Please fill your answers using an editor in the spaces provided in the PDF template. Then upload the result in PDF form to gradescope.

Name: Mansi Sharma **PID:** A59020070

Questions about "Duality"

Q1. What is epistemology?

Epistemology is the branch of philosophy concerned with the theory of knowledge. It deals with questions regarding the nature of knowledge, how it is acquired, and the extent to which any given subject or entity can be known. In the context of probability, epistemological probability refers to assessing reasonable degrees of belief in propositions quite devoid of statistical background. This aspect of probability theory deals with questions of knowledge, belief, and the justification of belief, rather than with the statistical or stochastic laws governing random processes.

Q2. How is weight different from mass?

Weight is dependent on both the mass of the object and the gravitational field strength. Mass is an intrinsic attribute that is independent of gravity and other external factors.
The chapter uses this distinction to draw a parallel with the difficulty in distinguishing between the aleatory (chance-based) and epistemological (belief-based) aspects of probability. Just as weight and mass are related but distinct concepts requiring different terms, the text argues that the dual nature of probability might similarly benefit from clear terminological distinctions to avoid confusion and conflation of fundamentally different concepts.

Q3. What does the word "probable" mean when used in the legal term "probable cause"?

When the word "probable" is used in the legal term "probable cause," it leans towards the epistemological aspect of probability rather than the aleatory (or statistical) aspect. In legal contexts, "probable cause" refers to a reasonable basis for believing that a crime may have been committed or that certain facts are likely to be true. This concept is fundamentally about assessing the credibility and reasonableness of belief based on evidence or circumstances, rather than calculating statistical probabilities.

Q4. Suppose the game described in class is stopped at a point where there are 5 heads and 9 tails. How should the money be divided? Explain how you get to the answer.

```
prob_heads_winning_directly = (0.5 ** 5)
prob_tails_winning = 1 - prob_heads_winning_directly
```

Based on the calculation, the probability of the heads player winning by getting 5 consecutive heads before the tails player gets 1 tail is approximately 3.125%, while the probability of the tails player winning is significantly higher at approximately 96.875%.
Thus, a fair division, based on the calculated probabilities, would be:
Tails player: 96.875% of $1 = $0.96875
Heads player: 3.125% of $1 = $0.03125

Q5. For each of the following, explain in one sentence whether a frequentist or a Bayesian approach is more appropriate and why:

(1) Estimating the probability of getting 5 heads in a row when flipping a fair coin.
(2) Estimating the probability that a specific jailed person, given bail, will attempt to flee from the country.
(3) Estimating the probability that a person in the ER has aids.

1) Frequentist approach is more appropriate because this situation involves repeated trials of a random process with well-defined probabilities.
2) Bayesian approach is more appropriate because this estimation requires incorporating prior knowledge and subjective judgement about the individual's behavior and circumstances.

(4) Estimating the probability that a roulette wheel will land on an even number.
(5) Estimating the probabilty that the next president would be a democrat.

3) Bayesian approach is more appropriate because it allows for the incorporation of prior probabilities (such as base rates of the condition and patient's risk factors) into the analysis.
4) Frequentist approach is more appropriate because the probability can be determined by the fixed odds of the game, based on the physical layout and rules of the roulette wheel.
5) Bayesian approach is more appropriate because this prediction requires incorporating prior information (such as historical election outcomes, current political climate, and polls) and updating beliefs as new information becomes available.

Questions about "Annuities"

Q6. What is perpetual annuity? Why is the value of a perpetual annuity not infinite?

A perpetual annuity is a type of financial arrangement where the annuitant receives annual payments indefinitely, without an end date. The payments continue for as long as the annuity agreement specifies, which, in the case of a perpetual annuity, means forever. Despite the potentially infinite duration of payments, the value of a perpetual annuity is not infinite due to the principle of the time value of money. This principle states that a dollar received today is worth more than a dollar received in the future because the money received today can be invested to earn interest over time. As a result, future payments are discounted to their present value, and the sum of these discounted payments (the present value of the annuity) is finite, even though payments are made indefinitely.

Q7. Why are life expectancy tables that are conservative for insurance companies, risky for companies that sell annuities.

Life expectancy tables that are conservative for insurance companies are designed to overestimate the likelihood of death, providing a statistical cushion that helps ensure the company remains profitable by charging higher premiums than might strictly be necessary to cover the actual risk of death. For companies that sell annuities, however, using these conservative tables can be risky because they underestimate the longevity of annuitants. Annuity sellers collect a lump sum upfront and pay out over the life of the annuitant; if people live longer than expected, the company may end up paying out more than it anticipated, eroding profits or causing losses.

Q8. What is the difference between mortality tables and life expectancy tables.

Mortality tables and life expectancy tables both relate to the study of demographics and the statistical analysis of death, but they serve different purposes. Mortality tables, also known as life tables, provide information about the death rates of a population at different ages. They show the likelihood of dying at each age and are used to calculate insurance premiums and pension benefits. Life expectancy tables, on the other hand, provide the average number of years a person of a certain age is expected to live. While mortality tables focus on the probability of death at each age, life expectancy tables summarize the overall survival outlook of individuals at different ages, often based on the data from mortality tables.

Q9. In 1660 it made economic sense to price annuities regardless of age, but it does not make sense these days, explain why.

In 1660, it made economic sense to price annuities regardless of age because the understanding and availability of detailed mortality and life expectancy data were limited, and the concept of adjusting annuity prices based on the age of the purchaser had not yet been widely accepted or implemented. Additionally, the population's general health and life expectancy were much lower than today, making the risk of significantly underestimating life expectancy less severe. In modern times, however, with advancements in medicine and public health, people are living much longer, making the cost of annuities more sensitive to the age of the annuitant. Pricing annuities without regard to age today would lead to significant financial losses for companies selling annuities to older individuals who, on average, are likely to live much longer than the populations of the 17th century. Therefore, adjusting the price of annuities based on age and life expectancy data is essential to maintain the financial viability of annuity products.