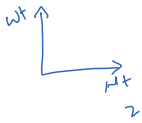


Linear Regression

Linear / Regression \Rightarrow o/p is continuous



$x \uparrow, y \uparrow$
 $x \uparrow, y \downarrow$ } Linear
 E.g., Ht v/s wt
 $Ht \uparrow \Rightarrow wt \uparrow$

* Sq.ft v/s Price
 $Sq.ft \uparrow \Rightarrow Price$
 * No. of years v/s Price
 $No. of years \uparrow, Price \downarrow$

$$y = mx + c$$

Assumption

Features are linearly varying with label

Linear $\Leftarrow y = 5x + 2 \rightarrow Order = 1$
 Quadratic $\Leftarrow y = 3x^2 - 8 \rightarrow Order = 2$
 Cubic $\Leftarrow y = x^3 + 5 \rightarrow Order = 3$
 $y = 3x^3 + 2x^2 + 5x - 2 \rightarrow Order = 3$

$$y = 2x + 5$$

$x = 1 \Rightarrow y = 2(1) + 5 = 7$
 $x = 2 \Rightarrow y = 2(2) + 5 = 9$
 $x = 3 \Rightarrow y = 2(3) + 5 = 11$

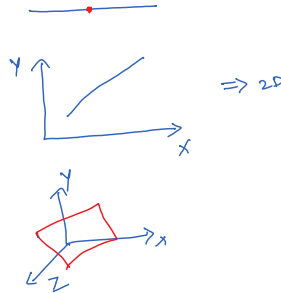
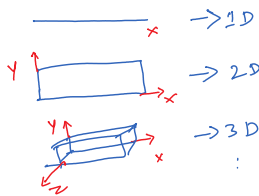
$y = x^2 \Rightarrow$ Eqⁿ of parabola.

$y = 1$
 $y = 4$
 $y = 9$

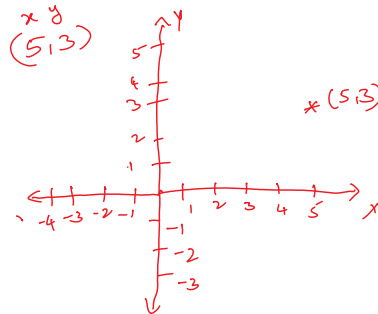
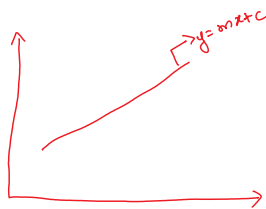
$x \uparrow, y \uparrow$

2D \rightarrow 1D

3D \rightarrow 2D



$$y = mx + c$$

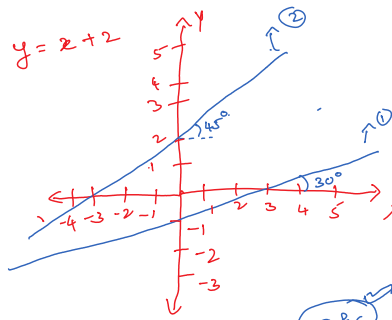


$y = mx + c$
 m \leftarrow Co-efficient of x / slope
 c \leftarrow y-intercept

$$m = \tan \theta$$

$m_1 = \tan 30^\circ$
 $= 0.57$
 $c_1 = -1$

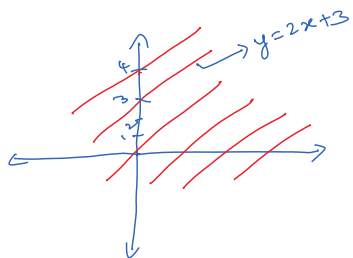
$m_2 = \tan 45^\circ$
 $m_2 = 1$
 $c_2 = 2$



$$m_2 = 1.$$

$$C_2 = 2$$

$$y = 1x + 2$$



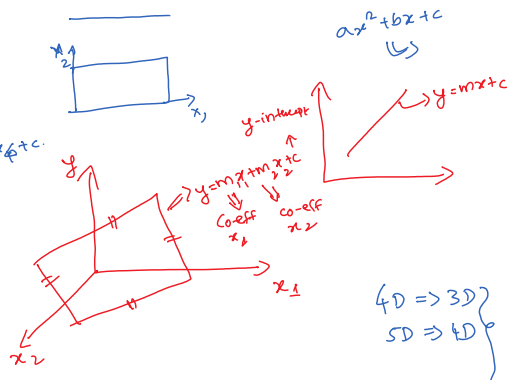
$$y = 2x + \boxed{3}$$

$$\begin{aligned} m &= 2. & C &= 3 \\ \tan \sigma &= 2 \\ \sigma &= \tan^{-1} 2 \\ \sigma &= 63.43^\circ \end{aligned}$$

Plane \leftarrow 2D $\leftarrow y = m_1 x_1 + m_2 x_2 + c$.

Hyperplane

$$\begin{aligned} y &= m_1 x_1 + m_2 x_2 + m_3 x_3 + c \\ y &= m_1 x_1 + m_2 x_2 + m_3 x_3 + m_4 x_4 + c \\ &\vdots \end{aligned}$$



$$y = m_1 x_1 + m_2 x_2 + m_3 x_3 + \dots + m_n x_n + c$$

$$y = \sum_{i=1}^n m_i x_i + c$$

$$n=3 \Rightarrow y = \sum_{i=1}^3 m_i x_i + c$$

$$\Rightarrow y = m_1 x_1 + m_2 x_2 + m_3 x_3 + c$$

$$y = m_1 x_1 + m_2 x_2 + m_3 x_3 + \dots + m_n x_n + c$$

$$m = [m_1 \ m_2 \ \dots \ m_n]_{1 \times n}$$

$$x = [x_1 \ x_2 \ \dots \ x_n]_{1 \times n}$$

$$x^T = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n \times 1}$$

$m^T \times n$

$$m^T X = m_1 x_1 + m_2 x_2 + m_3 x_3 + \dots + m_n x_n$$

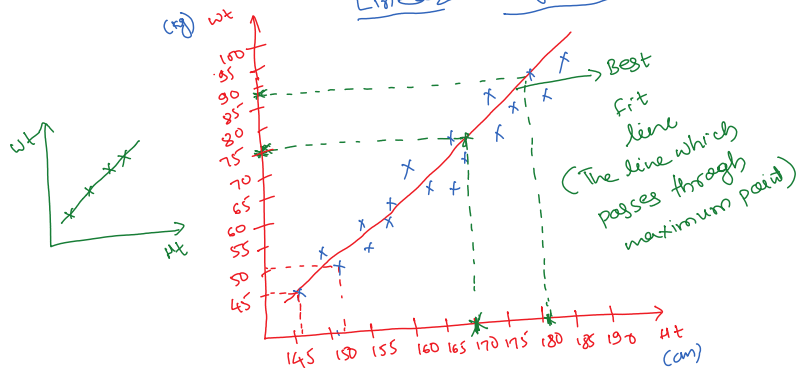
$$y = m^T x + c$$

$$m = w, \quad c = b$$

$$y = w^T x + b \longrightarrow \text{General eq}^n \text{ of Hyperplane}$$

- $y = w_1x + b \rightarrow \mathbb{R}^{1 \times n}$ of line
- $y = w_1x_1 + w_2x_2 + b \rightarrow$ " " plane
- $y = w_1x_1 + w_2x_2 + w_3x_3 + b \rightarrow$

Linear Regression

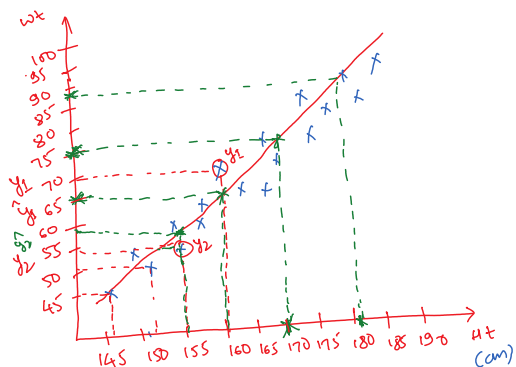
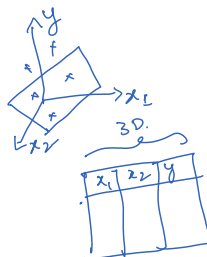


$$y = mx + c$$

2D →
3D →
4D →

Ht	Wt
145	45
150	52
...	...

170 → 75 kg
182 → 92 kg



Actual Ht → 160 cm
 y_1 = Actual Wt → 70 kg
 \hat{y}_1 = Predicted Wt → 65 kg

$$\text{Error}_1 = \text{Actual} - \text{Predicted}$$

$$= 70 - 65$$

$$= 5$$

$$\text{Error}_2 = y - \hat{y}$$

$$\text{Error}_2 = y_2 - \hat{y}_2$$

$$= 55 - 59$$

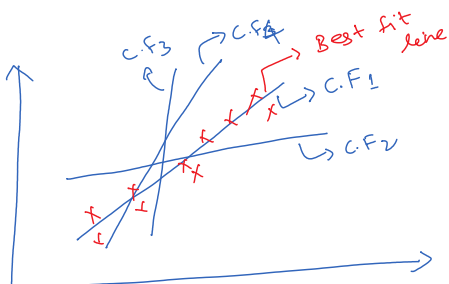
$$= -4$$

$$\text{Error} = y - \hat{y}$$

$$\text{Cost function} = \sum (y - \hat{y})^2$$

$$(y_1 - \hat{y}_1)^2 + (y_2 - \hat{y}_2)^2 + (y_3 - \hat{y}_3)^2 + \dots$$

$$= 5 + 4 + 3 + 2 = 14$$



Ht	Wt

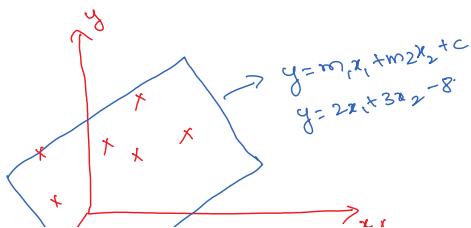
$$5 - 4 = 1$$

$$5 + 4 = 9$$

$$25 + 16 = 41$$

Square → Value change
Motivation to choose best fit line →

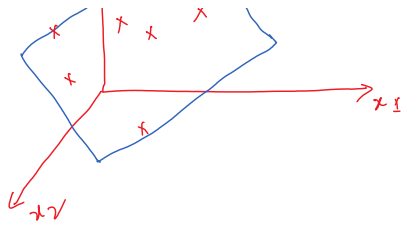
Choose that line which is giving least Cost function (Error)



1IP or

x_1	x_2	y
...
5	3	?

$$? \Rightarrow 11$$



$$y = 2 \times 5 + 3 \times 3 - 8$$

$$= 10 + 9 - 8$$

$$= 11$$

Cost function

① Square Error $= \sum (y - \hat{y})^2$

② Mean square Error (MSE) $= \frac{\sum (y - \hat{y})^2}{n}$

③ Root Mean square Error (RMSE) $= \sqrt{MSE} = \sqrt{\frac{\sum (y - \hat{y})^2}{n}}$

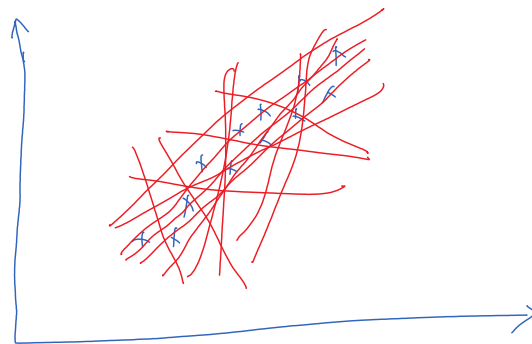
④ Mean Absolute deviation (MAD) $= \frac{\sum |y - \hat{y}|}{n}$

$$\frac{\sum (y - \hat{y})^2}{n}$$

$$4 \Rightarrow \boxed{16}$$

$$\frac{16}{4} = 4$$

* Choose that line which is giving least MSE.



In finite number of lines

Gradient Descent

step
by
step
by

40
39
38
37
36
35

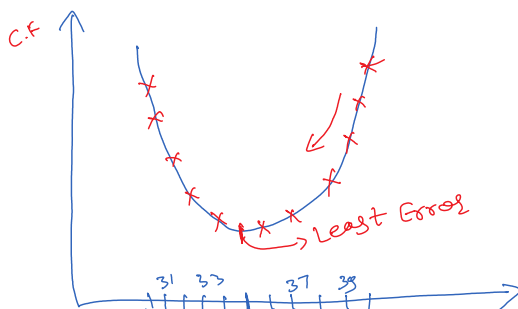
30
31
32
33
34
35

Random guess

40
30
32
38
33
37
34
26
35

$$40 - 35 = 5$$

$$39 - 35 = 4$$



$$\text{New} = \text{old} - \text{step size}$$

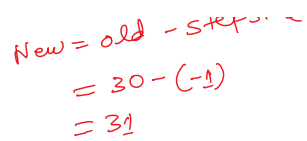
$$\text{New} = 40 - 1 = 39$$

$$\text{New} = \text{old} - \text{step size}$$

$$= 39 - 1$$

$$\text{New} = \text{old} - \text{step size}$$

$$= 30 - (-1)$$



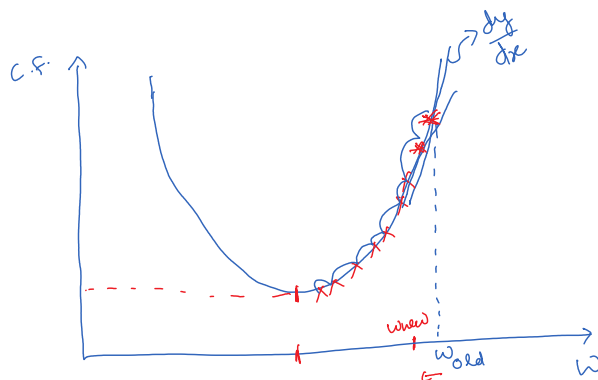
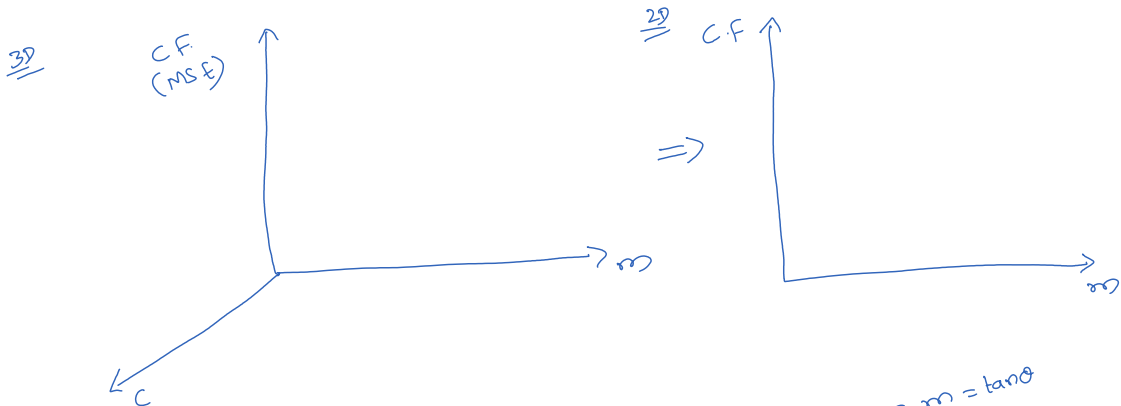
$y = mx + c$
 \downarrow slope
 \downarrow y-intercept
 Co-eff. at x / Gradient
 fixed \rightarrow $y = mx + c$
 \uparrow
 fixed

x y
 IIP OP

x_1	x_2	x_3	y

$$y = m_1x_1 + m_2x_2 + m_3x_3 + c$$

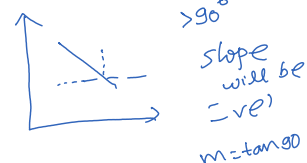
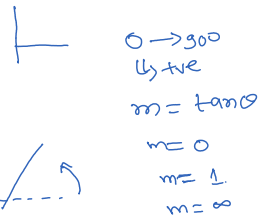
$m, c \rightarrow$ which are giving least MSE then I get a line which gives least MSE



slope, $m = \tan \theta$

$$\frac{dy}{dx} = \frac{d(CF)}{dw_{\text{total}}}$$

$$m = \text{slope} = \tan \theta = \frac{dy}{dx}$$



$\eta \rightarrow$ Learning Rate.
 $= 0.1, 0.01$

$$w_{\text{new}} = w_{\text{old}} - \text{step size} \\ = w_{\text{old}} - \eta \frac{d(\text{CF})}{dw_{\text{old}}}$$

$$w_{new} = w_{old} - (0.1)(+ve)$$

$$w_{\text{new}} < w_{\text{old}}$$

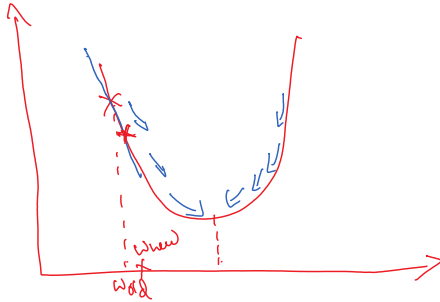
W_{old}

$$W_{new} = W_{old} - (0.1)(+ve)$$

$$W_{new} < W_{old}$$

$W_{new} \approx W_{old} \Rightarrow$ Stop the process

$W \Rightarrow$



$$W_{new} = W_{old} - \eta \frac{dL}{dW_{old}}$$

$$= W_{old} - (0.1)(-ve)$$

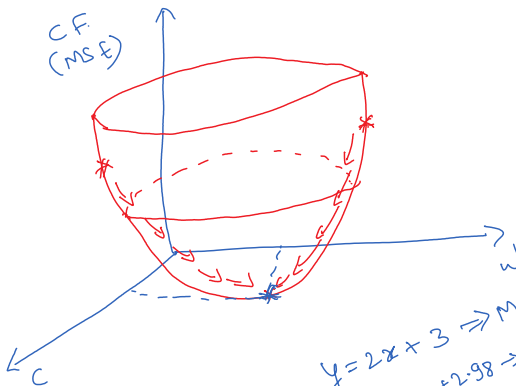
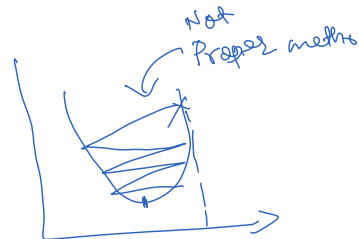
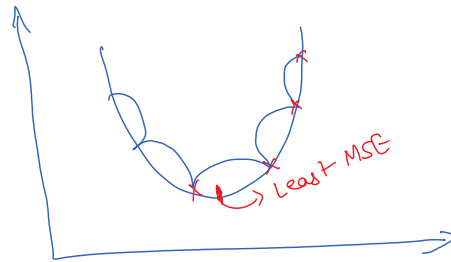
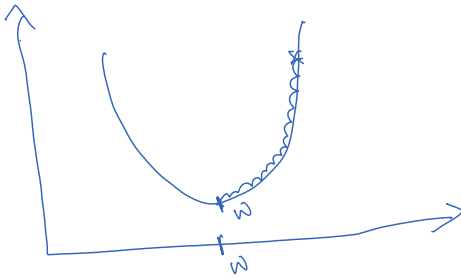
$$= W_{old} + 0.1$$

$$W_{new} > W_{old}$$

Stepsize

$$0.1 \times 50 = 5$$

\Downarrow Learning Rate



$$W_{new} = W_{old} - \eta \frac{d(C.F.)}{dW_{old}}$$

$$C_{new} = C_{old} - \eta \frac{d(C.F.)}{dC_{old}}$$

W, C

$$W = [2, 1.98]$$

$$C = [3, 2.98]$$

$$y = 2x + 3 \Rightarrow MSE = CF$$

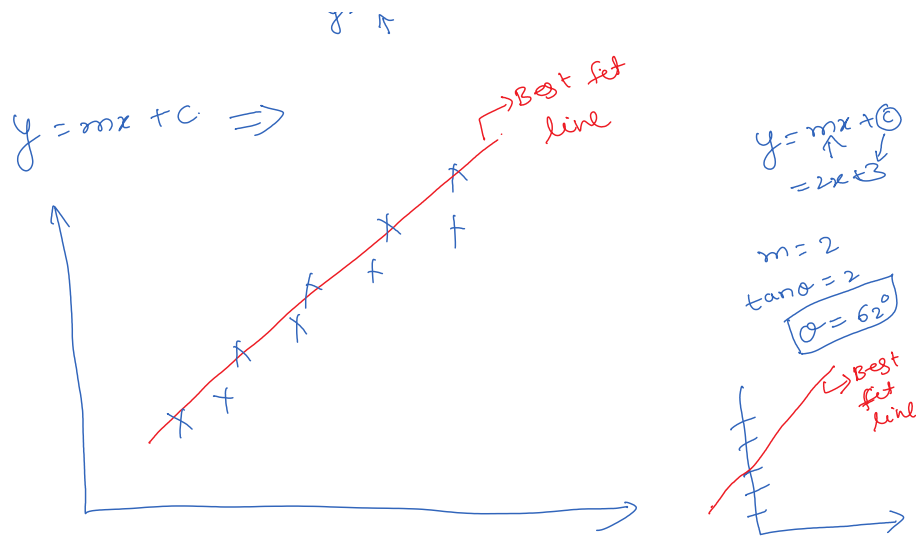
$$y = 1.98x + 2.98 \Rightarrow MSE = CF$$

\oplus, \ominus

$$W_{new} \approx W_{old}$$

$$y = 1.5x + 2 \Rightarrow \text{least MSE}$$

\rightarrow Best fit



Pipeline for Project

- * Understand Business Case & do domain Analysis
- * EDA (Exploratory data Analysis)
 - \rightarrow Statistical (Mean, Std, percentile)
 - \rightarrow Visualization
- * Data Preprocessing
 - \rightarrow Checks for null values & corrupted data
 - \rightarrow Converting categorical data into Numerical data
 - \rightarrow Handling with outliers
 - \rightarrow Scaling (If required)
 - \rightarrow Balancing (If required) (Classification pr. only)
- * Model Creation
- * Model Evaluation
- * Hyperparameter Tuning
- * Deploy the model

Model Creation

Model Creation
Model Evaluation

200
1 \rightarrow m

200
150
50
25:25
200

Model Creation

	X			Y
	TV	Radio	NP	Sales
Training		x-train		y-train
Testing		x-test		y-test

x-train & y-train

x-test \Rightarrow y-pred
Compare
y-test

TV = 200, R = 150, NP = 100 \Rightarrow $y = 200m_1 + 150m_2 + 100m_3 + c$
 Predicted \Leftarrow y-pred = 205
 Actual \Leftarrow y-test = 220
 Error = Actual - Predicted

Model Evaluation

1 column = 1D
 $y = mx + c$
 $y = m_1x_{TV} + m_2x_R + m_3x_{NP} + c$

200
150
50

	X			Y
		x-train		y-train
		x-test		y-test

x-train & y-train

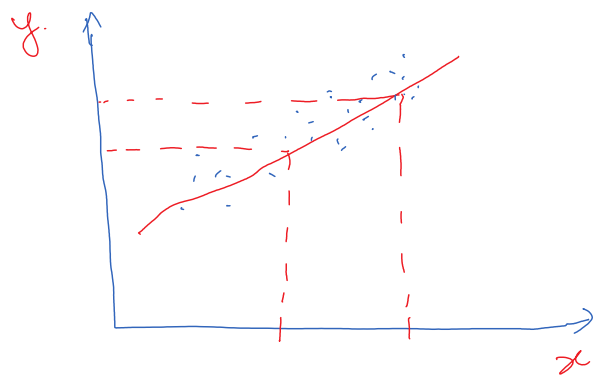
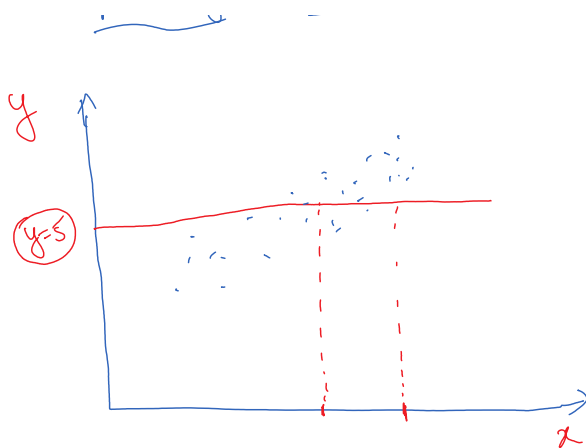
Compare
 Predict
 x-test \Rightarrow y-pred

x y
a a

x y-pred
a a
y-test

R² - Statistics

Average Model / Dumb Model



Residual \Rightarrow Error

$$R^2 = 1 - \frac{\text{Residual by your model}}{\text{Residual by dumb model}}$$

$$= 1 - \frac{\sum (y - \hat{y})^2}{\sum (y - \bar{y})^2}$$

Case 1: Res. by your model $>$ Res. by dumb. model.

$$R^2 = 1 - \frac{\sum (y - \hat{y})^2 \uparrow}{\sum (y - \bar{y})^2 \downarrow}$$

$$R^2 < 0 \text{ (-ve)}$$

Case 2: Res. by your model $<$ Res. by dumb. model.

$$R^2 = 1 - \frac{\sum (y - \hat{y})^2 \downarrow}{\sum (y - \bar{y})^2 \uparrow}$$

$$R^2 > 0 \text{ (+ve)}$$

Case 3: Res. by your model $=$ Res. by dumb. model.

$R^2 = 0$

Case 3: $\text{res. by your model} = \text{dumb model.}$

$$R^2 = 1 - \frac{\cancel{\sum (y - \hat{y})^2}}{\cancel{\sum (y - \bar{y})^2}}$$

$$R^2 = 0$$

Case 4: $\text{res. by your model} = 0$

$$R^2 = 1 - \frac{\cancel{\sum (y - \hat{y})^2} \rightarrow 0}{\sum (y - \bar{y})^2}$$

$$R^2 = 1$$



$R^2 < 0 \rightarrow$ Worst than dumb model

$R^2 = 0 \rightarrow$ Same as dumb model

$R^2 > 0 \rightarrow$ Better than dumb model

$R^2 = 1 \rightarrow$ Res. by your model is zero

$R^2 \in (0, 1) \rightarrow R^2$ should be nearer to 1

$\boxed{0.9}$ $\boxed{0.85}$ $\boxed{0.95}$
 $\boxed{0.2}$

$$R^2 = 1 - \frac{\sum (y - \hat{y})^2}{\sum (y - \bar{y})^2}$$

$\hat{y} = m_1 x_1 + m_2 x_2 + \dots + m_n x_n + c$
 (column) \rightarrow
 $\hat{y} = m_1 x_1 + m_2 x_2 + m_3 x_3 + c$
 $\hat{y} = m_1 x_1 + m_2 x_2 + m_3 x_3 + m_4 x_4 + c$

$$\frac{\sum (y - \hat{y})^2}{\sum (y - \bar{y})^2}$$

$$5 - 3 \uparrow = 2$$

$$5 - 4 = 1 \downarrow$$

$$\frac{3}{5} \quad \frac{2}{5}$$

← y 0

$$\hat{y} \uparrow \Rightarrow (y - \hat{y}) \downarrow \Rightarrow \frac{(y - \hat{y})^2}{(y - \bar{y})^2} \downarrow$$

$$\frac{3}{5} \quad \frac{2}{5} \\ = 0.6 \quad 0.4$$

$$1 - \frac{(y - \hat{y})^2}{(y - \bar{y})^2} \downarrow \uparrow$$

R^2 -score increasing. $\Rightarrow R^2$ -score is unstable.

$$\text{Adjusted } R^2\text{-score} = 1 - \frac{(1 - R^2)(n - 1)}{(n - p - 1)}$$

R^2 = R^2 -score
 n = No. of datapoints in testing
 p = No. of features