,a),(b,b)\}$

$\{(a,a),(a,b),(b,a),(b,b)\} \times \{a,b\}$

Ans $\rightarrow \{((a,a),a),((a,a),b),((a,b),a),((a,b),b),((b,a),a),((b,a),b),((b,b),a),((b,b),b)\}$

$\rightarrow$ If $A \times B = \{(4,8),(4,13)\}$

Ans $\rightarrow A = \{4\}$ $B = \{8,13\}$

Problem 10

$A = \{x,y,z,u,v,w\}$

$R = \{(x,x),(y,y),(z,z),(u,u),(v,v),(w,w),(x,v),(v,x),(y,u),(u,y),(z,v),(v,z)\}$

k) R is reflexive and symmetric but not transitive

for R to be transitive it should also have elements like (u,w), (z,x) as there is not direct connecti shown so it is not transitive.

Problem 11

$\rightarrow$ If A is set of element $A = \{1,2\}$

$R = \{(a,a) \mid a \in A\} = \{(1,1),(2,2)\}$

so

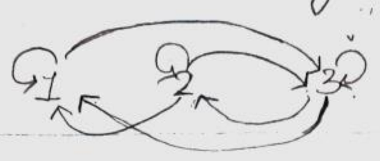
so R is both symmetric & antisymmetric.

$\rightarrow$ If A is a set of element $\{1,2,3\}$

Relation R to A will be

If A is a set of elements $\{1,2,3\}$ then Relation R on A where R is neither symmetric nor anti symmetric is

l)



$R = \{(1,1),(2,1),(3,1),(2,3),(2,2),(3,3),(1,3),(3,2)\}$

m)

Problem 12:

$A = \{1, 2, 3\}$

Suppose $R = \{(1,2), (2,3)\}$

R_1 will be $\{(1,2), (2,3), (1,3)\}$

R_2 will be $\{(1,2),(2,3),(1,3),(1,1),(2,2),(3,3)\}$

R_3 will be $R_2 \cup \{(2,1), (3,2)\}$

but R_3 is not transitive as we do not have $(3,1)$

$R_3 \to 1 \quad 2 \quad 3$

$R = \{(x,y) \in A \times A \mid x \neq y, y > x\}$

$R = 1 \to 2 \to 3$

Answer $\Rightarrow R = \{(1,2),(2,3)\}$

n)

Problem 6

$\{2, 4, \dots\}$

(i) $\{x = 2k : k \in N\}$

(ii) $\{x = y+1 : y \in \{1,3,5,7 \dots\}\}$

(iii) $\{x = z^2 : z \in \{\sqrt{2}, \sqrt{4}, \sqrt{6}, \sqrt{8} \dots\}\}$

Problem 13

$(0,0), 4|$
$(1,0)$

$S = \{(x,y) \in Z \times Z : 4 \mid (x+3y)\}$

for S to be reflexive

suppose take any $x \in Z$

$x + 3x = 4x$ which is multiple of 4

so S is reflexive

For S to be symmetric
suppose we have $(x,y) \in S$ then $(y,x) \in S$

Suppose $x+3y=4k$ for some k.
We can say that $y+3x = x+3y$ i.e. $x=y$
so $y+3x = 4l$ for some l which is multiple of 4
so S is symmetric

Transitive :-

o) Suppose we have $(x,y), (y,z) \in S$ so
$x+3y=4k$ for some k
$y+3z=4l$ for some l

$3y = 4k-x \quad 3z = 4l-y$

$$S(x,z) \Rightarrow x+3z = x+4l-y$$
$$= x+4l-\left(\frac{4k-x}{3}\right)$$
$$= 3x+12l-4k+x$$
$$= 4x+12l-4k$$
$$= 4(x+3l-4k)$$

So $x+3z = 4(m)$ for some m
Therefore it is transitive

As S is reflexive, Symmetric & transitive.
Hence it is equivalent.