

4th order Butterworth highpass filter
 having cut-off freq 300 Hz & 200 Hz
 sampling freq 1 kHz
 Using Bilinear transformation

→ $\Omega_c = 1 \text{ rad/s}$
 $f_s = 1 \text{ kHz} \Rightarrow T = 1 \text{ ms}$

Poles at, $s_k = \Omega_c e^{j(2k+N+1)\pi/2N}$

$N = 4, k = 0, 1, 2, 3$

$s_0 = e^{j(2 \cdot 0 + 4 + 1)\pi/2 \times 4}$
 $= e^{j5\pi/8}$

$= \cos(5\pi/8) + j \sin(5\pi/8)$
 $s_0 = -0.3827 + j0.9239$

$s_1 = e^{j(2 + 4 + 1)\pi/8}$

$= e^{j7\pi/8}$
 $= -0.9239 + j0.3827$

$s_2 = e^{j(4 + 4 + 1)\pi/8}$

$= e^{j9\pi/8}$
 $= -0.9239 - j0.3827$

$s_3 = e^{j(6 + 4 + 1)\pi/8}$

$= e^{j11\pi/8}$
 $= -0.3827 - j0.9239$

Now $H(s) = \frac{(\Omega_c)^N}{\prod_{k=0}^{N-1} (s - s_k)}$

$$H(s) = \frac{1}{[s + 0.3827 - j0.9239][s + 0.9239 - j0.3827][s + 0.9239 + j0.3827][s + 0.3827 + j0.9239]}$$

$$= \frac{1}{[(s + 0.3827)^2 + (0.9239)^2][(s + 0.9239)^2 + (0.3827)^2]}$$

$$= \frac{1}{[s^2 + 2s(0.3827) + (0.3827)^2 + (0.9239)^2][s^2 + 2s(0.9239) + (0.9239)^2 + (0.3827)^2]}$$

$$= \frac{1}{s^4 + 2s^3(0.9239) + s^2(0.9239)^2 + (0.3827)^2 + 2s^3(0.3827) + 4s^2(0.9239)(0.3827) + 2s(0.3827)(0.9239)^2 + 2s(0.3827)^3 + s^2(0.3827)^2 + 2s(0.9239)(0.3827)^2 + (0.9239)^2(0.3827)^2 + (0.3827)^4 + s^2(0.9239)^2 + 2s(0.9239)^3 + (0.9239)^4 + (0.3827)^2(0.9239)^2}$$

$$= \frac{1}{s^4 + s^3[1.8478 + 0.7654] + s^2[0.8536 + 0.1465 + 1.4143 + 0.1465 + 0.8536] + s[0.6533 + 0.1121 + 0.2706 + 1.5773] + [0.125 + 0.0215 + 0.7286 + 0.125]}$$

$$H(s) = \frac{1}{s^4 + s^3[2.6132] + s^2[3.4145] + s[2.6133] + 1}$$

from LowP filter to hp pass filter

$$s \rightarrow \frac{\Omega_c \times 2\pi}{s} \text{ rad/s}$$

$$s \Rightarrow \frac{600\pi}{s}$$

$$H(s) = \frac{1}{\left(\frac{600\pi}{s}\right)^4 + (2.6132) \left(\frac{600\pi}{s}\right)^3 + (3.4145) \left(\frac{600\pi}{s}\right)^2 + (2.6133) \left(\frac{600\pi}{s}\right) + 1}$$

$$= \frac{s^4}{(600\pi)^4 + s(2.6132)(600\pi)^3 + s^2(3.4145)(600\pi)^2 + s^2(2.6133)(600\pi) + s^4}$$

$$H(s) = \frac{s^4}{s^4 + s^3(4.9234 \times 10^3) + s^2(1.212 \times 10^7) + s(1.747 \times 10^{10}) + (1.2598 \times 10^{13})}$$

Applying bilinear transformation

$$s \rightarrow \frac{2}{T} \left(\frac{1 - z^{-1}}{1 + z^{-1}} \right)$$

$$s \Rightarrow \frac{2}{10^{-3}} \left(\frac{1 - z^{-1}}{1 + z^{-1}} \right) = 2000 \left(\frac{1 - z^{-1}}{1 + z^{-1}} \right)$$

$$H(z) = (2000)^4 \left(\frac{1-z^{-1}}{1+z^{-1}} \right)^4$$

$$\left\{ (2000)^4 \left(\frac{1-z^{-1}}{1+z^{-1}} \right)^4 + (2000)^3 \left(\frac{1-z^{-1}}{1+z^{-1}} \right)^3 \right.$$

$$(4.9234 \times 10^3) + (2000)^2 \left(\frac{1-z^{-1}}{1+z^{-1}} \right)^2 (1.212 \times 10^7)$$

$$\left. + 2000 \left(\frac{1-z^{-1}}{1+z^{-1}} \right) (1.747 \times 10^{10}) + 0.2598 \times 10^{13} \right\}$$

$$= \frac{1.6 \times 10^{13} (1-z^{-1})^4}{1}$$

$$\left\{ 1.6 \times 10^{13} (1-z^{-1})^4 + 8 \times 10^9 (1-z^{-1})^3 (1+z^{-1}) \right. \\ (4.9234 \times 10^3) + (4 \times 10^6) (1-z^{-1})^2 (1+z^{-1})^2 \\ (1.212 \times 10^7) + (2 \times 10^3) (1-z^{-1}) (1+z^{-1})^3 \\ \left. (1.747 \times 10^{10}) + (0.2598 \times 10^{13}) (1+z^{-1})^4 \right\}$$

$$H(z) = 1.6 \times 10^{13} (1 + 4z^{-2} + z^{-4} - 4z^{-1} - 4z^{-3} + 2z^{-2})$$

$$\left\{ 1.6 \times 10^{13} (1 + 4z^{-2} + z^{-4} - 4z^{-1} - 4z^{-3} + 2z^{-2}) \right. \\ + 0.8 \times 10^{10} (1 - z^{-3} - 3z^{-1} + 3z^{-2}) (1+z^{-1}) (4.9 \times 10^3) \\ + 4 \times 10^6 (1 - 2z^{-1} + z^{-2}) (1 + 2z^{-1} + z^{-2}) (2.212 \times 10^7) \\ + 2 \times 10^3 (1 - z^{-1}) (1 + z^{-3} + 3z^{-1} + 3z^{-2}) (1.747 \times 10^{10}) \\ \left. + 2.2598 \times 10^{13} (1 + 4z^{-2} + z^{-4} + 4z^{-1} + 4z^{-3} + 2z^{-2}) \right\}$$

$$= \frac{1.6 - 6.4z^{-1} + 9.6z^{-2} - 6.4z^{-3} + 1.6z^{-4}}{1}$$

$$\left\{ 1.6 - 6.4z^{-1} + 9.6z^{-2} - 6.4z^{-3} + 1.6z^{-4} + 3.9387 \right. \\ (1 - z^{-3} - 3z^{-1} + 3z^{-2} + z^{-1} - z^{-4} - 3z^{-2} + 3z^{-3}) \\ + 9.848 (1 - 2z^{-1} + z^{-2} + 2z^{-1} - 4z^{-2} + 2z^{-3} + z^{-2} - \\ 2z^{-3} + z^{-4}) + 3.949 (1 + z^{-3} + 3z^{-1} + 3z^{-2} - \\ z^{-1} - z^{-4} - 3z^{-2} - 3z^{-3}) + 2.2598 (1 + 4z^{-2} + \\ z^{-4} + 4z^{-1} + 4z^{-3} + 2z^{-2}) \left. \right\}$$

$$= 1.6 - 6.4 z^{-1} + 9.6 z^{-2} - 6.4 z^{-3} + 1.6 z^{-4}$$

$$15.042 - 2.2067 z^{-1} + 7.41736 z^{-2}$$

$$- 0.46811 z^{-3} + 0.2735 z^{-4}$$

normalized by 15.042,

$$H(z) = \frac{0.1067 - 0.4267 z^{-1} + 0.64 z^{-2} - 0.4266 z^{-3} + 0.1067 z^{-4}}{1 - 0.1467 z^{-1} + 0.49311 z^{-2} - 0.03112 z^{-3} + 0.018184 z^{-4}}$$

$$b = [0.1067 \quad -0.4267 \quad 0.64 \quad -0.4266 \quad 0.1067]$$

$$a = [1 \quad -0.1467 \quad 0.4931 \quad -0.03112 \quad 0.018184]$$