4^{th} order Butterworth highpass filter having cut-off freq 300 Hz & sampling freq 1 kHz Using Bilinear transformation

→ $\Omega_c = 1$ rad/s

$f_s = 1$ KHz $\Rightarrow T = 1$ mS

Poles at, $S_k = \Omega_c e^{j(2k+N+1)\pi/2N}$

$N = 4$, $k = 0, 1, 2, 3$

$S_0 = e^{j(2\cdot 0+4+1)\pi/2\times 4}$

$= e^{j5\pi/8}$

$= \cos(5\pi/8) + j\sin(5\pi/8)$

$S_0 = -0.3827 + j\,0.9239$

$S_1 = e^{j(2+4+1)\pi/8}$

$= e^{j(7\pi/8)}$

$= -0.9239 + j0.3827$

$S_2 = e^{j(4+4+1)\pi/8}$

$= e^{j(9\pi/8)}$

$= -0.9239 - j0.3827$

$S_3 = e^{j(6+4+1)\pi/8}$

$= e^{j11\pi/8}$

$= -0.3827 - j\,0.9239$

Now $H(s) = \dfrac{(\Omega_c)^N}{\prod_{k=0}^{N-1}(s - s_k)}$

$$H(s) = \frac{1}{[s+0.3827-j0.9239][s+0.9239-j0.3827][s+0.9239+j0.3827][s+0.3827+j0.9239]}$$

$$= \frac{1}{[(s+0.3827)^2+(0.9239)^2][(s+0.9239)^2+(0.3827)^2]}$$

$$= \frac{1}{[s^2+2s(0.3827)+(0.3827)^2+(0.9239)^2][s^2+2s(0.9239)+(0.9239)^2+(0.3827)^2]}$$

$$= \frac{1}{\begin{aligned}&s^4+2s^3(0.9239)+s^2(0.9239)^2+s^2(0.3827)^2 \\ &+2s^3(0.3827)+4s^2(0.9239)(0.3827)+ \\ &2s(0.3827)(0.9239)^2+2s(0.3827)^3 \\ &+s^2(0.3827)^2+2s(0.9239)(0.3827)^2+ \\ &(0.9239)^2(0.3827)^2+(0.3827)^4 \\ &+s^2(0.9239)^2+2s(0.9239)^3+(0.9239)^4 \\ &+(0.3827)^2(0.9239)^2\end{aligned}}$$

$$= \frac{1}{\begin{aligned}&s^4+s^3[1.8478+0.7654]+s^2[0.8536+0.1465 \\ &+1.4143+0.1465+0.8536]+s[0.6533+0.1121 \\ &+0.2706+1.5773]+[0.125+0.0215+ \\ &0.7286+0.125]\end{aligned}}$$

$$H(s) = \frac{1}{s^4+s^3[2.6132]+s^2[3.4145]+s[2.6133]+1}$$

from Lowpfilter to hp pass filter

$$s \to \frac{\Omega_c \times 2\pi}{s} \text{ rad/s}$$

$$s \Rightarrow \frac{600\pi}{s}$$

$$H(s) = \frac{1}{\left(\frac{600\pi}{s}\right)^4 + (2.6132)\left(\frac{600\pi}{s}\right)^3 + (3.4145)\left(\frac{600\pi}{s}\right)^2 + (2.6133)\left(\frac{600\pi}{s}\right) + 1}$$

$$= \frac{s^4}{(600\pi)^4 + s(2.6132)(600\pi)^3 + s^2(3.4145)(600\pi)^2 + s^3 2.6133(600\pi) + s^4}$$

$$H(s) = \frac{s^4}{s^4 + s^3(4.9234 \times 10^3) + s^2(1.212 \times 10^7) + s(1.747 \times 10^{10}) + (1.2598 \times 10^{13})}$$

Applying bilinear transformation

$$s \to \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$$

$$s \Rightarrow \frac{2}{10^{-3}}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) = 2000\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$$

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$$H(z) = \frac{(2000)^4 \left(\frac{1-z^{-1}}{1+z^{-1}}\right)^4}{\left\{ (2000)^4 \left(\frac{1-z^{-1}}{1+z^{-1}}\right)^4 + (2000)^3 \left(\frac{1-z^{-1}}{1+z^{-1}}\right)^3 (4.9234 \times 10^3) + (2000)^2 \left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2 (1.212 \times 10^7) + 2000 \left(\frac{1-z^{-1}}{1+z^{-1}}\right) (1.747 \times 10^{10}) + 0.2598 \times 10^{13} \right\}}$$

$$= \frac{1.6 \times 10^{13} (1-z^{-1})^4}{\left\{ 1.6 \times 10^{13} (1-z^{-1})^4 + 8 \times 10^9 (1-z^{-1})^3 (1+z^{-1}) (4.9234 \times 10^3) + (4 \times 10^6)(1-z^{-1})^2 (1+z^{-1})^2 (1.212 \times 10^7) + (2 \times 10^3)(1-z^{-1})(1+z^{-1})^3 (1.747 \times 10^{10}) + (1.2598 \times 10^{13})(1+z^{-1})^4 \right\}}$$

$$H(z) = \frac{1.6 \times 10^{13} (1 + 4z^{-2} + z^{-4} - 4z^{-1} - 4z^{-3} + 2z^{-2})}{\left\{ 1.6 \times 10^{13} (1 + 4z^{-2} + z^{-4} - 4z^{-1} - 4z^{-3} + 2z^{-2}) + 0.8 \times 10^{10} (1 - z^{-3} - 3z^{-1} + 3z^{-2})(1+z^{-1})(4.9 \times 10^3) + 4 \times 10^6 (1 - 2z^{-1} + z^{-2})(1 + 2z^{-1} + z^{-2})(2.212 \times 10^7) + 2 \times 10^3 (1 - z^{-1})(1 + z^{-3} + 3z^{-1} + 3z^{-2})(1.747 \times 10^{10}) + 2.2598 \times 10^{13} (1 + 4z^{-2} + z^{-4} + 4z^{-1} + 4z^{-3} + 2z^{-2}) \right\}}$$

$$= \frac{1.6 - 6.4z^{-1} + 9.6z^{-2} - 6.4z^{-3} + 1.6z^{-4}}{\left\{ 1.6 - 6.4z^{-1} + 9.6z^{-2} - 6.4z^{-3} + 1.6z^{-4} + 3.9387 (1 - z^{-3} - 3z^{-1} + 3z^{-2} + z^{-1} - z^{-4} - 3z^{-2} + 3z^{-3}) + 8.848 (1 - 2z^{-1} + z^{-2} + 2z^{-1} - 4z^{-2} + 2z^{-3} + z^{-2} - 2z^{-3} + z^{-4}) + 3.949 (1 + z^{-3} + 3z^{-1} + 3z^{-2} - z^{-1} - z^{-4} - 3z^{-2} - 3z^{-3}) + 2.2598 (1 + 4z^{-2} + z^{-4} + 4z^{-1} + 4z^{-3} + 2z^{-2}) \right\}}$$

$$= \frac{1.6 - 6.4 z^{-1} + 9.6 z^{-2} - 6.4 z^{-3} + 1.6 z^{-4}}{15.042 - 2.2067 z^{-1} + 7.41736 z^{-2} - 0.46811 z^{-3} + 0.2735 z^{-4}}$$

normalized by 15.042.

$$H(z) = \frac{0.1067 - 0.4267 z^{-1} + 0.64 z^{-2} - 0.4266 z^{-3} + 0.1067 z^{-4}}{1 - 0.1467 z^{-1} + 0.49311 z^{-2} - 0.03112 z^{-3} + 0.018184 z^{-4}}$$

b = [0.1067 −0.4267 0.64 −0.4266 0.1067]

a = [1 −0.1467 0.4931 −0.03112 0.018184]