5 Time Value of Money

Learning Goals

- LG 1 Discuss the role of time value in finance, the use of computational tools, and the basic patterns of cash flow.
- LG 2 Understand the concepts of future value and present value, their calculation for single amounts, and the relationship between them.
- LG 3 Find the future value and the present value of both an ordinary annuity and an annuity due, and find the present value of a perpetuity.
- LG 4 Calculate both the future value and the present value of a mixed stream of cash flows.
- LG 5 Understand the effect that compounding interest more frequently than annually has on future value and on the effective annual rate of interest.
- LG 6 Describe the procedures involved in (1) determining deposits needed to accumulate a future sum, (2) loan amortization, (3) finding interest or growth rates, and (4) finding an unknown number of periods.

Why This Chapter Matters to You

In your *professional* life

ACCOUNTING You need to understand time-value-of-money calculations to account for certain transactions such as loan amortization, lease payments, and bond interest rates.

INFORMATION SYSTEMS You need to understand time-value-of-money calculations to design systems that accurately measure and value the firm's cash flows.

MANAGEMENT You need to understand time-value-of-money calculations so that you can manage cash receipts and disbursements in a way that will enable the firm to receive the greatest value from its cash flows.

MARKETING You need to understand time value of money because funding for new programs and products must be justified financially using time-value-of-money techniques.

OPERATIONS You need to understand time value of money because the value of investments in new equipment, in new processes, and in inventory will be affected by the time value of money.

In your *personal* life

Time-value-of-money techniques are widely used in personal financial planning. You can use them to calculate the value of savings at given future dates and to estimate the amount you need now to accumulate a given amount at a future date. You also can apply them to value lump-sum amounts or streams of periodic cash flows and to the interest rate or amount of time needed to achieve a given financial goal.

City of Cincinnati

Pay Me Now or Pay Me Later

In part due to a rather anemic economic recovery, many state and local governments across the United States have recently faced budget shortfalls. The city of Cincinnati found an interesting way to help plug the hole in its budget. In March 2013, the Cincinnati City Council voted five to four to hand over the management of 5,000 metered parking spaces and seven parking lots and garages to a private company. In the deal, the city would give up the revenues it had been collecting from parking fees. The private company would staff the lots and garages, and it would maintain and collect fees from the parking meters. In exchange for the right to run the city's parking operations, the company paid an up-front lump sum of $92 million to the city and promised to pay $3 million per year for the life of the deal (30 years).

Why did Cincinnati agree to such an arrangement? Perhaps it was because city leaders believe that the private company could run parking operations more efficiently than the city could. However, the deal was appealing, at least in part, because it allowed the city to accelerate its collection of parking revenues. Rather than collecting parking receipts as citizens paid them, Cincinnati received a large up-front payment along with a smaller annual stream of income. Cincinnati was not alone in making this decision. A few years earlier, Chicago had struck a similar deal, as had Indianapolis. New York had considered the idea but had rejected it.

How should city leaders evaluate such a proposal, which involves giving up a substantial stream of future income for (primarily) a lump sum payment? The answer lies in a concept known as the time value of money. The time value of money refers to a set of analytical tools that allows investors and other individuals to evaluate cash flows that arrive at different times in the future. At its most basic level, the time value of money simply recognizes that a dollar tomorrow is worth less than a dollar today because of the opportunity to earn interest. In other words, if you have a dollar today, you can invest it, and it will grow to more than a dollar in the future. The key issue for Cincinnati leaders to consider is whether the lump sum they received from the parking deal was worth more than the right to collect parking fees over time. In this chapter, you'll learn how to make such a comparison.

LG 1

5.1 The Role of Time Value in Finance

The *time value of money* refers to the observation that it is better to receive money sooner than later. Money that you have in hand today can be invested to earn a positive rate of return, producing more money tomorrow. For that reason, a dollar today is worth more than a dollar in the future. In business, managers constantly face trade-offs in situations in which actions that require outflows of cash today may produce inflows of cash later. Because the cash that comes in the future is worth less than the cash that firms spend up front, managers need a set of tools to help them compare cash inflows and outflows that occur at different times. This chapter introduces you to those tools.

FUTURE VALUE VERSUS PRESENT VALUE

Suppose that a firm has an opportunity to spend $15,000 today on some investment that will produce $17,000 spread out over the next 5 years as follows:

Year 1	$3,000
Year 2	$5,000
Year 3	$4,000
Year 4	$3,000
Year 5	$2,000

Is this investment a wise one? It might seem that the obvious answer is yes because the firm spends $15,000 and receives $17,000. Remember, though, that the value of the dollars the firm receives in the future is less than the value of the dollars that they spend today. Therefore, it is not clear whether the $17,000 inflows are enough to justify the initial investment.

Time-value-of-money analysis helps managers answer questions like this one. The idea is that managers need a way to compare cash today versus cash in the future. There are two ways of doing so. One way is to ask the question, What amount of money in the future is equivalent to $15,000 today? In other words, what is the *future value* of $15,000? The other approach asks, What amount today is equivalent to $17,000 paid out over the next 5 years as outlined above? In other words, what is the *present value* of the stream of cash flows coming in the next 5 years?

time line
A horizontal line on which time zero appears at the leftmost end and future periods are marked from left to right; can be used to depict investment cash flows.

A **time line** depicts the cash flows associated with a given investment. It is a horizontal line on which time zero appears at the leftmost end and future periods are marked from left to right. A time line illustrating our hypothetical investment problem appears in Figure 5.1. The cash flows occurring at time zero (today) and

FIGURE 5.1
Time Line
Time line depicting an investment's cash flows

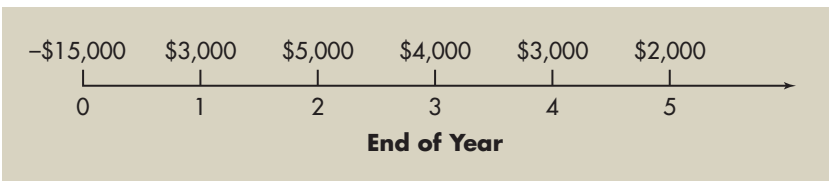
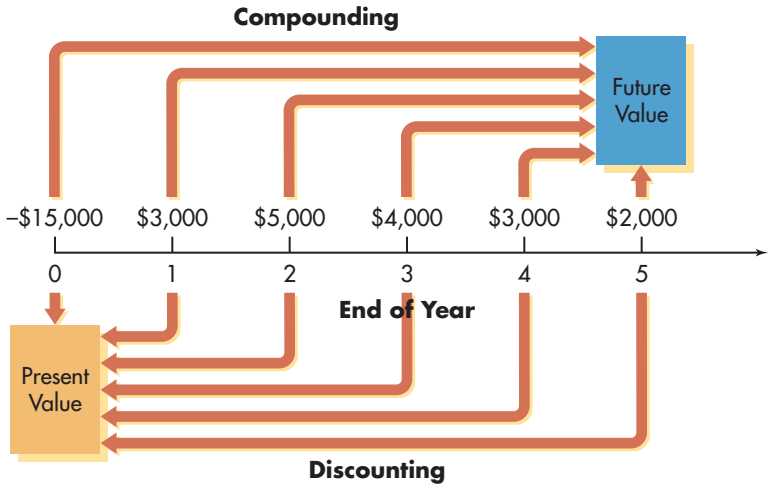


FIGURE 5.2

Compounding and Discounting
Time line showing compounding to find future value and discounting to find present value



at the end of each subsequent year are above the line; the negative values represent *cash outflows* ($15,000 invested today at time zero), and the positive values represent *cash inflows* ($3,000 inflow in 1 year, $5,000 inflow in 2 years, and so on).

To make the correct investment decision, managers need to compare the cash flows depicted in Figure 5.1 at a single point in time. Typically, that point is either the end or the beginning of the investment's life. The future value technique uses *compounding* to find the *future value* of each cash flow at the end of the investment's life and then sums these values to find the investment's future value. This approach is depicted above the time line in Figure 5.2. The figure shows that the future value of each cash flow is measured at the end of the investment's 5-year life. Alternatively, the present value technique uses *discounting* to find the *present value* of each cash flow at time zero and then sums these values to find the investment's value today. Application of this approach is depicted below the time line in Figure 5.2. In practice, when making investment decisions, *managers usually adopt the present value approach*.

COMPUTATIONAL TOOLS

Finding present and future values can involve time-consuming calculations. Although you should understand the concepts and mathematics underlying these calculations, financial calculators and spreadsheets streamline the application of time value techniques.

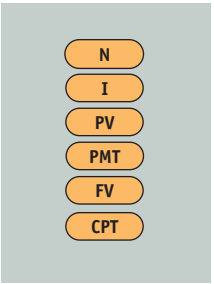
Financial Calculators

Financial calculators include numerous preprogrammed financial routines. Learning how to use these routines can make present and future values calculations a breeze.

We focus primarily on the keys pictured in Figure 5.3. We typically use four of the first five keys shown in the left column, along with the compute (**CPT**) key. One of the four keys represents the unknown value being calculated. The keystrokes on some of the more sophisticated calculators are menu-driven: After you

FIGURE 5.3

Calculator Keys
Important financial keys on the typical calculator



N — Number of periods
I — Interest rate per period
PV — Present value
PMT — Amount of payment (used only for annuities)
FV — Future value
CPT — Compute key used to initiate financial calculation once all values are input

select the appropriate routine, the calculator prompts you to input each value. Regardless, any calculator with the basic future and present value functions can simplify time-value-of-money calculations. The keystrokes for financial calculators are explained in the reference guides that accompany them.

Once you understand the underlying concepts, you probably will want to use a calculator to streamline calculations. With a little practice, you can increase both the speed and the accuracy of your financial computations. Remember that *conceptual understanding of the material is the objective*. An ability to solve problems with the aid of a calculator does not necessarily reflect such an understanding, so don't just settle for answers. Work with the material until you are sure that you also understand the concepts.

Electronic Spreadsheets

Like financial calculators, electronic spreadsheets have built-in routines that simplify time-value calculations. We provide in the text a number of spreadsheet solutions that identify the cell entries for calculating time values. The value for each variable is entered in a cell in the spreadsheet, and the calculation is programmed using an equation that links the individual cells. Changing any of the input variables automatically changes the solution as a result of the equation linking the cells.

Cash Flow Signs

To provide a correct answer, financial calculators and electronic spreadsheets require that a calculation's relevant cash flows be entered accurately as either cash inflows or cash outflows. Cash inflows are indicated by entering positive values, and cash outflows are indicated by entering negative values. By entering the cash flows correctly, you are providing the financial calculator or electronic spreadsheet the calculation's time line. With accurate cash flows entered, answers provided by financial calculators or electronic spreadsheets will indicate the proper result.

BASIC PATTERNS OF CASH FLOW

The cash flow—both inflows and outflows—of a firm can be described by its general pattern. It can be defined as a single amount, an annuity, or a mixed stream.

Single amount: A lump-sum amount either currently held or expected at some future date. Examples include $1,000 today and $650 to be received at the end of 10 years.

Annuity: A level periodic stream of cash flow. For our purposes, we'll work primarily with *annual* cash flows. Examples include either paying out or receiving $800 at the end of each of the next 7 years.

Mixed stream: A stream of cash flow that is *not* an annuity; a stream of unequal periodic cash flows that reflect no particular pattern. Examples include the following two cash flow streams A and B.

	Mixed cash flow stream	
End of year	A	B
1	$ 100	−$ 50
2	800	100
3	1,200	80
4	1,200	−60
5	1,400	
6	300	

Note that neither cash flow stream has equal, periodic cash flows and that A is a 6-year mixed stream and B is a 4-year mixed stream.

In the next three sections of this chapter, we develop the concepts and techniques for finding future and present values of single amounts, annuities, and mixed streams, respectively. Detailed demonstrations of these cash flow patterns are included.

→ REVIEW QUESTIONS

5–1 What is the difference between *future value* and *present value*? Which approach is generally preferred by financial managers? Why?

5–2 Define and differentiate among the three basic patterns of cash flow: (1) a single amount, (2) an annuity, and (3) a mixed stream.

LG 2 5.2 Single Amounts

Imagine that at age 25 you began investing $2,000 per year in an investment that earns 5 percent interest. At the end of 40 years, at age 65, you would have invested a total of $80,000 (40 years × $2,000 per year). How much would you have accumulated at the end of the fortieth year? $100,000? $150,000? $200,000? No, your $80,000 would have grown to $242,000! Why? Because the time value of money allowed your investments to generate returns that built on each other over the 40 years.

FUTURE VALUE OF A SINGLE AMOUNT

The most basic future value and present value concepts and computations concern single amounts, either present or future amounts. We begin by considering problems that involve finding the future value of cash that is on hand

immediately. Then we will use the underlying concepts to solve problems that determine the value today of cash that will be received or paid in the future.

We often need to find the value at some future date of a given amount of money placed on deposit today. For example, if you deposit $500 today into an account that pays 5 percent annual interest, how much would you have in the account in 10 years? **Future value** is the value at a given future date of an amount placed on deposit today and earning interest at a specified rate. The future value depends on the rate of interest earned and the length of time the money is left on deposit. Here we explore the future value of a single amount.

future value
The value at a given future date of an amount placed on deposit today and earning interest at a specified rate. Found by applying *compound interest* over a specified period of time.

compound interest
Interest that is earned on a given deposit and has become part of the *principal* at the end of a specified period.

principal
The amount of money on which interest is paid.

The Concept of Future Value

We speak of **compound interest** to indicate that the amount of interest earned on a given deposit has become part of the *principal* at the end of a specified period. The term **principal** refers to the amount of money on which the interest is paid. Annual compounding is the most common type.

The *future value* of a present amount is found by applying *compound interest* over a specified period of time. Savings institutions advertise compound interest returns at a rate of x percent, or x percent interest, compounded annually, semiannually, quarterly, monthly, weekly, daily, or even continuously. The concept of future value with annual compounding can be illustrated by a simple example.

Personal Finance Example 5.1 ► If Fred Moreno places $100 in a savings account paying 8% interest compounded annually, at the end of 1 year he will have $108 in the account, which is the initial principal of $100 plus 8% ($8) in interest. The future value at the end of the first year is

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$$\text{Future value at end of year 1} = \$100 \times (1 + 0.08) = \$108$$

If Fred were to leave this money in the account for another year, he would be paid interest at the rate of 8% on the new principal of $108. At the end of this second year, there would be $116.64 in the account. This amount would represent the principal at the beginning of year 2 ($108) plus 8% of the $108 ($8.64) in interest. The future value at the end of the second year is

$$\begin{aligned}\text{Future value at end of year 2} &= \$108 \times (1 + 0.08)\\ &= \$116.64\end{aligned}$$

Substituting the expression $\$100 \times (1 + 0.08)$ from the first-year calculation for the $108 value in the second-year calculation gives us

$$\begin{aligned}\text{Future value at end of year 2} &= \$100 \times (1 + 0.08) \times (1 + 0.08)\\ &= \$100 \times (1 + 0.08)^2\\ &= \$116.64\end{aligned}$$

The equations in the preceding example lead to a more general formula for calculating future value.

The Equation for Future Value

The basic relationship illustrated in Example 5.1 can be generalized to find the future value after any number of periods. We use the following notation for the various inputs:

FV_n = future value at the end of period n

PV = initial principal, or present value

r = annual rate of interest paid. (*Note*: On financial calculators, **I** is typically used to represent this rate.)

n = number of periods (typically years) that the money is left on deposit

The general equation for the future value at the end of period n is

$$FV_n = PV \times (1 + r)^n \tag{5.1}$$

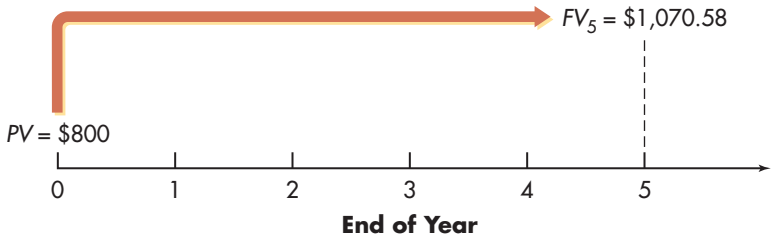
A simple example will illustrate how to apply Equation 5.1.

IRF Personal Finance Example 5.2 ▶ Jane Farber places $800 in a savings account paying 6% interest compounded annually. She wants to know how much money will be in the account at the end of 5 years. Substituting $PV = \$800$, $r = 0.06$, and $n = 5$ into Equation 5.1 gives the amount at the end of year 5:

$$FV_5 = \$800 \times (1 + 0.06)^5 = \$800 \times (1.33823) = \$1{,}070.58$$

This analysis can be depicted on a time line as follows:

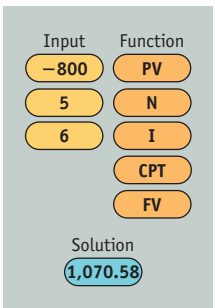
Time line for future value of a single amount ($800 initial principal, earning 6%, at the end of 5 years)



Solving the equation in the preceding example involves raising 1.06 to the fifth power. Using a financial calculator or electronic spreadsheet greatly simplifies the calculation.

Personal Finance Example 5.3 ▶ In Personal Finance Example 5.2, Jane Farber placed $800 in her savings account at 6% interest compounded annually and wishes to find out how much will be in the account at the end of 5 years.

MyFinanceLab Financial Calculator



Calculator use[1] The financial calculator can be used to calculate the future value directly. First enter –800 and depress **PV**; next enter 5 and depress **N**; then enter 6 and depress **I** (which is equivalent to "r" in our notation); finally, to calculate the future value, depress **CPT** and then **FV**. The future value of

1. Many calculators allow the user to set the number of payments per year. Most of these calculators are preset for monthly payments, or 12 payments per year. Because we work primarily with annual payments—one payment per year—it is important to *be sure that your calculator is set for one payment per year.* Although most calculators are preset to recognize that all payments occur at the end of the period, it is also important to *make sure that your calculator is correctly set on the END mode.* To avoid including previous data in current calculations, *always clear all registers of your calculator before inputting values and making each computation.* The known values *can be punched into the calculator in any order*; the order specified in this as well as other demonstrations of calculator use included in this text merely reflects convenience and personal preference.

$1,070.58 should appear on the calculator display as shown at the left. Remember that the calculator differentiates inflows from outflows by preceding the outflows with a negative sign. For example, in the problem just demonstrated, the $800 present value (PV), because it was keyed as a negative number, is considered an outflow. Therefore, the calculated future value (FV) of 1,070.58 is shown as a positive number to indicate that it is the resulting inflow. Had the $800 present value been keyed as a positive number (800), the future value of $1,070.58 would have been displayed as a negative number (–1,070.58). Simply stated, *the cash flows—present value (PV) and future value (FV)—will have opposite signs.* (*Note:* In future examples of calculator use, we will use only a display similar to that shown here. If you need a reminder of the procedures involved, review this paragraph.)

Spreadsheet use Excel offers a mathematical function that makes the calculation of future values easy. The format of that function is FV(rate,nper,pmt,pv,type). The terms inside the parentheses are inputs that Excel requires to calculate the future value. The terms *rate* and *nper* refer to the interest rate and the number of time periods, respectively. The term *pv* represents the lump sum (or present value) that you are investing today. For now, we will ignore the other two inputs, *pmt* and *type,* and enter a value of zero. The future value of the single amount also can be calculated as shown on the following Excel spreadsheet.

	A	B
1	FUTURE VALUE OF A SINGLE AMOUNT	
2	Present value	–$800
3	Annual rate of interest	6%
4	Number of years	5
5	Future value	$1,070.58

Entry in Cell B5 is =FV(B3,B4,0,B2,0).
The minus sign appears before the $800
in B2 because the cost of the investment
is treated as a cash outflow.

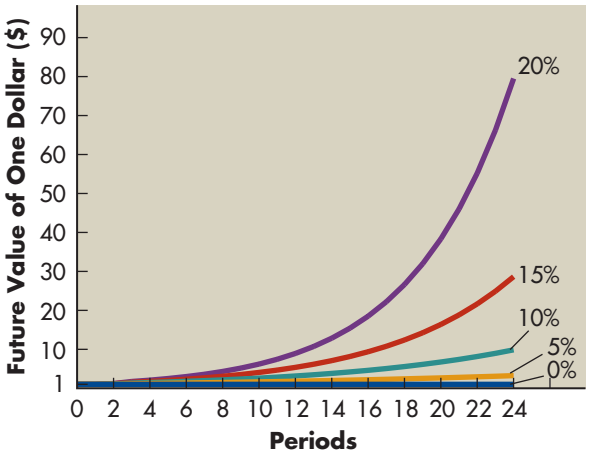
Changing any of the values in cells B2, B3, or B4 automatically changes the result shown in cell B5 because the formula in that cell links back to the others. As with the calculator, Excel reports cash inflows as positive numbers and cash outflows as negative numbers. In the example here, we have entered the $800 present value as a negative number, which causes Excel to report the future value as a positive number. Logically, Excel treats the $800 present value as a cash outflow, as if you are paying for the investment you are making, and it treats the future value as a cash inflow when you reap the benefits of your investment 5 years later.

A Graphical View of Future Value

Remember that we measure future value at the *end* of the given period. Figure 5.4 illustrates how the future value depends on the interest rate and the number of periods that money is invested. It shows that (1) the higher the interest rate, the higher the future value, and (2) the longer the period of time, the higher the future value. Note that for an interest rate of 0 percent, the future value always equals the present value ($1.00). For any interest rate greater than zero, however, the future value is greater than the present value of $1.00.

FIGURE 5.4

Future Value Relationship
Interest rates, time periods, and future value of one dollar



PRESENT VALUE OF A SINGLE AMOUNT

present value
The current dollar value of a future amount; the amount of money that would have to be invested today at a given interest rate over a specified period to equal the future amount.

It is often useful to determine the value today of a future amount of money. For example, how much would I have to deposit today into an account paying 7 percent annual interest to accumulate $3,000 at the end of 5 years? **Present value** is the current dollar value of a future amount, or the amount of money that would have to be invested today at a given interest rate over a specified period to equal the future amount. Like future value, the present value depends largely on the interest rate and the point in time at which the amount is to be received. This section explores the present value of a single amount.

The Concept of Present Value

discounting cash flows
The process of finding present values; the inverse of compounding interest.

The process of finding present values is often referred to as **discounting cash flows.** It is concerned with answering the following question: If I can earn r percent on my money, what is the most I would be willing to pay now for an opportunity to receive FV_n dollars n periods from today?

This process is actually the inverse of compounding interest. Instead of finding the future value of present dollars invested at a given rate, discounting determines the present value of a future amount, assuming an opportunity to earn a certain return on the money. This annual rate of return is variously referred to as the *discount rate, required return, cost of capital,* and *opportunity cost*. These terms will be used interchangeably in this text.

Personal Finance Example 5.4 ▸ Paul Shorter has an opportunity to receive $300 one year from now. If he can earn 6% on his investments in the normal course of events, what is the most he should pay now for this opportunity? To answer this question, Paul must determine how many dollars he would have to invest at 6% today to have $300 one year. Letting PV equal this unknown amount and using the same notation as in the future value discussion, we have

$$PV \times (1 + 0.06) = \$300$$

Solving for PV gives us

$$PV = \frac{\$300}{(1 + 0.06)}$$
$$= \$283.02$$

The value today ("present value") of $300 received 1 year from today, given an interest rate of 6%, is $283.02. That is, investing $283.02 today at 6% would result in $300 at the end of 1 year.

The Equation for Present Value

The present value of a future amount can be found mathematically by solving Equation 5.1 for PV. In other words, the present value, PV, of some future amount, FVn, to be received n periods from now, assuming an interest rate (or opportunity cost) of r, is calculated as

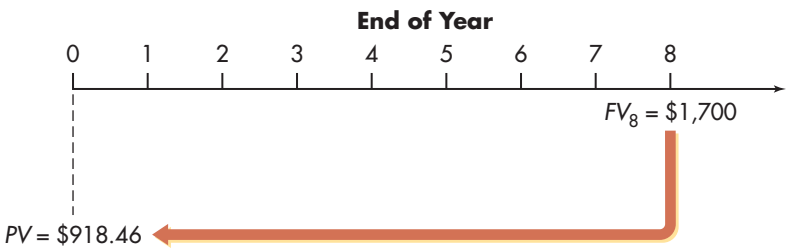
$$PV = \frac{FV_n}{(1 + r)^n} \quad (5.2)$$

IRF Personal Finance Example 5.5 ▶ Pam Valenti wishes to find the present value of $1,700 that she will receive 8 years from now. Pam's opportunity cost is 8%. Substituting $FV_8 = \$1{,}700$, $n = 8$, and $r = 0.08$ into Equation 5.2 yields

$$PV = \frac{\$1{,}700}{(1 + 0.08)^8} = \frac{\$1{,}700}{1.85093} = \$918.46$$

The following time line shows this analysis.

Time line for present value of a single amount ($1,700 future amount, discounted at 8%, from the end of 8 years)



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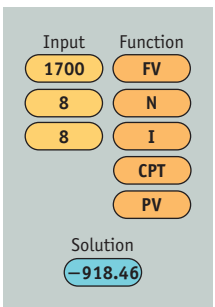


Calculator use Using the calculator's financial functions and the inputs shown at the left, you should find the present value to be $918.46. Notice that the calculator result is shown as a negative value to indicate that the present value is a cash outflow (that is, the investment's cost).

Spreadsheet use The format of Excel's present value function is very similar to the future value function covered earlier. The appropriate syntax is PV(rate,nper,pmt,fv,type). The input list inside the parentheses is the same as in Excel's future value function with one exception. The present value function contains the term *fv*, which represents the future lump sum payment (or receipt) whose present value you are trying to calculate. The present value of the single future amount also can be calculated as shown on the following Excel spreadsheet.

	A	B
1	PRESENT VALUE OF A SINGLE AMOUNT	
2	Future value	$1,700
3	Annual rate of interest	8%
4	Number of years	8
5	Present value	–$918.46

Entry in Cell B5 is =PV(B3,B4,0,B2,0).
The minus sign appears before the $918.46 in B5 because the cost of the investment is treated as a cash outflow.

A Graphical View of Present Value

Remember that present value calculations assume that the future values are measured at the *end* of the given period. The relationships among the factors in a present value calculation are illustrated in Figure 5.5. The figure clearly shows that, everything else being equal, (1) the higher the discount rate, the lower the present value, and (2) the longer the period of time, the lower the present value. Also note that given a discount rate of 0 percent, the present value always equals the future value ($1.00). But for any discount rate greater than zero, the present value is less than the future value of $1.00.

→ REVIEW QUESTIONS

5–3 How is the *compounding process* related to the payment of interest on savings? What is the general equation for future value?

5–4 What effect would a *decrease* in the interest rate have on the future value of a deposit? What effect would an *increase* in the holding period have on future value?

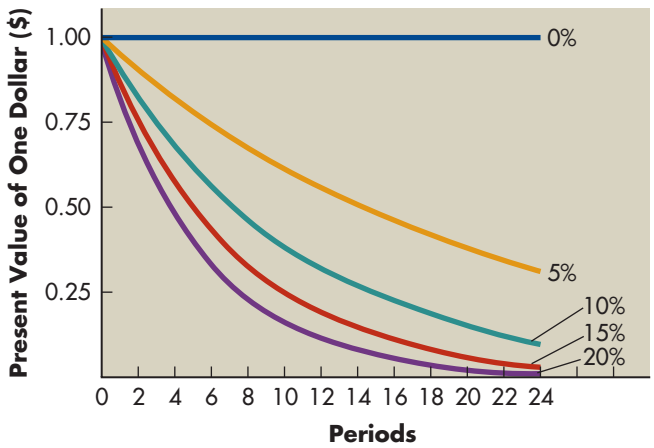
5–5 What is meant by "the present value of a future amount"? What is the general equation for present value?

5–6 What effect does *increasing* the required return have on the present value of a future amount? Why?

5–7 How are present value and future value calculations related?

FIGURE 5.5

Present Value Relationship
Discount rates, time periods, and present value of one dollar



→ EXCEL REVIEW QUESTIONS MyFinanceLab

5–8 It is tax time and you would like to make a tax deductible contribution to an Individual Retirement Account (IRA). Based on the information provided at MFL, find the future value of an IRA contribution grown until retirement.

5–9 It is never too soon to begin investing for a child's college education. Based on the information provided at MFL, determine the present value you would need to invest today to ensure that your child gets the college education she deserves.

LG 3 5.3 Annuities

annuity
A stream of equal periodic cash flows over a specified time period. These cash flows can be *inflows* of returns earned on investments or *outflows* of funds invested to earn future returns.

ordinary annuity
An annuity for which the cash flow occurs at the *end* of each period.

annuity due
An annuity for which the cash flow occurs at the *beginning* of each period.

How much would you pay today, given that you can earn 7 percent on low-risk investments, to receive a guaranteed $3,000 at the end of *each* of the next 20 years? How much will you have at the end of 5 years if your employer withholds and invests $1,000 of your bonus at the end of *each* of the next 5 years, guaranteeing you a 9 percent annual rate of return? To answer these questions, you need to understand the application of the time value of money to *annuities*.

An **annuity** is a stream of equal periodic cash flows, over a specified time period. These cash flows are usually annual but can occur at other intervals, such as monthly rent or car payments. The cash flows in an annuity can be *inflows* (the $3,000 received at the end of each of the next 20 years) or *outflows* (the $1,000 invested at the end of each of the next 5 years).

TYPES OF ANNUITIES

There are two general types of annuities. For an **ordinary annuity,** the cash flow occurs at the *end* of each period. For an **annuity due,** the cash flow occurs at the *beginning* of each period.

Personal Finance Example 5.6 ► Fran Abrams is evaluating two annuities. Both are 5-year, $1,000 annuities; annuity A is an ordinary annuity, and annuity B is an annuity due. To better understand the difference between these annuities, she has listed their cash flows in Table 5.1. The two annuities differ only in the timing of their cash flows: The cash flows occur sooner with the annuity due than with the ordinary annuity.

TABLE 5.1 Comparison of Ordinary Annuity and Annuity Due Cash Flows ($1,000, 5 Years)

	Annual cash flows	
Year	Annuity A (*ordinary*)	Annuity B (*annuity due*)
0	$ 0	$1,000
1	1,000	1,000
2	1,000	1,000
3	1,000	1,000
4	1,000	1,000
5	1,000	0
Totals	$5,000	$5,000

Although the cash flows of both annuities in Table 5.1 total $5,000, the annuity due would have a higher future value than the ordinary annuity because each of its five annual cash flows can earn interest for 1 year more than each of the ordinary annuity's cash flows. In general, as will be demonstrated later in this chapter, *the value (present or future) of an annuity due is always greater than the value of an otherwise identical ordinary annuity.*

Because ordinary annuities are more frequently used in finance, *unless otherwise specified, the term* annuity *is intended throughout this book to refer to ordinary annuities.*

FINDING THE FUTURE VALUE OF AN ORDINARY ANNUITY

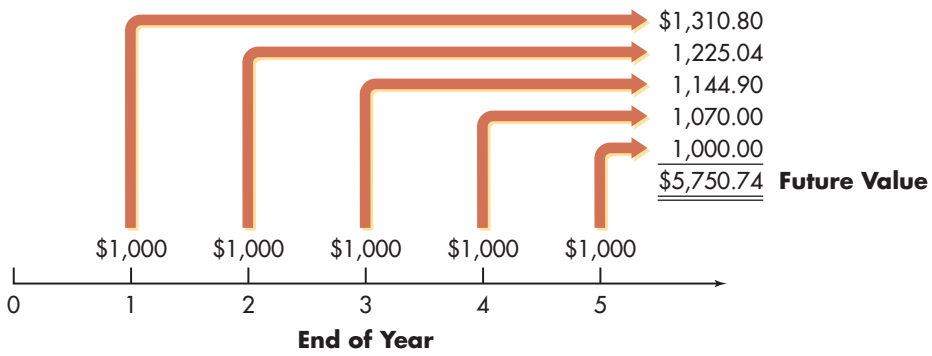
One way to find the future value of an ordinary annuity is to calculate the future value of each of the individual cash flows and then add up those figures. Fortunately, there are several shortcuts to get to the answer. You can calculate the future value of an ordinary annuity that pays an annual cash flow equal to *CF* by using Equation 5.3:

$$FV_n = CF \times \left\{ \frac{[(1 + r)^n - 1]}{r} \right\} \quad (5.3)$$

As before, in this equation r represents the interest rate, and n represents the number of payments in the annuity (or, equivalently, the number of years over which the annuity is spread). The calculations required to find the future value of an ordinary annuity are illustrated in the following example.

IRF Personal Finance Example 5.7 ► Fran Abrams wishes to determine how much money she will have at the end of 5 years if she chooses annuity A, the ordinary annuity. She will deposit $1,000 annually, at the *end of each* of the next 5 years, into a savings account paying 7% annual interest. This situation is depicted on the following time line.

Time line for future value of an ordinary annuity ($1,000 end-of-year deposit, earning 7%, at the end of 5 years)

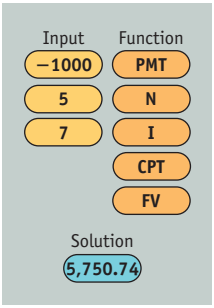


As the figure shows, at the end of year 5, Fran will have $5,750.74 in her account. Note that because the deposits are made at the end of the year, the first

deposit will earn interest for 4 years, the second for 3 years, and so on. Plugging the relevant values into Equation 5.3, we have

$$FV_5 = \$1{,}000 \times \left\{ \frac{[(1 + 0.07)^5 - 1]}{0.07} \right\} = \$5{,}750.74$$

MyFinanceLab Financial Calculator



Calculator use Using the calculator inputs shown at the left, you can confirm that the future value of the ordinary annuity equals $5,750.74. In this example, the $1,000 annuity payment is entered as a negative value because it is cash outflow, which in turn causes the calculator to correctly treat the resulting future value as a cash inflow (that is, the investment's payoff).

Spreadsheet use To calculate the future value of an annuity in Excel, we will use the same future value function that we used to calculate the future value of a lump sum, but we will add two new input values. Recall that the future value function's syntax is FV(rate,nper,pmt,pv,type). We have already explained the terms *rate*, *nper*, and *pv* in this function. The term *pmt* refers to the annual payment that the annuity offers. The term *type* is an input that lets Excel know whether the annuity being valued is an ordinary annuity (in which case the input value for *type* is 0 or omitted) or an annuity due (in which case the correct input value for *type* is 1). In this particular problem, the input value for *pv* is 0 or omitted because there is no up-front money received. The only cash flows are those that are part of the annuity stream. The future value of the ordinary annuity can be calculated as shown on the following Excel spreadsheet.

	A	B
1	FUTURE VALUE OF AN ORDINARY ANNUITY	
2	Annual annuity payment	–$1,000
3	Annual rate of interest	7%
4	Number of years	5
5	Future value	$5,750.74

Entry in Cell B5 is =FV(B3,B4,B2,0,0). The minus sign appears before the $1,000 in B2 because the annuity's payments are cash outflows.

FINDING THE PRESENT VALUE OF AN ORDINARY ANNUITY

Quite often in finance, there is a need to find the present value of a *stream* of cash flows to be received in future periods. An annuity is, of course, a stream of equal periodic cash flows. The method for finding the present value of an ordinary annuity is similar to the method just discussed. One approach would be to calculate the present value of each cash flow in the annuity and then add up those present values. Alternatively, the algebraic shortcut for finding the present value of an ordinary annuity that makes an annual payment of *CF* for *n* years looks like

$$PV_n = \left(\frac{CF}{r}\right) \times \left[1 - \frac{1}{(1 + r)^n}\right] \tag{5.4}$$

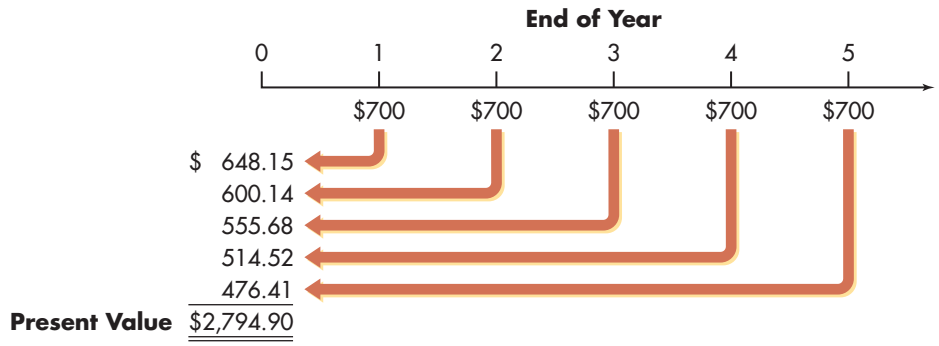
Of course, the simplest approach is to solve problems like this one with a financial calculator or spreadsheet program.

IRF Example 5.8 ▸

MyFinanceLab Solution Video

Braden Company, a small producer of plastic toys, wants to determine the most it should pay to purchase a particular ordinary annuity. The annuity consists of cash flows of $700 at the end of each year for 5 years. The firm requires the annuity to provide a minimum return of 8%. This situation is depicted on the following time line.

Time line for present value of an ordinary annuity ($700 end-of-year cash flows, discounted at 8%, over 5 years)



MyFinanceLab Financial Calculator

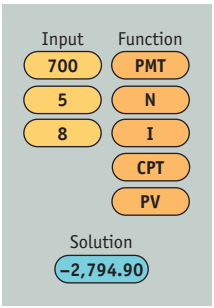


Table 5.2 shows that one way to find the present value of the annuity is to simply calculate the present values of all the cash payments using the present value equation (Equation 5.2 on page 218) and sum them. This procedure yields a present value of $2,794.90. Calculators and spreadsheets offer streamlined methods for arriving at this figure.

Calculator use Using the calculator's inputs shown at the left, you will find the present value of the ordinary annuity to be $2,794.90. Because the present value in this example is a cash outflow representing the cost of the annuity, it is shown as a negative value in the calculator display.

Spreadsheet use The present value of the ordinary annuity also can be calculated as shown on the following Excel spreadsheet.

TABLE 5.2 Long Method for Finding the Present Value of an Ordinary Annuity

Year (n)	Cash flow	Present value calculation	Present value
1	$700	$\frac{700}{(1 + 0.08)^1} =$	$ 648.15
2	700	$\frac{700}{(1 + 0.08)^2} =$	600.14
3	700	$\frac{700}{(1 + 0.08)^3} =$	555.68
4	700	$\frac{700}{(1 + 0.08)^4} =$	514.52
5	700	$\frac{700}{(1 + 0.08)^5} =$	476.41
		Present value of annuity	$2,794.90

	A	B
1	PRESENT VALUE OF AN ORDINARY ANNUITY	
2	Annual annuity payment	$700
3	Annual rate of interest	8%
4	Number of years	5
5	Present value	–$2,794.90

Entry in Cell B5 is =PV(B3,B4,B2,0,0).
The minus sign appears before the $2,794.90 in B5 because the annuity's present value is a cost and therefore a cash outflow.

FINDING THE FUTURE VALUE OF AN ANNUITY DUE

We now turn our attention to annuities due. Remember that the cash flows of an annuity due occur at the *start of the period*. In other words, if we are dealing with annual payments, each payment in an annuity due comes 1 year earlier than it would in an ordinary annuity, which in turn means that each payment can earn an extra year's worth of interest. That is why the future value of an annuity due exceeds the future value of an otherwise identical ordinary annuity.

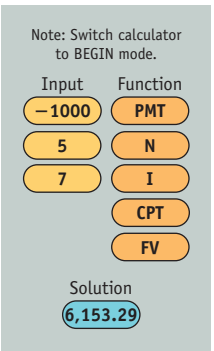
The algebraic shortcut for the future value of an annuity due that makes annual payments of CF for n years is

$$FV_n = CF \times \left\{ \frac{[(1 + r)^n - 1]}{r} \right\} \times (1 + r) \qquad (5.5)$$

Compare this equation with Equation 5.3 on page 221, which shows how to calculate the future value of an ordinary annuity. The two equations are nearly identical, but Equation 5.5 has an added term, $(1 + r)$, at the end. In other words, the value obtained from Equation 5.5 will be $(1 + r)$ times greater than the value in Equation 5.3 if the other inputs (CF and n) are the same, and that makes sense because all the payments in the annuity due earn 1 more year's worth of interest compared with the ordinary annuity.

IRF Personal Finance Example 5.9 ▶

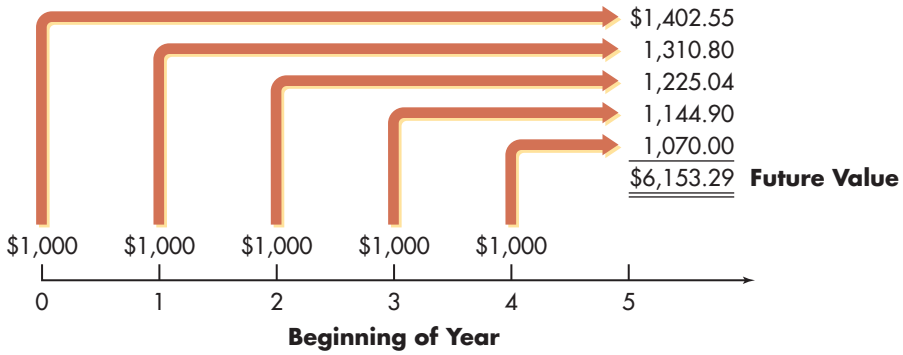
MyFinanceLab Financial Calculator



Recall from an earlier example, illustrated in Table 5.1 on page 220, that Fran Abrams wanted to choose between an ordinary annuity and an annuity due, both offering similar terms except for the timing of cash flows. We calculated the future value of the ordinary annuity in Example 5.7, but we now want to calculate the future value of the annuity due. This situation is depicted on the time line on the following page. We can calculate its future value using a calculator or a spreadsheet.

Calculator use Before using your calculator to find the future value of an annuity due, you must either switch it to BEGIN mode or use the DUE key, depending on the specific calculator. Then, using the inputs shown at the left, you will find the future value of the annuity due to be $6,153.29. (*Note:* Because we nearly always assume end-of-period cash flows, *be sure to switch your calculator back to END mode when you have completed your annuity-due calculations.*)

Time line for future value of an annuity due ($1,000 beginning-of-year deposit, earning 7%, at the end of 5 years)



Spreadsheet use The future value of the annuity due also can be calculated as shown on the following Excel spreadsheet. Remember that for an annuity due the *type* input value must be set to 1, and we must also specify the *pv* input value as 0 because the inputs are in an ordered series.

	A	B
1	FUTURE VALUE OF AN ANNUITY DUE	
2	Annual annuity payment	–$1,000
3	Annual rate of interest	7%
4	Number of years	5
5	Future value	$6,153.29

Entry in Cell B5 is =FV(B3,B4,B2,0,1).
The minus sign appears before the $1,000 in B2 because the annuity's payments are cash outflows.

Comparison of an Annuity Due with an Ordinary Annuity Future Value

The future value of an annuity due is *always greater* than the future value of an otherwise identical ordinary annuity. We can see that by comparing the future values at the end of year 5 of Fran Abrams's two annuities:

$$\text{Ordinary annuity} = \$5{,}750.74 \quad \text{versus} \quad \text{Annuity due} = \$6{,}153.29$$

Because the cash flow of the annuity due occurs at the beginning of the period rather than at the end (that is, each payment comes 1 year sooner in the annuity due), its future value is greater. How much greater? It is interesting to calculate the percentage difference between the value of the annuity and the value of the annuity due:

$$(\$6{,}153.29 - \$5{,}750.74) \div \$5{,}750.74 = 0.07 = 7\%$$

Recall that the interest rate in this example is 7 percent. It is no coincidence that the annuity due is 7 percent more valuable than the annuity. An extra year's interest on each of the annuity due's payments make the annuity due 7 percent more valuable than the annuity.

FINDING THE PRESENT VALUE OF AN ANNUITY DUE

We can also find the present value of an annuity due. This calculation can be easily performed by adjusting the ordinary annuity calculation. Because the cash flows of an annuity due occur at the beginning rather than the end of the period, to find their

present value each annuity due cash flow is discounted back 1 less year than for an ordinary annuity. The algebraic formula for the present value of an annuity due is

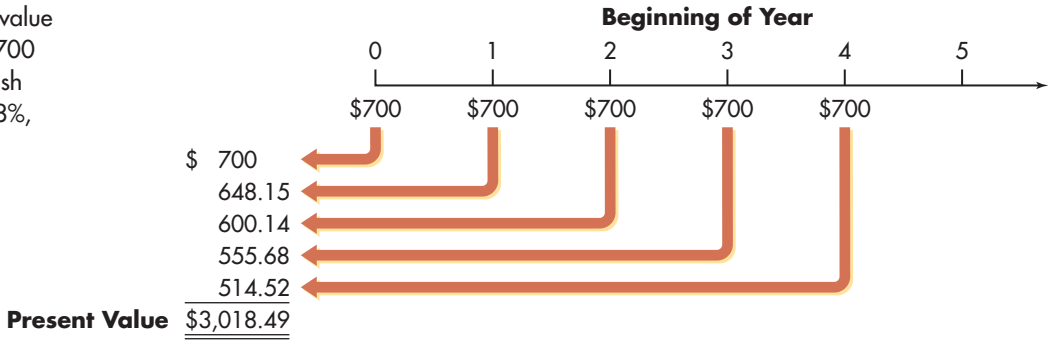
$$PV_n = \left(\frac{CF}{r}\right) \times \left[1 - \frac{1}{(1 + r)^n}\right] \times (1 + r) \quad (5.6)$$

Notice the similarity between this equation and Equation 5.4 on page 222. The two equations are identical except that Equation 5.6 has an extra term at the end, $(1+r)$. The reason for this extra term is the same as in the case when we calculated the future value of the annuity due. In the annuity due, each payment arrives 1 year earlier (compared to the annuity), so each payment is worth a little more, 1 year's interest more.

IRF Example 5.10 ▸

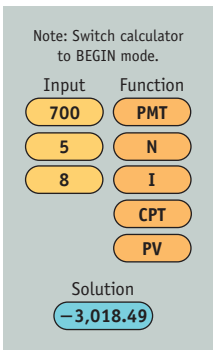
In Example 5.8 of Braden Company, we found the present value of Braden's \$700, 5-year ordinary annuity discounted at 8% to be \$2,794.90. If we now assume that Braden's \$700 annual cash flow occurs at the *start* of each year and is thereby an annuity due. This situation is depicted on the following time line.

Time line for present value of an annuity due ($700 beginning-of-year cash flows, discounted at 8%, over 5 years)



We can calculate its present value using a calculator or a spreadsheet.

MyFinanceLab Financial Calculator



Calculator use Before using your calculator to find the present value of an annuity due, you must either switch it to BEGIN mode or use the DUE key, depending on the specifics of your calculator. Then, using the inputs shown at the left, you will find the present value of the annuity due to be \$3,018.49 (*Note:* Because we nearly always assume end-of-period cash flows, *be sure to switch your calculator back to END mode when you have completed your annuity-due calculations.*)

Spreadsheet use The present value of the annuity due also can be calculated as shown on the following Excel spreadsheet.

	A	B
1	PRESENT VALUE OF AN ANNUITY DUE	
2	Annual annuity payment	\$700
3	Annual rate of interest	8%
4	Number of years	5
5	Present value	–\$3,018.49

Entry in Cell B5 is =PV(B3,B4,B2,0,1).
The minus sign appears before the \$3,018.49 in B5 because the annuity's present value is a cost and therefore a cash outflow.

Comparison of an Annuity Due with an Ordinary Annuity Present Value

The present value of an annuity due is always greater than the present value of an otherwise identical ordinary annuity. We can verify this statement by comparing the present values of the Braden Company's two annuities:

Ordinary annuity = \$2,794.90 versus Annuity due = \$3,018.49

Because the cash flow of the annuity due occurs at the beginning of the period rather than at the end, its present value is greater. If we calculate the percentage difference in the values of these two annuities, we will find that the annuity due is 8 percent more valuable than the annuity:

$$(\$3{,}018.49 - \$2{,}794.90) \div \$2{,}794.90 = 0.08 = 8\%$$

Matter of fact

Getting Your (Annuity) Due

Kansas truck driver Donald Damon got the surprise of his life when he learned that he held the winning ticket for the Powerball lottery drawing held November 11, 2009. The advertised lottery jackpot was \$96.6 million. Damon could have chosen to collect his prize in 30 annual payments of \$3,220,000 (30 × \$3.22 million = \$96.6 million), but instead he elected to accept a lump sum payment of \$48,367,329.08, roughly half the stated jackpot total.

FINDING THE PRESENT VALUE OF A PERPETUITY

perpetuity
An annuity with an infinite life, providing continual annual cash flow.

A **perpetuity** is an annuity with an infinite life. In other words, it is an annuity that never stops providing its holder with a cash flow at the end of each year (for example, the right to receive \$500 at the end of each year forever).

It is sometimes necessary to find the present value of a perpetuity. Fortunately, the calculation for the present value of a perpetuity is one of the easiest in finance. If a perpetuity pays an annual cash flow of CF, starting 1 year from now, the present value of the cash flow stream is

$$PV = CF \div r \tag{5.7}$$

IRF Personal Finance Example 5.11 ▶ Ross Clark wishes to endow a chair in finance at his alma mater. The university indicated that it requires \$200,000 per year to support the chair, and the endowment would earn 10% per year. To determine the amount Ross must give the university to fund the chair, we must determine the present value of a \$200,000 perpetuity discounted at 10%. Using Equation 5.7, we can determine that the present value of a perpetuity paying \$200,000 per year is \$2 million when the interest rate is 10%:

$$PV = \$200{,}000 \div 0.10 = \$2{,}000{,}000$$

In other words, to generate \$200,000 every year for an indefinite period requires \$2,000,000 today if Ross Clark's alma mater can earn 10% on its investments. If

the university earns 10% interest annually on the $2,000,000, it can withdraw $200,000 per year indefinitely.

→ REVIEW QUESTIONS

5-10 What is the difference between an *ordinary annuity* and an *annuity due*? Which is more valuable? Why?

5-11 What are the most efficient ways to calculate the present value of an ordinary annuity?

5-12 How can the formula for the future value of an annuity be modified to find the future value of an annuity due?

5-13 How can the formula for the present value of an ordinary annuity be modified to find the present value of an annuity due?

5-14 What is a *perpetuity*? Why is the present value of a perpetuity equal to the annual cash payment divided by the interest rate?

→ EXCEL REVIEW QUESTIONS MyFinanceLab

5-15 Since tax time comes around every year you smartly decide to make equal contributions to your IRA at the end of every year. Based on the information provided at MFL, calculate the future value of annual IRA contributions grown until retirement.

5-16 You have just graduated from college, begun your new career, and now it is time to buy your first home. Based on the information provided at MFL, determine how much you can spend for your new dream home.

5-17 Rather than making contributions to an IRA at the end of each year, you decide to make equal contributions at the beginning of each year. Based on the information provided at MFL, solve for the future value of beginning-of-year annual IRA contributions grown until retirement.

LG 4 5.4 Mixed Streams

mixed stream
A stream of unequal periodic cash flows that reflect no particular pattern.

Two types of cash flow streams are possible, the annuity and the mixed stream. Whereas an *annuity* is a pattern of equal periodic cash flows, a **mixed stream** is a stream of unequal periodic cash flows that reflect no particular pattern. Financial managers frequently need to evaluate opportunities that are expected to provide mixed streams of cash flows. Here we consider both the future value and the present value of mixed streams.

FUTURE VALUE OF A MIXED STREAM

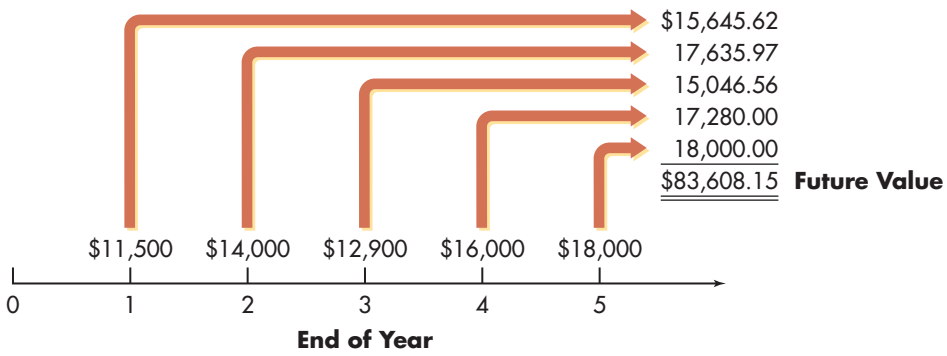
Determining the future value of a mixed stream of cash flows is straightforward. We determine the future value of each cash flow at the specified future date and then add all the individual future values to find the total future value.

IRF Example 5.12 ▸ Shrell Industries, a cabinet manufacturer, expects to receive the following mixed stream of cash flows over the next 5 years from one of its small customers.

End of year	Cash flow
1	$11,500
2	14,000
3	12,900
4	16,000
5	18,000

If Shrell expects to earn 8% on its investments, how much will it accumulate by the end of year 5 if it immediately invests these cash flows when they are received? This situation is depicted on the following time line.

Time line for future value of a mixed stream (end-of-year cash flows, compounded at 8% to the end of year 5)



Calculator use Unfortunately, unless you can program your calculator or have one of the more advanced financial calculators, most calculators lack a function that would allow you to input *all the cash flows*, specify the interest rate, and directly calculate the future value of the entire cash flow stream. Fortunately, there is a way to overcome this limitation. Use your calculator to find the future value of each individual cash flow, as demonstrated earlier (in Personal Finance Example 5.3 on page 215), and then sum the individual future values to get the future value of the entire cash flow stream. Summing the individual future values of Shrell Industries' mixed cash flow stream results in a future value of $83,608.15 at the end of year 5.

Spreadsheet use A relatively simple way to use Excel to calculate the future value of a mixed stream is to use the Excel net present value (NPV) function combined with the future value (FV) function discussed on page 216. The syntax of the NPV function is NPV(rate, value1, value2, value 3, . . .). The rate argument is the interest rate, and value1, value2, value3, . . . represent a stream of cash flows. The NPV function assumes that the first payment in the stream arrives 1 year in the future and that all subsequent payments arrive at 1-year intervals.

To find the future value of a mixed stream, the trick is to use the NPV function to first find the present value of the mixed stream and then find the future of this present value lump sum amount. The Excel spreadsheet at the top of the next page illustrates this approach (notice that the NPV is shown as an outflow because it represents the net present value of the stream of investment costs).

	A	B
1	FUTURE VALUE OF A MIXED STREAM	
2	Year	Cash Flow
3	1	–$11,500
4	2	–$14,000
5	3	–$12,900
6	4	–$16,000
7	5	–$18,000
8	Annual rate of interest	8%
9	NPV	–$56,902.30
10	Number of years	5
11	Future value	$83,608.15

Entry in Cell B9 is =NPV(B8,B3:B7).
Entry in Cell B11 is =FV(B8,B10,0,B9,0).
The minus sign appears before the values in B3:B7 because they are cash outflows.

PRESENT VALUE OF A MIXED STREAM

Finding the present value of a mixed stream of cash flows is similar to finding the future value of a mixed stream. We determine the present value of each future amount and then add all the individual present values together to find the total present value.

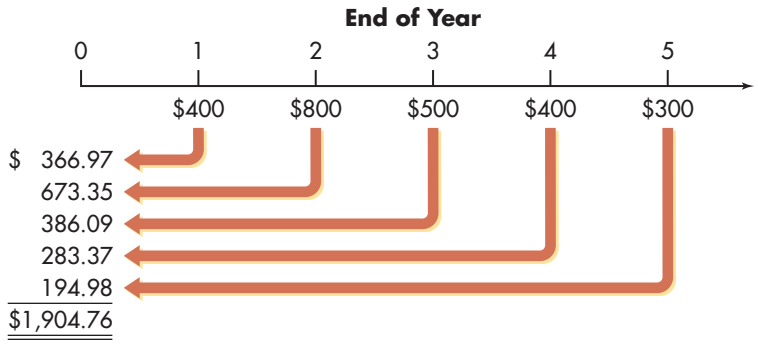
IRF Example 5.13 ▸

MyFinanceLab Solution Video

Frey Company, a shoe manufacturer, has been offered an opportunity to receive the following mixed stream of cash flows over the next 5 years.

End of year	Cash flow
1	$400
2	800
3	500
4	400
5	300

If the firm must earn at least 9% on its investments, what is the most it should pay for this opportunity? This situation is depicted on the following time line.



Time line for present value of a mixed stream (end-of-year cash flows, discounted at 9% over the corresponding number of years)

Calculator use You can use a calculator to find the present value of each individual cash flow, as demonstrated earlier (on page 218), and then sum the present values to get the present value of the stream. However, most financial calculators have a function that allows you to punch in *all cash flows,* specify the discount rate, and then directly calculate the present value of the entire cash flow stream. You can refer to your calculator's manual for the procedure to enter a stream of cash flows (the typical financial calculator will have a cash flow register). The present value of Frey Company's cash flow stream found using a calculator is $1,904.76.

Spreadsheet use To calculate the present value of a mixed stream in Excel, we will use the NPV function. The present value of the mixed stream of future cash flows can be calculated as shown on the following Excel spreadsheet.

	A	B
1	PRESENT VALUE OF A MIXED STREAM	
2	Year	Cash Flow
3	1	$400
4	2	$800
5	3	$500
6	4	$400
7	5	$300
8	Annual rate of interest	9%
9	Present value	$1,904.76
Entry in Cell B9 is =NPV(B8,B3:B7).		

→ REVIEW QUESTION

5-18 How is the future value of a mixed stream of cash flows calculated? How is the present value of a mixed stream of cash flows calculated?

→ EXCEL REVIEW QUESTION MyFinanceLab

5-19 To give yourself a financial head start after college you have decided to work summer jobs and invest the money you earn until after graduation. You expect that your earnings each summer will vary depending on the job you get. Based on the information provided at MFL, find the value of your financial head start after graduation.

LG 5 5.5 Compounding Interest More Frequently Than Annually

Interest is often compounded more frequently than once a year. Savings institutions compound interest semiannually, quarterly, monthly, weekly, daily, or even continuously. This section discusses various issues and techniques related to these more frequent compounding intervals.

SEMIANNUAL COMPOUNDING

semiannual compounding
Compounding of interest over two periods within the year.

Semiannual compounding of interest involves two compounding periods within the year. Instead of the stated interest rate being paid once a year, one-half of the stated interest rate is paid twice a year.

IRF Personal Finance Example 5.14 ▶ Fred Moreno has decided to invest $100 in a savings account paying 8% interest *compounded semiannually*. If he leaves his money in the account for 24 months (2 years), he will be paid 4% interest compounded over four periods, each of which is 6 months long. Table 5.3 shows that at the end of 12 months (1 year) with 8% semiannual compounding, Fred will have $108.16; at the end of 24 months (2 years), he will have $116.99.

TABLE 5.3 Future Value from Investing $100 at 8% Interest Compounded Semiannually over 24 Months (2 Years)

Period	Beginning principal	Future value calculation	Future value at end of period
6 months	$100.00	100.00 × (1 + 0.04) =	$104.00
12 months	104.00	104.00 × (1 + 0.04) =	108.16
18 months	108.16	108.16 × (1 + 0.04) =	112.49
24 months	112.49	112.49 × (1 + 0.04) =	116.99

QUARTERLY COMPOUNDING

quarterly compounding
Compounding of interest over four periods within the year.

Quarterly compounding of interest involves four compounding periods within the year. One-fourth of the stated interest rate is paid four times a year.

IRF Personal Finance Example 5.15 ▶ Fred Moreno has found an institution that will pay him 8% interest *compounded quarterly*. If he leaves his money in this account for 24 months (2 years), he will be paid 2% interest compounded over eight periods, each of which is 3 months long. Table 5.4 shows the amount Fred will have at the end of each period. At the end of 12 months (1 year), with 8% quarterly compounding, Fred will have $108.24; at the end of 24 months (2 years), he will have $117.17.

TABLE 5.4 Future Value from Investing $100 at 8% Interest Compounded Quarterly over 24 Months (2 Years)

Period	Beginning principal	Future value calculation	Future value at end of period
3 months	$100.00	100.00 × (1 + 0.02) =	$102.00
6 months	102.00	102.00 × (1 + 0.02) =	104.04
9 months	104.04	104.04 × (1 + 0.02) =	106.12
12 months	106.12	106.12 × (1 + 0.02) =	108.24
15 months	108.24	108.24 × (1 + 0.02) =	110.41
18 months	110.41	110.41 × (1 + 0.02) =	112.62
21 months	112.62	112.62 × (1 + 0.02) =	114.87
24 months	114.87	114.87 × (1 + 0.02) =	117.17

TABLE 5.5 Future Value at the End of Years 1 and 2 from Investing $100 at 8% Interest, Given Various Compounding Periods

	Compounding period		
End of year	Annual	Semiannual	Quarterly
1	$108.00	$108.16	$108.24
2	116.64	116.99	117.17

Table 5.5 compares values for Fred Moreno's $100 at the end of years 1 and 2 given annual, semiannual, and quarterly compounding periods at the 8 percent rate. The table shows that *the more frequently interest is compounded, the greater the amount of money accumulated.* This statement is true for *any interest rate* for *any period of time.*

A GENERAL EQUATION FOR COMPOUNDING MORE FREQUENTLY THAN ANNUALLY

The future value formula (Equation 5.1) can be rewritten for use when compounding takes place more frequently. If m equals the number of times per year interest is compounded, the formula for the future value of a lump sum becomes

$$FV_n = PV \times \left(1 + \frac{r}{m}\right)^{m \times n} \tag{5.8}$$

If $m = 1$, Equation 5.8 reduces to Equation 5.1. Thus, if interest compounds annually, Equation 5.8 will provide the same result as Equation 5.1. The general use of Equation 5.8 can be illustrated with a simple example.

IRF Personal Finance Example 5.16 ► The preceding examples calculated the amount that Fred Moreno would have at the end of 2 years if he deposited $100 at 8% interest compounded semiannually and compounded quarterly. For semiannual compounding, m would equal 2 in Equation 5.8; for quarterly compounding, m would equal 4. Substituting the appropriate values for semiannual and quarterly compounding into Equation 5.7, we find that

1. *For semiannual compounding:*

$$FV_2 = \$100 \times \left(1 + \frac{0.08}{2}\right)^{2 \times 2} = \$100 \times (1 + 0.04)^4 = \$116.99$$

2. *For quarterly compounding:*

$$FV_2 = \$100 \times \left(1 + \frac{0.08}{4}\right)^{4 \times 2} = \$100 \times (1 + 0.02)^8 = \$117.17$$

These results agree with the values for FV_2 in Tables 5.4 and 5.5.

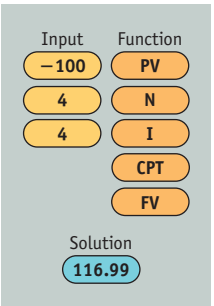
If the interest were compounded monthly, weekly, or daily, m would equal 12, 52, or 365, respectively.

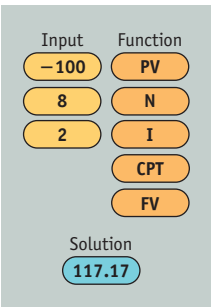
USING COMPUTATIONAL TOOLS FOR COMPOUNDING MORE FREQUENTLY THAN ANNUALLY

As before, we can simplify the process of doing the calculations by using a calculator or spreadsheet program.

Personal Finance Example 5.17 ▶ Fred Moreno wished to find the future value of $100 invested at 8% interest compounded both semiannually and quarterly for 2 years.

MyFinanceLab Financial Calculator





Calculator use If the calculator were used for the semiannual compounding calculation, the number of periods would be 4, and the interest rate would be 4%. The future value of $116.99 will appear on the calculator display as shown at the top left.

For the quarterly compounding case, the number of periods would be 8 and the interest rate would be 2%. The future value of $117.17 will appear on the calculator display as shown in the second display at the left.

Spreadsheet use The future value of the single amount with semiannual and quarterly compounding also can be calculated as shown on the following Excel spreadsheet.

	A	B
1	FUTURE VALUE OF A SINGLE AMOUNT WITH SEMIANNUAL AND QUARTERLY COMPOUNDING	
2	Present value	–$100
3	Annual rate of interest	8%
4	Compounding frequency - semiannual	2
5	Number of years	2
6	Future value with semiannual compounding	$116.99
7	Present value	–$100
8	Annual rate of interest	8%
9	Compounding frequency - quarterly	4
10	Number of years	2
11	Future value with quarterly compounding	$117.17

Entry in Cell B6 is =FV(B3/B4,B5*B4,0,B2,0).
Entry in Cell B11 is =FV(B8/B9,B10*B9,0,B7,0).
The minus sign appears before the $100 in B2 and B7 because the cost of the investment is treated as a cash outflow.

CONTINUOUS COMPOUNDING

continuous compounding Compounding interest literally all the time. Equivalent to compounding interest an infinite number of times per year.

In the extreme case, interest can be compounded continuously. In this case interest is compounded every second (or even every nanosecond)—literally, interest compounds all the time. In this case, m in Equation 5.8 would approach infinity. Through the use of calculus, we know that as m approaches infinity, Equation 5.8 converges to

$$FV_n = (PV) \times (e^{r \times n}) \tag{5.9}$$

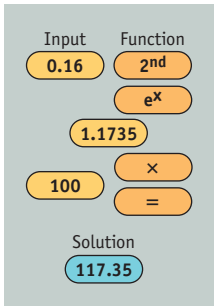
where e is the exponential function,[2] which has a value of approximately 2.7183.

2. Most calculators have the exponential function, typically noted by e^x, built into them. The use of this key is especially helpful in calculating future value when interest is compounded continuously.

IRF Personal Finance Example 5.18 ▶ To find the value at the end of 2 years ($n = 2$) of Fred Moreno's \$100 deposit ($PV = \$100$) in an account paying 8% annual interest ($r = 0.08$) compounded continuously, we can substitute into Equation 5.9:

$$FV_2 \text{ (continuous compounding)} = \$100 \times e^{0.08 \times 2}$$
$$= \$100 \times 2.7183^{0.16}$$
$$= \$100 \times 1.1735 = \$117.35$$

MyFinanceLab Financial Calculator



Calculator use To find this value using the calculator, you need first to find the value of $e^{0.16}$ by punching in 0.16 and then pressing **2nd** and then $\mathbf{e^x}$ to get 1.1735. Next multiply this value by \$100 to get the future value of \$117.35 as shown at the left. (*Note:* On some calculators, you may not have to press **2nd** before pressing $\mathbf{e^x}$.)

Spreadsheet use The future value of the single amount with continuous compounding of Fred's deposit also can be calculated as shown on the following Excel spreadsheet.

	A	B
1	FUTURE VALUE OF A SINGLE AMOUNT WITH CONTINUOUS COMPOUNDING	
2	Present value	\$100
3	Annual rate of interest, compounded continuously	8%
4	Number of years	2
5	Future value with continuous compounding	\$117.35
	Entry in Cell B5 is =B3*EXP(B3*B4).	

The future value with continuous compounding therefore equals \$117.35. As expected, the continuously compounded value is larger than the future value of interest compounded semiannually (\$116.99) or quarterly (\$117.17). In fact, continuous compounding produces a greater future value than any other compounding frequency.

NOMINAL AND EFFECTIVE ANNUAL RATES OF INTEREST

nominal (stated) annual rate
Contractual annual rate of interest charged by a lender or promised by a borrower.

effective (true) annual rate (EAR)
The annual rate of interest actually paid or earned.

Both businesses and investors need to make objective comparisons of loan costs or investment returns over different compounding periods. To put interest rates on a common basis, so as to allow comparison, we distinguish between nominal and effective annual rates. The **nominal, or stated, annual rate** is the contractual annual rate of interest charged by a lender or promised by a borrower. The **effective, or true, annual rate (EAR)** is the annual rate of interest actually paid or earned. The effective annual rate reflects the effects of compounding frequency, whereas the nominal annual rate does not.

Using the notation introduced earlier, we can calculate the effective annual rate, EAR, by substituting values for the nominal annual rate, r, and the compounding frequency, m, into the equation

$$EAR = \left(1 + \frac{r}{m}\right)^m - 1 \qquad (5.10)$$

We can apply Equation 5.10 using data from preceding examples.

Personal Finance Example 5.19 ▶ Fred Moreno wishes to find the effective annual rate associated with an 8% nominal annual rate ($r = 0.08$) when interest is compounded (1) annually ($m = 1$), (2) semiannually ($m = 2$), and (3) quarterly ($m = 4$). Substituting these values into Equation 5.10, we get

1. *For annual compounding:*

$$EAR = \left(1 + \frac{0.08}{1}\right)^1 - 1 = (1 + 0.08)^1 - 1 = 1 + 0.08 - 1 = 0.08 = 8\%$$

2. *For semiannual compounding:*

$$EAR = \left(1 + \frac{0.08}{2}\right)^2 - 1 = (1 + 0.04)^2 - 1 = 1.0816 - 1 = 0.0816 = 8.16\%$$

3. *For quarterly compounding:*

$$EAR = \left(1 + \frac{0.08}{4}\right)^4 - 1 = (1 + 0.02)^4 - 1 = 1.0824 - 1 = 0.0824 = 8.24\%$$

Calculator use To find the *EAR* using the calculator, you first need to enter the nominal annual rate and the compounding frequency per year. Most financial calculators have a NOM key for entering the nominal rate and either a P/Y or C/Y key for entering the compounding frequency per year. Once these inputs are entered, the EFF or CPT key is depressed to display the corresponding effective annual rate.

Spreadsheet use Interest rate conversions are easily done using Excel using the EFFECT and NOMINAL functions. To find the *EAR*, the EFFECT function requires you to input nominal annual rate and the compounding frequency, whereas if you input an *EAR* and the compounding frequency, the NOMINAL function provides the nominal annual rate or *APR*. Interest rate conversions from the 8% *APR* to the semiannual *EAR* and from the quarterly *EAR* back to the 8% *APR* are shown on the following Excel spreadsheet.

	A	B
1	INTEREST RATE CONVERSION NOMINAL VS. EFFECTIVE ANNUAL RATE	
2	Nominal annual rate of interest	8%
3	Compounding frequency - semiannual	2
4	Effective annual rate of interest	8.16%
5	Nominal annual rate of interest	8%
6	Compounding frequency - quarterly	4
7	Effective annual rate of interest	8.24%
	Entry in Cell B4 is =EFFECT(B2,B3). Entry in Cell B5 is =NOMINAL(B7,B6).	

These values demonstrate two important points. First, nominal and effective annual rates are equivalent for annual compounding. Second, the effective annual

rate increases with increasing compounding frequency, up to a limit that occurs with *continuous compounding*.[3]

For an EAR example related to the "payday loan" business, with discussion of the ethical issues involved, see the *Focus on Ethics* box.

annual percentage rate (APR) The *nominal annual rate* of interest, found by multiplying the periodic rate by the number of periods in one year, that must be disclosed to consumers on credit cards and loans as a result of "truth-in-lending laws."

annual percentage yield (APY) The *effective annual rate* of interest that must be disclosed to consumers by banks on their savings products as a result of "truth-in-savings laws."

At the consumer level, "truth-in-lending laws" require disclosure on credit card and loan agreements of the **annual percentage rate (APR).** The APR is the *nominal annual rate,* which is found by multiplying the periodic rate by the number of periods in 1 year. For example, a bank credit card that charges 1.5 percent per month (the periodic rate) would have an APR of 18 percent (1.5% per month × 12 months per year).

"Truth-in-savings laws," on the other hand, require banks to quote the **annual percentage yield (APY)** on their savings products. The APY is the *effective annual rate* a savings product pays. For example, a savings account that pays 0.5 percent per month would have an APY of 6.17 percent $[(1.005)^{12} - 1]$.

Quoting loan interest rates at their lower nominal annual rate (the APR) and savings interest rates at the higher effective annual rate (the APY) offers two advantages. First, it tends to standardize disclosure to consumers. Second, it enables financial institutions to quote the most attractive interest rates: low loan rates and high savings rates.

→ REVIEW QUESTIONS

5-20 What effect does compounding interest more frequently than annually have on (**a**) future value and (**b**) the *effective annual rate (EAR)*? Why?

5-21 How does the future value of a deposit subject to continuous compounding compare to the value obtained by annual compounding?

5-22 Differentiate between a *nominal annual rate* and an *effective annual rate (EAR)*. Define *annual percentage rate (APR)* and *annual percentage yield (APY)*.

→ EXCEL REVIEW QUESTIONS MyFinanceLab

5-23 You are responsible for managing your company's short term investments and you know that the compounding frequency of investment opportunities is quite important. Based on the information provided at MFL, calculate the future value of an investment opportunity based on various compounding frequencies.

5-24 What if your short term investments provide continuous compounding? Based on the information provided at MFL, determine the future value of an investment opportunity based on continuous compounding.

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3. The effective annual rate for this extreme case can be found by using the equation

$$EAR \text{ (continuous compounding)} = e^{r} - 1 \qquad (5.10a)$$

For the 8% nominal annual rate ($r = 0.08$), substitution into Equation 5.10a results in an effective annual rate of

$$e^{0.08} - 1 = 1.0833 - 1 = 0.0833 = 8.33\%$$

in the case of continuous compounding. This result is the highest effective annual rate attainable with an 8% nominal rate.

focus on **ETHICS**

How Fair Is "Check Into Cash"?

in practice In 1993, the first Check Into Cash location opened in Cleveland, Tennessee. Today, there are more than 1,100 Check Into Cash centers among an estimated 22,000 payday-advance lenders in the United States. There is no doubt about the demand for such organizations, but the debate continues on the "fairness" of payday-advance loans.

A payday loan is a small, unsecured, short-term loan ranging from $100 to $1,000 (depending on the state) offered by a payday lender such as Check Into Cash. A payday loan can solve temporary cash flow problems without bouncing a check or incurring late-payment penalties. To receive a payday advance, borrowers simply write a personal post-dated check for the amount they wish to borrow, plus the payday loan fee. Check Into Cash holds their checks until payday when the loans are either paid off in person or the check is presented to the borrowers' bank for payment.

Although payday-advance borrowers usually pay a flat fee in lieu of interest, it is the size of the fee in relation to the amount borrowed that is particularly aggravating to opponents of the payday-advance industry. A typical fee is $15 for every $100 borrowed. Payday advance companies that belong to the Community Financial Services Association of America, an organization dedicated to promoting responsible regulation of the industry, limit their member companies to a maximum of four rollovers of the original amount borrowed. Thus, a borrower who rolled over an initial $100 loan for the maximum of four times would accumulate a total of $75 in fees, all within a 10-week period. On an annualized basis, the fees would amount to a whopping 391 percent.

An annual rate of 391 percent is a huge cost in relation to interest charged on home equity loans, personal loans, and even credit cards. However, advocates of the payday-advance industry make the following arguments: Most payday loan recipients do so either because funds are unavailable through conventional loans or because the payday loan averts a penalty or bank fee, which is, in itself, onerous. According to Check Into Cash, the cost for $100 of overdraft protection is $26.90, a credit card late fee on $100 is $37, and the late/disconnect fee on a $100 utility bill is $46.16. **Bankrate.com** reports that nonsufficient funds (NSF) fees average $26.90 per occurrence.

A payday advance could be useful, for example, if you have six outstanding checks at the time you are notified that the first check has been returned for insufficient funds (NSF) and you have been charged an NSF fee of $26. A payday advance could potentially avert subsequent charges of $26 per check for each of the remaining five checks and allow you time to rearrange your finances. When used judiciously, a payday advance can be a viable option to meet a short-term cash flow problem despite its high cost. Used unwisely, or by someone who continuously relies on a payday loan to try to make ends meet, payday advances can seriously harm one's personal finances.

▶ *The 391 percent mentioned above is an annual nominal rate [15% × (365 ÷ 14)]. Should the 2-week rate (15 percent) be compounded to calculate the effective annual interest rate?*

5-25 Rather than comparing future values, you often compare the effective annual rates of various investment opportunities with differing compounding frequencies. Based on the information provided at MFL, solve for the effective annual rates of several investment opportunities with different compounding frequencies.

LG 6

5.6 Special Applications of Time Value

Future value and present value techniques have a number of important applications in finance. We'll study four of them in this section: (1) determining deposits needed to accumulate a future sum, (2) loan amortization, (3) finding interest or growth rates, and (4) finding an unknown number of periods.

DETERMINING DEPOSITS NEEDED TO ACCUMULATE A FUTURE SUM

Suppose that you want to buy a house 5 years from now, and you estimate that an initial down payment of $30,000 will be required at that time. To accumulate the $30,000, you will wish to make equal annual end-of-year deposits into an account paying annual interest of 6 percent. The solution to this problem is closely related to the process of finding the future value of an annuity. You must determine what size annuity will result in a single amount equal to $30,000 at the end of year 5.

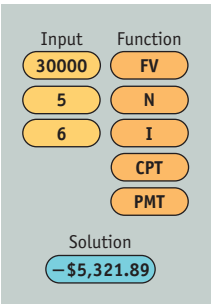
Earlier in the chapter, Equation 5.3 was provided for the future value of an ordinary annuity that made a payment, *CF*, each year. In the current problem, we know the future value we want to achieve, $30,000, but we want to solve for the annual cash payment that we'd have to save to achieve that goal. Solving Equation 5.3 for *CF* gives

$$CF = FV_n \div \left\{ \frac{[(1 + r)^n - 1]}{r} \right\} \tag{5.11}$$

As a practical matter, to solve problems like this one, analysts nearly always use a calculator or Excel as demonstrated in the following example.

Personal Finance Example 5.20 ▶ As just stated, you want to determine the equal annual end-of-year deposits required to accumulate $30,000 at the end of 5 years, given an interest rate of 6%.

MyFinanceLab Financial Calculator



Calculator use Using the calculator inputs shown at the left, you will find the annual deposit amount to be $5,321.89. Thus, if $5,321.89 is deposited at the end of each year for 5 years at 6% interest, there will be $30,000 in the account at the end of 5 years.

Spreadsheet use In Excel, solving for the annual cash flow that helps you reach the $30,000 means using the payment function. Its syntax is PMT (rate,nper,pv, fv,type). All the inputs in this function have been discussed previously. The following Excel spreadsheet illustrates how to use this function to find the annual payment required to save $30,000.

	A	B
1	ANNUAL DEPOSITS AMOUNT TO ACCUMULATE A FUTURE SUM	
2	Future value	$30,000
3	Annual rate of interest	6%
4	Number of years	5
5	Annual annuity payment	–$5,321.89

Entry in Cell B5 is =PMT(B3,B4,0,B2,0). The minus sign appears before the annuity payment in B5 because deposit amounts are cash outflows for the investor.

LOAN AMORTIZATION

loan amortization
The determination of the equal periodic loan payments necessary to provide a lender with a specified interest return and to repay the loan principal over a specified period.

loan amortization schedule
A schedule of equal payments to repay a loan. It shows the allocation of each loan payment to interest and principal.

The term **loan amortization** refers to the determination of equal periodic loan payments. These payments provide a lender with a specified interest return and repay the loan principal over a specified period. The loan amortization process involves finding the future payments, over the term of the loan, whose present value at the loan interest rate equals the amount of initial principal borrowed. Lenders use a **loan amortization schedule** to determine these payment amounts and the allocation of each payment to interest and principal. In the case of home mortgages, these tables are used to find the equal *monthly* payments necessary to *amortize*, or pay off, the mortgage at a specified interest rate over a 15- to 30-year period.

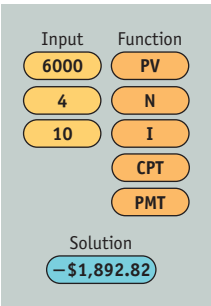
Amortizing a loan actually involves creating an annuity out of a present amount. For example, say you borrow $6,000 at 10 percent and agree to make equal annual end-of-year payments over 4 years. To find the size of the payments, the lender determines the amount of a 4-year annuity discounted at 10 percent that has a present value of $6,000.

Earlier in the chapter, Equation 5.4 demonstrated how to find the present value of an ordinary annuity given information about the number of time periods, the interest rate, and the annuity's periodic payment. We can rearrange that equation to solve for the payment, our objective in this problem:

$$CF = (PV \times r) \div \left[1 - \frac{1}{(1 + r)^n}\right] \tag{5.12}$$

Personal Finance Example 5.21 ▶ As just stated, you want to determine the equal annual end-of-year payments necessary to amortize fully a $6,000, 10% loan over 4 years.

MyFinanceLab Financial Calculator



Calculator use Using the calculator inputs shown at the left, you will find the annual payment amount to be $1,892.82. Thus, to repay the interest and principal on a $6,000, 10%, 4-year loan, equal annual end-of-year payments of $1,892.82 are necessary.

The allocation of each loan payment to interest and principal can be seen in columns 3 and 4 of the *loan amortization schedule* in Table 5.6. The portion of each payment that represents interest (column 3) declines over the repayment period, and the portion going to principal repayment (column 4) increases. This pattern is typical of amortized loans; as the principal is reduced, the interest component declines, leaving a larger portion of each subsequent loan payment to repay principal.

Spreadsheet use The annual payment to repay the loan also can be calculated as shown on the first Excel spreadsheet shown on page 241. The amortization schedule, shown in Table 5.6, allocating each loan payment to interest and principal can be calculated precisely as shown on the second Excel spreadsheet on page 241.

TABLE 5.6 Loan Amortization Schedule ($6,000 Principal, 10% Interest, 4-Year Repayment Period)

End of-year	Beginning of-year principal (1)	Loan payment (2)	Payments: Interest [0.10 × (1)] (3)	Payments: Principal [(2) − (3)] (4)	End-of-year principal [(1) − (4)] (5)
1	$6,000.00	$1,892.82	$600.00	$1,292.82	$4,707.18
2	4,707.18	1,892.82	470.72	1,422.10	3,285.08
3	3,285.08	1,892.82	328.51	1,564.31	1,720.77
4	1,720.77	1,892.82	172.08	1,720.74	—[a]

[a]Because of rounding, a slight difference ($0.03) exists between the beginning-of-year-4 principal (in column 1) and the year-4 principal payment (in column 4).

	A	B
1	ANNUAL PAYMENT AMOUNT TO REPAY A LOAN	
2	Present value	$6,000
3	Annual rate of interest	10%
4	Number of years	4
5	Annual loan payment	–$1,892.82

Entry in Cell B5 is =PMT(B3,B4,B2,0,0).
The minus sign appears before the loan payment in B5 because loan payments are cash outflows for the borrower.

	A	B	C	D	E
1	LOAN AMORTIZATION SCHEDULE				
2			Loan principal	$6,000	
3			Annual rate of interest	10%	
4			Number of years	4	
5			Annual annuity payments		
6	Year	Total	To Interest	To Principal	Year-End Principal
7	0				$6,000.00
8	1	–$1,892.82	–$600.00	–$1,292.82	$4,707.18
9	2	–$1,892.82	–$470.72	–$1,422.11	$3,285.07
10	3	–$1,892.82	–$328.51	–$1,564.32	$1,720.75
11	4	–$1,892.82	–$172.07	–$1,720.75	$0.00

Key Cell Entries
Cell B8 is =PMT(D3,D4,D2,0,0), copy to B9:B11
Cell C8 is =PMT(D3,D4,D2,0,0), copy to C9:C11
Cell D8 is =PMT(D3,D4,D2,0,0), copy to D9:D11
Cell E8 is =E7-D8, copy to E9:E11
The minus sign appears before the loan payments because these are cash outflows for the borrower.

To attract buyers who could not immediately afford 15- to 30-year mortgages of equal annual payments, lenders offered mortgages whose interest rates adjusted at certain points. The *Focus on Practice* box discusses how such mortgages have worked out for some "subprime" borrowers.

focus on **PRACTICE**

New Century Brings Trouble for Subprime Mortgages

in practice As the housing market began to boom at the end of the twentieth century and into the early twenty-first, the market share of subprime mortgages climbed from near 0 percent in 1997 to about 20 percent of mortgage originations in 2006. Several factors combined to fuel the rapid growth of lending to borrowers with tarnished credit, including a low interest rate environment, loose underwriting standards, and innovations in mortgage financing such as "affordability programs" to increase rates of homeownership among lower-income borrowers.

Particularly attractive to new home buyers was the hybrid adjustable rate mortgage (ARM), which featured a low introductory interest rate that reset upward after a preset period of time. Interest rates began a steady upward trend beginning in late 2004. In 2006, some $300 billion worth of adjustable ARMs were reset to higher rates. In a market with rising home values, a borrower has the option to refinance the mortgage, using some of the equity created by the home's increasing value to reduce the mortgage payment. After 2006, however, home prices started a 3-year slide, so refinancing was not an option for many subprime borrowers. Instead, borrowers in trouble could try to convince their lenders to allow a "short sale," in which the borrower sells the home for whatever the market will bear and the lender agrees to accept the proceeds from that sale as settlement for the mortgage debt. For lenders and borrowers alike, foreclosure is the last, worst option.

▶ *As a reaction to problems in the subprime area, lenders tightened lending standards. What effect do you think this change had on the housing market?*

FINDING INTEREST OR GROWTH RATES

It is often necessary to calculate the compound annual interest or *growth rate* (that is, the annual rate of change in values) of a series of cash flows. Examples include finding the interest rate on a loan, the rate of growth in sales, and the rate of growth in earnings. In doing so, we again make use of Equation 5.1. In this case, we want to solve for the interest rate (or growth rate) representing the increase in value of some investment between two time periods. Solving Equation 5.1 for r, we have

$$r = \left(\frac{FV_n}{PV}\right)^{1/n} - 1 \tag{5.13}$$

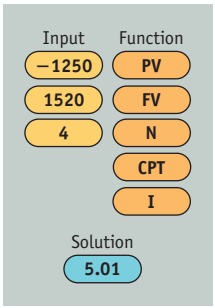
The simplest situation is one in which an investment's value has increased over time, and you want to know the annual rate of growth (that is, interest) that is represented by the increase in the investment.

Personal Finance Example 5.22 ▶ Ray Noble purchased an investment 4 years ago for $1,250. Now it is worth $1,520. What compound annual rate of return has Ray earned on this investment? Plugging the appropriate values into Equation 5.13, we have

$$r = (\$1{,}520 \div \$1{,}250)^{(1/4)} - 1 = 0.0501 = 5.01\% \text{ per year}$$

Calculator use Using the calculator to find the interest or growth rate, we treat the earliest value as a present value, PV, and the latest value as a future value, FVn. (*Note:* Most calculators require *either* the PV or the FV value to be input as a negative value to calculate an unknown interest or growth rate. That approach

MyFinanceLab Financial Calculator



is used here.) Using the inputs shown at the left, you will find the interest or growth rate to be 5.01%.

Spreadsheet use The interest or growth rate for the series of cash flows also can be calculated as shown on the following Excel spreadsheet.

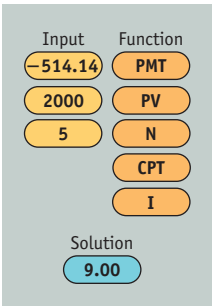
	A	B
1	SOLVING FOR INTEREST OR GROWTH RATE OF A SINGLE AMOUNT INVESTMENT	
2	Present value	– $1,250
3	Number of years	4
4	Future value	$1,520.00
5	Annual rate of interest	5.01%

Entry in Cell B5 is =RATE(B3,0,B2,B4,0).
The minus sign appears before the $1,250 in B2 because the cost of the investment is treated as a cash outflow.

Another type of interest-rate problem involves finding the interest rate associated with an *annuity*, or equal-payment loan.

Personal Finance Example 5.23 ▶ Jan Jacobs can borrow $2,000 to be repaid in equal annual end-of-year amounts of $514.14 for the next 5 years. She wants to find the interest rate on this loan.

MyFinanceLab Financial Calculator



Calculator use (*Note:* Most calculators require *either* the *PMT* or the *PV* value to be input as a negative number to calculate an unknown interest rate on an equal-payment loan. That approach is used here.) Using the inputs shown at the left, you will find the interest rate to be 9.00%.

Spreadsheet use The interest or growth rate for the annuity also can be calculated as shown on the following Excel spreadsheet.

	A	B
1	SOLVING FOR INTEREST OR GROWTH RATE OF AN ORDINARY ANNUITY	
2	Present value	$2,000
3	Number of years	5
4	Annual annuity amount	–$514.14
5	Annual rate of interest	9.00%

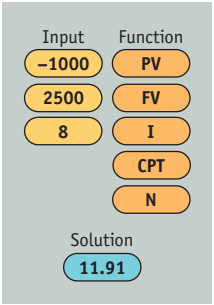
Entry in Cell B5 is =RATE(B3,B4,B2,0,0).
The minus sign appears before the $514.14 in B4 because the loan payment is treated as a cash outflow.

FINDING AN UNKNOWN NUMBER OF PERIODS

Sometimes it is necessary to calculate the number of time periods needed to generate a given amount of cash flow from an initial amount. Here we briefly consider this calculation for both single amounts and annuities. This simplest case is when a person wishes to determine the number of periods, n, it will take for an initial deposit, PV, to grow to a specified future amount, FV_n, given a stated interest rate, r.

Personal Finance Example 5.24 ▶ Ann Bates wishes to determine the number of years it will take for her initial $1,000 deposit, earning 8% annual interest, to grow to equal $2,500. Simply stated, at an 8% annual rate of interest, how many years, n, will it take for Ann's $1,000, PV, to grow to $2,500, FV_n?

MyFinanceLab Financial Calculator



Calculator use Using the calculator, we treat the initial value as the present value, PV, and the latest value as the future value, FV_n. (*Note:* Most calculators require *either* the PV or the FV value to be input as a negative number to calculate an unknown number of periods. That approach is used here.) Using the inputs shown at the left, we find the number of periods to be 11.91 years.

Spreadsheet use The number of years for the present value to grow to a specified future value can be calculated as shown on the following Excel spreadsheet.

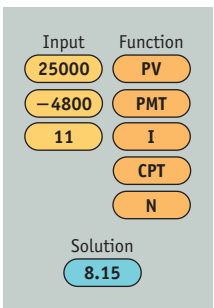
	A	B
1	SOLVING FOR THE YEARS OF A SINGLE AMOUNT INVESTMENT	
2	Present value	–$1,000
3	Annual rate of interest	8%
4	Future value	$2,500
5	Number of years	11.91

Entry in Cell B5 is =NPER(B3,0,B2,B4,0). The minus sign appears before the $1,000 in B2 because the initial deposit is treated as a cash outflow.

Another type of number-of-periods problem involves finding the number of periods associated with an *annuity*. Occasionally, we wish to find the unknown life, n, of an annuity that is intended to achieve a specific objective, such as repaying a loan of a given amount.

Personal Finance Example 5.25 ▶ Bill Smart can borrow $25,000 at an 11% annual interest rate; equal, annual, end-of-year payments of $4,800 are required. He wishes to determine how long it will take to fully repay the loan. In other words, he wishes to determine how many years, n, it will take to repay the $25,000, 11% loan, PV_n, if the payments of $4,800 are made at the end of each year.

MyFinanceLab Financial Calculator



Calculator use (*Note:* Most calculators require *either* the PV or the PMT value to be input as a negative number to calculate an unknown number of periods. That approach is used here.) Using the inputs shown at the left, you will find the number of periods to be 8.15 years. So, after making 8 payments of $4,800, Bill will still have a small outstanding balance.

Spreadsheet use The number of years to pay off the loan also can be calculated as shown on the following Excel spreadsheet.

	A	B
1	SOLVING FOR THE YEARS TO REPAY A SINGLE LOAN AMOUNT	
2	Present value	$25,000
3	Annual rate of interest	11%
4	Annual payment amount	–$4,800.00
5	Number of years	8.15
	Entry in Cell B5 is =NPER(B3,B4,B2,0,0). The minus sign appears before the $4,800 in B4 because the loan payments are treated as cash outflows.	

→ REVIEW QUESTIONS

5-26 How can you determine the size of the equal, annual, end-of-period deposits necessary to accumulate a certain future sum at the end of a specified future period at a given annual interest rate?

5-27 Describe the procedure used to amortize a loan into a series of equal periodic payments.

5-28 How can you determine the unknown number of periods when you know the present and future values—single amount or annuity—and the applicable rate of interest?

→ EXCEL REVIEW QUESTIONS MyFinanceLab

5-29 You want to buy a new car as a graduation present for yourself, but before finalizing a purchase you need to consider the monthly payment amount. Based on the information provided at MFL, find the monthly payment amount for the car you are considering.

5-30 As a savvy finance major you realize that you can quickly estimate your retirement age by knowing how much you need to retire, how much you can contribute each month to your retirement account, and what rate of return you can earn on your retirement investment and solving for the number of years it will take to get there. Based on the information provided at MFL, estimate the age at which you will be able to retire.

Summary

FOCUS ON VALUE

Time value of money is an important tool that financial managers and other market participants use to assess the effects of proposed actions. Because firms have long lives and some decisions affect their long-term cash flows, the effective application of time-value-of-money techniques is extremely important. These techniques enable financial managers to evaluate cash flows occurring at different times so as to combine, compare, and evaluate them and link them to the firm's

overall goal of share price maximization. It will become clear in Chapters 6 and 7 that the application of time value techniques is a key part of the value determination process needed to make intelligent value-creating decisions.

REVIEW OF LEARNING GOALS

LG 1 **Discuss the role of time value in finance, the use of computational tools, and the basic patterns of cash flow.** Financial managers and investors use time-value-of-money techniques when assessing the value of expected cash flow streams. Alternatives can be assessed by either compounding to find future value or discounting to find present value. Financial managers rely primarily on present value techniques. Financial calculators, electronic spreadsheets, and financial tables can streamline the application of time value techniques. The cash flow of a firm can be described by its pattern: single amount, annuity, or mixed stream.

LG 2 **Understand the concepts of future value and present value, their calculation for single amounts, and the relationship between them.** Future value (FV) relies on compound interest to measure future amounts. The initial principal or deposit in one period, along with the interest earned on it, becomes the beginning principal of the following period.

The present value (PV) of a future amount is the amount of money today that is equivalent to the given future amount, considering the return that can be earned. Present value is the inverse of future value.

LG 3 **Find the future value and the present value of both an ordinary annuity and an annuity due, and find the present value of a perpetuity.** An annuity is a pattern of equal periodic cash flows. For an ordinary annuity, the cash flows occur at the end of the period. For an annuity due, cash flows occur at the beginning of the period.

The future or present value of an ordinary annuity can be found by using algebraic equations, a financial calculator, or a spreadsheet program. The value of an annuity due is always $r\%$ greater than the value of an identical annuity. The present value of a perpetuity—an infinite-lived annuity—equals the annual cash payment divided by the discount rate.

LG 4 **Calculate both the future value and the present value of a mixed stream of cash flows.** A mixed stream of cash flows is a stream of unequal periodic cash flows that reflect no particular pattern. The future value of a mixed stream of cash flows is the sum of the future values of each individual cash flow. Similarly, the present value of a mixed stream of cash flows is the sum of the present values of the individual cash flows.

LG 5 **Understand the effect that compounding interest more frequently than annually has on future value and on the effective annual rate of interest.** Interest can be compounded at intervals ranging from annually to daily and even continuously. The more often interest is compounded, the larger the future amount that will be accumulated, and the higher the effective, or true, annual rate (EAR).

The annual percentage rate (APR)—a nominal annual rate—is quoted on credit cards and loans. The annual percentage yield (APY)—an effective annual rate—is quoted on savings products.

LG 6 **Describe the procedures involved in (1) determining deposits needed to accumulate a future sum, (2) loan amortization, (3) finding interest or growth rates, and (4) finding an unknown number of periods.** (1) The periodic deposit to accumulate a given future sum can be found by solving the equation for the future value of an annuity for the annual payment. (2) A loan can be amortized into equal periodic payments by solving the equation for the present value of an annuity for the periodic payment. (3) Interest or growth rates can be estimated by finding the unknown interest rate in the equation for the present value of a single amount or an annuity. (4) The number of periods can be estimated by finding the unknown number of periods in the equation for the present value of a single amount or an annuity.

Opener-in-Review

The chapter opener described an arrangement in which the city of Cincinnati gave up the right to collect parking fees over a 30-year period in exchange for a lump sum of $92 million plus a 30-year annuity of $3 million. Suppose that if the city had not entered into that arrangement, it would have collected parking fees the following year of $6 million (net of operating costs), and those fees would have grown at a steady 3% for the next 30 years. At an interest rate of 4%, what is the present value of the parking revenue that the city could have collected? Using the same 4% to value the payments that the city was set to receive in their privatization deal, do you think that the city made the correct decision? Why or why not?

Self-Test Problems (Solutions in Appendix)

LG 2 LG 5 IRF **ST5–1 Future values for various compounding frequencies** Delia Martin has $10,000 that she can deposit in any of three savings accounts for a 3-year period. Bank A compounds interest on an annual basis, bank B compounds interest twice each year, and bank C compounds interest each quarter. All three banks have a stated annual interest rate of 4%.

a. What amount would Ms. Martin have at the end of the third year, leaving all interest paid on deposit, in each bank?

b. What *effective annual rate (EAR)* would she earn in each of the banks?

c. On the basis of your findings in parts **a** and **b**, which bank should Ms. Martin deal with? Why?

d. If a fourth bank (bank D), also with a 4% stated interest rate, compounds interest continuously, how much would Ms. Martin have at the end of the third year? Does this alternative change your recommendation in part **c**? Explain why or why not.

LG 3 IRF

ST5–2 **Future values of annuities** Ramesh Abdul wishes to choose the better of two equally costly cash flow streams: annuity X and annuity Y. X is an *annuity due* with a cash inflow of $9,000 for each of 6 years. Y is an *ordinary annuity* with a cash inflow of $10,000 for each of 6 years. Assume that Ramesh can earn 15% on his investments.

a. On a purely subjective basis, which annuity do you think is more attractive? Why?

b. Find the future value at the end of year 6 for both annuities.

c. Use your finding in part **b** to indicate which annuity is more attractive. Why? Compare your finding to your subjective response in part **a**.

LG 2 LG 3 LG 4 IRF

ST5–3 **Present values of single amounts and streams** You have a choice of accepting either of two 5-year cash flow streams or single amounts. One cash flow stream is an ordinary annuity, and the other is a mixed stream. You may accept alternative A or B, either as a cash flow stream or as a single amount. Given the cash flow stream and single amounts associated with each (see the following table), and assuming a 9% opportunity cost, which alternative (A or B) and in which form (cash flow stream or single amount) would you prefer?

	Cash flow stream	
End of year	Alternative A	Alternative B
1	$700	$1,100
2	700	900
3	700	700
4	700	500
5	700	300
	Single amount	
At time zero	$2,825	$2,800

LG 6 IRF

ST5–4 **Deposits needed to accumulate a future sum** Judi Janson wishes to accumulate $8,000 by the end of 5 years by making equal, annual, end-of-year deposits over the next 5 years. If Judi can earn 7% on her investments, how much must she deposit at the *end of each year* to meet this goal?

Warm-Up Exercises

All problems are available in MyFinanceLab.

LG 2

E5–1 Assume that a firm makes a $2,500 deposit into its money market account. If this account is currently paying 0.7% (yes, that's right, less than 1%!), what will the account balance be after 1 year?

LG 2 LG 5

E5–2 If Bob and Judy combine their savings of $1,260 and $975, respectively, and deposit this amount into an account that pays 2% annual interest, compounded monthly, what will the account balance be after 4 years?

LG 3

E5–3 Gabrielle just won $2.5 million in the state lottery. She is given the option of receiving a total of $1.3 million now, or she can elect to be paid $100,000 at the end of each of the next 25 years. If Gabrielle can earn 5% annually on her investments, from a strict economic point of view which option should she take?

LG 4 **E5–4** Your firm has the option of making an investment in new software that will cost $130,000 today and is estimated to provide the savings shown in the following table over its 5-year life.

Year	Savings estimate
1	$35,000
2	50,000
3	45,000
4	25,000
5	15,000

Should the firm make this investment if it requires a minimum annual return of 9% on all investments?

LG 5 **E5–5** First Choice Bank wants to earn an effective interest rate of 18% per year. In order to suit different potential borrowers' needs, the bank offers two options. The first calculates interest on a weekly compounding basis while the second calculates interest on a monthly compounding basis. What interest rate is the bank required to report for the two options? Give one reason why a borrower might prefer monthly compounding over weekly compounding.

LG 6 **E5–6** Jack and Jill have just had their first child. If college is expected to cost $150,000 per year in 18 years, how much should the couple begin depositing annually at the end of each year to accumulate enough funds to pay the first year's tuition at the beginning of the nineteenth year? Assume that they can earn a 6% annual rate of return on their investment.

Problems

All problems are available in MyFinanceLab.

LG 1 **P5–1** **Using a time line** The financial manager at Starbuck Industries is considering an investment that requires an initial outlay of $25,000 and is expected to result in cash inflows of $3,000 at the end of year 1, $6,000 at the end of years 2 and 3, $10,000 at the end of year 4, $8,000 at the end of year 5, and $7,000 at the end of year 6.

a. Draw and label a time line depicting the cash flows associated with Starbuck Industries' proposed investment.

b. Use arrows to demonstrate, on the time line in part **a,** how compounding to find future value can be used to measure all cash flows at the end of year 6.

c. Use arrows to demonstrate, on the time line in part **b,** how discounting to find present value can be used to measure all cash flows at time zero.

d. Which of the approaches—*future value* or *present value*—do financial managers rely on most often for decision making? Why?

LG 2 **P5–2** **Future value calculation** *Without referring to the preprogrammed function on your financial calculator,* use the basic formula for future value along with the given interest rate, r, and the number of periods, n, to calculate the future value of $1 in each of the cases shown in the following table.

Case	Interest rate, r	Number of periods, n
A	12%	2
B	6	3
C	9	2
D	3	4

LG 1

P5–3 **Future value** You have $100 to invest. If you can earn 12% interest, about how long does it take for your $100 investment to grow to $200? Suppose that the interest rate is just half that, at 6%. At half the interest rate, does it take twice as long to double your money? Why or why not? How long does it take?

LG 2

P5–4 **Future values** For each of the cases shown in the following table, calculate the future value of the single cash flow deposited today at the end of the deposit period if the interest is compounded annually at the rate specified.

Case	Single cash flow	Interest rate	Deposit period (years)
A	$ 200	5%	20
B	4,500	8	7
C	10,000	9	10
D	25,000	10	12
E	37,000	11	5
F	40,000	12	9

Personal Finance Problem

LG 2

P5–5 **Time value** You have $1,500 to invest today at 7% interest compounded annually.

a. Find how much you will have accumulated in the account at the end of (1) 3 years, (2) 6 years, and (3) 9 years.

b. Use your findings in part **a** to calculate the amount of interest earned in (1) the first 3 years (years 1 to 3), (2) the second 3 years (years 4 to 6), and (3) the third 3 years (years 7 to 9).

c. Compare and contrast your findings in part **b**. Explain why the amount of interest earned increases in each succeeding 3-year period.

Personal Finance Problem

LG 2

P5–6 **Time value** As part of your financial planning, you wish to purchase a new car exactly 5 years from today. The car you wish to purchase costs $14,000 today, and your research indicates that its price will increase by 2% to 4% per year over the next 5 years.

a. Estimate the price of the car at the end of 5 years if inflation is (1) 2% per year and (2) 4% per year.

b. How much more expensive will the car be if the rate of inflation is 4% rather than 2%?

c. Estimate the price of the car if inflation is 2% for the next 2 years and 4% for 3 years after that.

Personal Finance Problem

LG 2 **P5–7** **Time value** You can deposit $10,000 into an account paying 9% annual interest either today or exactly 10 years from today. How much better off will you be at the end of 40 years if you decide to make the initial deposit today rather than 10 years from today?

Personal Finance Problem

LG 2 LG 5 **P5–8** **Time value** Peter just got his driver's license, and he wants to buy a new sports car for $70,000. He has $3,000 to invest as a lump sum today. Peter is a conservative investor and he only invests in safe products. After approaching different banks, he is offered the following investment opportunities:
(1) River Bank's savings account with an interest rate of 10.8% compounded monthly.
(2) First State Bank's savings account with an interest rate of 11.5% compounded annually.
(3) Union Bank's saving account with an interest rate of 9.3% compounded weekly.
How long will it take for Peter to accumulate enough money to buy the car in each of the above three cases?

Personal Finance Problem

LG 2 **P5–9** **Single-payment loan repayment** A person borrows $200 to be repaid in 8 years with 14% annually compounded interest. The loan may be repaid at the end of any earlier year with no prepayment penalty.
a. What amount will be due if the loan is repaid at the end of year 1?
b. What is the repayment at the end of year 4?
c. What amount is due at the end of the eighth year?

LG 2 **P5–10** **Present value calculation** *Without referring to the preprogrammed function on your financial calculator,* use the basic formula for present value, along with the given opportunity cost, r, and the number of periods, n, to calculate the present value of $1 in each of the cases shown in the following table.

Case	Opportunity cost, r	Number of periods, n
A	2%	4
B	10	2
C	5	3
D	13	2

LG 2 **P5–11** **Present values** For each of the cases shown in the following table, calculate the present value of the cash flow, discounting at the rate given and assuming that the cash flow is received at the end of the period noted.

Case	Single cash flow	Discount rate	End of period (years)
A	$ 7,000	12%	4
B	28,000	8	20
C	10,000	14	12
D	150,000	11	6
E	45,000	20	8

LG 2 **P5–12** **Present value concept** Answer each of the following questions.

a. What single investment made today, earning 12% annual interest, will be worth $6,000 at the end of 6 years?

b. What is the present value of $6,000 to be received at the end of 6 years if the discount rate is 12%?

c. What is the most you would pay today for a promise to repay you $6,000 at the end of 6 years if your opportunity cost is 12%?

d. Compare, contrast, and discuss your findings in parts **a** through **c**.

Personal Finance Problem

LG 2 **P5–13** **Time value** Jim Nance has been offered an investment that will pay him $500 three years from today.

a. If his opportunity cost is 7% compounded annually, what value should he place on this opportunity today?

b. What is the most he should pay to purchase this payment today?

c. If Jim can purchase this investment for less than the amount calculated in part **a**, what does that imply about the rate of return that he will earn on the investment?

LG 2 **P5–14** **Time value** Suppose you want to save money to pay for a down payment on an apartment in 5 years' time. One year from now, you will invest your $30,000 year-end bonus for the down payment. If you can invest at 15% per year, how much interest will you receive on your cash in 5 years? If you need $210,000 for the down payment, and you would like to top-up the remaining amount by investing a lump sum today, what is the amount you should invest?

Personal Finance Problem

LG 2 **P5–15** **Time value and discount rates** You just won a lottery that promises to pay you $1,000,000 exactly 10 years from today. Because the $1,000,000 payment is guaranteed by the state in which you live, opportunities exist to sell the claim today for an immediate single cash payment.

a. What is the least you will sell your claim for if you can earn the following rates of return on similar-risk investments during the 10-year period?

(1) 6%

(2) 9%

(3) 12%

b. Rework part **a** under the assumption that the $1,000,000 payment will be received in 15 rather than 10 years.

c. On the basis of your findings in parts **a** and **b**, discuss the effect of both the size of the rate of return and the time until receipt of payment on the present value of a future sum.

Personal Finance Problem

LG 2 **P5–16** **Time value comparisons of single amounts** In exchange for a $20,000 payment today, a well-known company will allow you to choose *one* of the alternatives shown in the following table. Your opportunity cost is 11%.

Alternative	Single amount
A	$28,500 at end of 3 years
B	$54,000 at end of 9 years
C	$160,000 at end of 20 years

a. Find the value today of each alternative.
b. Are all the alternatives acceptable? That is, are they worth $20,000 today?
c. Which alternative, if any, will you take?

Personal Finance Problem

LG 2 **P5–17** **Cash flow investment decision** Tom Alexander has an opportunity to purchase any of the investments shown in the following table. The purchase price, the amount of the single cash inflow, and its year of receipt are given for each investment. Which purchase recommendations would you make, assuming that Tom can earn 10% on his investments?

Investment	Price	Single cash inflow	Year of receipt
A	$18,000	$30,000	5
B	600	3,000	20
C	3,500	10,000	10
D	1,000	15,000	40

LG 2 **P5–18** **Calculating deposit needed** You put $10,000 in an account earning 5%. After 3 years, you make another deposit into the same account. Four years later (that is, 7 years after your original $10,000 deposit), the account balance is $20,000. What was the amount of the deposit at the end of year 3?

LG 3 **P5–19** **Future value of an annuity** For each case in the accompanying table, answer the questions that follow.

Case	Amount of annuity	Interest rate	Deposit period (years)
A	$ 2,500	8%	10
B	500	12	6
C	30,000	20	5
D	11,500	9	8
E	6,000	14	30

a. Calculate the future value of the annuity, assuming that it is
(1) An ordinary annuity.
(2) An annuity due.
b. Compare your findings in parts **a**(1) and **a**(2). All else being identical, which type of annuity—ordinary or annuity due—is preferable? Explain why.

LG 3 **P5–20** **Present value of an annuity** Consider the following cases.

Case	Amount of annuity	Interest rate	Period (years)
A	$ 12,000	7%	3
B	55,000	12	15
C	700	20	9
D	140,000	5	7
E	22,500	10	5

a. Calculate the present value of the annuity, assuming that it is
(1) An ordinary annuity.
(2) An annuity due.
b. Compare your findings in parts **a**(1) and **a**(2). All else being identical, which type of annuity—ordinary or annuity due—is preferable? Explain why.

Personal Finance Problem

LG 3 **P5–21** **Time value: Annuities** Marian Kirk wishes to select the better of two 10-year annuities, C and D. Annuity C is an *ordinary annuity* of $2,500 per year for 10 years. Annuity D is an *annuity due* of $2,200 per year for 10 years.
a. Find the *future value* of both annuities at the end of year 10 assuming that Marian can earn (1) 10% annual interest and (2) 20% annual interest.
b. Use your findings in part **a** to indicate which annuity has the greater future value at the end of year 10 for both the (1) 10% and (2) 20% interest rates.
c. Find the *present value* of both annuities, assuming that Marian can earn (1) 10% annual interest and (2) 20% annual interest.
d. Use your findings in part **c** to indicate which annuity has the greater present value for both (1) 10% and (2) 20% interest rates.
e. Briefly compare, contrast, and explain any differences between your findings using the 10% and 20% interest rates in parts **b** and **d.**

Personal Finance Problem

LG 3 **P5–22** **Retirement planning** Hal Thomas, a 25-year-old college graduate, wishes to retire at age 65. To supplement other sources of retirement income, he can deposit $2,000 each year into a tax-deferred individual retirement arrangement (IRA). The IRA will earn a 10% return over the next 40 years.
a. If Hal makes annual end-of-year $2,000 deposits into the IRA, how much will he have accumulated by the end of his sixty-fifth year?
b. If Hal decides to wait until age 35 to begin making annual end-of-year $2,000 deposits into the IRA, how much will he have accumulated by the end of his sixty-fifth year?
c. Using your findings in parts **a** and **b,** discuss the impact of delaying making deposits into the IRA for 10 years (age 25 to age 35) on the amount accumulated by the end of Hal's sixty-fifth year.
d. Rework parts **a, b,** and **c,** assuming that Hal makes all deposits at the beginning, rather than the end, of each year. Discuss the effect of beginning-of-year deposits on the future value accumulated by the end of Hal's sixty-fifth year.

Personal Finance Problem

LG 6 **P5–23** **Calculating the number of periods** You want to borrow $600,000 to buy an apartment, and you can only afford $4,000 a month to repay the loan. Suppose the bank charges you a fixed interest rate of 4% with monthly compounding. How long will it take you to pay off the loan?

Personal Finance Problem

LG 2 LG 3 **P5–24** **Funding your retirement** You plan to retire in exactly 20 years. Your goal is to create a fund that will allow you to receive $20,000 at the end of each year for the 30 years between retirement and death (a psychic told you that you would die exactly 30 years after you retire). You know that you will be able to earn 11% per year during the 30-year retirement period.

a. How large a fund will you need *when you retire* in 20 years to provide the 30-year, $20,000 retirement annuity?

b. How much will you need *today* as a single amount to provide the fund calculated in part **a** if you earn only 9% per year during the 20 years preceding retirement?

c. What effect would an increase in the rate you can earn both during and prior to retirement have on the values found in parts **a** and **b**? Explain.

d. Now assume that you will earn 10% from now through the end of your retirement. You want to make 20 end-of-year deposits into your retirement account that will fund the 30-year stream of $20,000 annual annuity payments. How large do your annual deposits have to be?

Personal Finance Problem

LG 2 LG 3 **P5–25** **Value of an annuity versus a single amount** Assume that you just won the state lottery. Your prize can be taken either in the form of $40,000 at the end of each of the next 25 years (that is, $1,000,000 over 25 years) or as a single amount of $500,000 paid immediately.

a. If you expect to be able to earn 5% annually on your investments over the next 25 years, ignoring taxes and other considerations, which alternative should you take? Why?

b. Would your decision in part **a** change if you could earn 7% rather than 5% on your investments over the next 25 years? Why?

c. On a strictly economic basis, at approximately what earnings rate would you be indifferent between the two plans?

LG 3 **P5–26** **Perpetuities** Consider the data in the following table.

Perpetuity	Annual amount	Discount rate
A	$ 20,000	8%
B	100,000	10
C	3,000	6
D	60,000	5

Determine the present value of each perpetuity.

Personal Finance Problem

LG 3 **P5–27** **Creating an endowment** On completion of her introductory finance course, Marla Lee was so pleased with the amount of useful and interesting knowledge she gained that she convinced her parents, who were wealthy alumni of the university she was attending, to create an endowment. The endowment is to allow three needy students to take the introductory finance course each year in perpetuity. The guaranteed annual cost of tuition and books for the course is $600 per student. The endowment will be created by making a single payment to the university. The university expects to earn exactly 6% per year on these funds.

a. How large an initial single payment must Marla's parents make to the university to fund the endowment?

b. What amount would be needed to fund the endowment if the university could earn 9% rather than 6% per year on the funds?

LG 4 **P5–28** **Value of a mixed stream** For each of the mixed streams of cash flows shown in the following table, determine the future value at the end of the final year if deposits are made into an account paying annual interest of 12%, assuming that no withdrawals are made during the period and that the deposits are made

a. At the *end* of each year.

b. At the *beginning* of each year.

	Cash flow stream		
Year	A	B	C
1	$ 900	$30,000	$1,200
2	1,000	25,000	1,200
3	1,200	20,000	1,000
4		10,000	1,900
5		5,000	

Personal Finance Problem

LG 4 **P5–29** **Value of a single amount versus a mixed stream** Gina Vitale has just contracted to sell a small parcel of land that she inherited a few years ago. The buyer is willing to pay $24,000 at the closing of the transaction or will pay the amounts shown in the following table at the *beginning* of each of the next 5 years. Because Gina doesn't really need the money today, she plans to let it accumulate in an account that earns 7% annual interest. Given her desire to buy a house at the end of 5 years after closing on the sale of the lot, she decides to choose the payment alternative—$24,000 single amount or the mixed stream of payments in the following table—that provides the higher future value at the end of 5 years. Which alternative will she choose?

Mixed stream	
Beginning of year	Cash flow
1	$ 2,000
2	4,000
3	6,000
4	8,000
5	10,000

LG 4 **P5–30** **Value of mixed streams** Find the present value of the streams of cash flows shown in the following table. Assume that the firm's opportunity cost is 12%.

A		B		C	
Year	Cash flow	Year	Cash flow	Year	Cash flow
1	–$2,000	1	$10,000	1–5	$10,000/yr
2	3,000	2–5	5,000/yr	6–10	8,000/yr
3	4,000	6	7,000		
4	6,000				
5	8,000				

LG 4 **P5–31** **Present value: Mixed streams** Consider the mixed streams of cash flows shown in the following table.

	Cash flow stream	
Year	A	B
1	$ 50,000	$ 10,000
2	40,000	20,000
3	30,000	30,000
4	20,000	40,000
5	10,000	50,000
Totals	$150,000	$150,000

a. Find the present value of each stream using a 15% discount rate.
b. Compare the calculated present values and discuss them in light of the undiscounted cash flows totaling $150,000 in each case.

LG 4 **P5–32** **Value of a mixed stream** In July 2012, Beijing had the heaviest rains in over six decades. More than 2 million people were affected by the rainfall, roads were flooded, and the whole transport system had to be suspended for days. The government now is offering a flood recovery project that requires the tender to draw the flood waters out within a week. CCTech is a large manufacturer of high-pressure industrial water pumps, and the firm decided to bid for the flood recovery project. The government will pay $5 million this year and $2 million for the following four years.
a. Draw the time line for the stream of cash flows.
b. If the discount rate is 8% per year, what is the present value of the project?
c. Suppose the project is expected to cost $10 million today. Should CCTech take the project if it is offered? Why or why not?

Personal Finance Problem

LG 4 **P5–33** **Funding budget shortfalls** As part of your personal budgeting process, you have determined that in each of the next 5 years you will have budget shortfalls. In other words, you will need the amounts shown in the following table at the end of the given year to balance your budget, that is, to make inflows equal outflows. You expect to be able to earn 8% on your investments during the next 5 years and wish to fund the budget shortfalls over the next 5 years with a single amount.

End of year	Budget shortfall
1	$ 5,000
2	4,000
3	6,000
4	10,000
5	3,000

a. How large must the single deposit today into an account paying 8% annual interest be to provide for full coverage of the anticipated budget shortfalls?
b. What effect would an increase in your earnings rate have on the amount calculated in part **a**? Explain.

LG 4 **P5–34** **Relationship between future value and present value: Mixed stream** Using the information in the accompanying table, answer the questions that follow.

Year (t)	Cash flow
1	$ 800
2	900
3	1,000
4	1,500
5	2,000

a. Determine the *present value* of the mixed stream of cash flows using a 5% discount rate.
b. How much would you be willing to pay for an opportunity to buy this stream, assuming that you can at best earn 5% on your investments?
c. What effect, if any, would a 7% rather than a 5% opportunity cost have on your analysis? (Explain verbally.)

LG 4 **P5–35** **Relationship between future value and present value: Mixed stream** The table below shows a mixed cash flow stream except that the cash flow for year 3 is missing.

Year 1	$10,000
Year 2	5,000
Year 3	
Year 4	20,000
Year 5	3,000

Suppose that somehow you know that the present value of the entire stream is $32,911.03 and that the discount rate is 4%. What is the amount of the missing cash flow in year 3?

LG 5 **P5–36** **Changing compounding frequency** Using annual, semiannual, and quarterly compounding periods for each of the following, (1) calculate the future value if $5,000 is deposited initially, and (2) determine the *effective annual rate (EAR)*.

a. At 12% annual interest for 5 years.
b. At 16% annual interest for 6 years.
c. At 20% annual interest for 10 years.

LG 5 **P5–37** **Compounding frequency, time value, and effective annual rates** For each of the cases in the following table:

a. Calculate the future value at the end of the specified deposit period.
b. Determine the *effective annual rate, EAR*.
c. Compare the nominal annual rate, r, to the effective annual rate, EAR. What relationship exists between compounding frequency and the nominal and effective annual rates?

Case	Amount of initial deposit	Nominal annual rate, r	Compounding frequency, m (times/year)	Deposit period (years)
A	$ 2,500	6%	2	5
B	50,000	12	6	3
C	1,000	5	1	10
D	20,000	16	4	6

LG 5 **P5–38** **Continuous compounding** For each of the cases in the following table, find the future value at the end of the deposit period, assuming that interest is compounded continuously at the given nominal annual rate.

Case	Amount of initial deposit	Nominal annual rate, r	Deposit period (years), n
A	$1,000	9%	2
B	600	10	10
C	4,000	8	7
D	2,500	12	4

Personal Finance Problem

LG 5 **P5–39** **Compounding frequency and time value** You plan to invest $2,000 in an individual retirement arrangement (IRA) today at a *nominal annual rate* of 8%, which is expected to apply to all future years.

a. How much will you have in the account at the end of 10 years if interest is compounded (1) annually, (2) semiannually, (3) daily (assume a 365-day year), and (4) continuously?
b. What is the *effective annual rate (EAR)* for each compounding period in part **a**?
c. How much greater will your IRA balance be at the end of 10 years if interest is compounded continuously rather than annually?
d. How does the compounding frequency affect the future value and effective annual rate for a given deposit? Explain in terms of your findings in parts **a** through **c.**

Personal Finance Problem

LG 5 LG 6 **P5–40** **Accumulating a growing future sum** You have $30,000, and you are making the decision between consumption and investment. You are considering either using all of the money to buy a new car or investing the whole amount. You have two investment options: You can either put the money into a savings account with a nominal interest rate of 5% compounded monthly, or invest the money in stocks with an expected return of 6% compounded continuously.
a. If you choose to invest, how much will you have 6 years later if you invested in (1) the savings account, or (2) stocks?
b. If you invested in the savings account, how long will it take for you to double your money?
c. Suppose the price of the car inflates by 2% per year. If you choose to invest your money in stocks, how long will it take for you to be able to afford 2 cars?

Personal Finance Problem

LG 3 LG 5 **P5–41** **Annuities and compounding** Janet Boyle intends to deposit $300 per year in a credit union for the next 10 years, and the credit union pays an annual interest rate of 8%.
a. Determine the future value that Janet will have at the end of 10 years, given that end-of-period deposits are made and no interest is withdrawn, if
(1) $300 is deposited annually and the credit union pays interest annually.
(2) $150 is deposited semiannually and the credit union pays interest semiannually.
(3) $75 is deposited quarterly and the credit union pays interest quarterly.
b. Use your findings in part **a** to discuss the effect of more frequent deposits and compounding of interest on the future value of an annuity.

LG 6 **P5–42** **Deposits to accumulate future sums** For each of the cases shown in the following table, determine the amount of the equal, annual, end-of-year deposits necessary to accumulate the given sum at the end of the specified period, assuming the stated annual interest rate.

Case	Sum to be accumulated	Accumulation period (years)	Interest rate
A	$ 5,000	3	12%
B	100,000	20	7
C	30,000	8	10
D	15,000	12	8

Personal Finance Problem

LG 6 **P5–43** **Creating a retirement fund** To supplement your planned retirement in exactly 42 years, you estimate that you need to accumulate $220,000 by the end of 42 years from today. You plan to make equal, annual, end-of-year deposits into an account paying 8% annual interest.

a. How large must the annual deposits be to create the $220,000 fund by the end of 42 years?
b. If you can afford to deposit only $600 per year into the account, how much will you have accumulated by the end of the forty-second year?

Personal Finance Problem

LG 6 **P5–44** **Finding interest rates** You want to borrow $24,000 for a tax payment. Your friend offers you a loan, and he claims that he will only charge you 10% interest. He calculates that the total interest of the loan will be $24,000 × 10% = $2,400, so he deducts this amount from the loan, gives you $21,600, and tells you to repay $24,000 in one year. Has your friend charged you 10% interest? What is the real interest rate charged on the loan?

Personal Finance Problem

LG 3 LG 6 **P5–45** **Deposits to create a perpetuity** You have decided to endow your favorite university with a scholarship. It is expected to cost $6,000 per year to attend the university into perpetuity. You expect to give the university the endowment in 10 years and will accumulate it by making equal annual (end-of-year) deposits into an account. The rate of interest is expected to be 10% for all future time periods.
a. How large must the endowment be?
b. How much must you deposit at the end of each of the next 10 years to accumulate the required amount?

Personal Finance Problem

LG 2 LG 3 LG 6 **P5–46** **Inflation, time value, and annual deposits** While vacationing in Florida, John Kelley saw the vacation home of his dreams. It was listed with a sale price of $200,000. The only catch is that John is 40 years old and plans to continue working until he is 65. Still, he believes that prices generally increase at the overall rate of inflation. John believes that he can earn 9% annually after taxes on his investments. He is willing to invest a fixed amount at the end of each of the next 25 years to fund the cash purchase of such a house (one that can be purchased today for $200,000) when he retires.
a. Inflation is expected to average 5% per year for the next 25 years. What will John's dream house cost when he retires?
b. How much must John invest at the *end* of each of the next 25 years to have the cash purchase price of the house when he retires?
c. If John invests at the *beginning* instead of at the end of each of the next 25 years, how much must he invest each year?

LG 6 **P5–47** **Loan payment** Determine the equal, annual, end-of-year payment required each year over the life of the loans shown in the following table to repay them fully during the stated term of the loan.

Loan	Principal	Interest rate	Term of loan (years)
A	$12,000	8%	3
B	60,000	12	10
C	75,000	10	30
D	4,000	15	5

Personal Finance Problem

LG 6 **P5–48** **Loan amortization schedule** Joan Messineo borrowed $15,000 at a 14% annual rate of interest to be repaid over 3 years. The loan is amortized into three equal, annual, end-of-year payments.

a. Calculate the annual, end-of-year loan payment.

b. Prepare a loan amortization schedule showing the interest and principal breakdown of each of the three loan payments.

c. Explain why the interest portion of each payment declines with the passage of time.

LG 6 **P5–49** **Loan interest deductions** Liz Rogers just closed a $10,000 business loan that is to be repaid in three equal, annual, end-of-year payments. The interest rate on the loan is 13%. As part of her firm's detailed financial planning, Liz wishes to determine the annual interest deduction attributable to the loan. (Because it is a business loan, the interest portion of each loan payment is tax-deductible to the business.)

a. Determine the firm's annual loan payment.

b. Prepare an amortization schedule for the loan.

c. How much interest expense will Liz's firm have in *each* of the next 3 years as a result of this loan?

Personal Finance Problem

LG 6 **P5–50** **Loan rates of interest** You want to buy a new car that costs $48,000. Dealer A offers to lend you the entire $48,000 for a zero-interest 2-year loan with monthly payments that start the next month. Dealer B requires you to pay $10,000 now, followed by installments of $1,500 for the next 24 months. You observe that the market interest rate is 6%.

a. What is the net cost today of the two options? Which option offers you the cheapest financing?

b. Use a financial calculator or spreadsheet to help you calculate what the interest rate would be if the financing cost from Dealer A was equal to that of Dealer B.

LG 6 **P5–51** **Growth rates** You are given the series of cash flows shown in the following table.

	Cash flows		
Year	A	B	C
1	$500	$1,500	$2,500
2	560	1,550	2,600
3	640	1,610	2,650
4	720	1,680	2,650
5	800	1,760	2,800
6		1,850	2,850
7		1,950	2,900
8		2,060	
9		2,170	
10		2,280	

a. Calculate the compound annual growth rate between the first and last payment in each stream.

b. If year-1 values represent initial deposits in a savings account paying annual interest, what is the annual rate of interest earned on each account?

c. Compare and discuss the growth rate and interest rate found in parts **a** and **b**, respectively.

Personal Finance Problem

LG 6 **P5–52** **Rate of return** Rishi Singh has $1,500 to invest. His investment counselor suggests an investment that pays no stated interest but will return $2,000 at the end of 3 years.

a. What annual rate of return will Rishi earn with this investment?

b. Rishi is considering another investment, of equal risk, that earns an annual return of 8%. Which investment should he make, and why?

Personal Finance Problem

LG 6 **P5–53** **Rate of return and investment choice** Clare Jaccard has $5,000 to invest. Because she is only 25 years old, she is not concerned about the length of the investment's life. What she is sensitive to is the rate of return she will earn on the investment. With the help of her financial advisor, Clare has isolated four equally risky investments, each providing a single amount at the end of its life, as shown in the following table. All the investments require an initial $5,000 payment.

Investment	Single amount	Investment life (years)
A	$ 8,400	6
B	15,900	15
C	7,600	4
D	13,000	10

a. Calculate, to the nearest 1%, the rate of return on each of the four investments available to Clare.

b. Which investment would you recommend to Clare, given her goal of maximizing the rate of return?

LG 6 **P5–54** **Rate of return: Annuity** What is the rate of return on an investment of $10,606 if the company will receive $2,000 each year for the next 10 years?

Personal Finance Problem

LG 6 **P5–55** **Monthly loan payments** Ricky is considering purchasing an apartment costing $700,000. He will pay a 30% down payment and take out a mortgage for the remainder. Since he just got married and wants to save some money for future use, he will choose the plan with the lowest monthly payment. After visiting several banks, he received the following mortgage offers:

Bank	Interest rate	Term (years)
A	3.5%	15
B	3	20
C	4	25
D	4.5	18

a. What are the monthly payments for plans offered by the four banks?
b. Which plan should Ricky choose?

Personal Finance Problem

LG 6 **P5–56** **Interest rate for an annuity** Anna Waldheim was seriously injured in an industrial accident. She sued the responsible parties and was awarded a judgment of $2,000,000. Today, she and her attorney are attending a settlement conference with the defendants. The defendants have made an initial offer of $156,000 per year for 25 years. Anna plans to counteroffer at $255,000 per year for 25 years. Both the offer and the counteroffer have a present value of $2,000,000, the amount of the judgment. Both assume payments at the end of each year.

a. What interest rate assumption have the defendants used in their offer (rounded to the nearest whole percent)?
b. What interest rate assumption have Anna and her lawyer used in their counteroffer (rounded to the nearest whole percent)?
c. Anna is willing to settle for an annuity that carries an interest rate assumption of 9%. What annual payment would be acceptable to her?

Personal Finance Problem

LG 6 **P5–57** **Loan rates of interest** You have credit card debt amounting to $50,000. The card charges you a 32% interest rate with monthly compounding. You believe that the interest rate of the existing debt is too high, so you decide to switch cards and move the outstanding balance on the old card to the new card. After doing some research, you find Cards A, B, and C as alternatives. If you move the existing debt to Card B or Card C, there will be extra charges (handling fees). Card B will charge $500, and Card C will charge $1,000 handling fees.

Card	Interest rate	Outstanding amount
A	30%	$50,000
B	29	50,500
C	28	51,000

a. If you pay off the debt in one year with equal monthly amounts, how much in interest payments have you saved by using (1) Card A, (2) Card B, and (3) Card C rather than keeping your existing card?
b. Which card should you choose?

LG 6 **P5–58** **Number of years to equal future amount** For each of the following cases, determine the number of years it will take for the initial deposit to grow to equal the future amount at the given interest rate.

Case	Initial deposit	Future amount	Interest rate
A	$ 300	$ 1,000	7%
B	12,000	15,000	5
C	9,000	20,000	10
D	100	500	9
E	7,500	30,000	15

Personal Finance Problem

P5–59 LG 6 **Time to accumulate a given sum** Manuel Rios wishes to determine how long it will take an initial deposit of $10,000 to double.

a. If Manuel earns 10% annual interest on the deposit, how long will it take for him to double his money?

b. How long will it take if he earns only 7% annual interest?

c. How long will it take if he can earn 12% annual interest?

d. Reviewing your findings in parts **a, b,** and **c,** indicate what relationship exists between the interest rate and the amount of time it will take Manuel to double his money.

P5–60 LG 6 **Number of years to provide a given return** In each of the following cases, determine the number of years that the given annual *end-of-year* cash flow must continue to provide the given rate of return on the given initial amount.

Case	Initial amount	Annual cash flow	Rate of return
A	$ 1,000	$ 250	11%
B	150,000	30,000	15
C	80,000	10,000	10
D	600	275	9
E	17,000	3,500	6

Personal Finance Problem

P5–61 LG 6 **Time to repay installment loan** Mia Salto wishes to determine how long it will take to repay a loan with initial proceeds of $14,000 where annual *end-of-year* installment payments of $2,450 are required.

a. If Mia can borrow at a 12% annual rate of interest, how long will it take for her to repay the loan fully?

b. How long will it take if she can borrow at a 9% annual rate?

c. How long will it take if she has to pay 15% annual interest?

d. Reviewing your answers in parts **a, b,** and **c,** describe the general relationship between the interest rate and the amount of time it will take Mia to repay the loan fully.

P5–62 LG 6 **ETHICS PROBLEM** A manager at a "Check Into Cash" business (see *Focus on Ethics* box on page 238) defends his business practice as simply "charging what the market will bear." "After all," says the manager, "we don't force people to come in the door." How would you respond to this ethical defense of the payday-advance business?

Spreadsheet Exercise

At the end of 2015, Uma Corporation is considering undertaking a major long-term project in an effort to remain competitive in its industry. The production and sales departments have determined the potential annual cash flow savings that could accrue to the firm if it acts soon. Specifically, they estimate that a mixed stream of future cash flow savings will occur at the end of the years 2016 through 2021. The years 2022 through 2026 will see consecutive and equal cash flow savings at the end of each year. The firm estimates that its discount rate over the first 6 years will be 7%. The expected discount rate over the years 2022 through 2026 will be 11%.

The project managers will find the project acceptable if it results in present cash flow savings of at least $860,000. The following cash flow savings data are supplied to the finance department for analysis.

End of year	Cash flow savings
2016	$110,000
2017	120,000
2018	130,000
2019	150,000
2020	160,000
2021	150,000
2022	90,000
2023	90,000
2024	90,000
2025	90,000
2026	90,000

TO DO

Create spreadsheets similar to Table 5.2, and then answer the following questions.

a. Determine the value (at the beginning of 2016) of the future cash flow savings expected to be generated by this project.

b. Based solely on the one criterion set by management, should the firm undertake this specific project? Explain.

c. What is the "interest rate risk," and how might it influence the recommendation made in part **b**? Explain.

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Track Software, Inc.

Seven years ago, after 15 years in public accounting, Stanley Booker, CPA, resigned his position as manager of cost systems for Davis, Cohen, and O'Brien Public Accountants and started Track Software, Inc. In the 2 years preceding his departure from Davis, Cohen, and O'Brien, Stanley had spent nights and weekends developing a sophisticated cost-accounting software program that became Track's initial product offering. As the firm grew, Stanley planned to develop and expand the software product offerings, all of which would be related to streamlining the accounting processes of medium- to large-sized manufacturers.

Although Track experienced losses during its first 2 years of operation—2009 and 2010—its profit has increased steadily from 2011 to the present (2015). The firm's profit history, including dividend payments and contributions to retained earnings, is summarized in Table 1.

Stanley started the firm with a $100,000 investment: his savings of $50,000 as equity and a $50,000 long-term loan from the bank. He had hoped to maintain his initial 100 percent ownership in the corporation, but after experiencing a $50,000 loss during the first year of operation (2009), he sold 60 percent of the stock to a group of investors to obtain needed funds. Since then, no other stock transactions have taken place. Although he owns only 40 percent of the firm, Stanley actively manages all aspects of its activities; the other stockholders are not active in management of the firm. The firm's stock was valued at $4.50 per share in 2014 and at $5.28 per share in 2015.

TABLE 1

**Track Software, Inc.,
Profit, Dividends, and Retained Earnings, 2009–2015**

Year	Net profits after taxes (1)	Dividends paid (2)	Contribution to retained earnings [(1) − (2)] (3)
2009	($50,000)	$ 0	($50,000)
2010	(20,000)	0	(20,000)
2011	15,000	0	15,000
2012	35,000	0	35,000
2013	40,000	1,000	39,000
2014	43,000	3,000	40,000
2015	48,000	5,000	43,000

Stanley has just prepared the firm's 2015 income statement, balance sheet, and statement of retained earnings, shown in Tables 2, 3, and 4, along with the 2014 balance sheet. In addition, he has compiled the 2014 ratio values and industry average ratio values for 2015, which are applicable to both 2014 and 2015 and are summarized in Table 5. He is quite pleased to have achieved record earnings of $48,000 in 2015, but he is concerned about the firm's cash flows. Specifically, he is finding it more and more difficult to pay the firm's bills in a timely manner and generate cash flows to investors, both creditors and owners. To gain insight into these cash flow problems, Stanley is planning to determine the firm's 2015 operating cash flow (OCF) and free cash flow (FCF).

Stanley is further frustrated by the firm's inability to afford to hire a software developer to complete development of a cost estimation package that is believed to have "blockbuster" sales potential. Stanley began development of this package 2 years ago, but the firm's growing complexity has forced him to devote more of his time to administrative duties, thereby halting the development of this product. Stanley's reluctance to fill this position stems from his concern that the added $80,000 per year in salary and benefits for the position would certainly lower the firm's earnings per share (EPS) over the next couple of years. Although the project's success is in no way guaranteed, Stanley believes that if the money were spent to hire the software developer, the firm's sales and earnings would significantly rise once the 2- to 3-year development, production, and marketing process was completed.

With all these concerns in mind, Stanley set out to review the various data to develop strategies that would help ensure a bright future for Track Software. Stanley believed that as part of this process, a thorough ratio analysis of the firm's 2015 results would provide important additional insights.

TABLE 2

Track Software, Inc., Income Statement ($000) for the Year Ended December 31, 2015	
Sales revenue	$ 1,550
Less: Cost of goods sold	$ 1,030
Gross profits	$ 520
Less: Operating expenses	
Selling expense	$ 150
General and administrative expenses	270
Depreciation expense	11
Total operating expense	431
Operating profits (EBIT)	$ 89
Less: Interest expense	29
Net profits before taxes	$ 60
Less: Taxes (20%)	12
Net profits after taxes	$ 48

TABLE 3

Track Software, Inc., Balance Sheet ($000)

	December 31	
Assets	**2015**	**2014**
Cash	$ 12	$ 31
Marketable securities	66	82
Accounts receivable	152	104
Inventories	191	145
Total current assets	$421	$362
Gross fixed assets	$195	$180
Less: Accumulated depreciation	63	52
Net fixed assets	$132	$128
Total assets	$553	$490
Liabilities and stockholders' equity		
Accounts payable	$136	$126
Notes payable	200	190
Accruals	27	25
Total current liabilities	$363	$341
Long-term debt	$ 38	$ 40
Total liabilities	$401	$381
Common stock (50,000 shares outstanding at $0.40 par value)	$ 20	$ 20
Paid-in capital in excess of par	30	30
Retained earnings	102	59
Total stockholders' equity	$152	$109
Total liabilities and stockholders' equity	$553	$490

TABLE 4

**Track Software, Inc.,
Statement of Retained Earnings ($000)
for the Year Ended December 31, 2015**

Retained earnings balance (January 1, 2015)	$ 59
Plus: Net profits after taxes (for 2015)	48
Less: Cash dividends on common stock (paid during 2015)	5
Retained earnings balance (December 31, 2015)	$102