

DIGITAL ANALYSIS AND ALGORITHM

EXPERIMENT - 03

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BATCH :- D1

Aim : Experiment based on divide and conquer approach: Strassen's Matrix Multiplication

Algorithm :

STRASSENS-MULTIPLICATION (A, B):

1. $n = A.rows$
2. Let C be a new $n \times n$ matrix 3. if $n == 2$:
 - a. $P1 \leftarrow A_{11} \times (B_{12} - B_{22})$
 - b. $P2 \leftarrow (A_{11} + A_{12}) \times B_{22}$
 - c. $P3 \leftarrow (A_{21} + A_{22}) \times B_{21}$
 - d. $P4 \leftarrow A_{22} \times (B_{21} - B_{11})$
 - e. $P5 \leftarrow (A_{11} + A_{22}) \times (B_{11} + B_{22})$
 - f. $P6 \leftarrow (A_{12} - A_{22}) \times (B_{21} + B_{22})$
 - g. $P7 \leftarrow (A_{11} - A_{21}) \times (B_{11} + B_{12})$
 - h. $C_{11} \leftarrow P5 + P4 - P2 + P6$
 - i. $C_{12} \leftarrow P1 + P2$
 - j. $C_{21} \leftarrow P3 + P4$
 - k. $C_{22} \leftarrow P5 + P1 - P3 - P7$
 - l. return C
4. Divide input matrices A and B and output matrix C into 4 submatrices of size $n/2 \times n/2$ each as follows:

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}, \quad B = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}, \quad C = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix},$$

5. $P1 \leftarrow \text{STRASSENS-MULTIPLICATION}(A_{11}, (B_{12} - B_{22}))$
6. $P2 \leftarrow \text{STRASSENS-MULTIPLICATION}(A_{11} + A_{12}, B_{22})$
7. $P3 \leftarrow \text{STRASSENS-MULTIPLICATION}(A_{21} + A_{22}, B_{21})$
8. $P4 \leftarrow \text{STRASSENS-MULTIPLICATION}(A_{22}, B_{21} - B_{11})$
9. $P5 \leftarrow \text{STRASSENS-MULTIPLICATION}(A_{11} + A_{22}, B_{11} + B_{22})$
10. $P6 \leftarrow \text{STRASSENS-MULTIPLICATION}(A_{12} - A_{22}, B_{21} + B_{22})$

11. $P7 \leftarrow \text{STRASSENS-MULTIPLICATION}(A_{11} - A_{21}, B_{11} + B_{12})$
12. $C_{11} \leftarrow P5 + P4 - P2 + P6$
13. $C_{12} \leftarrow P1 + P2$
14. $C_{21} \leftarrow P3 + P4$
15. $C_{22} \leftarrow P5 + P1 - P3 - P7$
16. return C

Code :

```
#include <stdio.h>
#include <stdlib.h>

// prototypes
int **addSquareMatrices(int **a, int **b, int n, int a_p, int a_q,
int b_p, int b_q);
int **subtractSquareMatrices(int **a, int **b, int n, int a_p, int
a_q, int b_p, int b_q);
int **strassensMultiplication(int **a, int **b, int n);
int **actualStrassensMultiplication(int **a, int **b, int n, int
a_p, int a_q, int b_p, int b_q); int **mallocSqaureMatrix(int n);
void freeSquareMatrix(int **mat, int n);

int **mallocSqaureMatrix(int n)
{
    int **new = malloc(n *
sizeof(int *));    for (int i = 0;
i < n; i++)        new[i] =
malloc(n * sizeof(int));    return
new;
}
void freeSquareMatrix(int **mat, int n)
{
    for (int i = 0; i < n; i++)
        free(mat[i]);
free(mat);
}
int **addSquareMatrices(int **a, int **b, int n, int a_p, int a_q,
int b_p, int b_q)
{
    int **sum =
mallocSqaureMatrix(n);    for (int
i = 0; i < n; i++)
    {
        for (int j = 0; j < n; j++)
            sum[i][j] = a[a_p + i][a_q + j] + b[b_p + i][b_q + j];
    }
    return sum;
}
int **subtractSquareMatrices(int **a, int **b, int n, int a_p, int
a_q, int b_p, int b_q)
```

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{
    int **diff =
mallocSquareMatrix(n);    for (int
i = 0; i < n; i++)
    {
        for (int j = 0; j < n; j++)
            diff[i][j] = a[a_p + i][a_q + j] - b[b_p + i][b_q + j];
    }
    return diff;
}
int **strassensMultiplication(int **a, int **b, int n)
{
    return actualStrassensMultiplication(a, b, n, 0, 0, 0,
0); }
int **actualStrassensMultiplication(int **a, int **b, int n, int a_p, int
a_q, int b_p, int b_q)
{
    int **prod = mallocSquareMatrix(n);
    if (n == 2)
    {
        int p1 = a[a_p][a_q] * (b[b_p][b_q + 1] - b[b_p + 1][b_q + 1]);
        int p2 = (a[a_p][a_q] + a[a_p][a_q + 1]) * b[b_p + 1][b_q + 1];
        int p3 = (a[a_p + 1][a_q] + a[a_p + 1][a_q + 1]) * b[b_p][b_q];
        int p4 = a[a_p + 1][a_q + 1] * (b[b_p + 1][b_q] - b[b_p][b_q]);
        int p5 = (a[a_p][a_q] + a[a_p + 1][a_q + 1]) * (b[b_p][b_q] + b[b_p +
1][b_q + 1]);
        int p6 = (a[a_p][a_q + 1] - a[a_p + 1][a_q + 1]) * (b[b_p +
1][b_q] + b[b_p + 1][b_q + 1]);
        int p7 = (a[a_p][a_q] - a[a_p + 1][a_q]) * (b[b_p][b_q] +
b[b_p][b_q + 1]);
        prod[0][0] = p5 + p4 - p2 + p6;
        prod[0][1] = p1 + p2;
        prod[1][0] = p3 + p4;
        prod[1][1] = p5 + p1 - p3 - p7;
    }
    else
    {
        int x = n / 2;
        int **temp = subtractSquareMatrices(b, b, x, b_p, b_q + x, b_p +
x, b_q + x);
        int **p1 = actualStrassensMultiplication(a, temp, x, a_p, a_q, 0,
0);

        freeSquareMatrix(temp, x);
        temp = addSquareMatrices(a, a, x, a_p, a_q, a_p, a_q + x);
        int **p2 = actualStrassensMultiplication(temp, b, x, 0, 0, b_p + x, b_q
+ x);

        freeSquareMatrix(temp, x);
        temp = addSquareMatrices(a, a, x, a_p + x, a_q, a_p + x, a_q + x);
        int **p3 = actualStrassensMultiplication(temp, b, x, 0, 0, b_p, b_q);
        freeSquareMatrix(temp, x);
        temp = subtractSquareMatrices(b, b, x, b_p + x, b_q, b_p, b_q);
        int **p4 = actualStrassensMultiplication(a, temp, x, a_p + x, a_q

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+ x, 0, 0);
    freeSquareMatrix(temp, x);
    temp = addSquareMatrices(a, a, x, a_p, a_q, a_p + x, a_q + x);
int **temp2 = addSquareMatrices(b, b, x, b_p, b_q, b_p + x, b_q + x);

    int **p5 = actualStrassensMultiplication(temp, temp2, x, 0, 0, 0,
0);
    freeSquareMatrix(temp, x);
freeSquareMatrix(temp2, x);
    temp = subtractSquareMatrices(a, a, x, a_p, a_q + x, a_p + x, a_q
+ x);
    temp2 = addSquareMatrices(b, b, x, b_p + x, b_q, b_p + x, b_q +
x);
    int **p6 = actualStrassensMultiplication(temp, temp2, x, 0, 0, 0,
0);
    freeSquareMatrix(temp, x);
freeSquareMatrix(temp2, x);
    temp = subtractSquareMatrices(a, a, x, a_p, a_q, a_p + x, a_q);
temp2 = addSquareMatrices(b, b, x, b_p, b_q, b_p, b_q + x);        int
**p7 = actualStrassensMultiplication(temp, temp2, x, 0, 0, 0,
0);
    freeSquareMatrix(temp, x);
freeSquareMatrix(temp2, x);
    temp = addSquareMatrices(p5, p4, x, 0, 0, 0, 0);
temp2 = addSquareMatrices(temp, p6, x, 0, 0, 0, 0);
freeSquareMatrix(temp, x);
    temp = subtractSquareMatrices(temp2, p2, x, 0, 0, 0, 0);
freeSquareMatrix(temp2, x);        for (int i = 0; i < x; i++)
    {
        for (int j = 0; j < x; j++)
prod[i][j] = temp[i][j];
    }
    freeSquareMatrix(temp, x);
    temp = addSquareMatrices(p1, p2, x, 0, 0, 0, 0);
for (int i = 0; i < x; i++)
    {
        for (int j = x; j < n; j++)
prod[i][j] = temp[i][j - x];
    }
    freeSquareMatrix(temp, x);
    temp = addSquareMatrices(p3, p4, x, 0, 0, 0, 0);
for (int i = x; i < n; i++)
    {
        for (int j = 0; j < x; j++)
prod[i][j] = temp[i - x][j];
    }
    freeSquareMatrix(temp, x);
    temp = addSquareMatrices(p5, p1, x, 0, 0, 0, 0);
temp2 = subtractSquareMatrices(temp, p3, x, 0, 0, 0, 0);
freeSquareMatrix(temp, x);
    temp = subtractSquareMatrices(temp2, p7, x, 0, 0, 0, 0);
for (int i = x; i < n; i++)
    {

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        for (int j = x; j < n; j++)
            prod[i][j] = temp[i - x][j - x];
    }
    freeSquareMatrix(temp, x);
freeSquareMatrix(temp2, x);
    }
    return prod;
}
int main()
{
    printf("Enter order of input matrices (should be a power of 2):
");    int n;    scanf("%d", &n);
    int **a = mallocSqaureMatrix(n);
    int **b = mallocSqaureMatrix(n);
    printf("Enter matrix elements of first matrix in row major order: ");
    for (int i = 0; i < n; i++)
    {
        for (int j = 0; j < n; j++)
scanf("%d", &a[i][j]);
    }
    printf("Enter matrix elements of second matrix in row major order: ");
    for (int i = 0; i < n; i++)
    {
        for (int j = 0; j < n; j++)
scanf("%d", &b[i][j]);
    }
    printf("Product of first and second
matrices:\n");    int **prod =
strassensMultiplication(a, b, n);    for (int i = 0;
i < n; i++)
    {
        for (int j = 0; j < n; j++)
printf("%15d", prod[i][j]);    printf("\n");
    }
}

```

Output :

Multiplication of 4x4 matrices:

```

Enter order of input matrices (should be a power of 2): 4
Enter matrix elements of first matrix in row major order: 1 2 3 4
5 6 7 8
9 10 11 12
13 14 15 -16
Enter matrix elements of second matrix in row major order: -17 18 -19 20
-21 22 -23 24
-25 26 -27 28
-29 30 -31 32
Product of first and second matrices:
      -250      260      -270      280
      -618      644      -670      696
      -986     1028     -1070     1112
      -426      452      -478      504

```

Multiplication of 8v8 matrix:

```

Enter order of input matrices (should be a power of 2): 8
Enter matrix elements of first matrix in row major order: 1 2 3 4 5 6 7 8
9 10 11 12 13 14 15 16
17 18 19 20 21 22 23 24
25 -26 -27 -28 -29 -30 31 32
33 34 35 36 37 38 39 40
-41 -42 -43 -44 -45 46 47 48
49 50 51 52 43 54 55 56
57 58 59 60 -61 -62 -63 -65
Enter matrix elements of second matrix in row major order: -100 -99 -98 -97 -96 -95 94 93
92 91 90 89 88 87 86 85
854 83 82 81 80 79 78 77
76 75 74 73 72 71 70 69
68 67 66 65 64 63 62 61
60 59 58 57 56 55 54 53
52 51 50 49 48 47 46 45
44 43 42 41 40 39 38 37
Product of first and second matrices:
      4366      2022      1988      1954      1920      1886      2040      2004
     13534      4982      4900      4818      4736      4654      6264      6164
     22702      7942      7812      7682      7552      7422      10488      10324
    -30030     -9938     -9836     -9734     -9632     -9530     -4728     -4676
     41038     13862     13636     13410     13184     12958     18936     18644
    -35574     -2472     -2480     -2488     -2496     -2504     -10220     -10146
     58694     19112     18800     18488     18176     17864     26764     26354
     40578     -4721     -4590     -4459     -4328     -4197      6650      6667

```

Conclusion : From this experiment we understand the concept of strassen multiplication and we try to multiply using strassen algorithm two 4x4 matrices and two 8v8 matrix