AMITY UNIVERSITY
MADHYA PRADESH, GWALIOR
QUESTION BANK
OF
APPLIED MATHEMATICS-I
MAT101

PREPARED BY

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Question Bank 2023-2027 Batch
Applied Mathematics-I
Calculus and Linear Algebra

AMITY SCHOOL OF ENGINEERING & TECHNOLOGY (ASET)
META DATA : QUESTION PAPER

Course	B.Tech.
Batch	2023-2027
Semester	Ist Sem
Course Subject	Applied Mathematics-I (Calculus and Linear Algebra)
Course Code	MAT101
Course Credits	04
Syllabus	Attached
Question bank	Attached
Name of the Faculty Member, Designation	Dr. Avaneesh Vaishwar, Assistant Prof.
Name of Assisting/ Re-checking Faculty Member	Dr. Santosh Kumar Sharma, Associate Prof.
Name of the HoD	Dr. Santosh Kumar Sharma

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SUMMARY OF QUESTIONS

S.N.	Module	Section – A (6 Marks)	Section – B (10 Marks)	Section – C (20 Marks)
1	I	22	13	07
2	II	9	06	03
3	III	10	07	05
4	IV	10	09	06
5	V	10	08	06
TOTAL		61	43	27

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APPLIED MATHEMATICS – I
(CALCULUS AND LINEAR ALGEBRA)

Course Code: MAT101 **Credit Units: 04**
Total Hours: 40

Course Objective:
The objective of this course is to familiarize the prospective engineers with techniques in calculus, multivariate analysis and linear algebra. It aims to equip the students with standard concepts and tools at an intermediate to advanced level that will serve them well towards tackling more advanced level of mathematics and applications that they would find useful in their disciplines.

Course Contents:
Module I: Differential Calculus: (08 Hours)
Successive differentiation, Leibnitz Theorem, Rolle's theorem, Mean value theorem, Taylor's and Maclaurin's theorems with remainders, Partial Differentiation, Total derivative; Maxima and minima for two variables.

Module II: Integral Calculus: (08 Hours)
Evaluation of definite and improper integrals: Beta and Gamma functions and their properties; Applications of definite integrals to evaluate surface, areas and volumes of revolutions, Multiple Integration: Double integrals (Cartesian and polar), Triple integrals (Cartesian).

Module III: Vector Calculus: (07 Hours)
Scalar and vector field, Gradient, Divergence and Curl, Directional Derivative, Evaluation of a Line Integral, Green's theorem in plane (without proof), Stoke's theorem (without proof) and Gauss Divergence theorem (without proof).

Module IV: Matrices: (07 Hours)
Inverse and Rank of a matrix, Linear systems of equations, Consistency of Linear Simultaneous Equations, linear Independence, Gauss elimination and Gauss-Jordan elimination, Eigen values, eigenvectors, Caley-Hamilton theorem, Diagonalization.

Module V: Linear algebra & Vector spaces: (10 Hours)
Linear algebra: Group, ring, field (Definition), Vector Space, linear dependence of vectors, basis, dimension; Linear transformations (maps), range and kernel of a linear map, Inverse of a linear transformation, rank- nullity theorem (without proof), composition of linear maps, Matrix associated with a linear map.

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Course Outcomes:

The objective of this course is to familiarize the prospective engineers with techniques in basic calculus and linear algebra. It aims to equip the students with standard concepts and tools at an intermediate to advanced level that will serve them well towards tackling more advanced level of mathematics and applications that they would find useful in their disciplines.

The students will learn:

- To apply differential and integral calculus tools to the notions of curvature and to improper integrals. Apart from various applications, they will have a basic understanding of Beta and Gamma functions.
- The mathematical tools needed in evaluating multiple integrals and their usage.
- The essential tools of matrices that are used in various techniques dealing with engineering problems.
- The tools of linear algebra including linear transformations, eigen values, diagonalization.

Examination Scheme:

Components	A	CT	S/V/Q/HA	EE
Weightage (%)	5	15	10	70

CT: Class Test, HA: Home Assignment, S/V/Q: Seminar/Viva/Quiz, EE: End Semester Examination; A: Attendance

Suggested Text/Reference Books:

- Erwin Kreyszig, Advanced Engineering Mathematics, 9th Edition, John Wiley & Sons, 2006.
- B.S. Grewal, Higher Engineering Mathematics, Khanna Publishers, 36th Edition, 2010.
- G.B. Thomas and R.L. Finney, Calculus and Analytic geometry, 9th Edition, Pearson, Reprint, 2002.
- V. Krishnamurthy, V.P. Mainra and J.L. Arora, An introduction to Linear Algebra, Affiliated East–West press, Reprint 2005.

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SECTION-A **6 Marks**

Module-I

Q01. Find the fifth derivative of $y = x^3 \log x$.

Q02. If $y = \dfrac{ax+b}{cx+d}$, prove that $2y_1y_3 = 3y_2^2$.

Q03. If $y = \sin^3 x$, find nth order derivative y_n.

Q04. If $y = e^{2x}\cos x \sin^2 2x$, find nth order derivative y_n.

Q05. If $y = \dfrac{x+1}{x^2-4}$, find nth order derivative y_n.

Q06. If $y = \dfrac{x^3}{x^2-1}$, find nth order derivative y_n.

Q07. If $y = e^x \sin x$, then prove that $y_2 - 2y_1 + 2y = 0$.

Q08. If $y = \tan x$, find the third order derivative y_3.

Q09. Verify Rolle's theorem for $f(x) = x(x-2)e^{3x/4}$ in $(0,2)$.

Q10. Verify Rolle's theorem for $f(x) = \dfrac{\sin x}{x}$ in $[0,\pi]$.

Q11. Verify Rolle's theorem for $f(x) = x^3 - 12x$ in $[0, 2\sqrt{3}]$.

Q12. Verify Rolle's theorem for $f(x) = \log\left(\dfrac{x^2+ab}{(a+b)x}\right)$ in $[a,b]$.

Q13. Verify Lagrange's theorem for $f(x) = x^2$ in $(1,5)$.

Q14. Verify Lagrange's theorem for $f(x) = x(x-1)(x-2)$ in $(0,1/2)$.

Q15. Verify Lagrange's theorem for $f(x) = \log x$ in (e^2, e^3).

Q16. Verify Lagrange's theorem for $f(x) = e^{-x}$ in $(-1, 1)$.

Q17. Expand in powers of (x-1) by Taylor's theorem: $f(x) = \log x$. Hence find $\log 1.1$

Q18. By Maclaurin's theorem prove that: $\dfrac{x}{\sin x} = 1 + \dfrac{x^2}{6} + \dfrac{x^4}{40} + \cdots$

Q19. By using Taylor's theorem expand $2x^3 + 7x^2 + x - 1$ in powers of $(x-2)$.

Q20. Expand $\sin x$ in powers of $\left(x - \frac{\pi}{2}\right)$ also find its general term.

Q21. Find the maximum and minimum value of $u = x^3 + y^3 - 3axy$.

Q22. Find the maximum and minimum value of $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$.

Module-II

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Q23. Evaluate the integral $\int_0^1 \int_0^{\sqrt{1+x^2}} (1+x^2+y^2)^{-1} dxdy$.

Q24. Evaluate the integral $\int_0^{\pi} \int_0^{x} x \sin y dxdy$.

Q25. Evaluate the integral $\iint_D (4xy - y^2) dxdy$, where D is rectangle $x = 1, x = 2, y = 0, y = 3$.

Q26. Evaluate the integral $\int_0^a \int_{x/a}^{x} \frac{xdydx}{x^2+y^2}$.

Q27. Evaluate the following integrals:

(i) $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dydx}{1+x^2+y^2}$. (ii) $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyzdxdydz$

Q28. Evaluate the following integrals:

(i) $\int_0^a \int_0^b \int_0^c (x^2+y^2+z^2) dxdydz$. (ii) $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} dzdxdy$.

Q29. Prove that $\beta(m,n) = \int_0^{\infty} \frac{y^{n-1}}{(1+y)^{m+n}} dy$.

Q30. Find the value of (i) $\int_0^{\infty} \frac{x^c}{c^x} dx$. (ii) $\int_0^1 (\log 1/y)^{n-1} dy, n > 0$.

Q31. (i) Express $\int_0^1 \frac{dx}{\sqrt{1-x^4}}$ in terms of gamma functions.

(ii) Show that $\beta(m,n) = \beta(n,m)$.

Module-III

Q32. If $\vec{a} = 5t^2\hat{\imath} + t\hat{\jmath} - t^3\hat{k}, \vec{b} = \sin t\hat{\imath} - \cos t\hat{\jmath}$, Find (i) $\frac{d}{dt}(\vec{a}.\vec{b})$ (ii) $\frac{d}{dt}(\vec{a} \times \vec{b})$.

Q33. A particle moves along a curve and the position vector of the particle at time t is $\vec{R} = e^{-t}\hat{\imath} + 2\cos 3t\,\hat{\jmath} + 2\sin 3t\,\hat{k}$. Determine its velocity and acceleration vectors and also the magnitudes of velocity and acceleration at t=0.

Q34. Find the divergence and the curl of the vector $\vec{F} = (xyz + y^2z)\hat{\imath} + (3x^2y + y^2z)\hat{\jmath} + (xz^2 - y^2z)\hat{k}$.

Q35. If $\vec{F} = (x+y+z)\hat{\imath} - (x+y)\hat{k}$, show that $\vec{F}.curl\vec{F} = 0$.

Q36. Find the $div\vec{F}$ and $curl\vec{F}$, where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$.

Q37. Prove that div (curl$\vec{A}$) =0.

Q38. If $\vec{R} = \vec{A}\cos\lambda t + \vec{B}\sin\lambda t$, where $\vec{A}$ and $\vec{B}$ are vectors independent of t and λ is a constant, show that

(i) $\vec{R} \times \frac{d\vec{R}}{dt} = \lambda\vec{A} \times \vec{B}$, (ii) $\frac{d^2\vec{R}}{dt^2} = -\lambda^2\vec{R}$.

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Q39. Consider the vector field defined by $\vec{V} = (x^2 - yz)\hat{\imath} + (y^2 - zx)\hat{\jmath} + (z^2 - xy)\hat{k}$. Show that the vector field is irrotational.

Q40. If $\vec{V} = 2xy^2\hat{\imath} + 3x^2y\hat{\jmath} - 3ayz\hat{k}$ is solenoidal at (1,1,1), find a.

Q41. If $\vec{r} = \sin t\,\hat{\imath} + \cos t\hat{\jmath} + t\hat{k}$, then find $\frac{d\vec{r}}{dt}, \frac{d^2\vec{r}}{dt^2}, \left|\frac{d\vec{r}}{dt}\right|$ and $\left|\frac{d^2\vec{r}}{dt^2}\right|$.

Module-IV

Q42. Find the rank of the following matrix by reducing normal form

$$\begin{bmatrix} 1 & 4 & 5 \\ 2 & 6 & 8 \\ 3 & 7 & 22 \end{bmatrix}$$

Q43. Find the rank of the following matrix

$$\begin{bmatrix} 1^2 & 2^2 & 3^2 & 4^2 \\ 2^2 & 3^2 & 4^2 & 5^2 \\ 3^2 & 4^2 & 5^2 & 6^2 \\ 4^2 & 5^2 & 6^2 & 7^2 \end{bmatrix}$$

Q44. Find non-singular matrix P & Q so that matrix reduced in normal form

$$\begin{bmatrix} 1 & 2 & 3 & -2 \\ 2 & -2 & 1 & 3 \\ 3 & 0 & 4 & 1 \end{bmatrix}$$

Q45. Using the Gauss-Jordan method, solve the following equation:
$3x + 2y + 7z = 4, 2x + 3y + z = 5, 3x + 4y + z = 7$.

Q46. Using the Gauss-Jordan method, solve the following equation:
$7x + 52y + 13z = 104, 83x + 11y - 4z = 95, 3x + 8y + 29z = 71$.

Q47. Using the Gauss-Jordan method, solve the following equation:
$2x + 4y + z = 3, 3x + 2y - 2z = -2, x - y + z = 6$.

Q48. Test for consistency and solve :
$5x + 3y + 7z = 4, 3x + 26y + 2z = 9, 7x + 2y + 10z = 5$.

Q49.Find for what value of $\lambda and \mu$ the system of linear equations:

$$x + y + z = 6, x + 2y + 5z = 10, 2x + 3y + \lambda z = \mu,$$

has (i) a unique solution (ii) no solution (iii) infinite solution.

Q50. Show that the linear equations $3x + 4y + 5z = a, 4x + 5y + 6z = b, 5x + 6y + 7z = c$ don't have

a solution unless $a + c = 2b$.

Q51. Solve by Gauss Elimination method:

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$x-y+z=1, -3x+2y-3z=-6, 2x-5y+4z=5.$

Module-V

Q52. Define the Group with example.

Q53. Define the Ring with example.

Q54. Define Field with example.

Q55. Define the terms with examples: Vector Space, Basis, linear transformations.
Q56. Give one example of linear transformations each in two and three dimensions.
Q57. Define the linearly dependent and linearly independent vectors with example.
Q58. Prove that $T: V_3 \to V_2$ given by $T(a,b,c)=(c,a+b)$ is a linear transformation.
Q59. Represent the linear transformation that maps (x,y) to $(2x-5y, 3x+4y)$ in matrix form. Also give the inverse transformation if exists.
Q60. Are the vectors $(1,3,4,2), (3,-5,2,2), (2,-1,3,2)$ linearly dependent? If so, express one of these as a linear combination of the others.
Q61. Are the vectors $(1,0,0), (0,1,0), (0,0,1)$ linearly dependent? If so, express one of these as a linear combination of others.

SECTION-B **10 Marks**

Module-I

Q62. If $y=x^2\sin x$, find the nth order derivative at $x=0$.

Q63. If $y=a\cos(\log x)+b\sin(\log x)$, prove that $x^2y_{n+2}+(2n+1)xy_{n+1}+(n^2+1)y_n=0$.

Q64. If $y=\sin(m\sin^{-1}x)$, prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2-m^2)y_n=0$.

Q65. Verify Cauchy's MVT for $f(x)=x^4, g(x)=x^2$ for $[a, b]$.

Q66. Verify Cauchy's MVT for $f(x)=\log x, g(x)=1/x$ for $[1, e]$.

Q67. Verify Cauchy's MVT for $f(x)=e^x, g(x)=e^{-x}$ for $[a, b]$.

Q68. Find the first order partial derivative of (i) $w=\tan^{-1}\left(\frac{y}{x}\right)$ and (ii) $w=\log\sqrt{x^2+y^2}$.

Q69. (i) If $u=e^{xyz}$, find $\frac{\partial^3 u}{\partial x\partial y\partial z}$. (ii) . If $v=\frac{1}{\sqrt{t}}e^{-x^2/4a^2t}$, prove that $\left(\frac{\partial v}{\partial t}\right)=a^2\frac{\partial^2 v}{\partial x^2}$.

Q70. If $\frac{x^2}{a^2+u}+\frac{y^2}{b^2+u}+\frac{z^2}{c^2+u}=1$, prove that, $\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial u}{\partial z}\right)^2=2\left(x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}\right)$

Q71. If $V=f(2x-3y, 3y-4z, 4z-2x)$, prove that $6V_x+4V_y+3V_z=0$.

Q72. If $u=x^2-y^2,\ x=2r-3s$ and $y=-r+3s$. Find $\frac{du}{ds}$ and $\frac{du}{dr}$.

Q73. Expand $e^{a\sin^{-1}x}$ by Maclaurin's theorem also find its general term.

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Q74. Show that
$\tan^{-1}(x+h) = \tan^{-1} x + (h\sin z)\sin z - (h\sin z)^2 \frac{\sin 2z}{2} + (h\sin z)^3 \frac{\sin 3z}{3} + \cdots$, where $z = \cot^{-1} x$.

Module-II

Q75. Find area bounded by the curves $y^2 = x$ and $x^2 = y$.

Q76. Find the area bounded by the curves $y = x$ and $x^2 = y$.

Q77. State and prove duplicate formula of Gamma function.

Q78. Find the volume of the reel-shaped solid formed by the revolution about the y-axis, of the part of the parabola $y^2 = 4ax$ cut off by the latusrectum.

Q79. Find the volume generated by revolving the portion of parabola $y^2 = 4ax$ cut off by its latusrectum about the axis parabola.

Q80. Prove that $\beta(m,n) = 2\int_0^{\frac{\pi}{2}} sin^{m-1}x\, cos^{n-1}x dx$ and hence show that

$$\int_0^{\frac{\pi}{2}} sin^p x\, cos^q x dx = \frac{\Gamma(\frac{p+1}{2})\Gamma(\frac{q+1}{2})}{2\Gamma(\frac{p+q+2}{2})}$$

Module-III

Q81. Find a unit vector normal to the surface $xy^3 + xz^2$ at the point (-1,-1, 2).

Q82. Find a unit vector normal to the vectors $\vec{a} = 3\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$ and $\vec{b} = \hat{\imath} + \hat{\jmath} - 2\hat{k}$. Also, find the sine of the angle between them.

Q83. A vector field is given by $\vec{F} = (x^2 - y^2 + x)\hat{\imath} - (2xy + y)\hat{\jmath}$. Show that the field is irrotational and find its scalar potential.

Q84. What is the directional derivative of $\varphi = xy^2 + yz^3$ at the point (2,-1,1) in the direction of the normal to the surface $x log z - y^2 = -4$ at (-1,2,1).

Q85. Suppose $\vec{F} = 2xy\hat{\imath} + 3y^2\hat{\jmath} + yz\hat{k}$ and the displacement it takes place from position $P(0,2,0)$ to position $Q(1,4,0)$ by moving first along the line $y = 2, z = 0$ and then along $= 1, z = 0$. Find the work done by the force $\vec{F}$ for this displacement.

Q86. Prove that $\nabla r^n = nr^{n-2}\vec{R}$, where $\vec{R} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and $r = |\vec{R}|$.

Q87. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at $(2,-1,2)$.

Q88. (a) Prove that $\nabla^2(\log r) = \frac{1}{r^2}$, where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and $r = |\vec{r}|$.

(b) If $\vec{R}$ is the position vector of any point $P(x,y,z)$. Show that $\nabla.\vec{R} = 3$.

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Module-IV

Q89. Find eigen values and eigen vectors of the following matrix

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

Q90. Find eigen values and eigen vectors of the following matrix

$$\begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ 2 & 4 & 1 \end{bmatrix}$$

Q91. Find eigen values and eigen vectors of the following matrix

$$\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

Q92. Find eigen values and eigen vectors of the following matrix

$$\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

Q93. Verify Cayley-Hamilton theorem and find A^{-1}

$$\begin{bmatrix} 1 & 1 & 3 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

Q94. Find A^4 with the help of Cayley-Hamilton theorem, if $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & -1 \\ 2 & 2 & 3 \end{bmatrix}$.

Q95. Find the matrix represented by the matrix $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$, where $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$.

Q96. Find the characteristic equation of the symmetric matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and verify that it issatisfied by 'A' and hence obtain A^{-1}. Express $A^6 - 6A^5 + 9A^4 - 2A^3 - 12A^2 + 23A - 9I$ in linear polynomial in 'A'.

Module-V

Q97. Explain: Rank- nullity theorem, Kernel and Range of a linear transformation.

Q98. Let us define $T: R^4 \to R^3$ by $T(x, y, z, w) = (x - w, y + z, y - w)$. Describe the null space (kernel) and the range of T.

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Q99. Let us define $T : R^2 \to R^3$ by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - x_2 \\ x_1 + x_2 \\ x_2 \end{bmatrix}$. Describe the null space (kernel) and the range of T.

Q100. Find the range, rank and nullity of the zero linear transformation.

Q101. Find the range, rank and nullity of the identity linear transformation.

Q102. Find the inverse transformation of $y_1 = x_1 + 2x_2 + 5x_3; y_2 = 2x_1 + 4x_2 + 11x_3; y_3 = -x_2 + 2x_3$.

Q103. Let $T(a, b)$ =reflextion on $x - axis. B_1 = \{(1,0), (0,1)\}$ and $B_2 = \{(1,1), (1,0)\}$ are the basis. The find $[T; B_1, B_1], [T; B_2, B_2], [T; B_1, B_2], and [T; B_2, B_1]$.

Q104. Let $T(a, b)$ =reflextion on $y - axis. B_1 = \{(1,0), (0,1)\}$ and $B_2 = \{(1,1), (1,0)\}$ are the basis. The find $[T; B_1, B_1], [T; B_2, B_2], [T; B_1, B_2], and [T; B_2, B_1]$.

SECTION-C **10 Marks**

Module-I

Q105. If $y = e^{m\cos^{-1}x}$, find the nth order derivative $y_n(0)$.

Q106. If $y = \cos(m\sin^{-1}x)$, find the nth order derivative $y_n(0)$.

Q107. If $y = (\sin^{-1}x)^2$, find the nth order derivative $y_n(0)$.

Q108. If $u = log(x^3 + y^3 + z^3 - 3xyz)$, show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$

Q109. If $u = f(r)$ where $r^2 = x^2 + y^2 + z^2$ then proves that $\left(\frac{\partial 2}{\partial x2} + \frac{\partial 2}{\partial y2} + \frac{\partial 2}{\partial z2}\right) f(r) = f''(r) - \frac{2}{r} f'(r)$.

Q110. (i) Differentiate totally to the function $u = f(x, y)$ where *x* and *y* are function of *t.* If $u = \sin^{-1}(x - y), x = 3t and y = 4t^3$, show that $\frac{du}{dt} = 3(1 - t^2)^{1/2}$.

(ii) Differentiate totally to the function $u = f(x, y)$ where *x* and *y* are function of *t.* If $u = \tan^{-1}\left(\frac{y}{x}\right)$, where $x = e^t - e^{-t} and y = e^t + e^{-t}$ find $\frac{du}{dt}$.

Q111. Differentiate totally to the function $u = f(x, y, z)$ where *x, y* and *z* are function of *t.* If $u = x^2 + y^2 + z^2 and x = e^{2t}, y = e^{2t}\cos 3t \, and z = e^{2t}\sin 3t$. Find $\frac{du}{dt}$ as a total derivative and verify the result by direct method.

Module-II

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Q112. Find $\iint_D (x^2+y^2)dxdy$, where the region of integration is bounded by $y=x,\ y=2x$ and $x=1$ in first quadrant.

Q113. Find the volume of the solid obtained by revolving the cissoid $y^2(2a-x)=x^3$.

Q114. Find the volume of the solid formed by the revolution, about x-axis, of the loop of the curve: $y^2(a-x)=x^2(a+x)$.

Module-III

Q115. (a) Prove that $\vec{F}=\frac{x\hat{\imath}+y\hat{\jmath}}{x^2+y^2}$ is both solenoidal and irrotational.

(b) Find the constants a and b so that $\vec{F}=(axy+z^3)\hat{\imath}+(3x^2-z)\hat{\jmath}+(bxz^2-y)\hat{k}$ is irrotational.

Q116. State the Divergence theorem. Verify the Divergence theorem for the expanding vector field $\vec{F}=x\hat{\imath}+y\hat{\jmath}+z\hat{k}$ over the sphere $x^2+y^2+z^2=a^2$.

Q117. State the Green's Theorem. Verify both forms of Green's Theorem for the vector field $\vec{F}=(x-y)\hat{\imath}+x\hat{\jmath}$ and the region R bounded by the unit circle $C: r(t)=\cos t\hat{\imath}+\sin t\hat{\jmath},\quad 0\le t\le 2\pi$.

Q118. State Stoke's Theorem. Verify the Stoke's Theorem for the hemisphere $S: x^2+y^2+z^2=9, z\ge 0$, its bounding circle $C: x^2+y^2=9, z=0$ and the field $\vec{F}=y\hat{\imath}-x\hat{\jmath}$.

Q119. Use Stoke's Theorem to evaluate $\int_C \vec{F}d\vec{r}$, if $\vec{F}=xz\hat{\imath}+xy\hat{\jmath}+3xz\hat{k}$ and C is the boundary of the portion of the plane $2x+y+z=2$ in the first octant, traversed counterclockwise.

Module-IV

Q120. Find a matrix P which transform the matrix $A=\begin{bmatrix}1 & 0 & -1\\ 1 & 2 & 1\\ 2 & 2 & 3\end{bmatrix}$ to diagonal form. Hence find A^4.

Q121. Find a matrix P which transform the matrix $A=\begin{bmatrix}1 & 1 & 3\\ 1 & 5 & 1\\ 3 & 1 & 1\end{bmatrix}$ to diagonal form. Hence find A^4.

Q122. Reduce the matrix 'A' into a diagonal matrix where $A=\begin{bmatrix}3 & -1 & 1\\ -1 & 5 & -1\\ 1 & -1 & 3\end{bmatrix}$. Hence calculate A^4.

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Applied Mathematics-I
Calculus and Linear Algebra

Q123. Determine the eigen values and corresponding eigen vectors of the matrix $A = \begin{bmatrix} 5 & 7 & -5 \\ 0 & 4 & -1 \\ 2 & 8 & -3 \end{bmatrix}$. Hence find the matrix 'P' such that $P^{-1}AP$ is diagonal matrix also find A^4.

Q124. Determine the eigen values and corresponding eigen vectors of the matrix $A = \begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 4 \end{bmatrix}$. Hence find the matrix 'P' such that $P^{-1}AP$ is diagonal matrix also find A^4.

Q125. Show that matrix 'A' is diagonalizable $= \begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$. If so obtain the matrix 'P' such that$P^{-1}AP$ is diagonal matrix also find A^4.

Module-V

Q126 Represent each of the transformations $x_1 = 3y_1 + 2y_2, y_1 = z_1 + 2z_2$ and $x_2 = -y_1 + 4y_2, y_2 = 3z_1$ by the use of matrices and find the composite transformation which expresses x_1, x_2 in terms of z_1, z_2.

Q127.If $T(a,b) = (a\ \cos 2\alpha + b \sin 2\alpha, a \sin 2\alpha - b \cos 2\alpha); B_1 = \{(1,0),(0,1)\}$ and $B_2 = \{(1,1),(1,0)\}$ are the basis. The find $[T;B_1,B_1],[T;B_2,B_2],[T;B_1,B_2], and [T;B_2,B_1]$.

Q128 A transformation from the variables $X = (x_1, x_2, x_3)$ to $Y = (y_1, y_2, y_3)$ is given by $Y = AX$, and another transformation from $Y = (y_1, y_2, y_3)$ to $Z = (z_1, z_2, z_3)$ is given by $Z = BY$, where

$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & -2 \\ -1 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{bmatrix}$. Obtain the transformation from x_1, x_2, x_3 to z_1, z_2, z_3.

Q129. Let T_1 and T_2 be a transformation from V to itself, where V be the vector space of polynomial of degree atmost 3 and

(i) $T_1(p(x)) = p'(x)$,

(ii) $T_2(p(x)) = \int_0^x p(t)dt$.

Then find $[T;B;B]$, where B is a standard basis of V.

Q130. Are the following vectors linearly dependent? If so, find the relation between them

(i) $(2,1,1),(2,0,-1),(4,2,1)$,

(ii) $(1,1,1,3),(1,2,3,4),(2,3,4,9)$.

Q131. Show that the transformation $T(x,y,z) = (3x, x - y, 2x + y + z)$ is linear and invertible. Also find T^{-1}.