SAL Institute of Technology and Engineering Research
Department of Computer Applications (306)
Bachelor Of Computer applications (BCA)

SUBJECT: - Mathematical Foundation for AI   **SEM: - 3**

Practice Sheet-2

1	Find the limit (if exists): a. $\lim_{(x,y)\to(1,2)} \dfrac{x^2+y}{3x+y^2}$ b. $\lim_{(x,y)\to(0,0)} \dfrac{x-y}{x+y}$ c. $\lim_{(x,y)\to(0,0)} \dfrac{x^4-y^2}{x^4+y^2}$ d. $\lim_{(x,y)\to(0,0)} \dfrac{xy}{y^2-x^2}$ e. $\lim_{(x,y)\to(0,0)} \dfrac{x^2y}{x^4+y^2}$ f. $\lim_{(x,y)\to(0,0)} \dfrac{x^2-y^2}{x^2+y^2}$ g. $\lim_{(x,y)\to(0,0)} \dfrac{3x^2y}{x^2+y^2}$ h. $\lim_{(x,y)\to(0,0)} \dfrac{2x^2y}{x^3+y^3}$

2	Find partial derivative $\left(\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z}\right)$: a. $u = x^2z + yz^3 + x$ b. $u = x^2y + xy^2$ c. $u = x^2y + y^3$ d. $u = x^2 + 3xy + y^2$ e. $u = x^2 + y^2$ f. $u = x^3 + y^3 - 3axy$ g. $u = x^2 - y^2$ h. $u = x^2y - y^2x$ i. $u = e^x + e^y$
3	Prove that $\dfrac{\partial^2 u}{\partial x\,\partial y} = \dfrac{\partial^2 u}{\partial y\,\partial x}$ a. $u = x^2 + y^2$ b. $u = e^{x+y}$ c. $u = x^3 + y^3 - 3axy$ d. $u = \log(x^2 + y^2)$ e. $u = x^3y + e^{xy^2}$ f. $u = x^y + y^x$
4	Find the value: a. If $u = x^2 + y^2 + z^2$ then $xy\dfrac{\partial u}{\partial z} + yz\dfrac{\partial u}{\partial x} + zx\dfrac{\partial u}{\partial y}$ b. If $u = x^2 + y^2 + z^2$ then $\dfrac{1}{x}\dfrac{\partial u}{\partial x} + \dfrac{1}{y}\dfrac{\partial u}{\partial y} + \dfrac{1}{z}\dfrac{\partial u}{\partial z}$

5	Find the value:
	
	  a. $u = y^2 - 4ax$ then $x = at^2, y = 2at$ find $\frac{du}{dt}$.
	
	  b. $u = x^2 y^3$ then $x = \log t, y = e^t$ find $\frac{du}{dt}$.
	
	  c. $z = x^2 y + xy^2, x = at^2, y = 2at$, find $\frac{dz}{dt}$.
	
	  d. $u = xy + yz + zx$ where $x = \frac{1}{t}, y = e^t$ and $z = e^{-t}$ find $\frac{du}{dt}$.

6	Find the maximum value of $x^2 y^3 z^4$, given that $2x + 3y + 4z = a$ using Lagrange's method.
8	Find the minimum value of $x^2 + y^2$, Subject to the condition $ax + by = c$.
9	Find the minimum value of $x^2 yz^3$, Subject to the condition $2x + y + 3z = a$.
10	Find ∇f at $(1, -2, 1)$ if $f = 3x^2 y - y^3 z^2$.