PROJECT REPORT

GREEKS ANALYSIS

YEAR OF REPORT 2025

Women mentorship program report : 2025

Prepared By:
Mansi Ghodke

Mentored by:
Pavan Hurgat

PART-B(ii) : Greeks analysis

Abstract

This report evaluates how different market parameters affect the price and Greeks of a floating strike lookback call option, using Monte Carlo simulation. Each parameter is varied systematically, and its impact on option price, Delta, Gamma, Vega, Rho, and Theta is recorded. The insights are supported by simulations on BTC-USD with antithetic variates and daily granularity over a 1-year horizon.

Introduction

The primary goal of this project is to accurately compute and analyze all key Greeks—**Delta, Gamma, Vega, Rho, and Theta**—under varying market conditions, and to study how changes in fundamental parameters **(spot price, volatility, interest rate, dividend yield, time to maturity, and strike price)** impact both the option's price and its sensitivity profile. For Simplicity the analysis done in this report is for Fixed strike lookback option to capture changes due to Fixed strike price(K) which is only relevant only for Fixed option.

The analysis is conducted by running monte carlo simulations and determining the options path and further calculating the payoffs associated with Fixed strike options.

For each parameter, we vary it across a predefined range while holding others constant and calculate the corresponding changes in the option price and its sensitivities. The results are visualized using line plots that clearly illustrate the directional behavior and curvature of each Greek with respect to the parameter in question.

The pre-determined parameter data used for bitcoin is:-

```
S0 = 102768.2    # Spot Price (calculated)
K = S0       # At-the-money strike
r = 0.05      # Risk-free rate
sigma = 0.41    # Volatility
T = 1.0      # Time to maturity
M = 252      # Time steps
```

Greeks Definitions

In options pricing, the "Greeks" refer to partial derivatives (or ratio of changes) of the option's value with respect to various input variables. They measure the sensitivity of the option's price to changes in key market parameters which to understand how an option's value reacts to the market, helping traders hedge risk, optimize strategies, and understand the behavior of complex derivatives.

Delta

$$\Delta = \frac{\text{Change in Option's Price}}{\text{Change in Underlying Price}}$$

Measures how much the option price changes for a small change in the spot price (S_0).

Gamma

$$\Gamma = \frac{\text{Change in Delta}}{\text{Change in Underlying Price}}$$

Measures how much Delta changes with respect to a small change in the underlying asset price. High Gamma means Delta is very sensitive to price changes. Hence it measures the "convexity" of change in option price.

Rho

$$\rho = \frac{\text{Change in Option's Price}}{\text{Change in Risk-Free Interest Rate}}$$

Measures how much the option value changes for a 1% change in the risk-free interest rate.

Vega

$$\text{Vega} = \frac{\text{Change in Option's Price}}{\text{Change in Volatility}}$$

Measures how much the option price changes for a 1% change in implied volatility. Vega is usually very high, because volatility increases the chance that the asset hits a deeper minimum, improving the payoff.

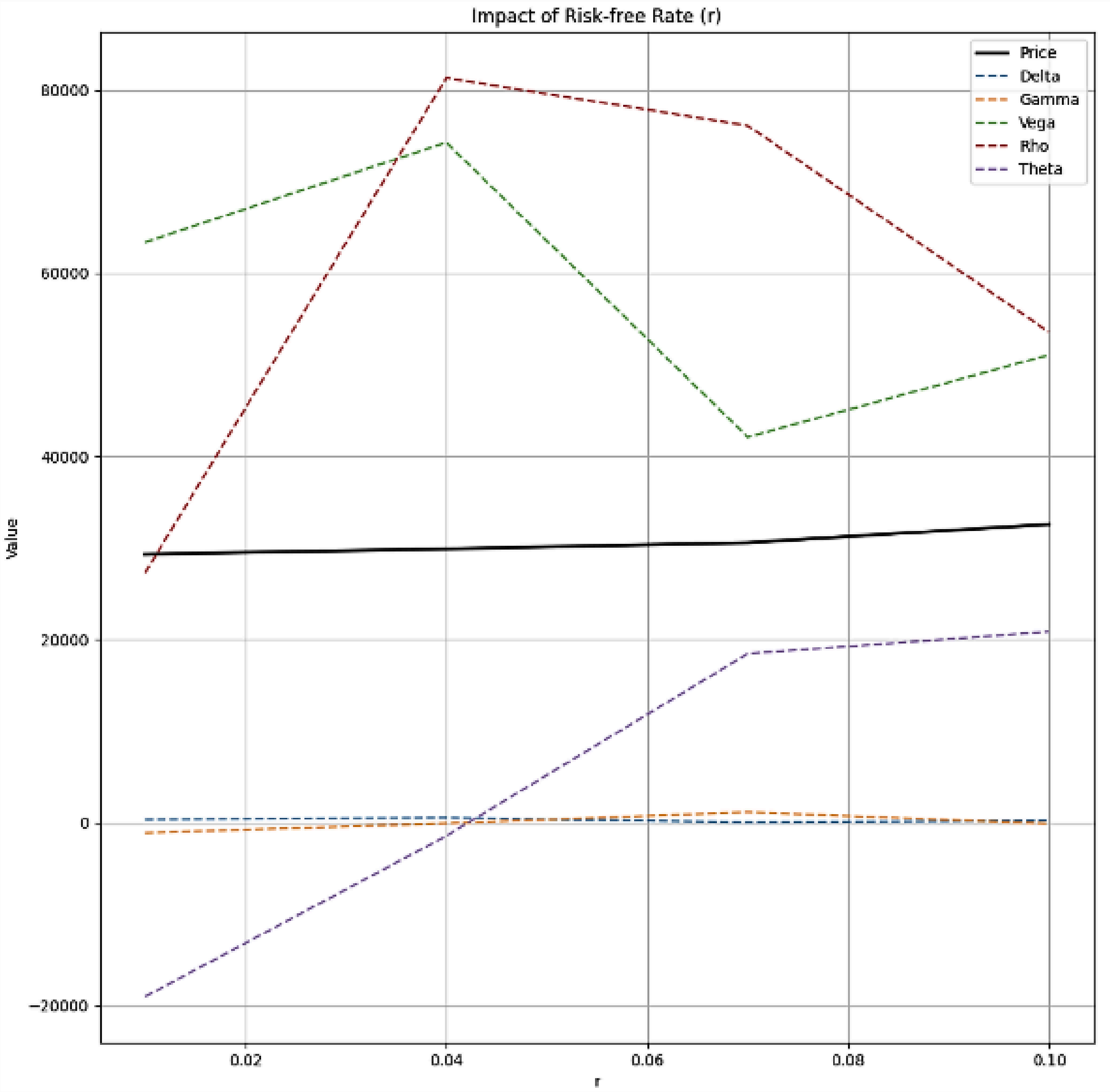
Theta

$$\theta = \frac{\text{Change in Option's Price}}{\text{Change in Time to Maturity}}$$

Sensitivity to Time (Time Decay) Measures how much the option price changes as time to maturity decreases (usually per day).

Parameter Variation Graphs

✦ Impact of Risk-Free Rate (r)



- r is varied from 0.01 to 0.1 (1% to 10%)

r	Price	Delta	Gamma	Vega	Rho	Theta
0.01	29108	15.41	258.06	69057	61022	24585
0.04	29693	211.24	1075.50	74820	68101	32930
0.07	30527	-157.38	2086.97	63114	40500	43710
0.10	32254	106.45	293.04	38271	53603	50884

Price:

- Price shows a steady increase in value as r increases.
- Higher r increases the expected growth rate of the underlying asset. This leads to higher simulated asset prices over time, increasing the likelihood of larger payoffs

Delta:

- Large fluctuation (15 → +211 → -157 → +106): Shows nonlinear relationship.
- In a lookback call, S(T) – min(S). Higher r increases drift, pushing S(T) up. But if min(S) also rises, Delta doesn't behave linearly.

Gamma:

- Sharp rise then fall: (258 → 1075 → 2086 → 293) i.e, nonlinear relationship
- Gamma measures convexity, i.e, change in delta → As r increases, variance in paths increases nonlinearly, amplifying curvature.

Rho:

- Classically increasing, except dip starting at r=0.04 (61022 → 68101 → 40500 → 53603)
- The change in risk free rate is constant, while options price sees a steady but varyingly. Usually, higher r increases price of calls (Rho ↑), but the path-dependent nature causes anomalies.

Vega:

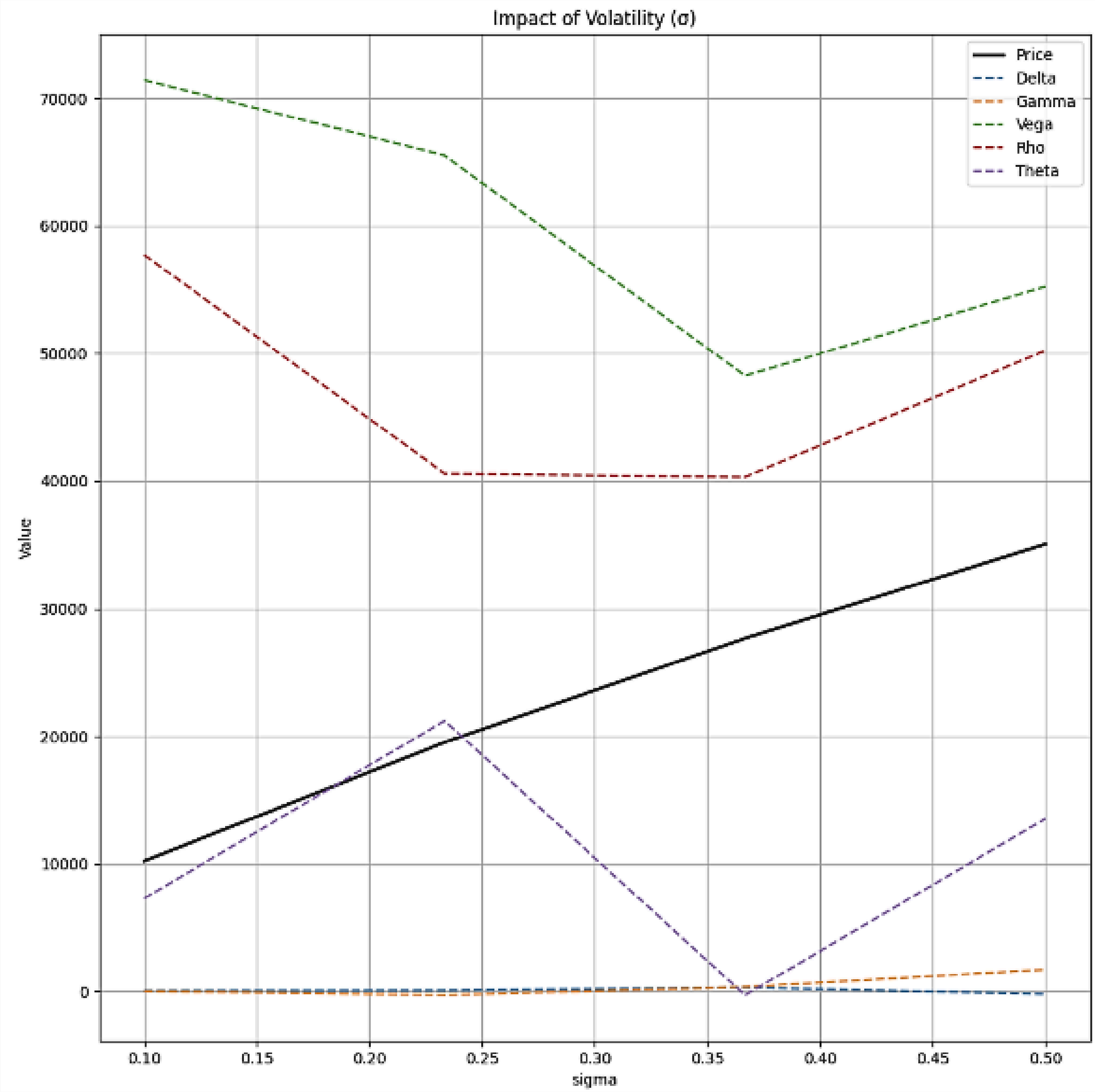
- High and peaking at r=0.04, then drops (69057 → 74820 → 63114 → 38271), then increases after r=0.07
- When interest rate is moderate, volatility has most influence Vega rises. At high r, drift dominates → volatility's role reduces → Vega falls.

Theta:

- Strongly positive and rising (24585 → 50884) but rises with a smaller slope at large r.
- As r increases the option price increases while also reducing the amount of time it takes to increase that much. Hence it is increasing

Parameters

✦ Impact of Volatility (σ)



- volatility is varied from 0.1 to 0.5

σ	Price	Delta	Gamma	Vega	Rho	Theta
0.10	10248	26.65	-114.45	64047	51117	2078
0.23	19106	-75.06	404.65	69671	57657	20461
0.37	27430	-124.18	159.27	69743	62368	33196
0.50	37062	-95.60	472.92	57702	64371	46783

Price:

- Price shows a steady increase in value as volatility increases.
- Higher volatility increases the difference between (min, max) and (final value, at the money strike price) of the asset hence increasing option price

Delta:

- Drops and turns negative→Price increase decouples from S_t at high volatility.
- Volatility creates more uncertainty, increasing the price but S_t sees a more variable path.

Gamma:

- Turns positive after $\sigma=0.10$ → increases → highly nonlinear convexity.
- Gamma measures convexity, i.e, change in delta → Reflects how payoff curvature increases due to wider spread in path endpoints.

Rho:

- Mildly increases → still positive.
- Because even in high volatility, the effect of r still boosts discount factor because r is constant but the price is increasing

Vega:

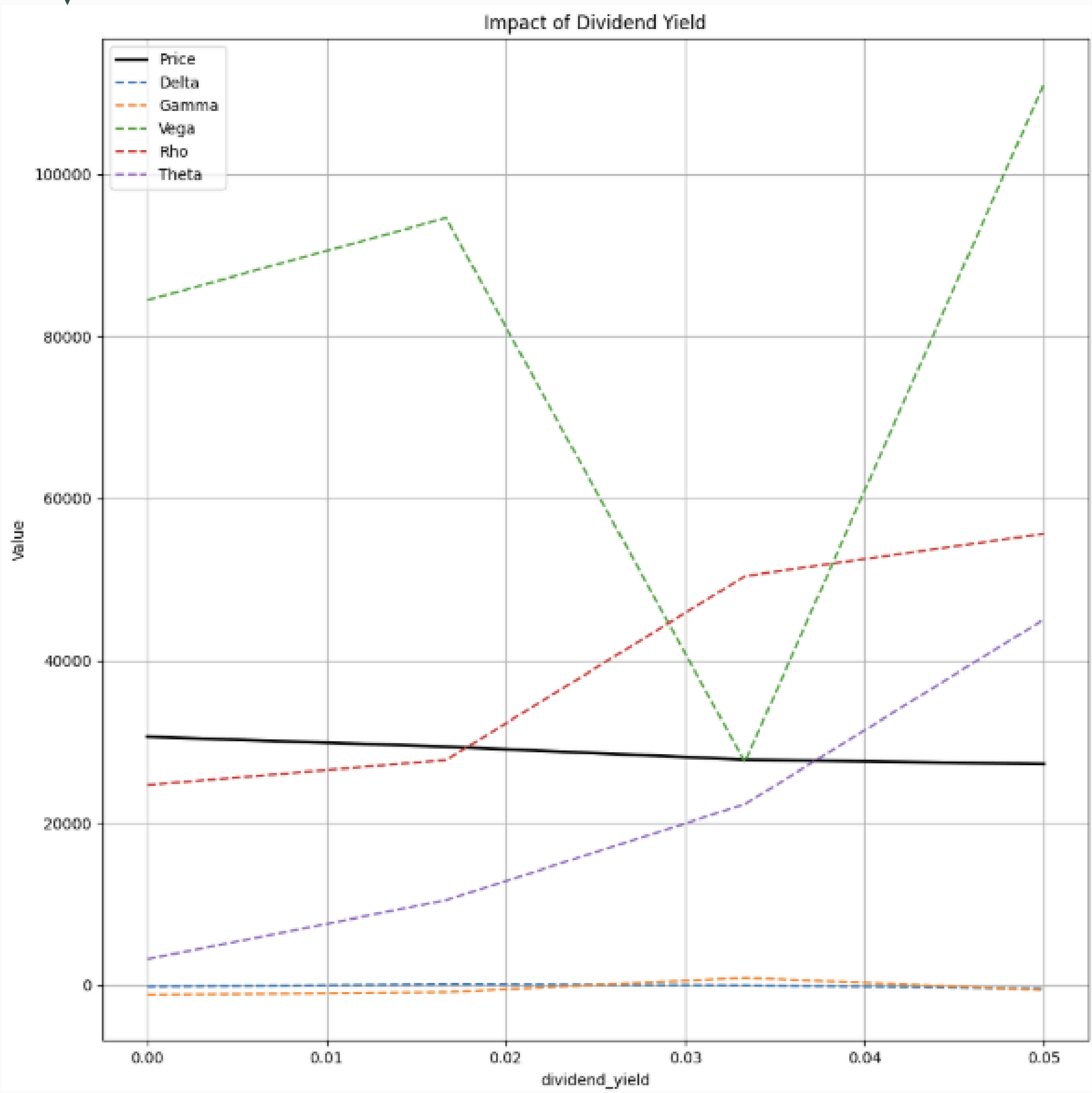
- Peaks around $\sigma=0.37$ then drops.
- Beyond a point, added volatility doesn't increase payoff much — min(S) already likely to be very low, indicates that volatility later has a faster increase than price increase

Theta:

- Grows in the beginning then drops then increases again
- As volatility creates more uncertainity in options price and value with which it grows (even if the growth is positive) is changing constantly with time. Hence we see such a non linear behaviour

Parameters

Impact of Dividend yield (q)



- **Dividend Yield is varied from 0.0 to 0.05 (0% to 5%)**

q	Price	Delta	Gamma	Vega	Rho	Theta
0.00	30483	-2.56	278.45	73604	62644	29049
0.017	29059	-186.90	-98.63	20761	69670	36458
0.033	28450	309.38	-1233.38	67170	70331	47880
0.050	25709	-21.99	493.34	36892	70731	60843

Price:

- Price shows a steady decrease in value as dividend yield increases.
- Dividend yield (q) reduces the drift term (r−q), slowing the asset’s expected growth.

Delta:

- Wild oscillation. From negative to large positive to negative.
- Dividends reduce spot growth mid-path, but if price rises later, effect may be masked. The change in drift affects underlying’s price differently at different values.

Gamma:

- Negative and large at mid q → recovers later. nonlinear just like gamma
- When dividend drag dominates, paths become more predictable → less convexity → Gamma negative. but has prices increase and prices n delta have reduced effect of dividend, gamma recovers.

Rho:

- Increases slightly → increases at lower slope then increases again
- Because if r is constant Rho demonstrates how dividend yield affects options price which is increasing, but fluctuating

Vega:

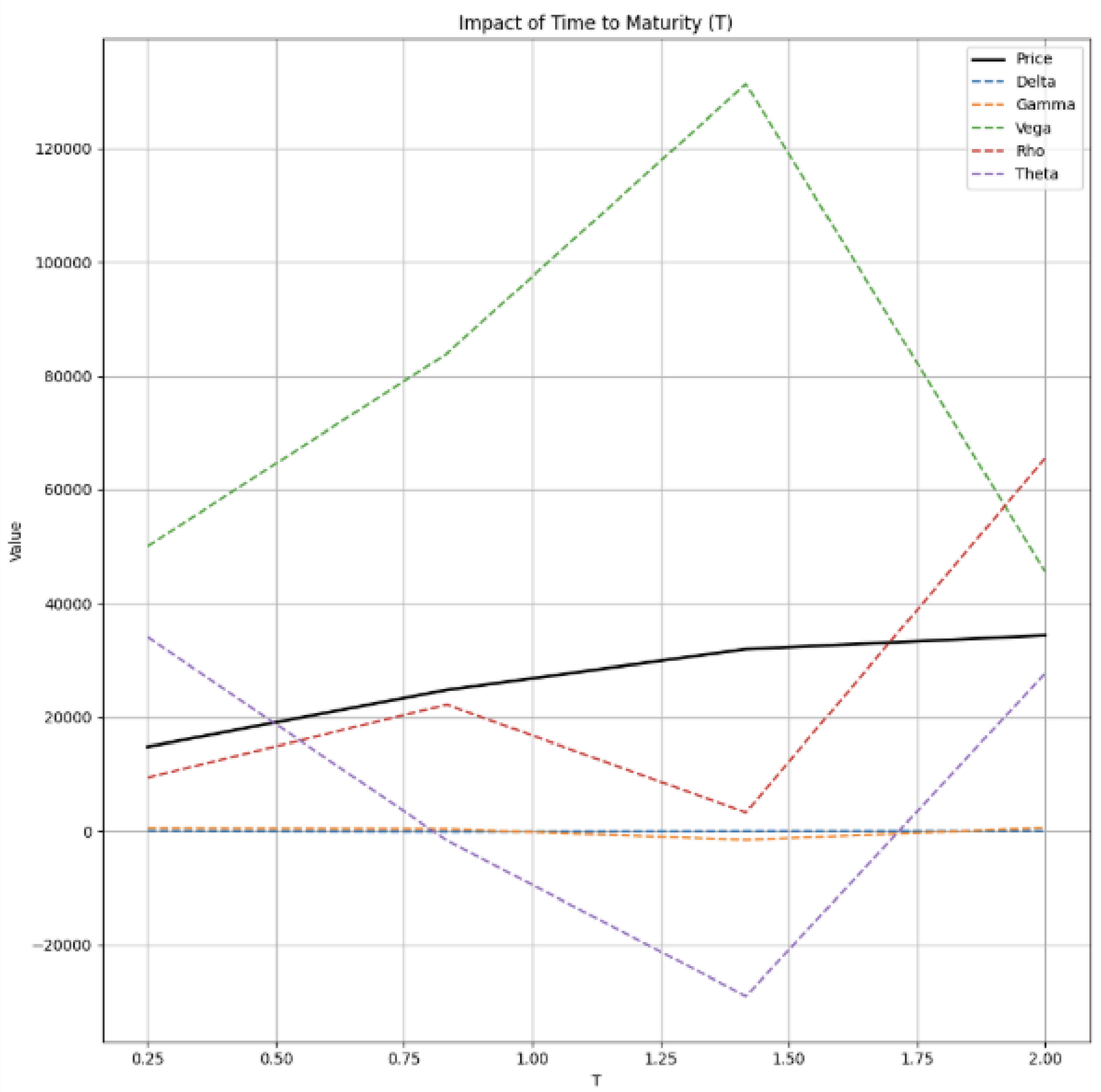
- Drops sharply then partially recovers.
- Since volatility is constant Vega demonstrates changes in option price, i.e, fluctuating, similar to rho.

Theta:

- Strongly increases
- Higher dividends reduce early payoff chances, so more time helps increase payoffs which increase at a later time, hence increasing Theta.

Parameters

Impact of Time to maturity (T)



- Time to maturity is varied from 0.25 years to 2 years)

T	Price	Delta	Gamma	Vega	Rho	Theta
0.25	14847	-75.94	301.83	18857	26315	13650
0.83	25124	-274.54	174.43	52054	45192	27966
1.42	30765	-90.99	255.26	59915	63258	44969
2.00	36610	173.42	39.31	61071	73349	63696

Price:

- Price rises and stays increasing but at a reducing rate
- As time to maturity decreases time step increases increasing the impact of drift term

Delta:

- Starts negative, rises to strong positive.
- At short maturity, the option is sensitive to current price going down—lowering the starting point means less chance for the min(S) to drop further → lower payoff. Later becomes positive: With more time, the option payoff is driven more by the gap S(T) − min(S), and less by where you start. So increasing S_0 increases S(T), lifting the payoff.

Gamma:

- Peaks early, then fades.
- With too much time, paths become “smooth” → reduced convexity → lower Gamma. Aligning with the delta experiencing a strong rise in the beginning (negative to positive) then stabilizing

Rho:

- Variable behaviour, increasing then decreasing then strongly increasing
- Since risk free interest rate is constant, rho demonstrates the variable change in options price, since with variable time S_t and min, max values deviate variably.

Vega:

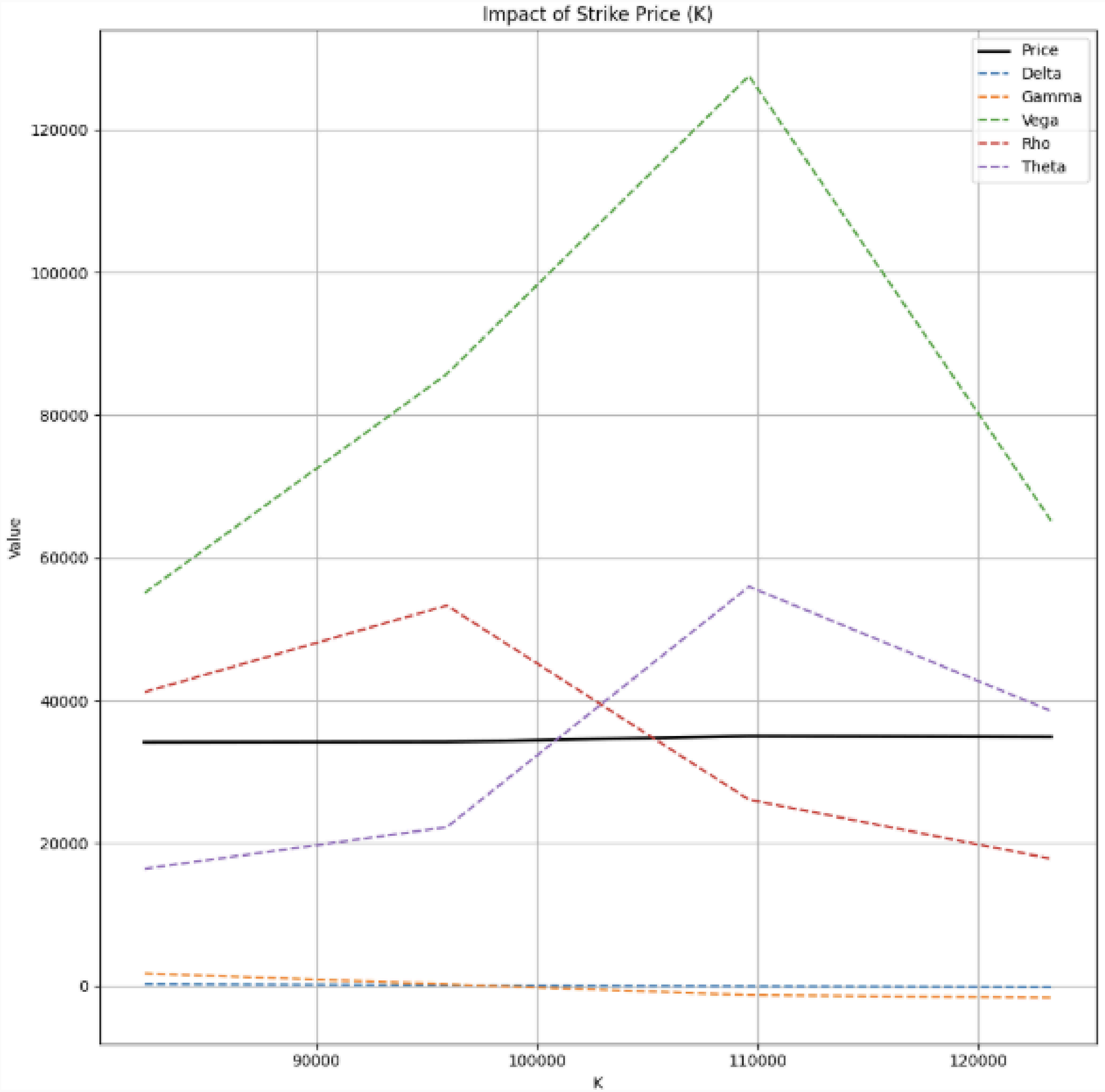
- Increases linearly then drops after a point
- Volatility is more powerful when there's more time—it has more space to operate. but after a while reduces as St gets just as much time to grow

Theta:

- Dips in the beginning to a negative value then increases
- When there is more time, there also more time for asset to grow, hence in the beginning theta dips to very low value but as time increases it starts increasing

Parameters

Impact of Strike price (K) (for fixed strike)



- **Strike price is varied from 80% of its value to 120% of its value**

Strike (K)	Price	Delta	Gamma	Vega	Rho	Theta
82,214.56	₹26,848.25	+248.02	+676.55	₹71,789.81	₹42,205.32	₹16,402.89
95,916.99	₹27,076.95	−321.87	+6.24	₹85,688.75	₹53,746.30	₹21,844.98
109,619.41	₹26,775.04	−83.99	+1100.59	₹40,997.05	₹25,664.10	₹56,656.69
123,322.64	₹28,671.91	−229.78	−433.63	₹12,943.80	₹16,560.29	₹38,808.62

Price:

- Almost stays the same, barely changes.
- As strike price has a major role in determining payoff the underlying price is barely affected.

Delta:

- Wildly fluctuates: +248 → −322 → −84 → −230
- At low K: Positive delta — the option's structure favors rising S_t more.
- At high K: Delta goes negative — possibly indicating that the simulation interprets higher K as detrimental.

Gamma:

- Strongly variable: +676 → +6 → +1100 → −433
- The curvature of the price vs. S_0 relationship is being distorted by changing K — again suggesting numerical volatility rather than a true economic effect..

Rho:

- Decreases with increasing K: 53,746 → 25,664 → 16,560
- Rho is highest when K is lower. This suggests interest rate sensitivity is higher when internal "strike-like" comparisons make the option more valuable. At higher K, the effect of r on drift may be muted in the simulation, hence lower Rho.

Vega:

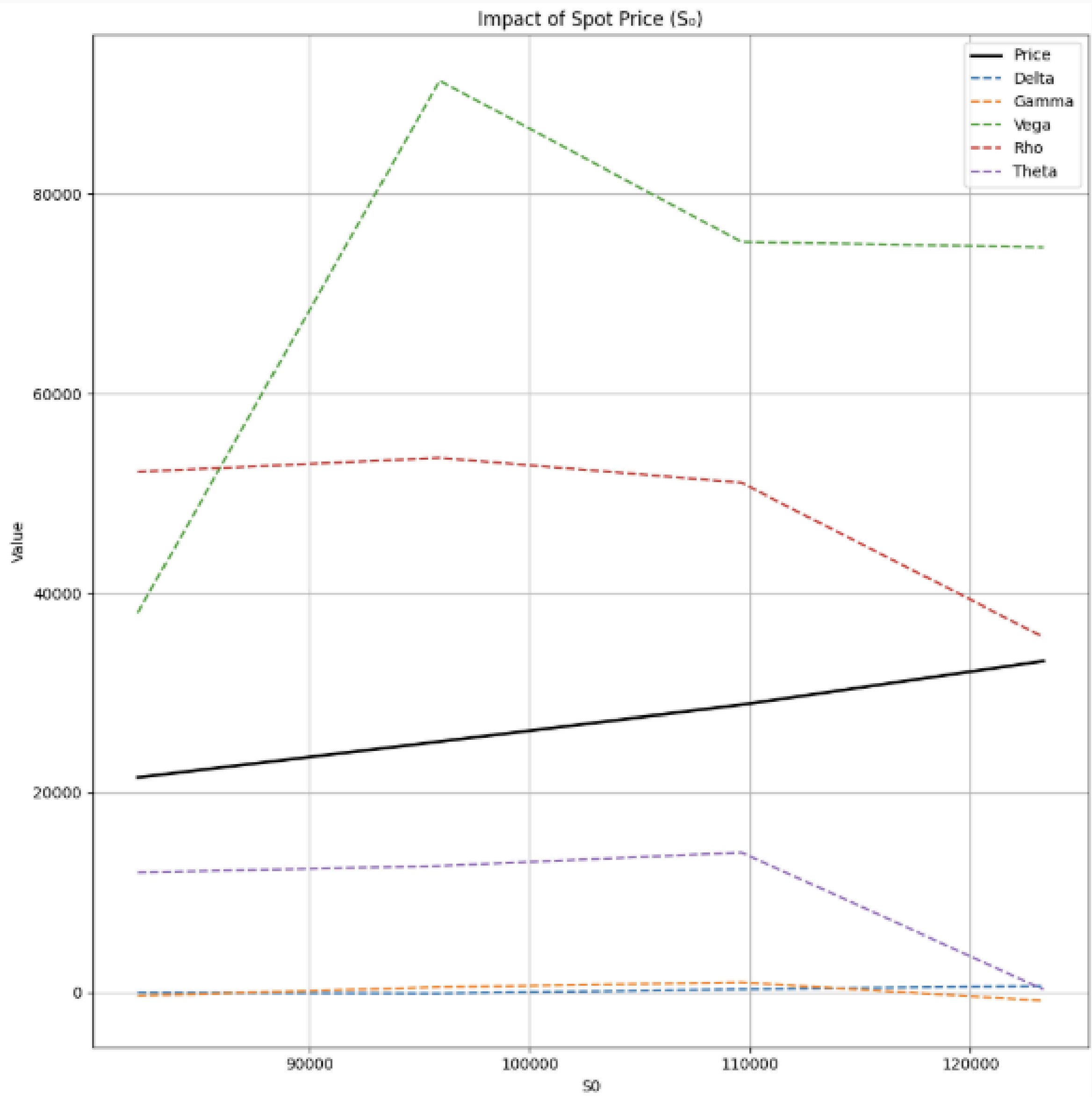
- Falls sharply: 85,688 → 40,997 → 12,943
- At high K, Vega drops → volatility has less effect, and S_t doesn't rise as fast as strike price is rising. That is why higher strike causes more out-of-money behavior in internal valuation.

Theta:

- Large increase: 16,402 → 56,657 at K ≈ 110k, then down to 38,809
- Strong positive Theta in the beginning reflects that the longer you wait, the better the payoff becomes, but as K rises and S_t cant rise so much theta falls.

Parameters

Impact of Spot price (S0)



- Spot price is varied from 80% of its value to 120% of its value

S_0	Price	Delta	Gamma	Vega	Rho	Theta
82215	21452	-32.99	156.91	50322	47587	22261
95917	25442	55.82	-660.70	48118	50360	27474
109619	29158	-197.86	-763.61	74792	57739	34706
123322	33953	-44.12	-185.99	88306	63736	42517

Price:

- Price rises at a steady rate
- As spot price is increased the underlying's price also increases since its value is derived from spot price itself (simply via drift)

Delta:

- Highly unstable: −32.99 → 55.82 → −197.86 → −44.12
- Because the change in S_0 affects both S(T) and min(S), and their relative behavior becomes chaotic under different starting points. For example, increasing S_0 lifts the whole path, but also means min(S) could still be relatively high or it might not also, hence the variability.

Gamma:

- Oscillates wildly, even negative: 156.91 → −660.70 → −763.61 → −185.99
- Gamma represents the curvature of the value wrt S_0: and this curvature flips sign depending on the region we're in aligning with gamma's instability.
- Higher S_0 means higher min(S) and S(T) doesn't rise enough, payoff decreases.

Rho:

- Decreases with a slightly increasing rate.
- Higher S_0 = higher S(T) (generally), which makes the final discounted payoff more sensitive to r. Therefore if max or min doesn't grow then rho id reduced

Vega:

- Increases then decreases then settles at a stable value.
- Higher S_0 = higher price path → more space for volatility to operate and dig valleys. But later when S_min starts increasing vega get reduced then gets stabilised for a favourable value

Theta:

- Experiences steady increase then decreases
- Aligning with vega's reasoning payoff is increasing as higher S_0 gives higher payoffs but then later when S_min effects starts dominating it loses value with time.

Conclusion

In this report, we performed a comprehensive analysis of the Greek sensitivities—Delta, Gamma, Vega, Rho, and Theta—for a floating-strike lookback call option using Monte Carlo simulations. By systematically varying key market parameters such as spot price, volatility, interest rate, dividend yield, time to maturity, and strike price, we explored how each Greek responds to changes in market conditions. The results highlight the complex, nonlinear behavior of exotic options: for example, Delta and Gamma exhibited erratic shifts due to path dependency, Vega remained highly influential due to the option's sensitivity to volatility, and Theta was notably positive in several cases, reflecting the added value of time in lookback structures. These insights underline the importance of detailed Greek analysis in pricing, hedging, and managing risk for exotic derivatives, where intuitive interpretations from vanilla options may no longer apply. The graphical and tabular results presented here serve as a practical toolkit for understanding and navigating the dynamic risk profile of lookback options.