1. ① C represents the capacitance across the double layer of charge that occurs at the electrode - electrolyte interface

② R_p is a parallel resistance that represents the leakage resistance to the DC current that may form across the double-layer of charge at the interface

C, R_p represent the overall impedance associated with electrode-electrolyte interface and any polarization effects

③ R_s is the series resistance that represents the resistance in the electrolyte. This is added for lower frequency cases, as without it, there would be infinite impedance. So, at low frequencies, there is a purely resistive impedance.

④ E_0 represents the half-cell potential of the electrode-electrolyte interface

2. In surface recordings, we generally have an initial electrode-electrolyte interface with a gel to establish contact with the skin. As a result, the electrode model in Q1 becomes the electrode-gel interface. Due to deep skin layers, we treat them as seperate individual interfaces, and model each layer as an additional interface circuit model. Thus, the circuit in Q1 becomes large.

In invasive recording, we do not have to worry about "additional layers". We can have a simple circuit model as in Q1, as there is no electrolyte-skin interface that needs to be considered. Then, we can expect the components to represent:

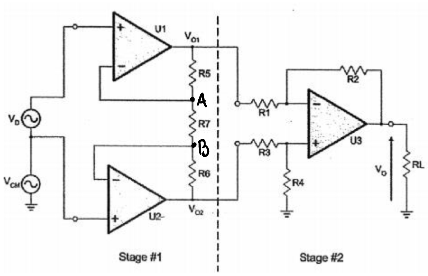
① C represents the capacitance across the double-layer of charge that would form between the invasive electrode and extracellular fluid within body interface (e.g. fluid around muscle fiber)

② R_p would be parallel resistance representing the resistance of moving charge across the double-layer at the electrode inside body and extracellular fluid (electrolyte) interface

③ R_s would be the series resistance, representing extracellular body fluid (electrolyte) resistance

④ E_0 would be half-cell potential of invasive electrode and extracellular fluid (electrolyte) within the body

3.



Stage #2:

From lecture we know V_- and V_+ at U3 (stage #2 is a differential amplifier):

$$V_- = \frac{V_{o1} R_2}{R_1+R_2} + \frac{V_o R_1}{R_1+R_2} \qquad V_+ = \frac{V_{o2} R_4}{R_3+R_4}$$

Remember: voltage divider at V_- and V_+ at U3

For ideal OA: $V_- = V_+ \Rightarrow \frac{V_{o1} R_2}{R_1+R_2} + \frac{V_o R_1}{R_1+R_2} = \frac{V_{o2} R_4}{R_3+R_4}$

$$\frac{V_o R_1}{R_1+R_2} = \frac{V_{o2} R_4}{R_3+R_4} - \frac{V_{o1} R_2}{R_1+R_2} \qquad — ①$$

Stage #1:

Apply KCL @ A: $\frac{V_{o1} - V_A}{R_5} = \frac{V_A - V_B}{R_7} \Rightarrow V_{o1} = \frac{V_A - V_B}{R_7} \cdot R_5 + V_A$ — ②

Apply KCL @ B: $\frac{V_A - V_B}{R_7} = \frac{V_B - V_{o2}}{R_6} \Rightarrow V_{o2} = V_B - \frac{V_A - V_B}{R_7} R_6$ — ③

Now, ②, ③ → ①:

$$\frac{V_o R_1}{R_1+R_2} = \frac{R_4}{R_3+R_4}\left[V_B - \frac{V_A - V_B}{R_7} R_6\right] - \frac{R_2}{R_1+R_2}\left[\frac{V_A - V_B}{R_7} R_5 + V_A\right]$$

$$\frac{V_o R_1}{R_1+R_2} = \frac{V_B R_4}{R_3+R_4} - \frac{R_4 R_6 (V_A - V_B)}{R_7 (R_3+R_4)} - \frac{R_2 R_5 (V_A - V_B)}{R_7 (R_1+R_2)} - \frac{V_A R_2}{R_1+R_2}$$

Now, for differential input: $V_o = V_{oD}$, $V_A = \frac{-V_D}{2}$, and $V_B = \frac{V_D}{2}$:

$$\frac{V_{oD} R_1}{R_1+R_2} = \frac{V_D}{2} \cdot \frac{R_4}{R_3+R_4} + \frac{V_D R_4 R_6}{R_7 (R_3+R_4)} + \frac{V_D R_2 R_5}{R_7 (R_1+R_2)} + \frac{V_D}{2} \cdot \frac{R_2}{R_1+R_2}$$

$$\frac{V_{oD} R_1}{R_1+R_2} = V_D \left[\frac{R_4}{2(R_3+R_4)} + \frac{R_4 R_6}{R_7 (R_3+R_4)} + \frac{R_2 R_5}{R_7 (R_1+R_2)} + \frac{R_2}{2(R_1+R_2)}\right]$$

$$A_D = \frac{V_{oD}}{V_D} = \frac{R_1+R_2}{R_1}\left[\frac{R_4}{2(R_3+R_4)} + \frac{R_4 R_6}{R_7 (R_3+R_4)} + \frac{R_2 R_5}{R_7 (R_1+R_2)} + \frac{R_2}{2(R_1+R_2)}\right]$$

4. First, we must compute A_{cm}: $V_O = V_{OCM}$, $V_A = V_{cm}$, $V_B = V_{cm}$

$$\frac{V_{ocm}R_1}{R_1+R_2} = \frac{V_{cm}R_4}{R_3+R_4} - \frac{V_{cm}R_2}{R_1+R_2}$$

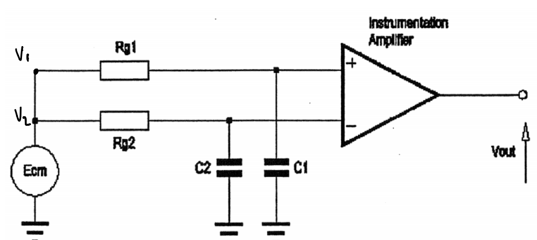
$$\frac{V_{ocm}R_1}{R_1+R_2} = V_{cm}\left[\frac{R_4}{R_3+R_4} - \frac{R_2}{R_1+R_2}\right]$$

$$A_{cm} = \frac{V_{ocm}}{V_{cm}} = \frac{R_1+R_2}{R_1}\left[\frac{R_4}{R_3+R_4} - \frac{R_2}{R_1+R_2}\right]$$

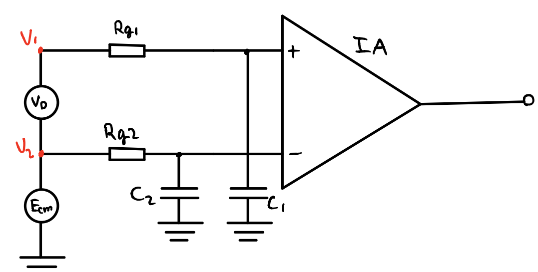
Now: $$CMRR = \frac{A_D}{A_{cm}} = \frac{\cancel{\frac{R_1+R_2}{R_1}}^{\,1}\left[\frac{R_4}{2(R_3+R_4)} + \frac{R_4R_6}{R_7(R_3+R_4)} + \frac{R_2R_5}{R_7(R_1+R_2)} + \frac{R_2}{2(R_1+R_2)}\right]}{1\;\cancel{\frac{R_1+R_2}{R_1}}\left[\frac{R_4}{R_3+R_4} - \frac{R_2}{R_1+R_2}\right]}$$

$$CMRR = \frac{\frac{R_4}{2(R_3+R_4)} + \frac{R_4R_6}{R_7(R_3+R_4)} + \frac{R_2R_5}{R_7(R_1+R_2)} + \frac{R_2}{2(R_1+R_2)}}{\frac{R_4}{R_3+R_4} - \frac{R_2}{R_1+R_2}}$$

5. Common-mode (only) case:



Differential Input Case:



Observe that for an IA, V_o will depend on A_D and A_{cm}:

$V_o = A_D V_D + \cancel{A_{cm}V_{cm}}^{\,0}$ Considering an ideal IA with ∞ impedance, $A_{cm} \approx 0$

Due to potential mismatches between V_+ and V_- inputs, there can be a small differential input. Let us compute V_+ and V_-:

$$\left.\begin{aligned} V_+ &= \frac{V_1 Z_{c1}}{Z_{c1} + R_{g1}} \\ V_- &= \frac{V_2 Z_{c2}}{Z_{c2} + R_{g2}} \end{aligned}\right\} \quad V_o = A_D(V_+ - V_-) = A_D\left[\frac{V_1 Z_{c1}}{Z_{c1}+R_{g1}} - \frac{V_2 Z_{c2}}{Z_{c2}+R_{g2}}\right] \quad \text{where } Z_C = \frac{1}{sC},\ s = j\omega$$

$$= A_D\left[\frac{V_1}{1+sC_1R_{g1}} - \frac{V_2}{1+sC_2R_{g2}}\right]$$

For overall differential gain, A_{OD}: $V_1 = \frac{V_D}{2}$, $V_2 = \frac{-V_D}{2}$, and $V_o = V_{OD}$

$$V_{OD} = A_D \frac{V_D}{2}\left[\frac{1}{1+sC_1R_{g1}} + \frac{1}{1+sC_2R_{g2}}\right]$$

$$A_{OD} = \frac{V_{OD}}{V_D} = \frac{A_D}{2}\left[\frac{1}{1+sC_1R_{g1}} + \frac{1}{1+sC_2R_{g2}}\right]$$

For overall common-mode gain, A_{OCM}: $V_1 = V_2 = V_{cm}$ and $V_o = V_{OCM}$

$$V_{OCM} = A_D V_{cm}\left[\frac{1}{1+sC_1R_{g1}} - \frac{1}{1+sC_2R_{g2}}\right]$$

$$A_{OCM} = \frac{V_{OCM}}{V_{CM}} = A_D\left[\frac{1}{1+sC_1R_{g1}} - \frac{1}{1+sC_2R_{g2}}\right]$$

Now, $$CMRR = \frac{A_{OD}}{A_{OCM}} = \frac{\frac{\cancel{A_D}'}{2}\left(\frac{1}{1+sC_1R_{g1}} + \frac{1}{1+sC_2R_{g2}}\right)}{\cancel{A_D}'\left(\frac{1}{1+sC_1R_{g1}} - \frac{1}{1+sC_2R_{g2}}\right)}$$

$$= \frac{1}{2}\,\frac{\dfrac{1+sC_2R_{g2}+1+sC_1R_{g1}}{(1+sC_1R_{g1})(1+sC_2R_{g2})}}{\dfrac{1+sC_2R_{g2}-1-sC_1R_{g1}}{(1+sC_1R_{g1})(1+sC_2R_{g2})}}$$

$$= \frac{1}{2}\left[\frac{1+sC_2R_{g2}+1+sC_1R_{g1}}{1+sC_2R_{g2}-1-sC_1R_{g1}}\right]$$

$$CMRR = \frac{1}{2}\left[\frac{2+s(C_2R_{g2}+C_1R_{g1})}{s(C_2R_{g2}-C_1R_{g1})}\right] \quad \text{where } s = j\omega$$

$$|CMRR| = \frac{1}{2}\left[\frac{\left(4+\omega^2(C_2R_{g2}+C_1R_{g1})^2\right)^{1/2}}{\omega(C_2R_{g2}-C_1R_{g1})}\right]$$

Now, to compute cutoff frequency and steady state CMRR, let us put the CMRR into standard form:

$$CMRR = \frac{C_2R_{g2}+C_1R_{g1}}{2(C_2R_{g2}-C_1R_{g1})}\left[\frac{\frac{2}{C_2R_{g2}+C_1R_{g1}} + s}{s}\right]$$

$$= \frac{1}{C_2R_{g2}-C_1R_{g1}}\left[\frac{\dfrac{s}{\left(\frac{2}{C_2R_{g2}+C_1R_{g1}}\right)} + 1}{s}\right]$$

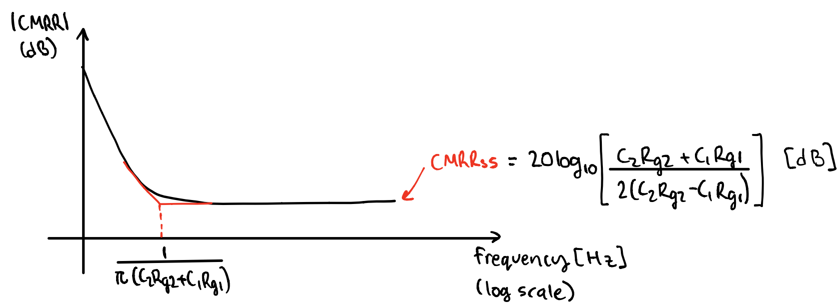
So, the break frequency is: $\frac{2}{C_2R_{g2}+C_1R_{g1}} \times \frac{1}{2\pi} = \frac{1}{\pi(C_2R_{g2}+C_1R_{g1})}$ [Hz]

The steady state CMRR is: $CMRR_{ss} = \lim_{s\to\infty} \dfrac{1}{C_2R_{g2}-C_1R_{g1}} \left[\dfrac{\left(\dfrac{s}{\frac{2}{C_2R_{g2}+C_1R_{g1}}}\right) + 1}{s} \right]$

$$= \lim_{s\to\infty} \frac{1}{C_2R_{g2}-C_1R_{g1}} \left[\frac{\cancel{s}(C_2R_{g2}+C_1R_{g1})}{2\cancel{s}} + \frac{1}{s} \right]$$

$$= \frac{C_2R_{g2}+C_1R_{g1}}{2(C_2R_{g2}-C_1R_{g1})}$$

Now, using bode plot rules, since there is a pole at 0, and zero at $\dfrac{-2}{C_2R_{g2}+C_1R_{g1}}$, we can plot the following CMRR-frequency graph:



6. <u>Amplifier range for CMRR < 80dB:</u>

$C = 58(10^{-12})(3) = 1.74(10^{-10})$ F

$20\log_{10}(CMRR) = 80\text{dB} \Rightarrow \log_{10}(CMRR) = 4 \Rightarrow CMRR = 10^4 = 10000$

Now: $|CMRR| = 10000 = \dfrac{1}{2(1.914(10^{-6}) - 1.74(10^{-6}))}\left[\dfrac{\left(4 + \omega^2(1.914(10^{-6})+1.74(10^{-6}))^2\right)^{1/2}}{\omega}\right]$

$$10000 = \frac{1}{3.48(10^{-7})}\left[\frac{(4+1.3352(10^{-11})\omega^2)^{1/2}}{\omega}\right]$$

$$0.00348\omega = (4+1.3352(10^{-11})\omega^2)^{1/2}$$

$$1.21104(10^{-5})\omega^2 = 4 + 1.3352(10^{-11})\omega^2$$

$$1.211038(10^{-5})\omega^2 = 4$$

$\omega = 574.713$ rad./s $\Rightarrow$ 91.47 Hz

$\therefore$ The amplifier's CMRR falls below 80 dB when $\omega >$ 91.47 Hz.

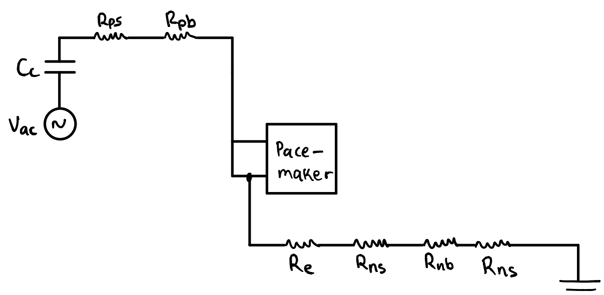
CMRR @ 60Hz :

$$|CMRR| = \frac{1}{3.48(10^{-7})}\left[\frac{\left(4+(2\pi\cdot 60)^2(1.3352(10^{-11}))\right)^{1/2}}{2\pi\cdot 60}\right]$$

$$|CMRR| = 15244.73 \rightarrow 20\log_{10}(15244.73) = 83.66\text{ dB}$$

∴ The CMRR at 60 Hz is 83.66 dB.

7. a)



b) By touching the pacemaker, the nurse will complete the circuit. We can then compute the current going through the nurse and the patient:

$$Z_c = \frac{1}{j(2\pi(60))(2500(10^{-12}))} = \frac{1}{j9.4247(10^{-7})}$$

$$R = Z_c + R_{ps} + R_{pb} + R_e + R_{nb} + 2R_{ns}$$

$$= \frac{1}{j9.4247(10^{-7})} + 100000 + 500 + 1000 + 500 + 2(100000)$$

$$= \left[\left(\frac{1}{j9.4247(10^{-7})}\right)^2 + (302000)^2\right]^{1/2}$$

$$= 1103174.931\ \Omega$$

Now, $I = \frac{V_{ac}}{R} = \frac{120}{1103174.931} \doteq 108.78\ \mu A$

∴ Since current is very small (less than perception level), the nurse will not be impacted and will not endure any electrical damage. Also, since current will flow from one arm to the other, the nurse will experience macroshock. This means current will get divided across her body, and only a small amount of the 108.78 μA will flow across her heart.

c) Since the circuit consists of all series resistors, the patient will have the same $I \doteq 108.78\ \mu A$ current go through them. Due to the trans-venous catheter, the patient will endure microshock and all all the current will pass through the heart. Since $I > 60\ \mu A$, the patient will experience fibrillation in their heart.