1. $E_{Cl^-} = 2.303 \frac{RT}{zF} \log \frac{[Cl^-]_o}{[Cl^-]_i} = 2.303 \frac{(8.314)(37+273.15)}{(-1)(96.5)} \log \frac{150}{13} \doteq -65.36 \text{ mV}$

2. Let us first compute the resting membrane potential (E_m) when it is impermeable to Cl^-:

$$E_m = 61.54 \log \frac{P_{K^+}[K^+]_o + P_{Na^+}[Na^+]_o}{P_{K^+}[K^+]_i + P_{Na^+}[Na^+]_i} = 61.54 \log \frac{40(5) + 1(150)}{40(100) + 1(15)} = -65.2 \text{ mV}$$

It can be observed that the E_m when it is impermeable to Cl^- is very similar to the equilibrium potential of Cl^- (E_{Cl^-}). Since $E_m \approx E_{Cl^-}$, considering the permeability of Cl^- ions would insignificantly impact the E_m. If we consider Cl^- ion permeability (P_{Cl^-}), as P_{Cl^-} increases, the E_m would shift towards the $E_{Cl^-} = -65.36$ mV, which would be a small, negligible change (see q3).

3. From lecture, we know $P_{K^+} = 40 P_{Na^+}$. Let us model $P_{Cl^-} = x P_{Na^+}$.

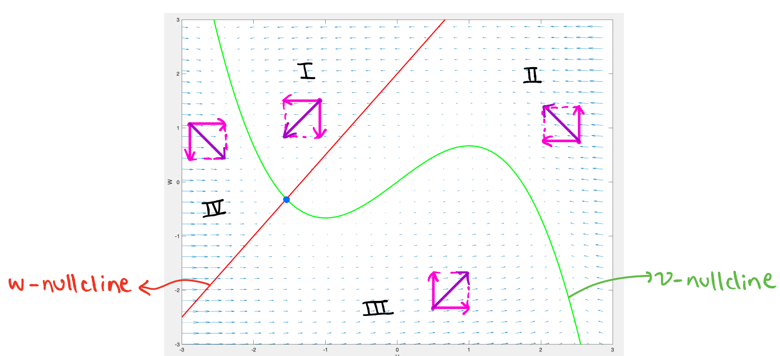
Then, $V_m = 61.54 \log \frac{40(5) + 1(150) + x(13)}{40(100) + 1(15) + x(150)} = 61.54 \log \frac{350 + 13x}{4015 + 150x}$

Now, as $x \to 0$: $V_m = 61.54 \log \frac{350}{4015} \doteq -65.21 \text{ mV}$

as $x \to \infty$: $V_m = 61.54 \log \frac{350 + 13(\infty)}{4015 + 150(\infty)} \doteq -65.36 \text{ mV}$

$\therefore$ V_m is insignificantly influenced by the permeability of Cl^- ions at very low and high levels of Cl^- ion permeability, since V_m changes very slightly.

4. Using MATLAB (a1q4.m), the phase plot of the dynamic system was generated:



Distinct Regions

I: $\frac{dv}{dt} < 0$, $\frac{dw}{dt} < 0$ III: $\frac{dv}{dt} > 0$, $\frac{dw}{dt} > 0$

II: $\frac{dv}{dt} < 0$, $\frac{dw}{dt} > 0$ IV: $\frac{dv}{dt} > 0$, $\frac{dw}{dt} < 0$

v-nullcline

When $\frac{dv}{dt} = 0 \rightarrow f(v,w) = v - \frac{1}{3}v^3 - w + I_{app}$

So, v-nullcline is $\quad w = v - \frac{1}{3}v^3 + I_{app} \quad$ — ①

w-nullcline

When $\frac{dw}{dt} = 0 \rightarrow f(v,w) = \epsilon(b_0 + b_1 v - w)$

So, w-nullcline is $\quad w = b_0 + b_1 v \quad$ — ②

Fixed Points

② → ①: $\quad b_0 + b_1 v = v - \frac{1}{3}v^3 + \cancel{I_{app}}^{\,0}$

$$0 = -\frac{1}{3}v^3 - 0.5v - 2$$

Using MATLAB roots method, we find $v = -1.5444$

Now, $v = -1.5444 \rightarrow$ ②: $w = 2 + 1.5(-1.5444) = -0.3166$

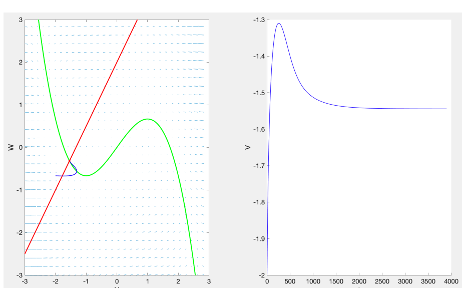
So, there is a fixed point at $(v,w) = (-1.5444, -0.3166)$. It is a stable fixed point, that attracts solutions. This is because the direction of solutions for initial points in the distinct regions tend towards this fixed point (based on vector directions). This can also be observed in the MATLAB simulation script (a1q4.m).

Resting membrane potential

Based on the fixed point, the resting membrane potential of this theoretical neuron is -1.5444 mV.

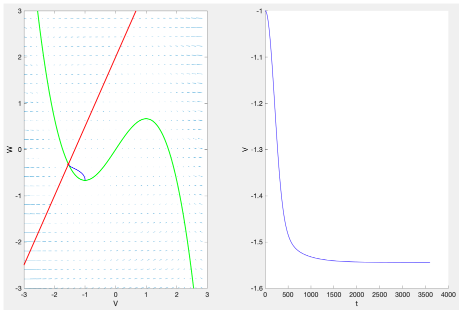
5. Using MATLAB (a1q5q6.m) the following scenarios were simulated:

$[v,w] = [-2, -2/3]$



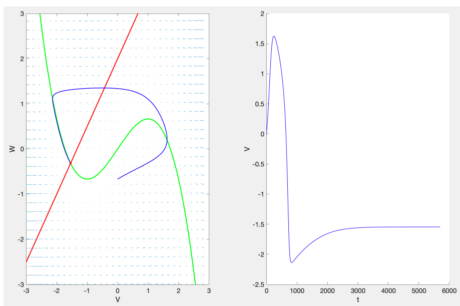
- Initially, v increases rapidly and rises above $v_{rest} = -1.5444$ mV, while w remains virtually unchanged
- Then, as the solution approaches the v-nullcline, the rate of change of v slows down, and w increases
  - The value of v decreases, but does not fall below v_{rest} (no repolarization phase)
- Finally, the solution settles at $[v,w] = [-1.5444, -0.3166]$

$[v, w] = [-1, -2/3]$



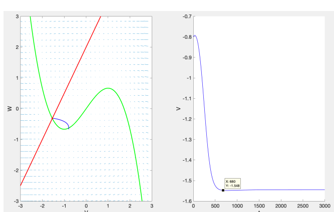
- Initially, v decreases rapidly, but does not drop below $v_{rest} = -1.5444\,mV$, while w increases
- Then, the solution approaches the fixed point, and settles at $[v, w] = [-1.5444, -0.3166]$
- No depolarization or repolarization phases are observed

$[v, w] = [0, -2/3]$



- Initially, there is a rapid increase in v, w as solution approaches v-nullcline, where $v > v_{rest} = -1.5444\,mV$
- After, v slightly decreases as w rapidly increases
- Then, v rapidly decreases and w remains virtually unchanged as v drops below v_{rest}
  - The solution approaches the w-nullcline at this point
- Finally, w rapidly decreases, and v slightly increases to v_{rest} and solution settles at the fixed point of $[v, w] = [-1.5444, -0.3166]$

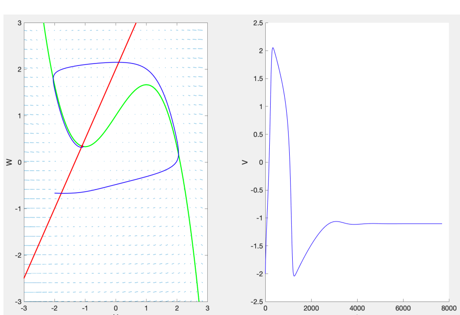
Based on the action potential description in the question, an action potential is observed when $[v, w] = [0, -2/3]$. The threshold voltage is $\sim -0.8\,mV$, computed experimentally from the MATLAB simulation (alq5q6.m).



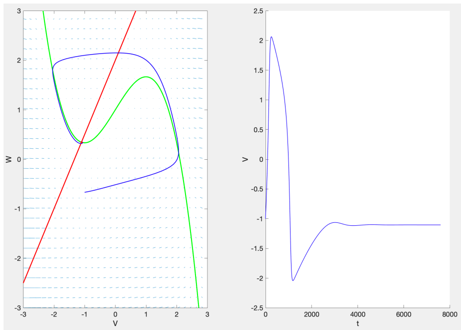
Simulation when $[v, w] = [-0.8, -2/3]$ where $v = -0.8\,mV$ is the threshold voltage.

6. Using MATLAB (alq5q6.m), the following scenarios with $I_{app} = 1$ were simulated:

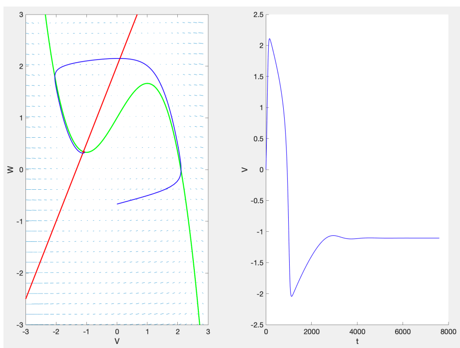
$[v, w] = [-2, -2/3]$



$[v,w]=[-1,-2/3]$

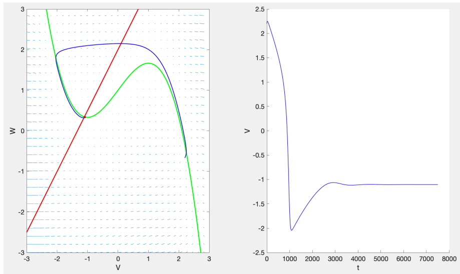


$[v,w]=[0,-2/3]$



In all the simulations, the following observations can be made:
- Initially, there is a rapid increase in v, w as solution approaches v-nullcline, where $v > v_{rest} = -1.104$ mV
- After, v slightly decreases as w rapidly increases
- Then, v rapidly decreases and w remains virtually unchanged as v drops below v_{rest}
  - The solution approaches the w-nullcline at this point
- Finally, w rapidly decreases, and v slightly increases above v_{rest} (following near the v-nullcline) and solution settles at the fixed point of $[v,w]=[-1.104, 0.3444]$

In general, when $I_{app}=1$, the fixed point of the system shifted to $[v,w]=[-1.104, 0.3444]$ and the resting membrane potential changed to $v_{rest}=-1.104$ mV. Also, the v-nullcline shifts up. In all simulation scenarios, the trajectory of the solutions in the phase-plane mimics a "loop" - however the solution does converge to the system's fixed point (so technically there are no limit cycles that form). Action potentials are also observed in all simulation scenarios - based on q5 action potential definition. The threshold voltage also changes to ~2.2mV. Overall, APs are triggered more easily with an applied current.



Simulation when $[v, w] = [2.2, -2/3]$ where $v = 2.2\,mV$ is the empirically computed threshold voltage.