School of Engineering and Applied Science (SEAS)
Ahmedabad University

BTech(ICT) Semester VI:Digital Signal Processing

Laboratory Assignment-8

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AIM :: LAB8 helps to understand the concept of chebyshev filter Using **impz and freqz** functions. In addition to this, I can use buttord,butter function for finding n,cutoffFreq,numerator and denominator coefficients and after that use of freqz function for plot magnitude and phase .

1. Solution Problem-2(a)

(a) Matlab Script:

```
clc ;
close all ;
clear ;
%b :: coefficients of numerator part as [...b4(S^4) b3(S^3) b1(S^2) b1(S^1) b0(S^0) b(-1)(S^-1) b(-2)(S^-2) b(-3)(S^-3) ..]
%a :: coefficients of denominator part as [...b4(S^4) b3(S^3) b1(S^2) b1(S^1) b0(S^0) b(-1)(S^-1) b(-2)(S^-2) b(-3)(S^-3) ..]
b = [0 0 86.5801 0 0];
a = [1 10.2904 452.2273 1826.2675 31496.595];
%finds the residues, poles, and direct term of a Partial Fraction Expansion of the ratio of two polynomials using b,a coefficients of Ha(s)
[r,p,k] = residue(b,a);
disp(r);
disp(p);

%passing signal from Laplace domain to time domain using ilaplace() func and passing output of this funtion to ztans() func which gives Z-trassform function of s-domain T1 here
%k/(s+a) => kz/z-exp(-aT)
T1(s)= ( (0.4347 - 3.3840i)/(s-(-3.3596 +17.9615i)) ) + ( (0.4347 + 3.3840i)/(s-(-3.3596 -17.9615i)) ) + ( (-0.4347 + 1.7605i)/(s-(-1.7856 + 9.5467i)) ) + ( (-0.4347 - 1.7605i)/(s-(-1.7856 - 9.5467i)) )
T2(t)=ilaplace(T1)
T3(z)=ztrans(T2)
%>>T3(z)=(z*(4347/10000 + 423i/125))/(z - exp(- 8399/2500 - 35923i/2000)) + (z*(4347/10000 - 423i/125))/(z - exp(- 8399/2500 + 35923i/2000)) + (z*(-4347/10000 - 3521i/2000))/(z - exp(- 1116/625 - 95467i/10000)) + (z*(-4347/10000 + 3521i/2000))/(z - exp(- 1116/625 + 95467i/10000))

% %T=0.1 ,sampling time
% e1 = exp(0.1*(-3.3596 + 17.9615i))
% e2 = exp(0.1*(-3.3596 - 17.9615i))
% e3 = exp(0.1*(-1.7856 + 9.5467i))
% e4 = exp(0.1*(-1.7856 - 9.5467i))
%output
% e1 =-0.1597 + 0.6966i
% e2 =-0.1597 - 0.6966i
% e3 =0.4834 + 0.6827i
% e4 =0.4834 - 0.6827i    ,putting this values in T3z eq

%instead of syms z
z = tf('z');   %z = tf('z',ts) creates special variable z that you can use in a rational expression to create a discrete-time transfer function model. To leave the sample time unspecified, set ts input argument to -1.

```

```
Hz = ((0.4347 - 3.3840i)*z)/(z + 0.1597 - 0.6966i) + ((0.4347 + 3.3840i)*z)/(z +
    0.1597 + 0.6966i) + ((-0.4347 + 1.7605i)*z)/(z - 0.4834 - 0.6827i) + ((-0.4347
     - 1.7605i)*z)/(z - 0.4834 + 0.6827i);   % Calculated H(z), putting value of
    exp

[b, a] = tfdata(Hz,'v');    %[num,den] = tfdata(sys,'v') returns the numerator and
     denominator coefficients as row vectors rather than cell arrays for a SISO
    transfer function represented by sys.
display(b);
display(a);

figure(1)
fs=1/0.1;                %sampling freq
freqz(b,a,fs);           %Plots magnitude and phase response

figure(2)
zplane(b,a)    %zplane(b,a), where b and a are row vectors, first uses roots to
    find the zeros and poles of the transfer function represented by the numerator
     coefficients b and the denominator coefficients a.
```

(b) Approach:

$\omega_L = 0.3\pi$

$\omega_H = 0.6\pi$

$T = 0.1$

$\delta_p = 0.11$

Here chebyshev(I) $N=2$ Lowpass

$$\varepsilon^2 = \frac{1}{(1-\delta_p)^2} - 1$$

$$= \sqrt{\frac{1}{(1-0.11)^2} - 1}$$

$$= \sqrt{1.2625 - 1}$$

$$\boxed{\varepsilon = 0.5123}$$

$$\beta = \left[\frac{\sqrt{1+\varepsilon^2} + 1}{\varepsilon}\right]^{1/N}$$

$$= \left[\frac{\sqrt{1.2625} + 1}{0.5123}\right]^{1/2}$$

$$= \left[\frac{2.1236}{0.5123}\right]^{1/2}$$

$$\beta = \sqrt{4.1452}$$

$$\boxed{\beta = 2.0359}$$

$$r_1 = \left(\frac{\beta^2+1}{2\beta}\right) = \left(\frac{(2.0359)^2+1}{2\times 2.0359}\right) = \frac{5.1449}{4.0718}$$

$$\boxed{r_1 = 1.2635}$$

$$r_2 = \left(\frac{\beta^2-1}{2\beta}\right) = \left(\frac{(2.0359)^2-1}{2\times 2.0359}\right) = \frac{3.1449}{4.0718}$$

$$\boxed{r_2 = 0.7724}$$

$$\phi_k = \frac{(2k+N+1)\pi}{2N}$$, Here $N=2$ so $k=0,1$

$$\phi_0 = \left(\frac{0+2+1}{2\times 2}\right)\pi = \boxed{\frac{3\pi}{4}}$$

$$\phi_1 = \left(\frac{2+2+1}{2\times 2}\right)\pi = \boxed{\frac{5\pi}{4}}$$

$$x_k = r_2 \cos\phi_k$$

$$y_k = r_1 \sin\phi_k$$

$$x_0 = r_2 \cos\phi_0 = 0.7724 \times \cos\left(\frac{3\pi}{4}\right)$$

$$= 0.7724 \times (-0.7071)$$

$$\boxed{x_0 = -0.546}$$

$$x_1 = r_2 \cos\phi_1 = 0.7724 \times \cos\left(\frac{5\pi}{4}\right)$$

$$= 0.7724 \times (-0.7071)$$

$$\boxed{x_1 = -0.546}$$

$$y_0 = r_1 \sin\phi_0 = 1.2635 \sin(3\pi/4)$$

$$= 1.2635 \times 0.7071$$

$$\boxed{y_0 = 0.8934}$$

$$y_1 = r_1 \sin\phi_1 = 1.2635 \times \sin(5\pi/4)$$

$$= 1.2635 \times (-0.7071)$$

$$\boxed{y_1 = -0.8934}$$

$\Rightarrow$ $s_k = x_k + j\, y_k$

$s_0 = x_0 + j\, y_0$

$$\boxed{\begin{aligned} s_0 &= -0.546 + j\,0.8934 \\ s_1 &= -0.546 - j\,0.8934 \end{aligned}}$$

system function,

$$H(s) = \frac{b_0}{(s-s_0)(s-s_1)}$$

Here, $b_0 = \frac{s_0 \cdot s_1}{\sqrt{1+\varepsilon^2}}$

$$= \frac{(-0.546+j\,0.8934)(-0.546-j\,0.8934)}{1.1236}$$

$$= \frac{(-0.546)^2 - (j\,0.8934)^2}{1.1236}$$

$$= \frac{0.2981 + 0.7982}{1.1236}$$

$$= \frac{1.0963}{1.1236}$$

$$\boxed{b_0 = 0.9757}$$

so,

$$H(s) = \frac{0.9757}{(s-s_0)(s-s_1)}$$

$$= \frac{0.9757}{[s-(-0.546+j\,0.8934)][s-(-0.546-j\,0.8934)]}$$

$$= \frac{0.9757}{[s + 0.546 - j0.8934][s + 0.546 + j0.8934]}$$

$$= \frac{0.9757}{(s + 0.546)^2 + (0.8934)^2}$$

$$\boxed{H(s) = \frac{0.9757}{s^2 + s(1.0924) + 1.0963}} \quad \text{--- } (1)$$

lowpass to bandpass

$$\Omega_l = \frac{0.3\pi}{0.1} = 3\pi$$

$$\Omega_H = \frac{0.6\pi}{0.1} = 6\pi$$

$$s \rightarrow \frac{s^2 + \Omega_l \Omega_H}{s(\Omega_H - \Omega_l)}$$

$$= \frac{s^2 + (3\pi)(6\pi)}{s(6\pi - 3\pi)}$$

$$= \frac{s^2 + 177.4728}{s \times 9.42} \quad \text{--- } (2)$$

putting eq (2) in eq (1)

$$H(s) = \frac{0.9757}{\left(\frac{s^2+177.4728}{s\times 9.42}\right)^2 + \left(\frac{s^2+177.4728}{s\times 9.42}\right)(1.0924) + (1.0963)}$$

$$= \frac{0.9757 \times s^2 \times (9.42)^2}{(s^2+177.4728)^2 + (s^2+177.4728)(1.0924)(s\times 9.42) + 1.0963\,(s\times 9.42)^2}$$

$$= \frac{s^2 \times 86.5801}{s^4 + 354.9456\times s^2 + 31496.595 + s^3\times 10.2904 + s\times 1826.2675 + s^2\times 97.2817}$$

$$H(s) = \frac{s^2 \times 86.5801}{s^4 + s^3\times 10.2904 + s^2\, 452.2273 + 1826.2675\, s + 31496.595}$$

After partial fraction expansion

$$H(s) = \frac{r_0}{(s-P_0)} + \frac{r_1}{(s-P_1)} + \frac{r_2}{(s-P_2)} + \frac{r_3}{(s-P_3)}$$

using residue( ) in MATLAB using coeff of H(s) b, a

$r_0 = 0.4347 - 3.3840i$

$r_1 = 0.4347 + 3.3840i$

$r_2 = -0.4347 + 1.7605i$

$r_3 = -0.4347 - 1.7605i$

$p_0 = -3.3596 + 17.9615i$

$p_1 = -3.3596 - 17.9615i$

$p_2 = -1.7856 + 9.5467i$

$p_3 = -1.7856 - 9.5467i$

$$H(s) = \frac{(0.4347 - 3.3840i)}{(s - [-3.3596 + 17.9615i])} + \frac{(0.4347 + 3.3840i)}{(s - [-3.3596 - 17.9615i])} + \frac{(-0.4347 + 1.7605i)}{(s - [-1.7856 + 9.546i])} + \frac{(-0.4347 + -1.7605i)}{(s - [-1.7856 - 9.5467i])}$$

S-domain to z-domain

$$\frac{\text{constant}}{s+a} \longrightarrow \text{constant}\ \frac{z}{z - e^{-aT}}$$

here $$\frac{\text{constant}}{s-a} \longrightarrow \text{constant}\ \frac{z}{z - e^{aT}}$$

T is sampling freq. = 0.1

In MATLAB,

After applying Impulse Invariance method,

$$H(z) = \frac{(0.4347 - 3.3840\,i) \times z}{z - \exp(0.1 \times (-3.3596 + 17.9615\,i))}$$

$$+ \frac{(0.4347 + 3.3840\,i) \times z}{z - \exp(0.1 \times (-3.3596 - 17.9615\,i))}$$

$$+ \frac{(-0.4347 + 1.7605\,i) \times z}{z - \exp(0.1 \times (-1.7856 + 9.546\,i))}$$

$$+ \frac{(-0.4347 - 1.7605\,i) \times z}{z - \exp(0.1 \times (-1.7856 - 9.546\,i))}$$

after calculating exp in matlab,

$$H(z) = \frac{(0.4347 - 3.3840\,i)\,z}{(z + 0.1597 - 0.6966\,i)}$$

$$+ \frac{(0.4347 + 3.3840\,i)\,z}{(z + 0.1597 + 0.6966\,i)}$$

$$+ \frac{(-0.4347 + 1.7605\,i)\,z}{(z - 0.4834 - 0.6827\,i)}$$

$$+ \frac{(-0.4347 - 1.7605\,i)\,z}{(z - 0.4834 + 0.6827\,i)} \quad \text{--- eq } 3$$

→ After simplifying eq (3) in mathematica,

$$H(z) = \frac{1.75164 z^{-1} - 5.161 z^{-2} + 2.38 z^{-3}}{1 z^{0} - 0.646 z^{-1} + 0.90172 z^{-2} - 0.269 z^{-3} + 0.3574 z^{-4}} \quad \text{--- (4)}$$

→ and using tfdata (Hz) in matlab which gives coefficients b and a of eq (3) which is same as eq (4),

b = 0+0i 1.7517+0i −5.1615−0i 2.3831−0i 0+0i

a = 1+0i −0.6474+0i 0.9017+0i −0.2703+0i 0.3574+0i

b = numerator coeff.

a = denominator coeff

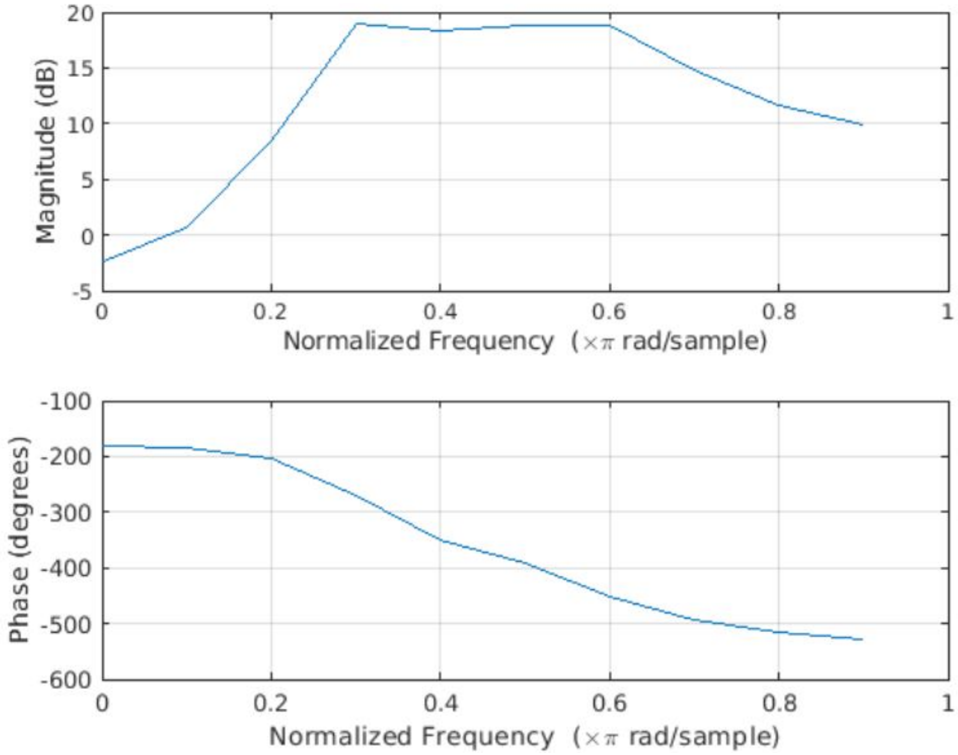
plotted i) phase & magnitude of Hz &
ii) poles & zeros of Hz

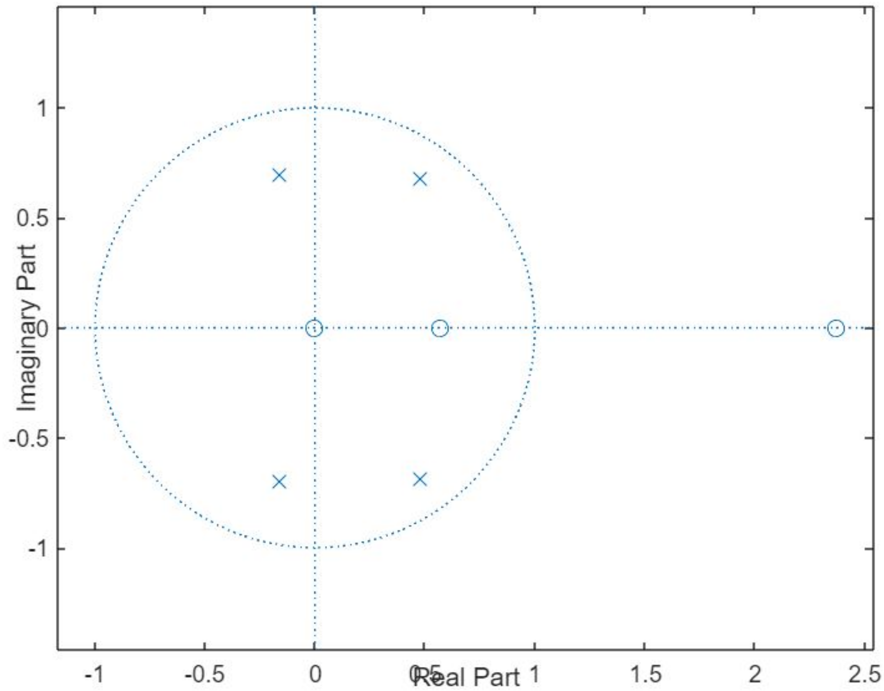
$$\frac{b(s)}{a(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} + \ldots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0} = \frac{r_n}{s - p_n} + \ldots + \frac{r_2}{s - p_2} + \frac{r_1}{s - p_1} + k(s).$$

r and p values

```
 0.4347 - 3.3840i
 0.4347 + 3.3840i
-0.4347 + 1.7605i
-0.4347 - 1.7605i

-3.3596 +17.9615i
-3.3596 -17.9615i
-1.7856 + 9.5467i
-1.7856 - 9.5467i
```

```
b =

   0.0000 + 0.0000i   1.7517 + 0.0000i  -5.1615 - 0.0000i   2.3831 - 0.0000i   0.0000 + 0.0000i

a =

   1.0000 + 0.0000i  -0.6474 + 0.0000i   0.9017 + 0.0000i  -0.2703 + 0.0000i   0.3574 + 0.0000i
```





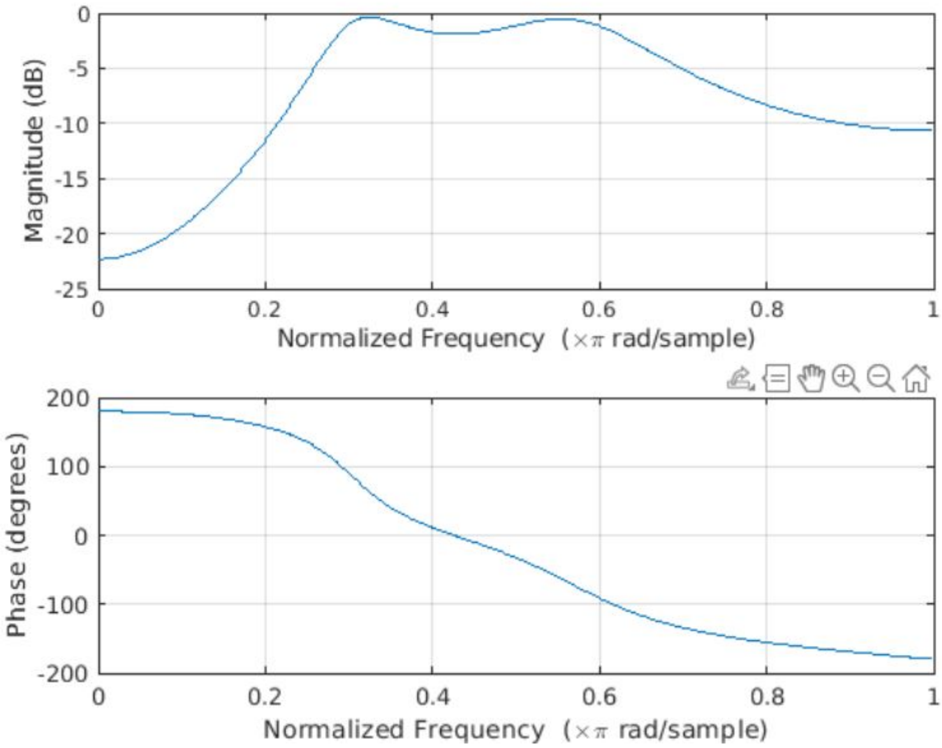
2. **Solution Problem-2(b)**

   (a) Matlab Script:

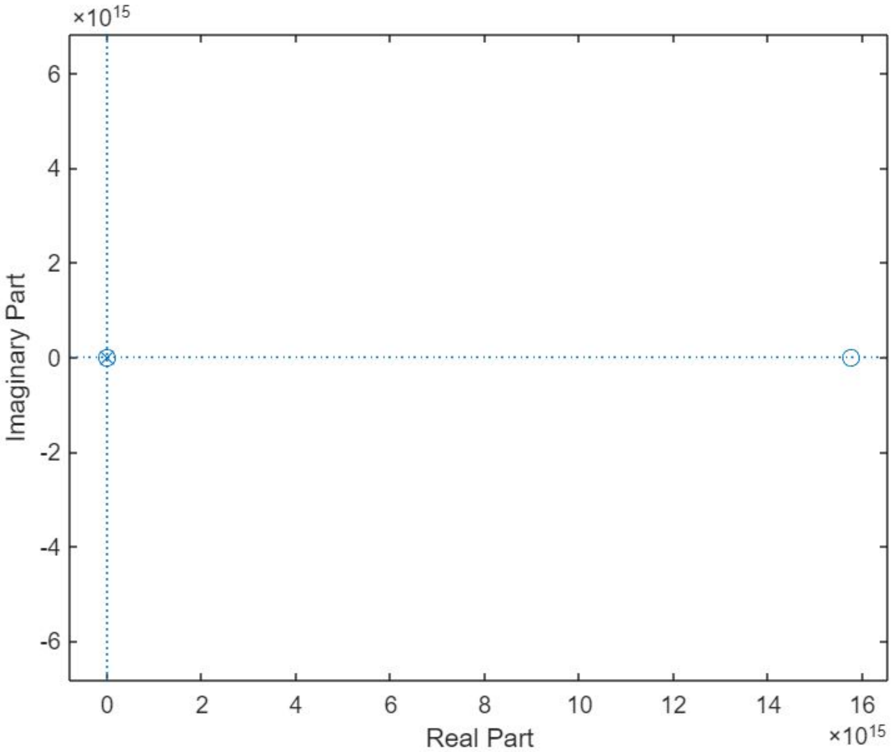
```
clc ;
close all ;
clear ;
N =2;                                          % order of chebyshev filter
```

```
w_L = 0.3* pi;                                         % lower cut-off frequency w
w_H = 0.6* pi;                                         % higher cut-off frequency w
ripple_p = 0.11;                                       % passband ripple
T = 0.1;                                               % sampling time
fs = 1/ T ;                                            % sampling frequency
ripple_p = -20* log10 (1 - ripple_p );                 % convert it into dB

[b , a] = cheby1 ( N , ripple_p ,1 ,  's') ;     %[b,a] = cheby1(n,Rp,Wp,ftype)
    designs a lowpass, highpass, bandpass, or bandstop Chebyshev Type I filter,
    depending on the value of ftype and the number of elements of Wp. The
    resulting bandpass and bandstop designs are of order 2n.

omega_c = sqrt ( w_L * w_H )/T;                        % center freq
bandwidth =( w_H - w_L )/ T;                           % bandwidth
[bt , at ] = lp2bp (b ,a , omega_c , bandwidth ); %[bt,at] = lp2bp(b,a,Wo,Bw)
    transforms an analog lowpass filter prototype given by polynomial coefficients
     (specified by row vectors b and a) into a bandpass filter with center
    frequency Wo and bandwidth Bw.
%The input system must be an analog filter prototype.

[bz , az ] = impinvar ( bt ,at , fs ) ;          %[bz,az] = impinvar(b,a,fs)
    creates a digital filter with numerator and denominator coefficients bz and az
    , respectively, whose impulse response is equal to the impulse response of the
     analog filter with coefficients b and a, scaled by 1/fs, where fs is the
    sample rate.
figure(1);
freqz (bz , az );
figure(2);
zplane ( bz , az );
```

(b) Approach:
With order 4(2*2) lower freq=0.3pi ,higher freq=0.6pi and converting ripple into dB for simple calculation ,sampling time =0.1, Cheby1 function returns coefficients depending upon ripple.Calculating center freq and bandwidth by formulas and using this as arguments in lp2bp function ,Passing output of this in impulse invariant function impinvar to get Z-domain function.



Magnitude (dB)
0
-5
-10
-15
-20
-25
0
0.2
0.4
0.6
0.8
1
Normalized Frequency (×π rad/sample)
Phase (degrees)
200
100
0
-100
-200
0
0.2
0.4
0.6
0.8
1
Normalized Frequency (×π rad/sample)



×10^15
6
4
2
0
-2
-4
-6
Imaginary Part
0
2
4
6
8
10
12
14
16
Real Part
×10^15