School of Engineering and Applied Science (SEAS), Ahmedabad University

B.Tech(ICT) Semester V: Wireless Communication (CSE 311)

- Group No : **DSA_S11**
- Group Members:
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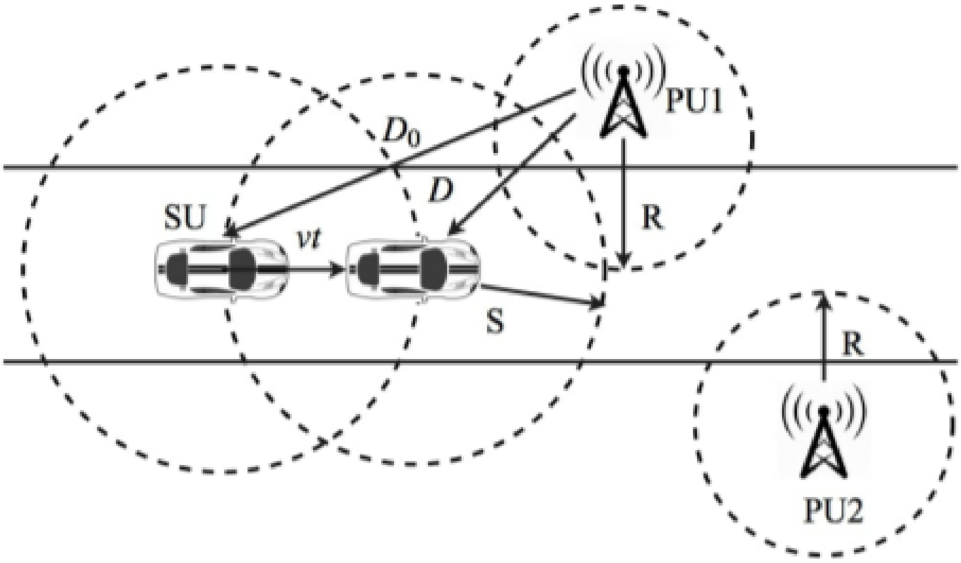
- Base Article Title: **On the Joint Impact of SU Mobility and PU Activity in Cognitive Vehicular Networks with Improved Energy Detection** [1]
- Task 2 : Results Reproduction of Base Article

1 Performance Analysis of Base Article

- Dynamic Spectrum Access (DSA)/Cognitive Radio(CR) systems aim at increasing the efficiency of spectrum use by allowing unli censed (Secondary-SU) users to opportunistically access licensed spectrum bands temporarily unused by the licensed (Primary-PU) users in a non-interfering manner.SU's mobility and often yields a detection performance loss as compared to static scenarios.
- A joint impact of velocity of vehicles PU's activities, transmission range of PUs and sensing range of SUs to evaluate the performance of spectrum sensing in CVNs

Symbol	Description
P_d	Probability of detection
P_{fa}	Probability of false alarm
P_{md} *or* P_m	Probability of miss-detection
CED	Classical energy detection
IED	Improved energy detection
$P(I)$	Probability of PU being inside SU's sensing range
$P(O)$	Probability of PU is outside sensing range of SU.
P	Number of previous sensing events considered
S	Sensing range of SU
R	Protection range of PU
M	Number of sensing events

- **Network Model :PU Traffic Model and Signal Model for Spectrum Sensing**



- One vehicle representing SU with sensing range S and multiple PUs with their protection range R (S>R for avoiding interference)

- SU vehicle moves forward on the straight road with velocity v with initial distance between PU and SU being equal to D_0, As PU is considering stationary. After time t = t', SU vehicle travels a distance equal to vt', altering the instantaneous distance between PU and SU to be equal to D

- S overlaps with R with SU being outside R, hence providing spectrum opportunities. This overlap duration creating spectrum opportunities depends on speed and direction of SU, R and S

- D is crucial parameter So we take 2 Events As,
  Event "I" : PU is inside sensing range of SU.
  Event "O" : PU is outside sensing range of SU.

- As SU's viewpoint ,The channel alternates between two states: "PU as Idle" and "PU as Busy". SO taking two hypothesis,
  for Event "I" ,

$$y_I(t) = \begin{cases} n(t) & , H_0 \\ h(t) * x(t) + n(t) & , H_1 \end{cases} \tag{1}$$

where,
y_I(t) is received signal at SU given Event "I"
h(t) is sensing channel gain
x(t) is transmitted signal by PU
n(t) is Addictive White Gaussian Noise

H_0 as PU is not transmitting to SU

H_1 as PU is transmitting to SU

for Event "O" ,

$$y_O(t) = n(t) \qquad\qquad , H_0, H_1 \tag{2}$$

where,

y_O(t) is received signal at SU given Event "O"

- CED as the sensing technique in CVNs is not a wise approach due to its limitations of sensing highly variable signals and instantaneous energy drops. IED was proposed in order to avoid false alarms or rather reduce their frequency of occurrence as an improvement of Modified Energy Detection (MED)

- Test-statistic as defined for CED,

  PU's state as busy if the measured energy is greater than a predefined threshold λ, or idle otherwise

$$\Gamma_i(y_i) = \Sigma_{n=1}^{N} |y_i(t)|^2 \tag{3}$$

Using central limit theorem,

$$\Gamma_{i-1}(y_{i-1}) > \lambda \; indicates \; \Gamma_i(y_i) < \lambda \; , In \; such \; case \; hypothesis \; H_1 \; consider$$
$$\Gamma_{i-1}(y_{i-1}) < \lambda \; indicates \; \Gamma_i(y_i) < \lambda \; , In \; such \; case \; hypothesis \; H_0 \; consider$$

- **Performance Matrix I - P(I)**

- Assuming that the SU and PUs are stationary, hence having a fixed separation between them. The probability distribution of the distance between SU and PU separated by distance D is assumed to follow log-normal distribution,

$$P(I) = P(R < D \leq S) \tag{4}$$

$$= F_S(s) - F_R(r)$$
$$\text{P(I)}=\frac{1}{2}[\text{erf}(\frac{s-\mu_s}{\sigma_s})\text{-erf}(\frac{r-\mu_r}{\sigma_r})]$$

where,

μ_s and σ_s represent mean and standard deviation of sensing range and μ_r and σ_r represent mean and standard deviation of protection range

we can simply subtract the probability of Event "I" from 1, considering the fact that Events"I" and "O" are mutually exclusive and collectively exhaustive events.

$$P(O) = 1 - P(I) \tag{5}$$

- Using the theory of random variables, we obtain distribution of time T = $\frac{D}{V}$ which evaluates to a lognormal distribution .Hence, the final distance between PU and SU at any time t is D_0 + vt ,D_0 is intial distance betn SU and PU at t=0 ,so Probability of PU being inside SU's sensing range P(I),

$$P(I) = \frac{1}{2}[erf(\frac{\frac{S-D_0}{v} - \mu_t}{\sigma_t}) - erf(\frac{\frac{R-D_0}{v} - \mu_t}{\sigma_t})] \tag{6}$$

where,μ_t and σ_t represent mean and standard deviation of sensing range at time t .P(O) is same as ,

P(O) = 1 - P(I)

- **Performance Matrix II III - P(fa) and P(m)**

PU state sensing by SU	SU sensing	Occurrence
Idle(Actually Idle)	can sense	Detection
Idle(Actually Busy)	can't sense	Miss Detection
Busy(Actually Idle)	can sense	False Alarm

Accordingly, False Alarm probability

P_f^{CED} = P($R_E > \lambda$| H_0)*P(OFF)

=P($R_E > \lambda$| H_0,I)*P(I)*P(OFF) + P($R_E > \lambda$| H_0,O)*P(O)*P(OFF)

$$P_f^{CED} = P(f|I) * P(OFF) \tag{7}$$

which leads to wastage of spectrum resource,where,

λ denotes decision threshold

R_E denotes Energy of received signal

P(OFF) as PU is actually Idle

P(f|I) is conditional False alarm Probability given PU is inside SU's Sensing range.

Accordingly,Miss detection probability

P_m^{CED} = P($R_E \leq \lambda$| H_1)*P(ON)

=P($R_E \leq \lambda$| H_1,I)*P(I)*P(ON) + P($R_E \leq \lambda$| H_1,O)*P(O)*P(ON)

$$P_m^{CED} = [P(m|I) * P(I) + P(m|O) * P(O)] * P(ON) \tag{8}$$

creating interference to the PU,where,

P(m|I) denotes conditional miss detection prob given PU is inside SU's sensing range

P(m|O) denotes conditional miss detection prob given PU is outside SU's sensing range

P(ON) as PU is actually Busy

So , Detection Probablity is ,

$$P_d^{CED} = 1 - \{[P(m|I) * P(I) + P(m|O) * P(O)] * P(ON)\} \tag{9}$$

- IED technique has been incorporated as the sensing scheme instead of using the CED technique. A detailed analysis of the improvement in performance metrics has been carried out along with a comparison of increase in time and memory complexity of IED and CED

  We're writing conditional probability as Q-function to easily plot So as follows

$$P(m|I) = 1 - Q(\frac{\lambda - \mu_{i|H_1,I}}{\sigma_{i|H_1,I}}) \tag{10}$$

$$P(f|I) = Q(\frac{\lambda - \mu_{i|H_0,I}}{\sigma_{i|H_0,I}}) \tag{11}$$

where, $\mu_{i|H_0,I}$ = Nσ_n^2 ,

$\sigma^2_{i|H_0,I}$ =2Nσ_n^4 ,

$\mu_{i|H_1,I}$ = N$(\sigma_n^2 + \sigma_s^2)$,

$\sigma^2_{i|H_1,I}$ =2N$(\sigma_n^2 + \sigma_s^{2^2}$) are from Test statistic

- Next equations for Improved Energy Detection at SU So taking Derivation of μ_{avg}, σ^2_{avg}

  ★ μ_{avg}=$\frac{M}{P}$ $\mu_{i|H_1}$ + $\frac{P-M}{P}$ $\mu_{i|H_0}$

  =$\frac{M}{P}$ N$(\sigma_n^2 + \sigma_s^2)$ + $\frac{P-M}{P}$ Nσ_n^2

  ★ σ^2_{avg}=$\frac{M}{P^2}$ $\sigma^2_{i|H_1}$ + $\frac{P-M}{P^2}$ $\sigma^2_{i|H_0}$

  =$\frac{M}{P^2}$ 2N$(\sigma_n^2 + \sigma_s^{2^2}$) + $\frac{P-M}{P^2}$ 2Nσ_n^4

  Now,As per definition of Probability of IED,

  $P(\Gamma_i(y_i) > \lambda)_{H_1}$ and $P(\Gamma_{i-1}(y_{i-1}) > \lambda)_{H_1}$ for P_d^{CED}

  $P(\Gamma_i(y_i) \leq \lambda)_{H_1}$ denotes P_m^{CED} as subtracting 1 from Detection Probability,

$$\therefore \mathrm{P}(\Gamma_i^{avg}(T_i))_{H_1} = Q(\tfrac{\lambda-\mu_{avg}}{\sigma_{avg}})$$

$$P_f^{IED} = P_f^{CED} + P_f^{CED} * (1 - P_f^{CED}) * Q(\frac{\lambda - \mu_{avg}}{\sigma_{avg}}) \tag{12}$$

Similarly IED for P(d),

$$P_d^{IED} = P_d^{CED} + P_d^{CED} * (1 - P_d^{CED}) * Q(\frac{\lambda - \mu_{avg}}{\sigma_{avg}}) \tag{13}$$

2 Numerical Results

2.1 Simulation Framework

- **For Analytical part, In Figure 1** , According to IEEE 802.11p DSRC (Dedicated Short-Range Communications) standards, the maximum value of sensing range is 1000 meters . Here, we take the value of protection range, R = 100, d = 200,λ= ,μ_t = 15, σ_t = 5.5, v = 20 and since S > R, the value of S from 200 to 1000 meters,so the probability of inside (P(I)) increases with increase in sensing range using eq(6).

- **In Figure 2** ,Taking threshold 0 to 120, The effect of SU Mobility on trade-off between P(f) and P(m) is evaluated for both CED and IED algorithms. using eq(7) , eq(8) , eq(12) and eq(13) .As we considering velocity constant at particular sensing value.Results show that lower threshold level results in higher false alarm probability, but lower miss-detection probability.Here, P(ON) = P(OFF) = 0.5 ,R = 100m, D = 200m, S = {300m, 700m, 900m} v = 40kmph, M = 1, P = 3, P(ON) = 0.5, P(OFF) = 0.5. so, the value of false alarm increases for IED algorithm as a cost of paying for reduced miss-detection probability .

- **Figure 3 ,**Comparing P(f) for both CED and IED algorithm And taking lower ON probability than OFF, {i.e. P(ON)=[0.25,0.40]} , { P(OFF)=[0.75,0.60]} and λ = 46.573 ,it provides total four cases,where PU protection range, R = 100 meters, initial separation between PU and SU, D = 200 meters and SU's sensing ranges (S = 300, 700 meters) using eq(8) and M=1,P=3.Figure showing that the latter P(m) are quite smaller than earlier. Similarly, P(m) decreases with increase in value of sensing range, keeping P(ON) and P(OFF) constant .

- Values of S,D,M,λ and P are same as Figure3 and { P(ON)=[0.75,0.60]} , {i.e. P(OFF)=[0.25,0.40]} **In Figure 4 ,** the variation of P(m) for both CED and IED versus SU velocity (in km/h) has been analyzed. But change in P(ON) is larger than P(OFF) ,with increase in value of sensing range, the value of P(m) increases for constant value of P(ON) and P(OFF).

- **In Figure 5** ,P(ON) = 0.25, P(OFF) = 0.75, R = 100m, S = 300m, D = 200m, M = 1, P = 3 with Analyzing the overall percentage gain in P(d) obtained by IED and CED for different velocities. As evident, a significant percentage improvement is achieved simply by changing the local sensing technique at SU vehicle.IED overcomes limitations of CED technique so the percentage gain of approximately 10% obtained by using IED technique for V2V networks

- **For Simulation part, In Figure 6** ,The impact of mobility of SU on Pr(I) is illustrated with S is taking between 0 to 30 meters, r = 100m, a=40 m . The random distance between a fixed PU and a mobile SU [2]. The SU moves randomly in a square area, Cartesian coordinate of SU can be expressed in terms of random variable .

The protected radius is r, which means no active SUs can stay in a protected area of πr^2. The position of the PU is assumed at the center of the square P(X,Y) = (a/2, a/2),Now the random Euclidian distance $D = \sqrt{X + Y}$,Here $X = |x - a/2|^2$ at max value of distance between PU and SU , Therefore Cumulative Density Function of X and Y is,

$$
\begin{aligned}
&\therefore F_X(x) = P(X \leq x)\\
&\therefore \qquad = P(-\sqrt{X} \leq (x - a/2) \leq \sqrt{X})\\
&\therefore \qquad = F_X(\sqrt{x} + a/2) - F_X(-\sqrt{x} + a/2)\\
&\therefore F_X(x) = 2\sqrt{x}/a
\end{aligned}
$$

Deriving Probability Density Function from CDF,

$f_X(x) = \frac{d}{dx}(F_X(x)) = \frac{1}{a\sqrt{x}}; \qquad 0 \leq X \leq (a^2/4)$

Similarly, $f_Y(y) = \frac{1}{a\sqrt{y}}; \qquad 0 \leq X \leq (a^2/4)$

The PDF of Z is the convolution of X and Y,

$f_Z = \int_{-\infty}^{\infty} f_X(x) f_Y(z - x)dx$ where $Z = D^2$,

$$
f_D(\mathrm{d}) = \begin{cases} 0, & D \leq \mathrm{r} \\ \frac{2d}{a^2}\{\frac{\pi}{2} - arcsin(\frac{2r - d^2}{d^2})\}, & r \leq \mathrm{D} < (\mathrm{a}/2) \\ \frac{4d}{a^2}\{arcsin(\frac{2r - d^2}{d^2})\}, & (a/2) \leq \mathrm{D} < (\mathrm{a}/\sqrt{2}) \end{cases}
$$

Same as eq(4) P(I) but practically,

P(I)=P(R < D≤ S)

P(I)=$\int_r^S f_D(d)dd$

P(I)=$\frac{1}{a^2}$ [a$\sqrt{4S^2 - a^2}$ - $2S^2$arcsin(1 - $\frac{a^2}{2S^2}$) - a$\sqrt{4r^2 - a^2}$ - $2r^2$arcsin(1 - $\frac{a^2}{2r^2}$)] ; a/2 ≤ S < a/$\sqrt{2}$

Where one monte-carlo loop for different sensing Range of SU . velocity is taken constant and SU moves with $\theta = 0$.

2.2 Reproduced Figures

- Reproduced Figure-1

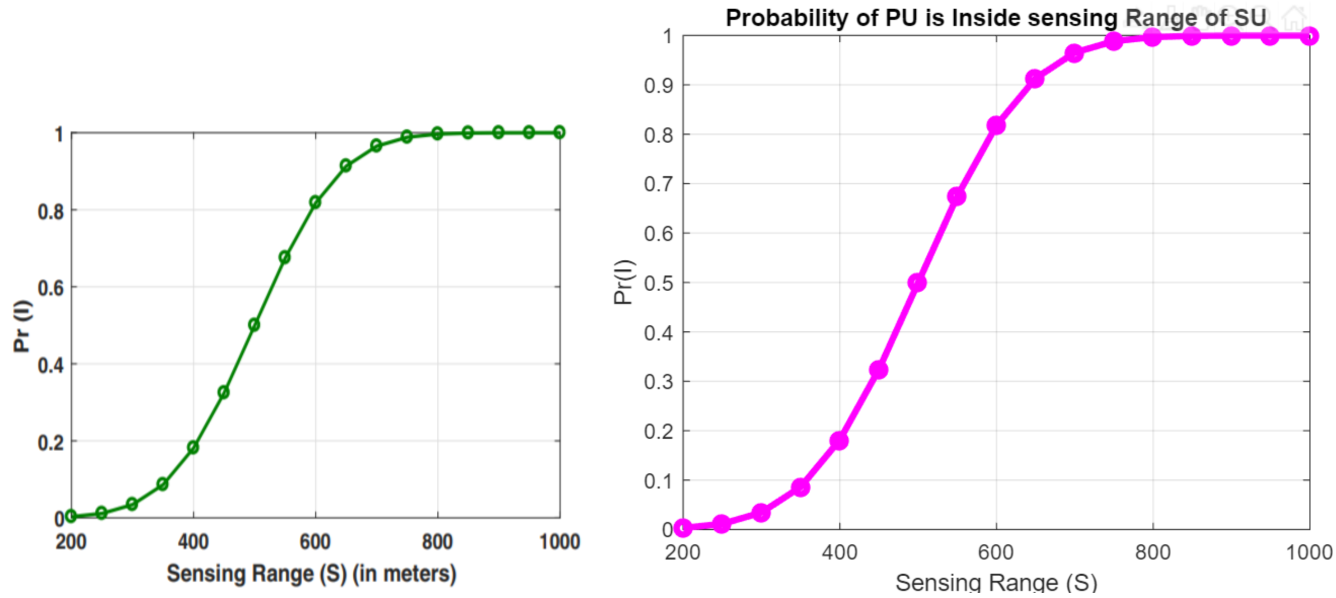


Fig 1: Probability of PU being inside SU's sensing range vs various values of sensing range (R = 100m, v = 20kmph)

- Reproduced Figure-2

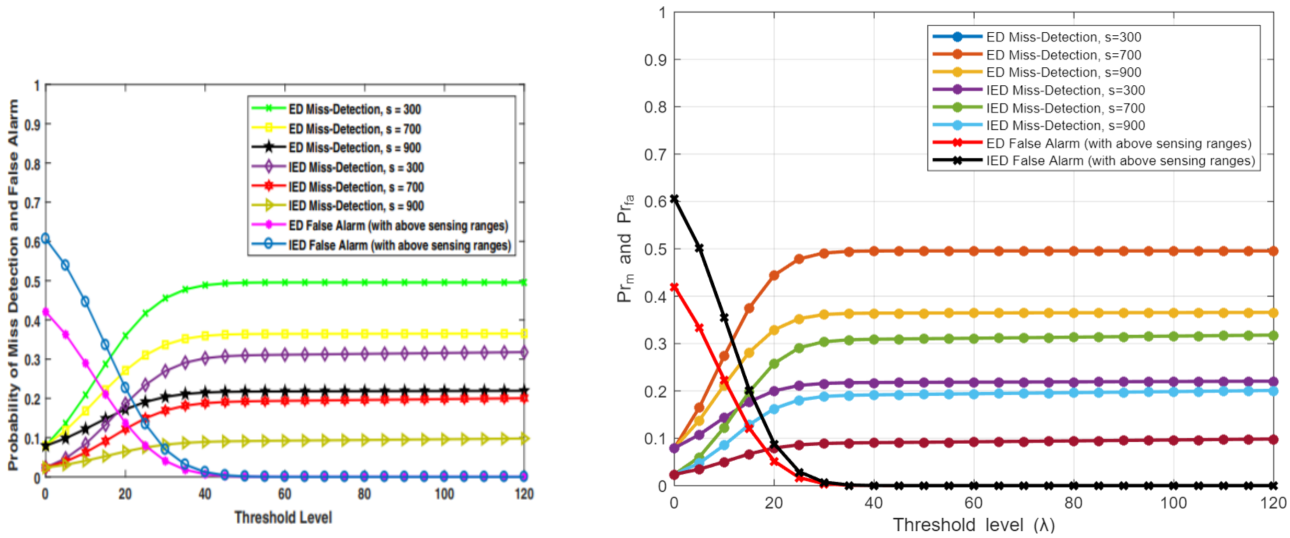


Fig 2 : Probabilities of Miss-Detection and False Alarm vs Various Sensing Ranges and Thresholds (R = 100m; D = 200m; S = {300m, 700m, 900m} v = 40kmph; M = 1; P = 3; P(ON) = 0.5, P(OFF) = 0.5)

- Reproduced Figure-3

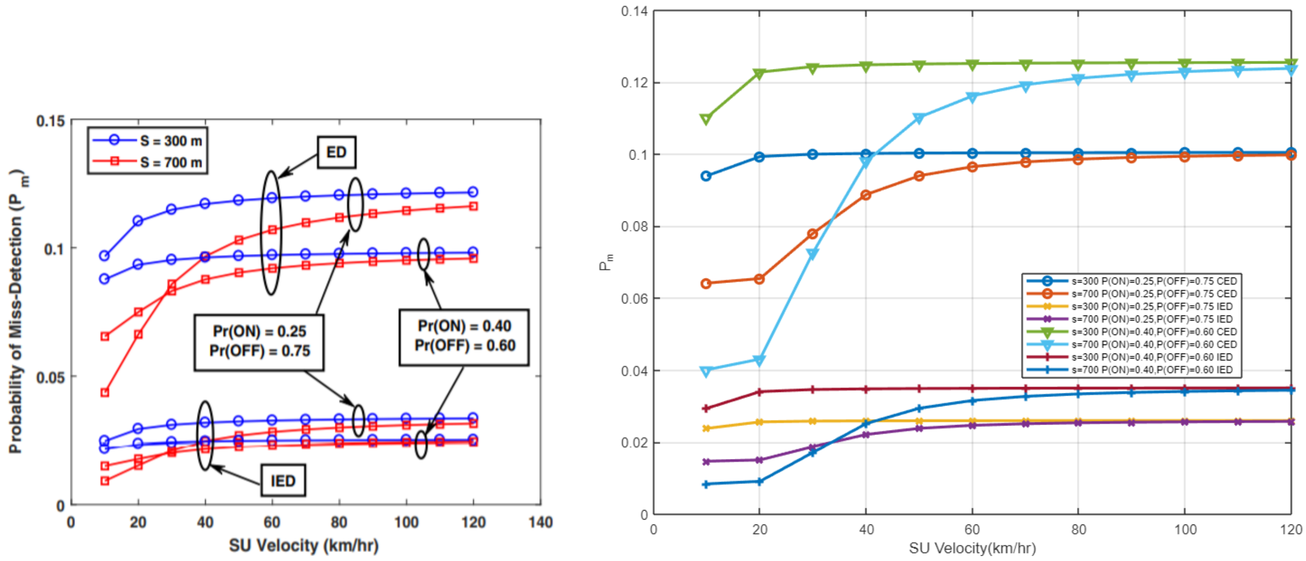


Fig 3 : Comparison of Miss-detection probabilities using CED and IED for different values of SU velocities with different sensing ranges for cases when P(OFF) > P(ON), (P(ON) = { 0.25, 0.40} , P(OFF) = {0.75, 0.60} , R = 100m, D = 200m, S = {300m, 700m}, M = 1, P = 3)

- Reproduced Figure-4

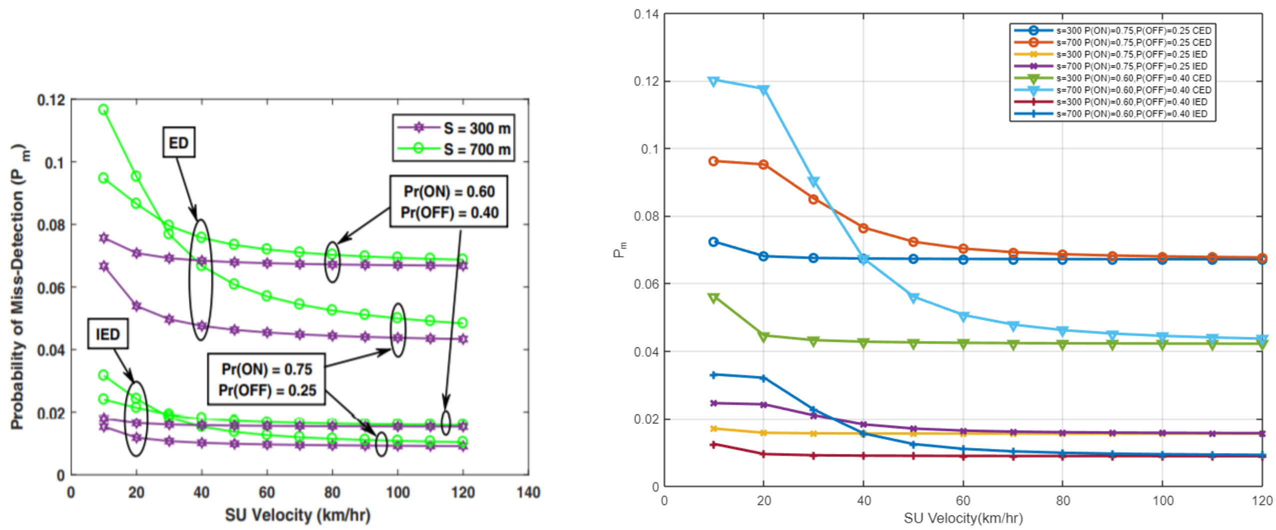


Fig 4 : Comparison of Miss-detection probabilities using CED and IED for different values of SU velocities with different sensing ranges for cases when P(OFF) < P(ON), (P(ON) = { 0.75, 0.60} , P(OFF) = {0.25, 0.40} , R = 100m, D = 200m, S = {300m, 700m}, M = 1, P = 3)

- Reproduced Figure-5

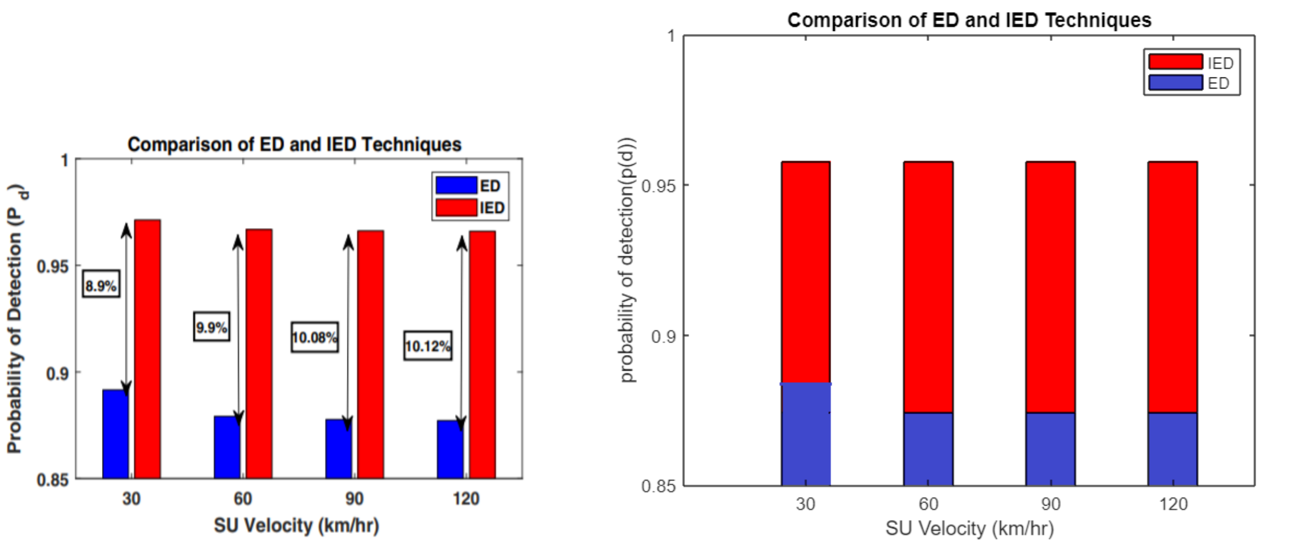


Fig 5 : Percentage gain comparison of CED and IED Techniques in terms of detection probabilities for different velocities, P(ON) = 0.25, P(OFF) = 0.75, R = 100m, S = 300m, D = 200m, M = 1, P = 3

2.3 Simulation Part of graph

- Simulated Figure 1

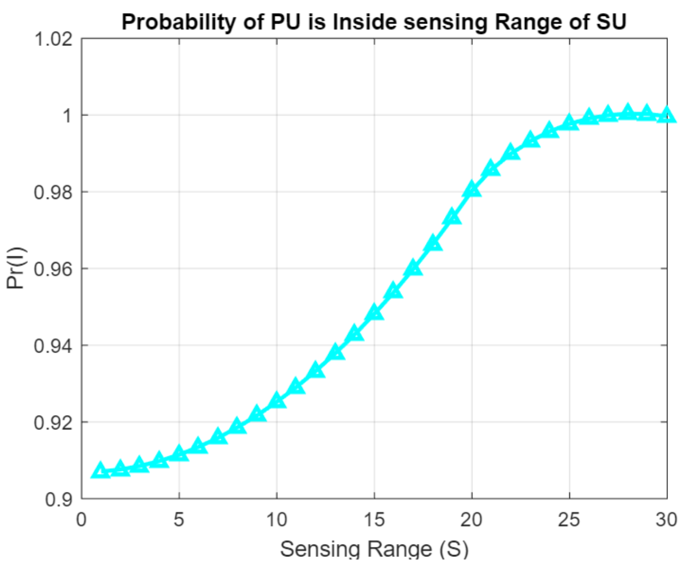


Fig 6 : Simulation result of P(I) vs Sensing Range of SU in meters

2.4 Inferences

Figure 1 , As we increase sensing range(S) of SU, Probability of PU being inside S will tend to increase. So ,Detection Probability will increase. It seems from
S=450 m and P(I)=0.32458 , S=600 m and P(I)=0.818211

In Figure 2 , The value of false alarm increases for IED algorithm as a cost of paying for reduced miss-detection probability. As increasing threshold value miss-detection Probability will increase due to get large receiving energy at SU at particular λ . Clearly, results show that lower threshold level results in higher false alarm probability, but lower miss-detection probability.
It seems from at threshold level(λ) 10,

S	P_m^{CED}	P_m^{IED}
300	0.274012	0.123254
600	0.212541	0.0857024
900	0.143737	0.504637

at (λ) 5,

S	P_m^{CED}	P_m^{IED}
300	0.495489	0.310244
600	0.36458	0.192908
900	0.21805	0.0916997

λ	P_f^{CED}	P_f^{IED}
10	0.22387	0.355548
5	1.39752e-6	1.39752e-6

Figure 3 shows that when less probability of PU is transmitting ,then P_m will increase (P_d decrease) with velocity of SU increase . Moreover,IED technique has smaller P_m than CED whether Velocity is 300 km/hr or 700 km/hr .
For fig.3 At velocity 20m/s

S	P(OFF)	P(ON)	P_m^{CED}	P_m^{IED}
300	0.75	0.25	0.0993503	0.0256019
300	0.60	0.40	0.122809	0.0340214
700	0.75	0.25	0.0654355	0.0150332
700	0.60	0.40	0.0430763	0.00910256

At velocity 60m/s

S	P(OFF)	P(ON)	P_m^{CED}	P_m^{IED}
300	0.75	0.25	0.100368	0.0259483
300	0.60	0.40	0.125201	0.034931
700	0.75	0.25	0.0965404	0.0246542
700	0.60	0.40	0.116203	0.0315587

Similarly **Figure 4** , shows higher Probability of PU is transmitting then miss-detection probability decrease with detection probability increase where spectrum opportunity exists.Same as above IED technique has smaller P_m than CED. Clearly,IED technique has been incorporated as the sensing scheme instead of using the CED technique.
For fig.4 At velocity 20m/s

S	P(ON)	P(OFF)	P_m^{CED}	P_m^{IED}
300	0.75	0.25	0.0680721	0.0157869
300	0.60	0.40	0.044613	0.0094839
700	0.75	0.25	0.095247	0.024222
700	0.60	0.40	0.11760	0.0320762

At velocity 60m/s

S	P(ON)	P(OFF)	P_m^{CED}	P_m^{IED}
300	0.75	0.25	0.0672567	0.0155526
300	0.60	0.40	0.0424234	0.00894178
700	0.75	0.25	0.0703236	0.0164395
700	0.60	0.40	0.050661	0.011022

Figure 5 Clearly shows,Usage of Improved Energy Detection (IED) technique in CVNs results more than 10% increment of detection probability. Even changing in SU velocity overall difference between IED and CED is approx 10% .

Figure 6 ,It illustrates the impact of sensing range. The maximum sensing range can reach a/$\sqrt{2}$. P(I) increases dramatically while the sensing range (S) increases. This can be interpreted as that the PU has a higher possibility to fall into the SU's sensing range, if the sensing range is sufficiently large, though the SU is mobile .

3 Contribution of team members

3.1 Technical contribution of all team members

Tasks	Mansi	Khushi
Reproduction of Fig 1	✓	✓
Reproduction of Fig 2	✓	✓
Reproduction of Fig 3	✓	
Reproduction of Fig 4		✓
Reproduction of Fig 5		✓
Simulation of Fig 6	✓	

3.2 Non-Technical contribution of all team members

Tasks	Mansi	Khushi
Brainstorming	✓	✓
Miro-1,2,3	✓	✓
Report - Network Model	✓	
Report- Performance Matrix - I		✓
Report- Performance Matrix - II (P_{fa})	✓	✓
Report - Performance Matrix - III (P_m)	✓	✓
Numerical Results (Analytical)	✓	
Numerical Results (Simulation)	✓	✓

References

[1] O. Thakkar, D. K. Patel, Y. L. Guan, S. Sun, Y. C. Chang, and J. M. Lim, "On the joint impact of su mobility and pu activity in cognitive vehicular networks with improved energy detection," pp. 1–6, 2019.

[2] Y. Zhao, P. Paul, C. Xin, and M. Song, "Performance analysis of spectrum sensing with mobile sus in cognitive radio networks," pp. 2761–2766, 2014.