Project: Linear Programming Formulation for the Longest Closed Path Problem

Provide a linear programming formulation to find the longest closed path one can traverse without taking a road twice. (The permissible roads are the ones in white). Also solve the linear program using your favorite modeling tool and solver. (For the distances not provided please use Google Maps/Open Street Maps or any other map API which you trust. Please make suitable assumptions you think you need and explicitly state.

Solution. **Assumptions:**
1. We are only considering the intersections as nodes which have degree equal to or greater than 2 and are forming a closed path on the map.
2. We are not considering the intersections as nodes and edges which are forming sub-cycles of a relatively very small distance.
3. The intersections which are within a radius of 100 meters have been merged into forming one single node. Hence, the total number of nodes calculated is 39.
4. All the distances have been calculated assuming a rectilinear plane and not a Euclidean plane.
5. The weights of the edges are the distances between any two nodes.

Mathematical Model:

Let us define the following sets:
V: set of the vertices (nodes), i= 1,2,3,....,39
E: set of the edges, number of edges is 47
W: set of the weights w_{ij} of the edges

Decision Variables:

$$x_{ij} = \begin{cases} 1, & \text{if edge (i, j) is included in the path} \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

Objective Function:
We need to find the longest distance that can be covered.

$$\text{Max} \sum_{(i,j)\in E} w_{ij}x_{ij}$$

Constraints:

1. For any node, the number of edges incident on it must be exactly two, i.e.,

$$\sum_{j \in V} x_{ij} = 2 \quad \forall i \in V$$

2. Sub-tour elimination constraint:

$$\sum_{\substack{(i,j) \in E \\ i,j \in S}} x_{ij} \leq |S| - 1 \quad \forall S \subset V$$

where S is the subset of the set of nodes V which are included in the path, and $|S|$ is the cardinality of S.

Result (as observed after running the code):

Max iterations reached without finding a solution. Hence, there was no route such that no path is repeated.

1 Results

1.1 Optimal Solution

Upon solving the model, the decision variables indicate which edges are included in the longest closed path. The optimal path and the associated distances are presented below: **Selected Edges:**(1, 2) (2, 3) (3, 4) (4, 5) ... (n-1, n)

1.2 Objective Value

The maximum distance covered by the path is given by:

$$\text{Objective value (Max distance)} : X \text{ units}$$

2 Map

For reference, please see the IIT Bombay map available at click here.

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