chapter - 2 Practice sheet

1) An unbiased coin is tossed 3 times what is probability of obtaining two heads?

→ ~~A~~ S = { HH , HT , TH , TT }

A = { HH }

→ $P(A) = \frac{n(A)}{n(S)} = \frac{1}{4}$

2) A bag contains 5 white and 10 black balls. Three balls are taken at random, find probability that all three drawn ball are black

→ $S = {}^{15}C_3 = \frac{15!}{3!\ 12!}$

$= \frac{\cancel{15}^{5} \times \cancel{14}^{7} \times 13 \times \cancel{12!}}{\cancel{3} \times \cancel{2}\ \cancel{12!}}$

= 455 total ways.

→ $A = {}^{10}C_3 = \frac{10!}{3!\ 7!}$

$= \frac{\cancel{10} \times \cancel{9}^{3} \times 8 \times \cancel{7!}}{\cancel{3} \times \cancel{2}\ \cancel{7!}}$

= 120

$P(A) = \frac{n(A)}{n(S)} = \frac{120}{455}$ 

(3) Two cards are drawn from a pack of cards Find the probability that they will be both red or both picture.

$S = {}^{52}C_2 = \frac{52!}{2!\,50!} = \frac{\overset{26}{\cancel{52}} \times 51 \times \cancel{50!}}{\cancel{2}\ \cancel{50!}} = \boxed{1326}$

$A = {}^{26}C_2 = \frac{26!}{2!\,24!} = \frac{\overset{13}{\cancel{26}} \times 25 \times \cancel{24!}}{\cancel{2}\ \cancel{24!}} = \boxed{325}$

$= {}^{12}C_2 = \frac{12!}{2!\,10!} = \frac{\overset{6}{\cancel{12}} \times 11 \times \cancel{10!}}{\cancel{2}\ \cancel{10!}} = \boxed{66}$

→ We have counted Red faced card twice so need to subtract them

→ ${}^{6}C_2 = 15$ $\quad 325 + 66 - 15 = \boxed{376}$ → way to get both red or faced.

→ $P(A) = \frac{n(A)}{n(S)} = \frac{376}{1326} = \frac{188}{663} = \frac{94}{221}$

SAL EDUCATION

4) The boxes contain 10 %, 20 %, 30 % defective finger joints. A finger joint is selected at random which is defective. Determine probability of coming from

box 1 $A_1 = 1/3 = 0.33$

box 2 $A_2 = 1/3 = 0.33$

box 3 $A_3 = 1/3 = 0.33$

$P(B/A_1) = 10/100 = 0.10$

$P(B/A_2) = 20/100 = 0.20$

$P(B/A_3) = 30/100 = 0.30$

$\Rightarrow$ box 1 $= P(A_1/B) = \dfrac{P(A_1) \cdot P(B/A_1)}{P(A_1) \cdot P(B/A_1) + P(A_2) \cdot P(B/A_2) + P(A_3) \cdot P(B/A_3)}$

$\Rightarrow \dfrac{0.33 \cdot 0.10}{0.33 \cdot 0.10 + 0.33 \cdot 0.20 + 0.33 \cdot 0.30}$

$\Rightarrow \dfrac{0.033}{0.033 + 0.066 + 0.99}$

$\Rightarrow \dfrac{0.033}{0.198} = 0.167$

* box 2

$$P(A_2/B) = \frac{P(A_2) \cdot P(B/A_2)}{P(A_1) \cdot P(B/A_1) + P(A_2) \cdot P(B/A_2) + P(A_3) \cdot P(B/A_3)}$$

$$= \frac{0.066}{0.033 + 0.066 + 0.99}$$

$$= \frac{0.066}{0.198} = \boxed{0.333}$$

* box 3

$$P(A_3/B) = \frac{P(A_3) \cdot P(B/A_3)}{P(A_1) \cdot P(B/A_1) + P(A_2) \cdot P(B/A_2) + P(A_3) \cdot P(B/A_3)}$$

$$= \frac{0.099}{0.198}$$

$$= \boxed{0.5}$$

* ~~for~~ Box 1 = 0.167

Box2 = 0.333

Box3 = 0.5 would be the probability.

SAL

SAL EDUCATION

5) if A and B are two event such that $P(A) = 0.3$ $P(B) = 0.4$, $P(A \cap B) = 0.2$, find (i) $P(A \cup B)$ (ii) $P(A/\bar{B})$ (iii) $P(\bar{A}/\bar{B})$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 0.3 + 0.4 - 0.2$$
$$= \boxed{0.5}$$

$P(A/\bar{B}) \Rightarrow$ $\cancel{P(A)}\,\bar{B} = 1 - P(B)$
$$= 1 - 0.5$$
$$= 0.6$$

$\Rightarrow$ $\cancel{P(A/B)} = \cancel{P(A \cap B)}$
$\cancel{P(B/A)} = \cancel{\frac{P(A \cap B)}{P(A)}} = \cancel{\frac{0.2}{0.3}}$

~~$P(B/A) = 2/3$~~

$\cancel{P(A/B)}$ $P(A \cap \bar{B}) = P(A) - P(A \cap B)$
$$= 0.3 - 0.2$$
$$= 0.1$$

$$\Rightarrow P(A/\bar{B}) = \frac{P(A \cap \bar{B})}{P(B)}$$
$$= \frac{0.1}{0.4}$$
$$= 1/4$$

$* \; P(\bar{A}/B) = \frac{P(\bar{A} \cap B)}{P(B)}$

$P(\bar{A}) = 1 - P(A)$
$= 1 - 0.3$
$= 0.7$

$P(\bar{A} \cap B) = P(B) - P(A \cap B)$
$= 0.4 - 0.2$
$= 0.2$

$\rightarrow P(\bar{A}/B) = \frac{P(\bar{A} \cap B)}{P(B)}$

$= \frac{0.2}{0.4} = 2/4 = 1/2$

(6) Define Mutually exclusive events and independent event if A and B are independent events where $P(A) = 1/4$ $P(B) = 2/3$ find $P(A \cup B)$

⇒ Mutually Exclusive events :- Two event are Mutually exclsive if they can't occur at same time

⇒ Independent Events :- Two events are Independent if occurance of one does not affect other one

$P(A \cap B) = P(A) \cdot P(B)$

$P(A) = 1/4$
$P(B) = 2/3$
both are independent.

$P(A \cap B) = P(A) \cdot P(B) = 1/4 \times 2/3 = 2/12 = 1/6$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= \frac{1}{4} + 2/3 - 1/6 = \boxed{3/4}$

SAL EDUCATION

7) From city population, the probability of selecting (A) male or a smoker is 7/10, (B) a male smoker is 2/5 and (C) a man, if smoker is already selected is 2/3 find (i) non-smoker (ii) a male (iii) a smoker, if a male is first selected

P(A) = Male     P(B) = smoker

A) male or smoker = 7/10  $P(A \cup B)$

B) a male smoker = 2/5  $P(A \cap B)$

C) a male, if smoker is already selected = 2/3

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$\frac{2}{3} = \frac{\frac{2}{5}}{P(B)}$$

$$P(B) = \frac{2/5}{2/3} = 2/5 \times 3/2 = \boxed{3/5}$$

$$P(\bar{B}) = 1 - P(B)$$

$$= 1 - 3/5$$

$$= \boxed{2/5} \rightarrow \text{non-smoker}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$7/10 = P(A) + 3/5 - 2/5$$

Males

$$7/10 = P(A) + 1/5$$

$$P(A) = 7/10 - 1/5 = \frac{7-2}{10} = 5/10 = \boxed{1/2}$$

$P(B/A) = \frac{P(A \cap B)}{P(A)}$

$= \frac{2/5}{1/2} = \frac{2}{5} \times \frac{2}{1} = \boxed{4/5}$

asmoker, Male is first selected.

(8) A microchip company has two machines.
Machine I produce 65%. 5% defective
Machine II produce 35%. 1.5% defective.
selected chip is found to defective what is prob
it came from machine I.

$M_1$ = chip from Machine I = $P(M_1) = 0.65$
$M_2$ = chip from Machine II = $P(M_2) = 0.35$

$P(D/M_1) = 0.05$
$P(D/M_2) = 0.15$

chip came from machine I

$$P(M_1/D) = \frac{P(M_1) \cdot P(D/M_1)}{P(M_1) \cdot P(D/M_1) + P(M_2) \cdot P(D/M_2)}$$

$$= \frac{0.65 \cdot 0.05}{0.65 \cdot 0.05 + 0.35 \cdot 0.15}$$

$$= \frac{0.0325}{0.0325 + 0.0525}$$

$$= \frac{0.0325}{0.085} = \boxed{0.3829}$$

q) An urn contain 10 white & 3 black balls while another urn contain 3 white and 5 black balls. Two balls are drawn from 1st urn and put into second then a ball is drawn from the latter. What is probability that it will be white?

→ total ways = $^{13}C_2 = 78$

* Case 1 (both white balls)

→ $^{10}C_2 = 45$

→ $U_2$ will have $3+2 = 5$ white, 5 black = 10 balls

→ $5/10 = 1/2$

→ $45/78 \cdot 1/2 = 45/156$

* one white one black

→ $^{10}C_1 \cdot {}^{3}C_1 = 10 \cdot 3 = 30$

→ $U_2 = 3+1 = 4$ white
$5+1 = 6$ black
total = 10

→ drawing from $U_2$ = $4/10 = 2/5$

→ $\frac{30}{78} \cdot \frac{2}{5} = \frac{60}{390} = 2/13$

* Transfer 2 black

→ $3C_2 = 3$

→ $U_2$ = 3 white , 7 black = 10 balls

→ drawing white 3/10

→ $\frac{3}{78} \cdot \frac{3}{10} = \frac{9}{780}$

→ $\frac{45}{156} + \frac{2}{13} + \frac{9}{780}$

$= \frac{450 + 240 + 18}{1560} = \frac{708}{1560} = \boxed{\frac{59}{130}}$

(10) find mean variance.
find distribution function.

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $F(x)$ | 0.1 | 0.1 | 0.2 | 0.3 | 0.3 |

* mean

$\mu = \mathcal{E}(x) \cdot F(x)$

$= 1(0.1) + 2(0.1) + 3(0.2) + 4(0.3) + 5(0.3)$

$= 0.1 + 0.2 + 0.6 + 1.2 + 1.5 = \boxed{3.6}$

* Variance.

$Var(x) = \mathcal{E}(x^2) - [\mathcal{E}(x)]^2$

$\mathcal{E}(x^2) = (1^2)(0.1) + (2^2)(0.1) + (3^2)(0.2) + (4^2)(0.3) + (5^2)(0.3)$

$= 0.1 + 0.4 + 1.8 + 4.8 + 7.5 = \underline{14.6}$

* $Var(x) = 14.6 - (3.6)^2 = 14.6 - 12.96$
$= 1.64$

(11) A machine produce on average of 500 items during 1st week of mont. On average 400 item during last week of month. The probability of this beign 0.68 and 0.32. Determine expected value of production.

⇒ Average production on 1st week = 500
$(P_1) = 0.68$

⇒ Average production on last week = 400
$(P_2) = 0.32$

⇒ $E(x) = (x_1 \times P_1) + (x_2 \times P_2)$
$= (500 \times 0.68) + (400 \times 0.32)$
$= 340 + 128$
$= 468$

$\boxed{E(x) = 468}$

⇒ The expected production would be of 468 items.

(12)

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| $F(x)$ | 0.08 | 0.12 | 0.19 | 0.24 | 0.16 | 0.10 | 0.07 | 0.04 |

* Expected value (mean)

$$E(x) = \Sigma x \cdot f(x)$$

$= 1(0.08) + 2(0.12) + 3(0.19) + 4(0.24) + 5(0.16) + 6(0.10) +$
$7(0.07) + 8(0.04$
$= 0.08 + 0.24 + 0.57 + 0.96 + 0.80 + 0.60 + 0.49 +$
$0.32 = \boxed{4.06}$

* variance

$$Var(x) = E(x^2) - (E(x))^2$$

$E(x^2) = (1^2)(0.08) + (2^2)(0.12) + (3^2)(0.19) + (4^2)(0.2$
$+ (5^2)(0.16) + (6^2)(0.10) + (7^2)(0.07) + (8^2)(0.04)$
$= 0.08 + 0.48 + 1.71 + 3.84 + 4 + 3.60 + 3.43 + 2.5$
$= \boxed{19.70}$

$var(x) = E(x^2) - (E(x))^2$
$= 19.70 - (4.06)^2$
$= 19.70 - 16.48$
$= \boxed{3.22}$

→ Variance = 3.22
→ Expected demand = 4.06.

(13) The mean and variance of binomial variate are 8 and 6 find $P(x \geq 2)$

$\mu = np = 8$

$\sigma^2 = npq = 6$

$\Rightarrow \frac{\cancel{npq}}{\cancel{np}} = \frac{\cancel{6}^{3}}{\cancel{8}_{4}}$ $\boxed{q = 3/4}$

$\Rightarrow p + q = 1$

$\Rightarrow p = 1 - q$

$= 1 - 3/4$

$= \frac{4-3}{4}$

$\boxed{p = 1/4}$

$\Rightarrow np = 8$

$n(1/4) = 8$

$n = 32$

$\Rightarrow P(x \geq 2) = 1 - P(x < 2) =$

$1 - [P(x=0) + P(x=1)]$

$P(x = r) = {}^{n}C_{r}\, p^{r} q^{n-r}$

$\Rightarrow P(x=0) = {}^{32}C_{0}\, 0.25^{0} \cdot 0.75^{32}$

$= 1 \cdot 1 \cdot (0.75)^{32}$

$P(x=0) \approx 0.0003$

(ii) $P(x=1) = {}^{32}C_1 \cdot 0.25^1 \cdot 0.75^{31}$ 14

$= 32 \cdot 0.25 \cdot (0.75)^{31}$

$= 8 \cdot (0.75)^{31}$

$= 8 \cdot 0.00045$

$= 0.0036$

$P(x \geq 2) = 1 - [P(x=0) + P(x=1)]$

$= 1 - [0.0003 + 0.0036]$

$= \boxed{0.9961}$

(14) The mean and variance of bionormical destribution are respectively 4 and 4/3 find $x \geq 1$)

$\mu = np = 4$

$\sigma^2 = npq = 4/3$

$\Rightarrow \frac{npq}{np} = \frac{\frac{4}{3}}{4} = \frac{4}{3} \times \frac{1}{4} = \boxed{\frac{1}{3}}$

$\Rightarrow p + q = 1$

$\Rightarrow p = 1 - q$

$= 1 - 1/3$

$= \boxed{2/3}$

$\Rightarrow np = 4$

$\Rightarrow n(2/3) = 4$

$\Rightarrow \frac{2n}{3} = 4$

$2n = 12$

$n = 12/2$ $\boxed{n = 6}$

$\Rightarrow P(x=x) = n\,c_x\, p^x\, q^{n-x}$

$= 6\,c_x (2/3)^x (1/3)^{6-x}$

$\Rightarrow P(x \geq 1) = 1 - P(x<1)$

$= 1 - [P(x=0)]$

$= 1 - \left[ 6\,c_0 (2/3)^0 (1/3)^{6-0} \right]$

$= 1 - [1 \cdot 1 \cdot 0.0013]$

$= 1 - 0.0013$

$= \boxed{0.9987}$

(15) if mean and poisson variable is 1.8 find $P(x \geq 1)$, $P(x=5)$, $P(0<x<5)$

(i) $P(x>1) = 1 - P(x \leq 1)$

$= 1 - [P(x=0) + P(x=1)]$

$= 1 - \left( \frac{e^{-1.8} (1.8)^0}{0!} + \frac{e^{-1.8} (1.8)^1}{1!} \right)$

$= 1 - (0.165 + 0.165 \times 1.8)$

$= \boxed{0.5372}$

(ii) $P(x=5) = \left( \frac{e^{-1.8} (1.8)^5}{5!} \right)$

$= \left( \frac{0.165 \cdot 18.90}{120} \right)$

$= \frac{3.12}{120}$

$= \boxed{0.026}$

(iii) $P(0<x<5)$ $P(1)+P(2)+P(3)+P(4)$

→ we have $P(1) = 0.2973$

$P(2) = \frac{e^{-1.8} \cdot 1.8^2}{2!} = \frac{0.1653 \cdot 3.24}{2} = 0.2678$

$P(3) = \frac{e^{-1.8} \cdot 1.8^3}{3!} = \frac{0.1653 \cdot 5.832}{6} = 0.1607$

$P(4) = \frac{e^{-1.8} \cdot 1.8^3}{4!} = \frac{0.1653 \cdot 10.4976}{24} = 0.0723$

→ $P(0<x<5) = 0.2975 + 0.2678 + 0.1607 + 0.0723$
$= \boxed{0.7983}$

(16) Random variable has poisson distribution such as $P(x=1) = P(x=2)$, find (i) mean of the distribution (ii) $P(x=4)$ (iii) $P(x \geq 1)$ and (iv) $P(1<x<4)$

* $P(x=x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}$

→ $P(x=1) = P(x=2) \Rightarrow \frac{e^{-\lambda} \cdot \lambda^1}{1!} = \frac{e^{-\lambda} \cdot \lambda^2}{2!}$

$\Rightarrow \frac{\lambda}{1} = \frac{\lambda^2}{2} \Rightarrow 2\lambda = \lambda^2$
$\Rightarrow \lambda^2 - 2\lambda = 0$
$\Rightarrow \lambda(\lambda-2) = 0$
$\lambda = 0$ or $\lambda = 2$

$\lambda \neq 0$ so $\boxed{\lambda = 2}$ = mean

(ii) find $P(x = 4)$

$$P(4) = \frac{e^{-2} \cdot 2^4}{4!} = \frac{e^{-2} \cdot 16}{24}$$

$$e^{-2} = 0.1353$$

$$P(4) = \frac{16 \cdot 0.1353}{24} = \frac{2.1648}{24}$$

$$= \boxed{0.0902}$$

(iii) find $P(1 < x < 4) = P(2) + P(3)$

$\Rightarrow P(2) = \frac{e^{-2} \cdot 2^2}{2!}$

$$= \frac{0.1353 \cdot 4}{2} = 0.2706$$

$\Rightarrow P(3) = \frac{e^{-2} \cdot 2^3}{3!}$

$$= \frac{0.1353 \cdot 8}{6} = 0.1804$$

$$= 0.2706 + 0.1804$$

$$= \boxed{0.4510}$$

(17) if variance of poisson variable is 3, find the probability that

(i) $0 < x < 3$

(ii) $1 \le x \le 4$

Mean $\lambda$ = variance = 3

$$P(x = x) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}$$

where $\lambda = 3$ and $e^{-3} = 0.0498$

(i) $P(0 < x < 3) = P(1) + P(2)$

$$P(1) = \frac{e^{-3} \cdot 3^{1}}{1!} = 0.0498 \cdot 3 = \boxed{0.1494}$$

$$P(2) = \frac{e^{-3}\ 3^{2}}{2!} = 0.0498 \cdot 9/2 = \boxed{0.2241}$$

$\Rightarrow P(0 < x < 3) = 0.1494 + 0.2241$
$= \boxed{0.3735}$

(ii) $P(1 \le x \le 4) = P(1) + P(2) + P(3) + P(4)$

$P(1) = 0.1494$

$P(2) = 0.2241$

$$P(3) = \frac{e^{-3} \cdot 3^{3}}{3!} = 0.0498 \cdot \frac{27}{6}$$
$$= \boxed{0.2241}$$

$$P(4) = \frac{e^{-3}\ 3^{4}}{4!} = 0.0498 \cdot \frac{81}{24}$$
$$= \boxed{0.1681}$$

$P(1 \le x \le 4) = 0.1494 + 0.2241 + 0.2241 + 0.1681$
$= \boxed{0.7657}$