Tutorial-5 Solution

① Design a digital IIR low pass Butterworth Prototype filter with a passband edge of 500 HZ, a stopband edge of 2 KHZ, Passband ripple of 0.1, stopband ripple of 0.1 and Sampling frequency of 8 KHZ using Impulse Invariance method. [20 marks]

(Note :- ~~Le~~ Right side bracket shows stepwise marks)

Solution :-

→ $\Omega_p = 1000\pi$ rad/sec

$\Omega_s = 4000\pi$ rad/sec

$\delta_p = 0.1 \Rightarrow \varepsilon_1 = \sqrt{\frac{1}{(1-\delta_p)^2} - 1} = 0.4844$

$\delta_s = 0.1$

[2 marks]

→ first we have to determine the order of filter

$$N = \frac{\log\left[\dfrac{\sqrt{(1/\delta_s^2) - 1}}{\varepsilon}\right]}{\log(\Omega_s/\Omega_p)} = 2.1802$$

we take nearest value of order $N = 3$.

[2 marks]

→ Determine 3dB cutoff frequency

$$N = \frac{\log\left[\left(1/\delta_s^2\right)-1\right]}{2\log\left(\Omega_s/\Omega_c\right)} = 3$$

$$\Rightarrow \frac{\Omega_s}{\Omega_c} = 2.1509 \Rightarrow \boxed{\Omega_c = 5842.6 \text{ rad/sec}}$$

[2 marks]

→ Now we find the poles of low pass filter.

$$S_K = \Omega_c \, e^{j(2K+N+1)\pi/2N}, \quad K = 0,1,2,3 \ldots$$

for here, $S_K = 5842.6 \, e^{j(2K+4)\pi/6}$, $K = 0,1,2$

[1 mark]

The poles are

$$S_1 = -2921.3 + j\,5059.8$$

$$S_2 = -5842.6$$

$$S_3 - 2921.3 - j\,5059.8$$

[2 marks]

→ The System function become

$$H(S) = \frac{\Omega_c^N}{\prod_{K=0}^{N-1}(S - S_K)}$$

$$H(S) = \frac{1.9944 \times 10^{11}}{(S + 2921.3 - j5059.8)\cdot(S + 5842.6)\,(S + 2921.3 + j5059.8)}$$

→ After partial fraction expansion we can find

$$H(s) = \frac{-2921.3 - j1686.6}{(s+2921.3 - j5059.8)} + \frac{-5842.6}{(s+5842.6)} + \frac{-2921.3 + j1686.6}{(s+2921.3 + j5059.8)}$$

[3 marks]

→ After Applying Impulse Invariance method we can find

$$H(z) = \frac{-2921.3 - j1686.6}{(1-e^{-(2921.3-j5059.8)T}z^{-1})} + \frac{5842.6}{(1-e^{-(5842.6)T}z^{-1})} + \frac{(-2921.3)+j1686.6}{(1-e^{-(2921.3+j5059.8)T}z^{-1})}$$

[2 marks]

→ with $T = \frac{1}{8000} = 1.25 \times 10^{-4}$ sec

$$H(z) = \frac{-2921.3 - j1686.6}{(1-(0.6598 + j0.4103)z^{-1})} + \frac{5842.6}{(1-0.4818 z^{-1})} + \frac{-2921.3 + j1686.6}{1-(0.5598 - j0.4103)z^{-1}}$$

[2 marks]

→ After Simplification

$$\boxed{H(z) = \frac{928.3075 z^{-1} + 571.8612 z^{-2}}{1 - 1.6014 z^{-1} + 1.0211 z^{-2} - 0.2321 z^{-3}}}$$

[1 mark]

→ In MATLAB for converting into normalized magnitude response, the numerator filter coefficients are scaled by Sampling Interval 'T'.

Thus, the filter system function (Normalized) becomes

$$\boxed{H(z) = \frac{0.1160 z^{-1} + 0.0715 z^{-2}}{1 - 1.6014 z^{-1} + 1.0211 z^{-2} - 0.2321 z^{-3}}}$$

[1 mark]