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Bias-preserving gates with stabilized cat qubits

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Question

What do we need on the road to fully universal quantum computing ?

Background

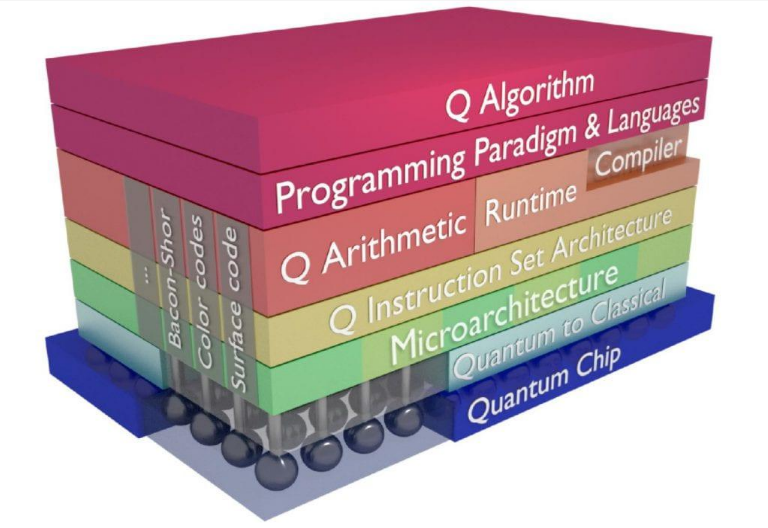


Fig. 1. Quantum computing stack

7-layer configuration

- **Q Algorithm**
- Programming Languages
- Q Arthmetic
- Q Instruction Set
- Microarchitecture
- Quantum to Classical
- **Quantum Chip**

- The basis of this architecture is **quantum chip** (**superconduncting quantum circuits**, **quantum dots**)
- The middle part between the chip and the algorithm translates the quantum algorithm into control pulses acting on the physical chip.
- Quantum bits are extremely **fragile**. The way forward is to correct the errors faster than they appear, a notion called **fault-tolerance**.
- The approach to achieve fault-tolerant quantum computing is based on **Quantum Error Correction.**

Quantum Error Correction

The cause of errors

Interaction of quantum system with environment

Energy loss & Decoherence

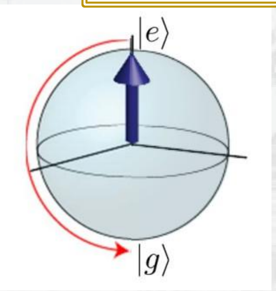
Loss of quantum information

Energy Loss (Decay)

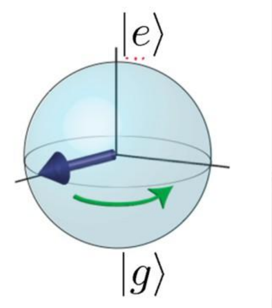
Decoherence

Errors

Bit-flip



Dephasing



Quantum Error Correction

Review: Classical error correction

- Redundancy encoding: Storing information multiple times
- Error correction: If these copies disagree, take the majority of the correct values

Quantum Errors

- Bit flip
- Dephasing

QEC strategies

Bit flip code:
- Analogous to classical repetition code, this method uses three **entangled** physical qubits to encode one logical qubit.

Sign flip code:
- Constructing three-body entangled state in the Hardmard basis.

Shor code:
- Shor code corrects arbitrary **single**-qubit errors.
- Coding in groups of three

Bosonic codes:
- Storing error-correctable quantum information in **bosonic** modes
- **Cat**/Gottesman-Kitaev-Preskill (GKP)/Binomial

Current Situations

- The overhead to realize QEC is too **large** to afford at this era.
- In the widely studied depolarizing noise model, assuming that the stochastic error occurs in X, Y and Z channel is **equal**.
- In many physical systems, noise is **asymmetrical** (fluxonium、quantum-dot spin qubits、nuclear spins in diamond).
- In the asymmetric noise system, it is better to design QEC strategies aimed to suppress the dominant error with **lower overhead**.

Bias-noise

Current Problems

- There has a surface code tailored to biased Z channel noise. However, this approach is very limited beacuse the qubit could not be protected if the gate, such as X and Y, is **not commuted** with biased noise channel.
- QEC with noise-biased channel is **impossible** in a native two-level system.

Proof 1: 2-level system cannot build universal biased-preserving gate sets

- Assuming that the biased noise is Z channel error
- We want to implement a CX gate

Physical Realization

- CX gate: $CX = \left(\frac{I_1 + Z_1}{2} \otimes I_2\right) + \left(\frac{I_1 - Z_1}{2} \otimes X_2\right)$
- Hamltonian: $\hat{H}_{CX} = -V[\left(\frac{I_1 + Z_1}{2} \otimes I_2\right) + \left(\frac{I_1 - Z_1}{2} \otimes X_2\right)]$
- Time evolution: $U(t) = e^{-i\hat{H}t} = e^{iV\left[\left(\frac{I_1+Z_1}{2}\otimes I_2\right)+\left(\frac{I_1-Z_1}{2}\otimes X_2\right)\right]t}$
- Unitary CX: $U_{CX}(T) = e^{iV\left[\left(\frac{I_1+Z_1}{2}\otimes I_2\right)+\left(\frac{I_1-Z_1}{2}\otimes X_2\right)\right]T} = CX$

Bias-noise

- Unitary CX: $U_{CX}(T) = e^{iV\left[\left(\frac{I_1+Z_1}{2}\otimes I_2\right)+\left(\frac{I_1-Z_1}{2}\otimes X_2\right)\right]T} = CX$

Using the exponential function of Pauli operators

$$e^{i\theta\hat{A}} = cos(\theta)I + isin(\theta)\hat{A}$$

We can get

$$U_{CX}(T) = cos(VT)I - isin(VT)CX = CX \Longrightarrow \boxed{VT = \frac{\pi}{2}}$$

$if\ V = 1,$
$\pi/2\ pulse$

Noisy CX gate (phase-flip error occurs in **qubit 2** at time **τ**) $(0 < \tau < T)$

$$U_{CX}^{error}(T) = U_{CX}(T-\tau)(\hat{I}_1 \otimes \hat{Z}_2)U_{CX}(\tau)$$
$$= \hat{I}_1 \otimes \hat{Z}_2\, e^{iV(T-\tau)(\hat{I}_1-\hat{Z}_1)\otimes X_2} U(T)$$

Bias-noise

Noise analysis

- When an **phase-flip** error occurs in **target qubit**, an **phase-flip** error also occurs in **control qubit.**
- When the unitray operation is complete, **target qubit** will have **both phase-flip** error and **bit-flip** error.
- The **uncertainty** of gate operation (**fluctudation**) can also cause bit-flip error.
- After the gate operation, the **noise** does not remain in channel **Z**.

Conclusion

In the strict 2-level system, bias-preserving CX gate is **impossible**.

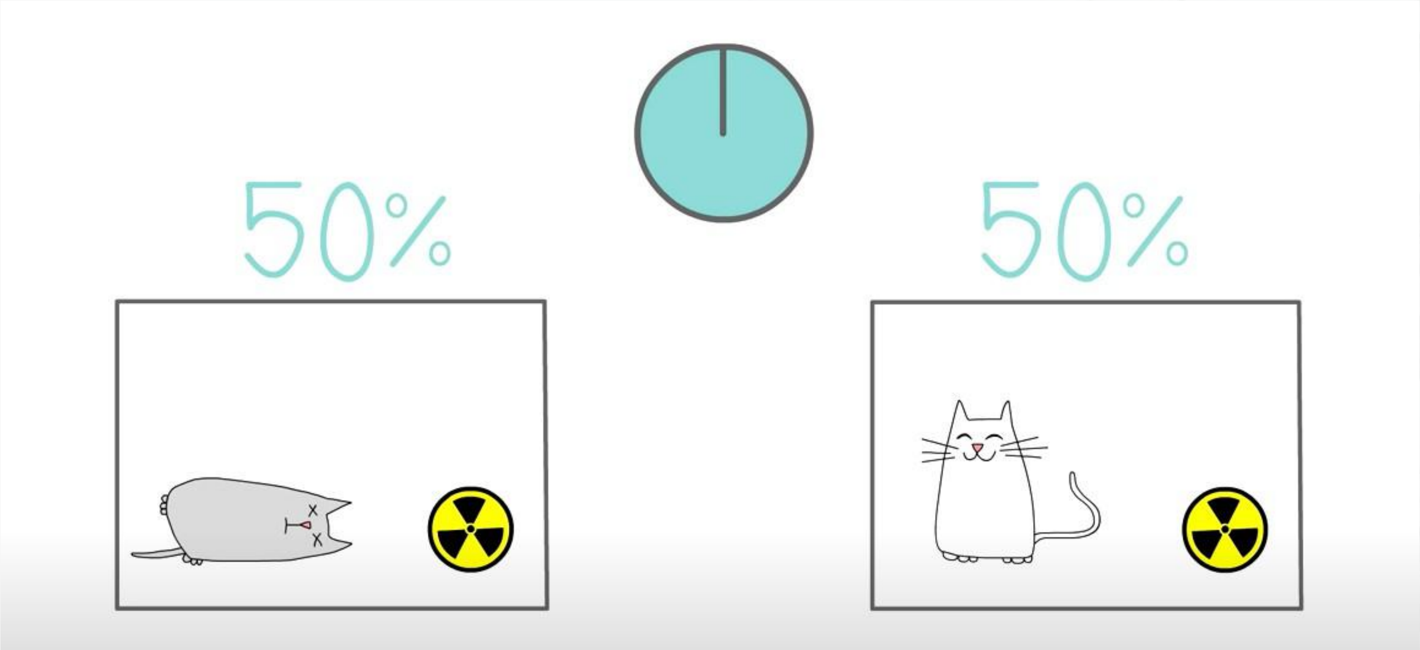
Cat qubit

How can we address these challenges ?

In order to take advantage of biased noise, we need something that acting all universal gate sets in this biased channel.

A better choice is

Cat qubit



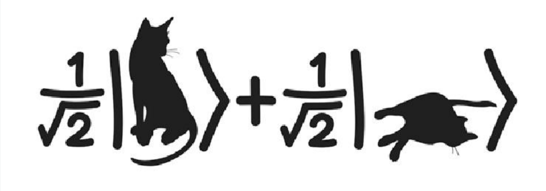
Cat state

What is cat state ?

Cat state, generally, is a quantum state that is a superposition of two completely opposite states.

e.g.

Cats are both alive and dead at the same time



Cat state in quantum optics

Cat state is a superposition of two opposite-phase **coherent states** of a single mode

$$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2} \sum_{n=0} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

Cat state

Cat state

Even cat state

- Define

$$|C_e\rangle = \mathbb{N}(|\alpha\rangle + |-\alpha\rangle)$$

- Why **even** ?

$$|C_e\rangle \propto e^{-\frac{1}{2}|\alpha|^2} \sum_{n=0} \frac{\alpha^n + (-\alpha)^n}{\sqrt{n!}} |n\rangle$$

$$= 2e^{-\frac{1}{2}|\alpha|^2} \left(\frac{\alpha^0}{\sqrt{0!}} |0\rangle + \frac{\alpha^2}{\sqrt{2!}} |2\rangle + \frac{\alpha^4}{\sqrt{4!}} |4\rangle + \cdots \right)$$

$|0\rangle$ $|1\rangle$

Odd cat state

- Define

$$|C_o\rangle = \mathbb{N}(|\alpha\rangle - |-\alpha\rangle)$$

- Why **odd** ?

$$|C_o\rangle \propto e^{-\frac{1}{2}|\alpha|^2} \sum_{n=0} \frac{\alpha^n - (-\alpha)^n}{\sqrt{n!}} |n\rangle$$

$$= 2e^{-\frac{1}{2}|\alpha|^2} \left(\frac{\alpha^1}{\sqrt{1!}} |1\rangle + \frac{\alpha^3}{\sqrt{3!}} |3\rangle + \frac{\alpha^5}{\sqrt{5!}} |5\rangle + \cdots \right)$$

Cat state

Features

- The larger $\boldsymbol{\alpha}$ is, the less **overlap** the two coherent states $|\pm\boldsymbol{\alpha}\rangle$ have, and the closer they are to the **ideal cat state**.
- It is difficult to realize large mean photon number ($|\alpha|^{\mathbf{2}}$). A typical method is to approximate cat state by **photon subtraction** from a squeezed vacuum state.
- We can using "**kitten state**" to generate larger cat state:
  - Entangling two "kittens" with size $\boldsymbol{\alpha}$ on a beamsplitter
  - Performing a **homodyne measurement** on one output
  - Measurement of **Q = 0**, the remaining output is projected to a larger cat state with size increased to $\sqrt{\mathbf{2}}\boldsymbol{\alpha}$

Cat qubit

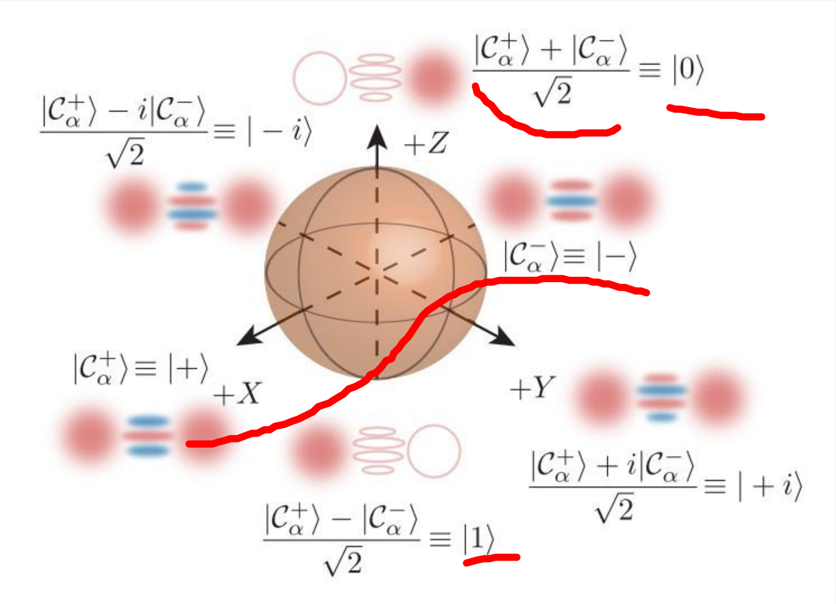


Fig. 2. Bloch sphere of the cat qubit

Physical qubit:

2-component cat state

- $C_\alpha^\pm\rangle = N_\pm(|\alpha\rangle \pm |-\alpha\rangle)$
- $\langle C_\alpha^\pm | C_{-\alpha}^\pm \rangle = 0$
- $N_\pm = 1/\sqrt{2(1 \pm e^{-2|\alpha|^2})}$

$\frac{1}{\sqrt{2}}$

Logical qubit configuration

X-axis: $|\pm\rangle = |C_\alpha^\pm\rangle$

Z-axis: $|0\rangle = \frac{|+\rangle + |-\rangle}{\sqrt{2}} = \frac{|C_\alpha^+\rangle + |C_\alpha^-\rangle}{\sqrt{2}}$

$|1\rangle = \frac{|+\rangle - |-\rangle}{\sqrt{2}} = \frac{|C_\alpha^+\rangle - |C_\alpha^-\rangle}{\sqrt{2}}$

For large **α** , $N_\pm \approx 1/\sqrt{2}$

$|0\rangle \approx |\alpha\rangle$

$|1\rangle \approx |-\alpha\rangle$

Cat qubit

Physical realization: Two-photon driven nonlinear oscillator

Kerr nonlinear Resonator

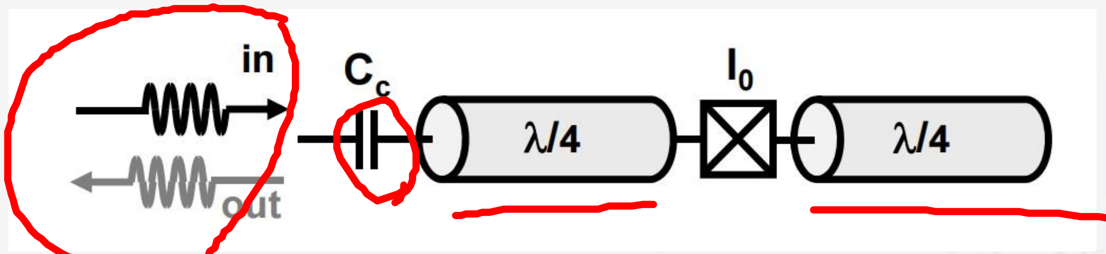


Fig. 3. Scheme of the nonlinear resonator

- Kerr nonlinear resonator is composed of a nonlinear device, **Josephson junction**, inserted into a $\lambda/2$ waveguide.
- The waveguide cavity is capacitatively (C_c) coupled with the transmission line for signal readout.
- Impedance matching is 50Ω(Avoid multiple signal oscillations)

Cat qubit

Equivalent circuit model

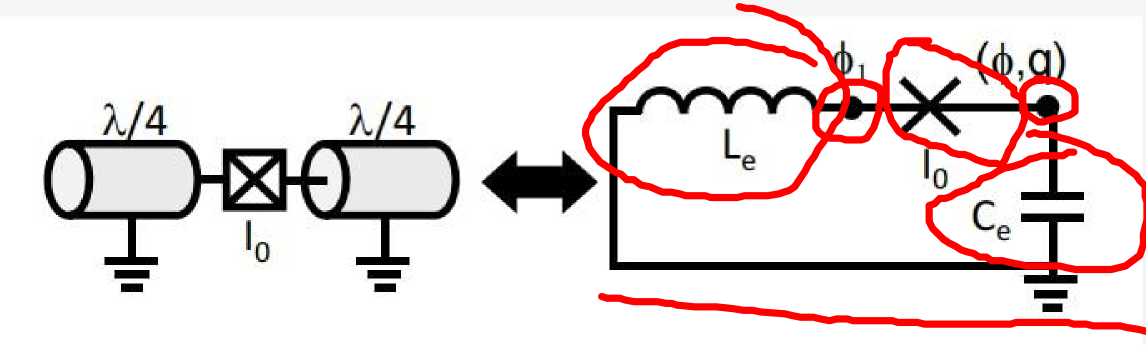


Fig. 4. Equivalent circuit of the nonlinear resonator

Hamlitonian

$$H = \frac{\phi_1^2}{2L_e} + \frac{q^2}{2C_e} - E_J \cos\left(\frac{\phi - \phi_1}{\phi_0}\right)$$

Where E_J is the Josephson energy, It is determined by the property of material itself

$$\phi_0 = \frac{\hbar}{2e}$$ is the **reduced flux quantum**

Cat qubit

According to the Josephson relation as below

$$I = I_0 \sin\left(\frac{\phi - \phi_1}{\phi_0}\right) = \frac{\phi_1}{L_e}$$

I_0 is the Josephson critical current

We can get the relationship between ϕ and ϕ_1

$$\phi = \phi_1 + \phi_0 \arcsin\frac{\phi_1}{L_e I_0}$$

Using equation above, we can expand the Hamiltonian in terms of ϕ

$$H = \frac{\phi^2}{2L_t} + \frac{q^2}{2C_e} - \frac{1}{24}p^3\frac{\phi^4}{L_t\phi_0^2} + O(\phi^6)$$

Cat qubit

$$H = \frac{\phi^2}{2L_t} + \frac{q^2}{2C_e} - \frac{1}{24}p^3\frac{\phi^4}{L_t\phi_0^2} + O(\phi^6)$$

Where $L_t = L_J + L_e$ is the total inductance, $p = L_J/L_t$ is the participation ratio of the Josephson inductance to total.

Second quantization

Let $\quad \phi = i\sqrt{\hbar Z_e/2}(a - a^\dagger), \quad q = \sqrt{\hbar/2Z_e}(a + a^\dagger)$

Where $\quad Z_e = \sqrt{\frac{L_t}{C_e}}$

Then, after a **rotation wave approximation (RWA)**, we will get

$$H_{NL}/\hbar = \omega_r a^\dagger a + \frac{\chi}{2} a^{\dagger 2} a^2$$

Cat qubit

Two-photon driven

$$H_d/\hbar = \epsilon a^{\dagger 2} e^{-2i\omega t} + \mathrm{h.c.}$$

Two-photon driven nonlinear oscillator

Hamiltonian in the interaction picture

$$\begin{aligned} H &= -K a^{\dagger 2} a^2 + P(a^{\dagger 2} e^{2i\phi} + a^2 e^{-2i\phi}) \\ &= -K(a^{\dagger 2} - \alpha^2 e^{-2i\phi})(a^2 - \alpha^2 e^{2i\phi}) + \frac{P^2}{K} \end{aligned}$$

Where

- P is the driven amplitude
- ϕ is the phase of driving
- K is the strength of the nonlinearity
- $\alpha = \sqrt{P/K}$

Cat qubit

Features

- The cat states with the size $\alpha e^{i\phi}$ is the degenerate eigenstate of this hamiltonian

$$C^{\pm}_{\alpha e^{i\phi}}\rangle = N_{\pm}(\ \alpha e^{i\phi}\rangle \pm\ -\alpha e^{i\phi}\rangle)$$

- The hamiltonian is commuted with photon number parity operator $\Pi = (-1)^{a^\dagger a}$, so the eigenspace can be divided into **odd** (**blue**) and **even** (**red**) subspace

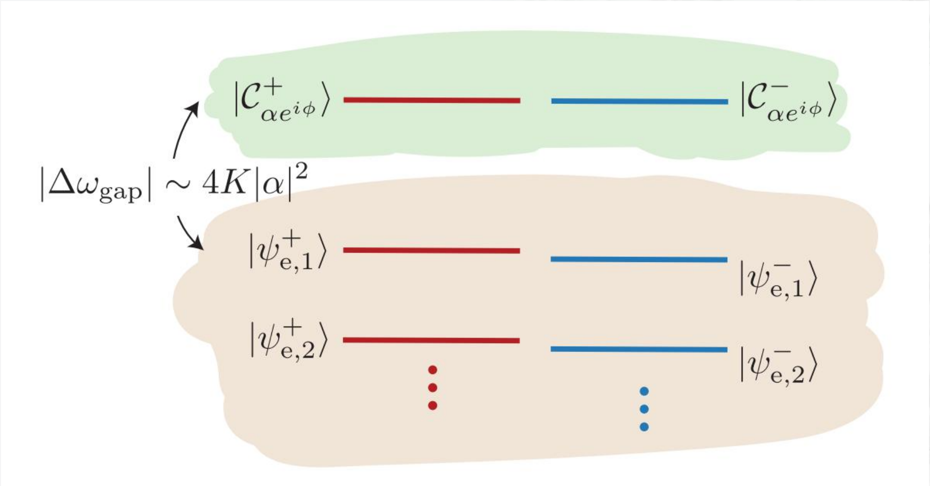


Fig. 5. Eigenstates of 2-photon driven KNR

Cat qubit

- Cat subspace is separeted from remaining state space by the large energy gap $\Delta\omega_{gap} \sim -4K\alpha^2$
- As $\boldsymbol{\alpha}$ goes up, the **energy gap** between a pair of eigenstate decreases exponentially, and the cat states are completely degenerate.
- This Hilbert space symmetry is important for the exponential suppression of **bit-flip errors**.

Preparing the cat states

Initial state
$\mathrm{P} = 0$

Vaccum state

Single-photon
Fock state

Increase driving
strength P adiabatically

Keeping the rate far less
than $1/|\Delta\omega_{gap}|$

$\left. C^{+}_{\alpha e^{i\phi}} \right\rangle$

$\left. C^{-}_{\alpha e^{i\phi}} \right\rangle$

CX gate based on cat qubits

Initial state

$$
\begin{aligned}
|\psi(0)\rangle &= (c_0|0\rangle + c_1|1\rangle) \otimes (d_0|0\rangle + d_1|1\rangle) \\
&= (c_0|0\rangle + c_1|1\rangle) \otimes \frac{1}{\sqrt{2}}[(d_0 + d_1)|C_\alpha^+\rangle + (d_0 - d_1)|C_\alpha^-\rangle]
\end{aligned}
$$

Two-photon drive applied to the target oscillator

At time t

$$
\begin{aligned}
|\psi(t)\rangle = c_0|0\rangle \otimes \frac{1}{\sqrt{2}}[(d_0 + d_1)|C_\alpha^+\rangle + (d_0 - d_1)|C_\alpha^-\rangle] \\
+ c_1|1\rangle \otimes \frac{1}{\sqrt{2}}[(d_0 + d_1)|C_{\alpha e^{i\phi(t)}}^+\rangle + (d_0 - d_1)|C_{\alpha e^{i\phi(t)}}^-\rangle]
\end{aligned}
$$

CX gate

When $\phi(T) = \pi$

$$
\begin{aligned}
|\psi(T)\rangle &= c_0|0\rangle \otimes \frac{1}{\sqrt{2}}[(d_0+d_1)|C_\alpha^+\rangle + (d_0-d_1)|C_\alpha^-\rangle] \\
&\quad + c_1|1\rangle \otimes \frac{1}{\sqrt{2}}[(d_0+d_1)|C_{\alpha e^{i\pi}}^+\rangle + (d_0-d_1)|C_{\alpha e^{i\pi}}^-\rangle] \\
&= c_0|0\rangle \otimes \frac{1}{\sqrt{2}}[(d_0+d_1)|C_\alpha^+\rangle + (d_0-d_1)|C_\alpha^-\rangle] \\
&\quad + c_1|1\rangle \otimes \frac{1}{\sqrt{2}}[(d_0+d_1)|C_\alpha^-\rangle + (d_0-d_1)|C_\alpha^+\rangle] \\
&= c_0|0\rangle \otimes (d_0|0\rangle + d_1|1\rangle) + c_1|1\rangle \otimes (d_0|1\rangle + d_1|0\rangle) \\
&= U_{CX}|\psi(0)\rangle
\end{aligned}
$$

CX gate with errors

Predominant stochastic errors are of the form

$$\text{Control error}: \quad \hat{O}_c = f(\alpha)\hat{Z}_c$$
$$\text{Target error}: \quad \hat{O}_t^{\mathcal{T}} = f(\alpha e^{i\phi(\tau)})\hat{Z}_t^{\mathcal{T}}$$

CX gate with control error

$$\begin{aligned}|\psi(\tau)\rangle_{error}^{ctrl} = \hat{O}_c \otimes \hat{I}_t\{c_0|0\rangle \otimes [(d_0+d_1)|C_\alpha^+\rangle + (d_0-d_1)|C_\alpha^-\rangle] \\ + c_1|1\rangle \otimes [(d_0+d_1)|C_{\alpha e^{i\phi(\tau)}}^+\rangle + (d_0-d_1)|C_{\alpha e^{i\phi(\tau)}}^-\rangle]\}\end{aligned}$$

After a time T making $\phi(t) = \pi$, we will get a CX gate

$$\begin{aligned}|\psi(T)\rangle_{error}^{ctrl} &= c_0|0\rangle \otimes [(d_0+d_1)|C_\alpha^+\rangle + (d_0-d_1)|C_\alpha^-\rangle] \\ &\quad - c_1|1\rangle \otimes [(d_0+d_1)|C_{\alpha e^{i\phi(\tau)}}^+\rangle - (d_0-d_1)|C_{\alpha e^{i\phi(\tau)}}^-\rangle] \\ &= \hat{Z}_c \otimes \hat{I}_t^{\mathcal{T}}\{c_0|0\rangle \otimes [(d_0+d_1)|C_\alpha^+\rangle + (d_0-d_1)|C_\alpha^-\rangle] \\ &\quad + c_1|1\rangle \otimes [(d_0+d_1)|C_\alpha^+\rangle - (d_0-d_1)|C_\alpha^-\rangle]\} \\ &= \hat{Z}_c \otimes \hat{I}_t^{\mathcal{T}} U_{CX}|\psi(0)\rangle\end{aligned}$$

CX gate

CX gate with target error

$$
\begin{aligned}
|\psi(\tau)\rangle^{targ}_{error} &= \hat{I}_c \otimes \hat{O}_t^{\tau} \{c_0|0\rangle \otimes [(d_0+d_1)|C_\alpha^+\rangle + (d_0-d_1)|C_\alpha^-\rangle] \\
&\quad + c_1|1\rangle \otimes [(d_0+d_1)|C^+_{\alpha e^{i\phi(\tau)}}\rangle + (d_0-d_1)|C^-_{\alpha e^{i\phi(\tau)}}\rangle]\} \\
&= f(\alpha)\{c_0|0\rangle \otimes [(d_0+d_1)|C_\alpha^-\rangle + (d_0-d_1)|C_\alpha^+\rangle]\} \\
&\quad + f(\alpha e^{i\phi(\tau)})\{c_1|1\rangle \otimes [(d_0+d_1)|C^-_{\alpha e^{i\phi(\tau)}}\rangle + (d_0-d_1)|C^+_{\alpha e^{i\phi(\tau)}}\rangle]\}
\end{aligned}
$$

After a time T making $\phi(t) = \pi$

$$
\begin{aligned}
|\psi(T)\rangle^{targ}_{error} &= f(\alpha)\{c_0|0\rangle \otimes [(d_0+d_1)|C_\alpha^-\rangle + (d_0-d_1)|C_\alpha^+\rangle]\} \\
&\quad + f(\alpha e^{i\phi(\tau)})\{c_1|1\rangle \otimes [-(d_0+d_1)|C_\alpha^-\rangle + (d_0-d_1)|C_\alpha^+\rangle]\} \\
&= \hat{I}_c \otimes \hat{Z}_t[f(\alpha)c_0|0\rangle \otimes (d_0|0\rangle + d_1|1\rangle) \\
&\quad - f(\alpha e^{i\phi(\tau)})c_1|1\rangle \otimes (d_0|0\rangle + d_1|1\rangle)] \\
&= [\hat{Z}_c f(\alpha e^{i\phi(\tau)(1-\hat{Z}_c)/2}) \otimes \hat{Z}_t]\, U_{CX}|\psi(0)\rangle
\end{aligned}
$$

CX gate

Features

- A phase error on the control cat qubit at any time during the implementation of the CX is equivalent to a **phase-flip** on the **control qubit** after an ideal CX.
- A phase-flip error on the target qubit at any time during the CX evolution is equivalent to **phase errors** on the **control and target** qubits after the ideal CX gate.
- This method to implement CX gate keeps the **noise channel biased to Z**, which is in contrast to strict 2-level system.

Physical Realization

Hamiltonian of CX gate in interaction picture

$$
\begin{aligned}
H_{CX} = & - K(a_c^{\dagger 2} - \beta^2)(a_c^2 - \beta^2) \\
& - K\left[a_t^{\dagger 2} - \alpha^2 e^{-2i\phi(t)}\left(\frac{\beta - a_c^\dagger}{2\beta}\right) - \alpha^2\left(\frac{\beta + a_c^\dagger}{2\beta}\right)\right] \\
& \times \left[a_t^2 - \alpha^2 e^{2i\phi(t)}\left(\frac{\beta - a_c}{2\beta}\right) - \alpha^2\left(\frac{\beta + a_c}{2\beta}\right)\right] \\
& - \frac{\dot{\phi}(t)}{4\beta} a_t^\dagger a_t (2\beta - a_c^\dagger - a_c)
\end{aligned}
$$

Hamiltonian of CX gate

Analysis: How to realize CX gate using this Hamiltonian

If control qubit is $|0\rangle$

$$\begin{aligned} H_{CX}^{|0\rangle_c} = & - K(a_c^{\dagger 2} - \beta^2)(a_c^2 - \beta^2) \\ & - K(a_t^{\dagger 2} - \alpha^2)(a_t^2 - \alpha^2) \end{aligned}$$

If control qubit is $|1\rangle$

$$\begin{aligned} H_{CX}^{|1\rangle_c} = & - K(a_c^{\dagger 2} - \beta^2)(a_c^2 - \beta^2) \\ & - K\left(a_t^{\dagger 2} - \alpha^2 e^{-2i\phi(t)}\right)\left(a_t^2 - \alpha^2 e^{2i\phi(t)}\right) \\ & - \dot{\phi}(t) a_t^\dagger a_t \end{aligned}$$

Summary

The road to fault-tolerant quantum computing

- Quantum computing has natural advantages, we need quantum computing to accelerate algorithms
- Quantum systems are fragile, and error correction is needed to achieve fault-tolerant quantum computing
- The overhead of existing error correction schemes is higher, such as surface code
- Biased-noise system can be used to design error correction focusing on major errors, reducing costs and improve fidelity
- Cat qubits implemented with two-photon driven KNR enable biased noise systems
- Quantum gate in cat qubits is bised-preserving, which is beneficial to error correction code

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Thank you!

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